

The Influence of inclined magnetic field, heat and mass transfer on the peristaltic flow of blood in an inclined asymmetric channel with variable viscosity

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Abstract: The present study investigates the effects of inclined magnetic field, heat source/sink, and concentration on the peristaltic flow of blood in an inclined asymmetric channel with variable viscosity. Mathematical analysis has been carried out in the presence of aligned magnetic field. The flow is examined in a wavy frame of references moving with the velocity of the wave. The physical problem is the first model, and then, the analytic solution is carried out under long-wavelength and low-Reynolds number approximation. The expressions for velocity, temperature, concentration, pressure gradient, tangential stress and heat transfer coefficient are obtained. The results for the pressure rise and frictional force have also been computed numerically. Numerical results are graphically discussed for various values of physical parameters of interest.

Keywords: Peristaltic flow, Thermal radiation, Inclined asymmetric channel, Aligned magnetic field, Variable viscosity, Temperature jump, Concentration.

Nomenclature

a_1, b_1	Amplitudes of the wavy walls
a, b	Amplitude ratios
$d_1 + d_2$	Width of the channel
d	Mean half width of the channel
$\overline{H_1} + \overline{H_2}$	Geometry of the upper and lower surface
λ	Wavelength
c	Wave speed
\bar{t}	Time in the fixed frame
t	Time in the wave frame
ρ	Constant density
$\overline{U}, \overline{V}$	Velocity components in the fixed frame
\bar{u}, \bar{v}	Velocity components in the wave frame
\overline{P}	Pressure in the fixed frame
\bar{p}	Pressure in the wave frame

$\overline{\mu}(\overline{Y})$	Viscosity function
ε	Viscosity parameter
μ_0	Constant Viscosity
B_0	Intensity of external magnetic field
α	Aligned magnetic field
g	Acceleration due to gravity
$\overline{\alpha}_t$	Coefficient of thermal expansion
ζ	Inclination of asymmetric channel to the vertical
c_p	Specific heat
K	Thermal conductivity
q_r	Radiative heat flux
T_1, T_o	Temperature of the lower and upper walls of the channel
C_1, C_o	Concentration of the lower and upper walls of the channel
\overline{Q}	Heat generation coefficient
$\overline{\alpha}_c$	Coefficient of viscosity at constant concentration
D_m	Coefficient of mass diffusivity
K_T	Thermal-diffusion ratio
T_m	Temperature of the medium
C	Concentration of the fluid
θ	Temperature distribution
Θ	Concentration distribution
σ	Electrical conductivity of the fluid
$\overline{\sigma}$	Stefan-Boltzmann constant
\overline{k}	Mean absorption coefficient
ϕ	Phase difference
δ	Wave number
β	Heat source/sink parameter
R	Thermal radiation parameter
γ	Thermal slip
M	Hartmann number
Re	Reynolds number
Pr	Prandtl number
Gr	Grashof number
Fr	Froude number
Br	Brinkman number

