# Supporting Information for "Rapid Fluctuations of the Subsurface Chlorophyll Maximum in Response to Wind Forcing in a Long, Narrow Bay"

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**Dynamical equations for the barotropic response of the Ría**

Let's take the Ría as a rectangular channel extending in the $x$ direction (the $y$ direction is assumed to be irrelevant, as we neglect rotation) with a length $L$ and a height $h$ at equilibrium, and study the barotropic response to an along-channel wind stress ($\tau_w$). At the west end of the Ría, the height is fixed (we assume that the volume of the adjacent ocean is infinite), and at the inner eastern end the surface position can vary. We define the surface height anomaly with respect to the equilibrium as $\eta$ (Fig. S1).

The response is determined by the continuity equation and the momentum equation in the $x$ axis. We reduce the problem to two dimensions by resolving the eastward velocity, $u(z,t)$, of the Ría at its mouth ($x = 0$). The continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

is reduced to two dimensions and written in an integral form:

$$\int_0^h \frac{\partial u}{\partial x} dz + \frac{\partial \overline{\eta}}{\partial t} = h\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{\eta}}{\partial t} = 0 \tag{2}$$

where $\overline{\eta}$ is the mean height within the Ría, which corresponds to half of the height in the eastern-most point, $\overline{\eta} = \eta/2$. Because the eastward velocity is zero at the solid wall ($x = L$):

$$\frac{\partial \overline{u}}{\partial x} = \frac{\overline{u}(x = L) - \overline{u}(x = 0)}{L} = -\frac{\overline{u}(x = 0)}{L} \equiv -\frac{\overline{u}}{L} \tag{3}$$

So then we have as continuity equation:

$$\boxed{\frac{\partial \eta}{\partial t} = 2h\frac{\overline{u}}{L}} \tag{4}$$

On the other hand, the momentum equation in the $x$ direction:

$$\frac{Du}{Dt} = fv - \frac{1}{\rho}\left(\frac{\partial p}{\partial x} + \frac{\partial \tau_x}{\partial z}\right) \tag{5}$$

We neglect the non linear terms ($\frac{Du}{Dt} \approx \frac{\partial u}{\partial t}$), and also Coriolis acceleration ($fv \approx 0$), and we follow a hydrostatic approximation ($p = g\rho(z + \eta)$). Because at $x = 0$, $\eta = 0$, for every $z$:

$$\frac{\partial p}{\partial x} = \frac{p(x = L) - p(x = 0)}{L} = \frac{g\rho\eta}{L} \tag{6}$$

We model the shear stresses with a turbulent viscosity ($\kappa$):

$$\tau_x(z,t) = -\rho\kappa\frac{\partial u}{\partial z}(z,t) \tag{7}$$

With all this, the momentum equation is reduced to:

$$\boxed{\frac{\partial u}{\partial t} = -\frac{g\eta}{L} + \kappa\frac{\partial^2 u}{\partial z^2}} \tag{8}$$

with boundary conditions:

$$u(z=0) = 0 \tag{9}$$

and

$$\rho\kappa\frac{\partial u}{\partial z}(z=h) = \tau_w \tag{10}$$

where we calculated

$$\tau_w = \rho_{air}C_D W^2 \tag{11}$$

To illustrate this non-rotational barotropic response of the Ría to an along-channel wind pulse we performed a simulation using equations 4, and 8 to 11, by taking $h = 40$ m, $L = 30$ km, $g = 9.81$ m s$^{-2}$, $\rho_a = 1.2$ kg m$^{-3}$, $\rho = 1000$ kg m$^{-3}$, $C_D = 10^{-3}$ and $\kappa = 5 \times 10^{-4}$ m$^2$ s$^{-1}$. The wind was set to $W = -10$ m s$^{-1}$ (offshore) between days 2 and 5 of the simulation. A 30 day spin off was used to allow the system to equilibrate and damp oscillations. Figure S2 shows the result of the simulation. As soon as the wind starts blowing, the water level inside the Ría drops by $\sim 20$ cm and starts oscillating at relatively high frequency ($< 1$ h). At the same time a bidirectional flow, with outflowing surface layer and inflowing bottom layer starts to develop immediately, first with strong linear acceleration, and equilibrates slowly (due to the action of viscosity) over the duration of the wind pulse (3 days, a typical value for the system). However, full equilibrium seems not to be reached.