Sr	Soret number
Sc	Schmidt number

1. Introduction

Peristaltic is an important mechanism for mixing and transporting fluids produced by a gradual wave of contraction or expansion on the tube wall. The peristalsis process occurs for urine transportation in the gastrointestinal tract, the movements of the male spermatozoa in the male reproductive tract's efferent ducts, Ovum's movement in the fallopian tube and vasomotion are small blood vessels. The use of peristaltic pumping in biomedical devices like lung machine to pump the blood is also encountered. Peristaltic flows are exploited in industrial pumping as they provide an efficient means for sanitary fluid transport. The industrial use of peristaltic pumping in roller/finger pump is well known. Dar [1] clarified the effect of thermal radiation, temperature jump and inclined magnetic field on the peristaltic transport of blood flow in an asymmetric channel with variable viscosity and heat absorption. With inclined magnetic field and convective boundary conditions, Noreen and Qasim [2] examined the peristaltic flow. The rotation effect on the peristaltic transport of a Jeffrey fluid in an asymmetric channel with gravity field was discussed by Abd-Alla and Abo-Dahab [3]. Reddy [4] explored the heat and mass transfer on magneto hydrodynamic peristaltic flow in a porous medium with partial slip. Nadeem and Akram [5] debated the influence of inclined magnetic field on the peristaltic flow of a Williamson fluid model in an inclined symmetric or asymmetric channel. The effects of inclined magnetic field and porous medium on the peristaltic flow of a Bingham fluid with heat transfer were displayed by Divya et al. [6]. Ahmed [7] analyzed inclined magnetic field's effect on the peristaltic flow of Carreau fluid through a porous medium in an inclined tapered asymmetric channel. Jagadeesh et al. [8] were researching the influence of inclined magnetic field on the peristaltic flow of a Jeffrey fluid in an inclined porous channel. The magnetic field and gravity effects on the peristaltic transport of a Jeffrey fluid in an asymmetric channel were presented by Abd-Alla et al. [9]. Also, Abd-Alla et al. [10] studied the peristaltic transport of a Jeffrey fluid under the effect of the gravity field and rotation in an asymmetric channel with magnetic field. Rajesh and Gowd [11] illustrated the heat and mass transfer analysis on MHD peristaltic Prandtl fluid model through a tapered channel with thermal radiation. Ramesh and Devakar [12] evaluated the magneto-hydrodynamic peristaltic transport of couple stress fluid through a porous medium in an inclined asymmetric channel with heat transfer. Srinivas and Gayathri [13] investigated the peristaltic transport of a Newtonian fluid in a vertical asymmetric channel with heat transfer and porous medium. Hayat et al. [14] explained the effect of an inclined magnetic field on the peristaltic flow of Williamson fluid in an inclined channel with convective conditions. The peristaltic transport of MHD flow and heat transfer in an asymmetric channel was exposed by Sinha et al. [15]. Kumar and Ahamad [16] have shown the peristaltic hemodynamic Jeffery fluid through a tapered channel with heat and mass transfer under the influence of radiation – blood flow model. Ali et al. [17] were studying the slip effects on the peristaltic transport of MHD fluid with variable viscosity. The MHD mixed convection peristaltic flow with

variable viscosity and thermal conductivity was discussed by Hayat et al. [18]. The effects of slip condition, variable viscosity and inclined magnetic field were estimated by Khan et al. [19] on the peristaltic motion of a non-Newtonian fluid in an inclined asymmetric channel. Rao and Mishra [20] calculated the nonlinear and curvature effects on the peristaltic flow of a viscous fluid in an asymmetric channel. Kumar and Naidu [21] studied the streamlines and the velocity profiles in the laboratory frame suggest that the maximum mixing of the fluid is taking place near the stagnation points on the central axis. In the presence of gold nanoparticles, Asha and Sunitha [22] reconnoitered the influence of thermal radiation on the peristaltic blood flow of a Jeffrey fluid with double diffusion. Hasen and Abdulhadia [23] inspected the influence of a rotating frame on the peristaltic flow of a Rabinowitsch fluid model in an inclined channel. Joule heating and zeta potential effects on Peristaltic blood flow through porous micro vessels altered by electro-hydrodynamic were studied by Ranjit et al. [24]. Prakash et al. [25] deliberated the thermal radiation effects on the electro-osmosis modulated peristaltic transport of ionic nanoliquids in the biomicro-fluid channel. Tripathi et al. [26] studied the Electro-osmosis modulated peristaltic bio-rheological flow through an asymmetric micro-channel. Latha et al. [27] observed heat dissipation's effects on the peristaltic flow of Jeffery and Newtonian fluid through an asymmetric channel with a porous medium. The effects of the induced magnetic field for the peristaltic flow of Williamson fluid in a curved channel were exhibited by Rashid et al. [28]. Vaidyaa et al. [29] have checked the influence of transport properties of the peristaltic MHD Jeffrey fluid flow through a porous asymmetric tapered channel. Sinha and Shit [30] revealed the Electromagnetohydrodynamic flow of blood and heat transfer in a capillary with thermal radiation. Pulsatile flow of blood and heat transfer with variable viscosity was exhibited under magnetic and vibration environment by Shit and Majee [31].

The aim of this paper is to investigate the influence of inclined magnetic field, thermal radiation and heat and mass transfer on the peristaltic flow of blood in an inclined asymmetric channel with variable viscosity. The fluid is electrically conducted in the presence of an inclined magnetic field. The governing equations are modeled under long-wavelength and low-Reynolds number assumptions and then solved analytically subject to relevant boundary conditions. Expressions of the velocity, temperature, concentration, coefficients of heat transfer, pressure gradient, tangential stress, pressure rise and frictional forces are given and discussed. At the end, the behavior of different parameters of interest is shown graphically. Also, it is estimated that the results obtained here will serve as equally good theoretical estimates of various potential fluid mechanical flow governing parameters related to the peristaltic transport of blood.

2. Formulation of the problem

Include the peristaltic transportation of a variable viscosity electrical fluid (Blood) in an asymmetric channel of width $d_1 + d_2$. The peristaltic motion of a sinusoidal wave spreads at a constant speed c along the channel's walls (\overline{H}_1 is the upper wall and \overline{H}_2 is the lower wall). T_o and T_1 are the temperatures of the upper and lower channel walls, respectively. C_o and C_1 are

the concentration of the upper and lower channel walls, respectively. The geometry of the two wall surfaces is described by

$$\bar{H}_1 = d_1 + a_1 \cos\left[\frac{2\pi}{\lambda}(\bar{X} - c\bar{t})\right], \quad (1)$$

$$\bar{H}_2 = -d_2 - b_1 \cos\left[\frac{2\pi}{\lambda}(\bar{X} - c\bar{t}) + \phi\right]. \quad (2)$$

\bar{X} in the above terminology is defined to be the direction propagation and \bar{Y} is perpendicular to it. Phase difference ϕ varies in the range $0 < \phi < \pi$. It should be noted that $\phi = 0$ corresponds to symmetric channel with waves out of phase and $\phi = \pi$ for the waves are in phase. A uniform magnetic field is applied at an angle α of to the flow. The electric field takes zero, and the Reynolds number is taken small, so that the induced magnetic field is negligible in comparison with the applied magnetic field. Also a_1, b_1, d_1, d_2 satisfy the following relation

$$a_1^2 + b_1^2 + 2a_1b_1 \cos\phi \leq (d_1 + d_2)^2. \quad (3)$$

In the fixed frame, the leading equations are as follows:

$$\begin{aligned} \rho\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{Y}}\right)\bar{U} = & -\frac{\partial \bar{P}}{\partial \bar{X}} + 2\frac{\partial}{\partial \bar{X}}(\bar{\mu}(\bar{Y})\frac{\partial \bar{U}}{\partial \bar{X}}) + \frac{\partial}{\partial \bar{Y}}(\bar{\mu}(\bar{Y})\left(\frac{\partial \bar{V}}{\partial \bar{X}} + \frac{\partial \bar{U}}{\partial \bar{Y}}\right)) \\ & - \sigma \bar{B}_o^2 \cos\alpha(\bar{U} \cos\alpha - \bar{V} \sin\alpha) + \rho g \bar{\alpha}_i(\bar{T} - \bar{T}_0) + \rho g \bar{\alpha}_c(\bar{C} - \bar{C}_0) + \rho g \sin\zeta, \end{aligned} \quad (4)$$

$$\begin{aligned} \rho\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{Y}}\right)\bar{V} = & -\frac{\partial \bar{P}}{\partial \bar{Y}} + 2\frac{\partial}{\partial \bar{Y}}(\bar{\mu}(\bar{Y})\frac{\partial \bar{V}}{\partial \bar{Y}}) + \frac{\partial}{\partial \bar{X}}(\bar{\mu}(\bar{Y})\left(\frac{\partial \bar{V}}{\partial \bar{X}} + \frac{\partial \bar{U}}{\partial \bar{Y}}\right)) \\ & + \sigma \bar{B}_o^2 \sin\alpha(\bar{U} \cos\alpha - \bar{V} \sin\alpha) - \rho g \cos\zeta, \end{aligned} \quad (5)$$

$$\rho_{cp}\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{Y}}\right)\bar{T} = K\left(\frac{\partial^2}{\partial \bar{X}^2} + \frac{\partial^2}{\partial \bar{Y}^2}\right)\bar{T} - \frac{\partial q_r}{\partial \bar{Y}} + \bar{Q}, \quad (6)$$