There are two inherent time-scales to this response. First, the barotropic along-Ría time-scale which determines the propagation of the pressure perturbation signal along the channel, and an equilibration time-scale which depends on the damping effect of viscosity. The barotropic time scale can be determined by neglecting the viscous term in Eq. 8 (second term on the right hand side), and by derivating and substituting with Eq. 4, taking into account that for a barotropic response without friction or wind forcing the velocity profile is uniform, $u(z) = \overline{u}$:

$$\frac{\partial^2 u}{\partial t^2} = -\frac{g}{L}\frac{\partial \eta}{\partial t} \Rightarrow \frac{\partial^2 u}{\partial t^2} = -\frac{2hg}{L2}u \tag{12}$$

This is the equation for an harmonic oscillation with frequency $\omega = (\frac{2hg}{L^2})^{1/2}$. Hence, the barotropic period is:

$$T_{bt} = 2\pi\omega^{-1} \approx 1.90\,\mathrm{h} \tag{13}$$

So the barotropic response of the Ría starts to develop in less than a couple of hours. This is also the frequency of the oscillations observed in the water level. However, the acceleration time-scale (and the equilibrium exchange velocities) are dictated by the equilibrium between the pressure gradient and the frictional response. The equilibrium solution could be find analytically by taking $\frac{\partial u}{\partial t}, \frac{\partial \eta}{\partial t} = 0$ in Eq. 4 and 8. Because we are interested on the dynamic response (equilibration time), we performed instead three simulations with different values of $\kappa$ and for a wind pulse extending between days 2 and 10 of the simulation, in order to allow some extra time for equilibration (Fig. S3). This figure shows that the equilibrium exchange velocities are larger for weaker viscosities. Viscosity values of $5 - 10 \times 10^{-4}$ m$^2$ s$^{-1}$ produced realistic equilibrium exchange velocities (10-20 cm s$^{-1}$) (Barton et al., 2015), but those were unrealistically high for a weaker viscosity of $\kappa = 0.1 \times 10^{-4}$ m$^2$ s$^{-1}$. This figure also illustrates the linear response of the Ría until the viscosity effects become important. Until a time scale $t$ for which $u = u_{max}/2$, the response is almost linear. This time scale was of about half a day (smaller than the local inertial period of 0.75 days) for $\kappa = 10 \times^{-4}$ m$^2$ s$^{-1}$, and of about 1 day for $\kappa = 5 \times 10^{-4}$ m$^2$ s$^{-1}$. In any case, the bidirectional circulation is set-up almost immediately, while a rotational response would need to wait one inertial period or more such that the along-shore circulation equilibrates with the Coriolis force.

## References

Barton, E., Largier, J., Torres, R., Sheridan, M., Trasviña, A., Souza, A., ... Valle-Levinson, A. (2015). Coastal upwelling and downwelling forcing of circulation in a semi-enclosed bay: Ria de Vigo. *Progress in Oceanography*, *134*, 173–189. doi: 10.1016/J.POCEAN.2015.01.014

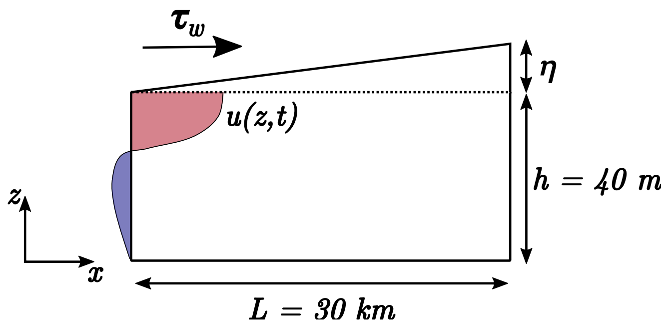


**Figure S1.** Schematics of the dynamical balance of barotropic response of the Ría to an along-channel wind stress ($\tau_w$).