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{Y}}\right)\bar{C} = D_m\left(\frac{\partial^2}{\partial \bar{X}^2} + \frac{\partial^2}{\partial \bar{Y}^2}\right)\bar{C} + \frac{D_m K_T}{T_m}\left(\frac{\partial^2}{\partial \bar{X}^2} + \frac{\partial^2}{\partial \bar{Y}^2}\right)\bar{T}, \quad (7)$$

$$\frac{\partial \bar{U}}{\partial \bar{X}} + \frac{\partial \bar{V}}{\partial \bar{Y}} = 0. \quad (8)$$

And the shear stresses in Cartesian coordinates can be given from the following relations

$$S_{\bar{X}\bar{X}} = \frac{2\mu}{1+\lambda_1}\left[1 + \lambda_2\left(\bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{Y}}\right)\right]\frac{\partial \bar{U}}{\partial \bar{X}}, \quad (9)$$

$$S_{\bar{X}\bar{Y}} = \frac{\mu}{1+\lambda_1}\left[1 + \lambda_2\left(\bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{Y}}\right)\right]\left(\frac{\partial \bar{U}}{\partial \bar{Y}} + \frac{\partial \bar{V}}{\partial \bar{X}}\right), \quad (10)$$

$$S_{\bar{Y}\bar{Y}} = \frac{2\mu}{1+\lambda_1}\left[1 + \lambda_2\left(\bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{Y}}\right)\right]\frac{\partial \bar{V}}{\partial \bar{Y}}. \quad (11)$$

In the above equations, λ_1 is the ratio of relaxation to retardation times and λ_2 is the retardation time. The flow in the fixed coordinates (\bar{X}, \bar{Y}) is unsteady and it becomes steady in the wave

frame (\bar{x}, \bar{y}) . The transformations between the two frames are given by:

$$\bar{x} = \bar{X} - c\bar{t}, \quad \bar{y} = \bar{Y}, \quad \bar{u} = \bar{U} - c, \quad \bar{v} = \bar{V}, \quad \bar{p} = \bar{P}, \quad \bar{T} = \bar{T}. \quad (12)$$

In the fixed frame, the applicable boundary conditions are of the following form:

$$\bar{U} = 0, \quad \bar{T} + \gamma \frac{\partial \bar{T}}{\partial \bar{Y}} = T_0, \quad \bar{C} = \bar{C}_0 \quad \text{at} \quad \bar{Y} = \bar{H}_1, \quad (13)$$

$$\bar{U} = 0, \quad \bar{T} - \gamma \frac{\partial \bar{T}}{\partial \bar{Y}} = T_1, \quad \bar{C} = \bar{C}_1 \quad \text{at} \quad \bar{Y} = \bar{H}_2. \quad (14)$$

In the wave frame, the leading equations are as follows:

$$\rho((\bar{u} + c) \frac{\partial}{\partial \bar{x}} + \bar{v} \frac{\partial}{\partial \bar{y}}) \bar{u} = - \frac{\partial \bar{p}}{\partial \bar{x}} + 2 \frac{\partial}{\partial \bar{x}} (\bar{\mu}(\bar{y}) \frac{\partial \bar{u}}{\partial \bar{x}}) + \frac{\partial}{\partial \bar{y}} (\bar{\mu}(\bar{y}) (\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}})) \quad (15)$$

$$- \sigma B_o^2 \cos \alpha ((\bar{u} + c) \cos \alpha - \bar{v} \sin \alpha) + \rho g \bar{\alpha}_t (\bar{T} - \bar{T}_0) + \rho g \bar{\alpha}_c (\bar{C} - \bar{C}_0) + \rho g \sin \zeta,$$

$$\rho((\bar{u} + c) \frac{\partial}{\partial \bar{x}} + \bar{v} \frac{\partial}{\partial \bar{y}}) \bar{v} = - \frac{\partial \bar{p}}{\partial \bar{y}} + 2 \frac{\partial}{\partial \bar{y}} (\bar{\mu}(\bar{y}) \frac{\partial \bar{v}}{\partial \bar{y}}) + \frac{\partial}{\partial \bar{x}} (\bar{\mu}(\bar{y}) (\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}})) \quad (16)$$

$$+ \sigma B_o^2 \sin \alpha ((\bar{u} + c) \cos \alpha - \bar{v} \sin \alpha) - \rho g \cos \zeta,$$

$$\rho_{Cp}((\bar{u} + c) \frac{\partial}{\partial \bar{x}} + \bar{v} \frac{\partial}{\partial \bar{y}}) \bar{T} = K (\frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \bar{y}^2}) \bar{T} - \frac{\partial q_r}{\partial \bar{y}} + \bar{Q}, \quad (17)$$

$$((\bar{u} + c) \frac{\partial}{\partial \bar{x}} + \bar{v} \frac{\partial}{\partial \bar{y}}) \bar{C} = D_m (\frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \bar{y}^2}) \bar{C} + \frac{D_m K_t}{T_m} (\frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \bar{y}^2}) \bar{T}, \quad (18)$$

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0. \quad (19)$$

The non-dimensional parameters and variables are defined as follows:

$$\begin{aligned} x &= \frac{\bar{x}}{\lambda}, \quad y = \frac{\bar{y}}{d_1}, \quad u = \frac{\bar{u}}{c}, \quad v = \frac{\bar{v}}{c\delta}, \quad t = \frac{c\bar{t}}{\lambda}, \quad h_1 = \frac{\bar{H}_1}{d_1}, \quad h_2 = \frac{\bar{H}_2}{d_2}, \\ a &= \frac{a_1}{d_1}, \quad b = \frac{b_1}{d_1}, \quad d = \frac{d_2}{d_1}, \quad \delta = \frac{d_1}{\lambda}, \quad p = \frac{d_1^2 \bar{p}}{c\lambda\mu_0}, \quad \mu(y) = \frac{\bar{\mu}(\bar{y})}{\mu_0}, \\ M &= \sqrt{\frac{\sigma}{\mu_0}} B_o d_1, \quad \text{Pr} = \frac{\mu_0 C_p}{K}, \quad \text{Re} = \frac{\rho c d_1}{\mu_0}, \quad \text{Gr} = \frac{\rho g \bar{\alpha}_t (T_1 - T_o) d_1^2}{\mu_0 c}, \\ \text{Sc} &= \frac{\mu}{D_m \rho}, \quad \text{Sr} = \frac{\rho D_m K_T (T_1 - T_o)}{\mu T_m (C_1 - C_0)}, \quad \text{Br} = \frac{\rho g \bar{\alpha}_c (C_1 - C_o) d_1^2}{\mu_0 c}, \\ \beta &= \frac{d_1^2 \bar{Q}}{K(T_1 - T_o)}, \quad R = \frac{16\sigma T_o^3}{3kK}, \quad \theta = \frac{\bar{T} - T_o}{T_1 - T_o}, \quad \Theta = \frac{\bar{C} - C_o}{C_1 - C_o}. \end{aligned} \quad (20)$$

The radiative flux q_r of radiation is sculpted as

$$q_r = -RK \frac{\partial \bar{T}}{\partial y}. \quad (21)$$

3. Solution of the problem

For the above-mentioned modifications and non-dimensional variables listed earlier, the preceding equations are reduced to:

$$\begin{aligned} \text{Re } \delta((u+1) \frac{\partial}{\partial x} + v \frac{\partial}{\partial y})u = & -\frac{\partial p}{\partial x} + 2\delta^2 \frac{\partial}{\partial x}(\mu(y) \frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(\mu(y)(\delta^2 \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y})) \\ & - M^2 \cos \alpha((u+1) \cos \alpha - \delta v \sin \alpha) + Gr\theta + Br\Theta + \frac{\text{Re}}{Fr} \sin \zeta, \end{aligned} \quad (22)$$

$$\begin{aligned} \text{Re } \delta^3((u+1) \frac{\partial}{\partial x} + v \frac{\partial}{\partial y})v = & -\frac{\partial p}{\partial y} + 2\delta^2 \frac{\partial}{\partial y}(\mu(y) \frac{\partial v}{\partial y}) + \delta^4 \frac{\partial}{\partial x}(\mu(y)(\frac{\partial v}{\partial x} + \frac{1}{\delta^2} \frac{\partial u}{\partial y})) \\ & + \delta M^2 \sin \alpha((u+1) \cos \alpha - \delta v \sin \alpha) - \delta \frac{\text{Re}}{Fr} \cos \zeta, \end{aligned} \quad (23)$$