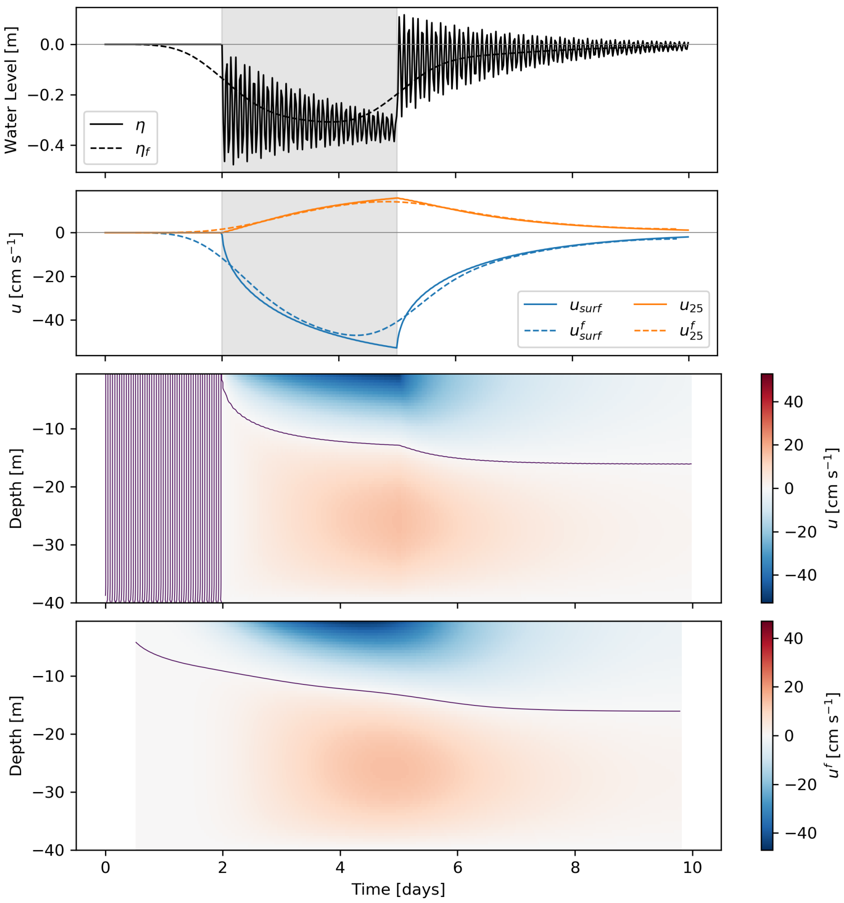


**Figure S2.** Simulation of the Ría response to a down-channel wind pulse of $W = 10$ m s$^{-1}$ during days 2-5 of the simulation using the barotropic model. Water level at the inner-most point ($\eta$) and eastward velocities ($u$) are shown. The $f$ index indicates quantities filtered with a Godin 24/25/24 filter. In the second panel, the velocity at the surface layer ($z = 0.5$ m) and at 25 m depth are displayed. In this simulation, the turbulent viscosity is set to $\kappa = 5 \times 10^{-4}$ m$^2$ s$^{-1}$.