$$\text{Re Pr } \delta((u+1) \frac{\partial}{\partial x} + v \frac{\partial}{\partial y})\theta = (\delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})\theta + R \frac{\partial^2 \theta}{\partial y^2} + \beta, \quad (24)$$

$$\text{Re } \delta((u+1) \frac{\partial}{\partial x} + v \frac{\partial}{\partial y})\Theta = \frac{1}{Sc} (\delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})\Theta + Sr(\delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})\theta, \quad (25)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad (26)$$

where,

$$S_{xx} = \frac{2\mu c \delta}{d_1(1+\lambda_1)} [1 + \frac{\lambda_2 c \delta}{d_1} (u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y})] \frac{\partial u}{\partial x}, \quad (27)$$

$$S_{xy} = \frac{\mu}{(1+\lambda_1)} [1 + \frac{\lambda_2 c \delta}{d_1} (u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y})] (\frac{\partial u}{\partial y} + \delta^2 \frac{\partial v}{\partial x}), \quad (28)$$

$$S_{yy} = \frac{2\mu c \delta}{d_1(1+\lambda_1)} [1 + \frac{\lambda_2 c \delta}{d_1} (u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y})] \frac{\partial v}{\partial y}. \quad (29)$$

with boundary conditions

$$u = -1, \quad \theta + \gamma \frac{\partial \theta}{\partial y} = 0, \quad \Theta = 0 \quad \text{at} \quad y = h_1 = 1 + a \cos(2\pi x), \quad (30)$$

$$u = -1, \quad \theta - \gamma \frac{\partial \theta}{\partial y} = 1, \quad \Theta = 1 \quad \text{at} \quad y = h_2 = -d - b \cos(2\pi x + \phi). \quad (31)$$

The non-dimensional expressions for volume flow rate F , pressure rise ΔP_λ and frictional forces F_λ^u, F_λ^l on both upper and lower walls are respectively generated by

$$F = \int_{h_2}^{h_1} u dy, \quad (32)$$

$$\Delta P_\lambda = \int_0^1 \left(\frac{dp}{dx} \right) dx, \quad (33)$$

$$F_\lambda^{(u)} = \int_0^1 h_1^2 \left(-\frac{dp}{dx} \right) dx, \quad (34)$$

$$F_\lambda^{(l)} = \int_0^1 h_2^2 \left(-\frac{dp}{dx} \right) dx. \quad (35)$$

In the present analysis, we also consider μ as a function of y in the exponential form as

$$\mu = e^{-\varepsilon y}, \quad (35a)$$

Or

$$\mu = 1 - \varepsilon y \quad \text{for} \quad \varepsilon \ll 1.$$

where ε is the viscosity parameter.

4. The analytical solution

The expressions of temperature distribution, Concentration distribution, velocity and pressure gradient can be obtained under the assumptions of long wavelength and low Reynolds as follow:

$$\theta = \frac{1}{2}(c_1 + c_2), \quad (36)$$

$$\Theta = \frac{c_4}{c_5}, \quad (37)$$

$$u = a_1 \times [a_8 - (h_1 - h_2) \times (a_{10} + a_{16} + a_{20})], \quad (38)$$

$$\frac{dp}{dx} = \frac{F - S_1}{S_2}. \quad (39)$$

where,

$$S_1 = \frac{b_2 \times [b_3 + b_4]}{b_1}, \quad (40)$$

$$S_2 = \frac{b_2 \times [b_5 + b_6 + b_9 + b_{12} + b_{13} \times (1 + \frac{\varepsilon^2}{B} - \varepsilon y) + A \times b_4 + b_{15} + b_{20}]}{b_1}. \quad (41)$$

The equations (36) –(41) contains expressions for the unknowns, $c_1, c_2, c_4, c_5, a_1,$

$a_8, a_{10}, a_{16}, a_{20}, s_1, s_2, b_1, b_2, b_3, b_4, b_5, b_6, b_9, b_{12}, b_{13}, b_{15}, b_{20}, A, B$ Given in the Appendix

The coefficients of heat transfer Zh_1 and Zh_2 at the walls $y = h_1$ and $y = h_2$ are given by

$$Zh_1 = \theta_y h_{1x}, \quad (42)$$

$$Zh_2 = \theta_y h_{2x}. \quad (43)$$

Hence, The solutions of heat transfer at the walls $y = h_1$ and $y = h_2$ are given by

$$Zh_1 = \left[\frac{k}{2}(h_1 + h_2 - 2y) + \frac{1}{(-h_1 + h_2 - 2\gamma)} \right] \times [-2a\pi \sin(2\pi x)], \quad (44)$$

$$Zh_2 = \left[\frac{k}{2}(h_1 + h_2 - 2y) + \frac{1}{(-h_1 + h_2 - 2\gamma)} \right] \times [2b\pi \sin(2\pi x + \phi)]. \quad (45)$$

5. Numerical results and discussion

In this section the temperature θ , concentration Θ , velocity u , pressure gradient $\frac{dp}{dx}$, pressure rise Δp_λ and friction forces on the upper and the lower walls F_λ^u, F_λ^l are analyzed carefully. For this object, the Figures 2-11 are displayed.

Figure 2 shows the variations of the temperature θ with respect to the distance y for different physical parameters of the thermal radiation R , heat source/sink β , thermal slip γ and phase difference ϕ . We observed that the increase in the values of thermal radiation causes the decreases in temperature in the interval $-1 \leq y \leq -0.6$, while it causes increases in temperature in the interval $-0.6 \leq y \leq 1$, as well it increases with increasing of heat source/sink in the interval $-1 \leq y \leq -0.6$, while it decreases with increasing of heat source/sink in the interval $-0.6 \leq y \leq 1$, otherwise with the increases in thermal slip and phase difference cause the decrease in temperature. It is observed that the bolus appear in the center region for $\phi = 0$. This result is in good agreement with the results obtained by Sinha and Shit [30].

Figure 3 demonstrates the variation of the concentration Θ with regards to the distance y for different physical parameters of the thermal radiation R , heat source/sink β , phase difference ϕ and Schmidt number Sc . Note that the concentration decreases with increasing thermal radiation in the interval $-1 \leq y \leq -0.8$, while it increases in the interval $-0.8 \leq y \leq 1$, as well it increases with increasing of heat source/sink in the interval $-1 \leq y \leq -0.4$, while it decreases with increasing heat source/sink in the interval $-0.4 \leq y \leq 1$, otherwise it decreases with increasing the phase difference in the interval $-1 \leq y \leq -0.8$, while it increases in the interval $-0.8 \leq y \leq 1$, Also it increases with increasing of Schmidt number in the interval $-1 \leq y \leq -0.4$, otherwise it decreases with increasing of Schmidt number in the interval $-0.4 \leq y \leq 1$. It is observed that the bolus appear in the center region for $\phi = 0$.

Figure 4 describes the variations of the velocity u against the distance y for the different values of thermal radiation R , Hartman number M , thermal slip γ , viscosity parameter ε , heat source/sink β and aligned magnetic field α . It should be noted that an inclination of the channel does not affect the velocity field. It affects only the pressure field. It is observed that the velocity decreases with increasing of thermal radiation, Hartman number and thermal slip, while it increases with increasing of the viscosity parameter, heat source/sink and aligned magnetic field near the walls of the artery, as well it satisfied boundary conditions. Whereas an opposite trend is observed at the center of the channel. This result is in good agreement with the results obtained by Shit and Majee [31] in the case of symmetric channel.