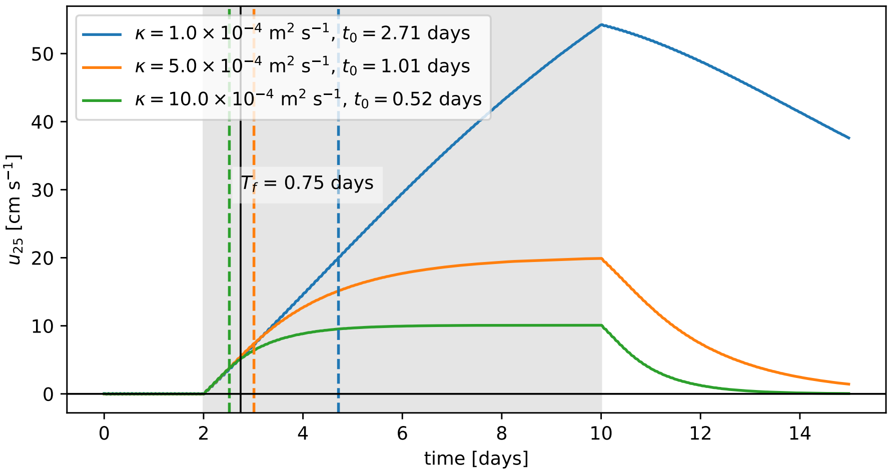


**Figure S3.** Along-channel velocities at 25 m depth for simulations of the barotropic non-rotational response of the Ría to a wind pulse $W = -10$ m s$^{-1}$ between days 2 and 12 of the simulation, with different values of the turbulent viscosity coefficient ($\kappa$). The response time ($t_0$) is the time required for $u$ to reach $1/e$ of its maximum value. The inertial period $T_f$ is shown for comparison.

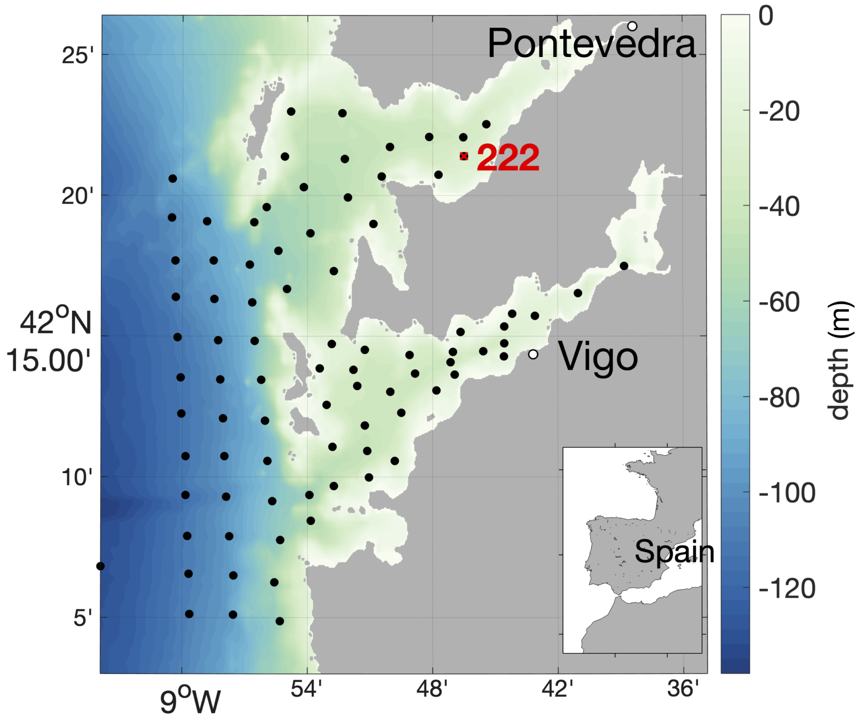


**Figure S4.** Bathymetry map of the two southernmost Galician Rías Baixas: Ría de Pontevedra and Ría de Vigo. Black dots indicate the sampling stations during REMEDIOS-TLP cruise. The red cross indicates the intensive sampling station, 222. Bathymetry data ta from GEBCO Compilation Group (2020) GEBCO 2020 Grid (doi:10.5285/a29c5465-b138-234d- e053-6c86abc040b9).