In **Figure 5** the distributions of the pressure gradient $\frac{dp}{dx}$ over the distance $y \in [0,10]$ are exhibited for different values of the thermal radiation R , Hartman number M , thermal slip γ and the viscosity parameter ε respectively. It can be clearly seen from Figure 5 that, on the one hand, in the wide part of the peristaltic channel, $y \in (0,5)$, the pressure gradient increases with increasing the thermal radiation and Hartman number, when $y \in (5,10)$ the pressure gradient decreases with increasing of thermal radiation and Hartman number, while it decreases with increasing of thermal slip at $y \in (0,10)$, as well it decreases with increasing of viscosity parameter at $y \in (0,5)$ and increases with increasing of viscosity parameter when $y \in (5,7.4)$, otherwise it decreases with increasing of viscosity parameter when $y \in (7.4,10)$. From these figures, we observe that a much large pressure gradient is required to maintain the same flux to pass it for the wide part of the channel $y \in [0,10]$.

The influence of heat source/sink β and thermal radiation R on the heat transfer coefficients Zh_1 of the upper wall and Zh_2 of lower wall are graphically displayed in **Figure 6** by choosing different values of the heat source/sink and thermal radiation. Obviously, the increasing the heat source/sink and thermal radiation decrease on the amplitude of the heat transfer coefficients in the whole range x . Such an effect may be expected, because under the conditions considered the thermal radiation and heat source/sink is a resistance to the flow, and its magnitude is proportional to the heat transfer coefficients. From Figure 6, one can observe that heat transfer coefficients are in oscillatory behavior, which may be due to peristalsis.

Figure 7 is plotted for various values of Hartman number M , thermal slip γ , viscosity parameter ε and aligned magnetic field α . From these figures, we observe that with the increase of Hartman number and viscosity parameter a tangential stress s_{xy} is increasing, while it decreases with increasing of thermal slip and aligned magnetic field. It is noticed that one can observe the tangential stress is in oscillatory behavior, which may be due to peristalsis.

The influences of the phase difference ϕ , thermal slip γ and viscosity parameter ε are illustrated in **Figure 8**. It is observed that the pressure rise Δp_λ increases rapidly with the increase of the phase difference, thermal slip and viscosity parameter when $F \in (-300,0)$, while it decreases

when $F \in (0, 300)$. As expected, that the pressure rise gives larger values for small volume flow rate F and it gives smaller values for large volume flow rate. Moreover, the peristaltic pumping occurs in the region $-300 \leq F \leq 300$, otherwise augmented pumping occurs.

Figure 9, 10 clarify the variation of frictional force F_λ^u, F_λ^l against volume flow rate F for different values phase difference ϕ , thermal slip γ and viscosity parameter ε . It is observed that there is a direct linear relation between frictional force and volume flow rate, i.e. the frictional force on the upper and lower walls decreases by increasing phase difference, thermal slip, and viscosity parameter when $F \in (-300, 0)$, while it increases by increasing phase difference, thermal slip, and viscosity parameter when $F \in (0, 300)$. The frictional force also have opposite behavior when compared with that of pressure rise.

Figure 11 is plotted in 3D schematics concern the temperature θ , concentration Θ , velocity u and pressure gradient $\frac{dp}{dx}$ with respect to x and y axes in the presence of phase difference ϕ , Schmidt number Sc , Hartman number M and thermal slip γ . It is observed that the temperature decreases by increasing of the phase difference, while the concentration increases by increasing of Schmidt number, as well the velocity decreases with increasing of Hartman number, otherwise the pressure gradient decreases by increasing thermal slip. We obtain for all physical quantities, the peristaltic flow in 3D overlapping and damping when x and y increase to reach the state of particle equilibrium. The vertical distance has more significant of the curves were obtained, most physical fields are moving in peristaltic flow.

6. Conclusion

The analytical solution has been computed for velocity, temperature, concentration. The expressions for velocity, temperature, concentration, pressure gradient, heat transfer coefficients, tangential stress, pressure rise and frictional forces have been discussed graphically. The following observations have been found

1. An increase in M while keeping all the other parameters fixed results in decrease of velocity.
2. The magnitude of $\frac{dp}{dx}$ increases when there is an increase in R and M .
3. It is observed that frictional force has an opposite behavior as compared to the pressure rise.
4. The behavior of heat source/sink parameter and thermal radiation parameter on the characteristics is totally opposite.
5. It is observed that the concentration field decreases with the increases in ϕ .
6. In case of an inclined angle $\alpha = 0$, our results are in agreement with Kumar and Ahmed [16].

7. The results presented in this paper should prove useful for researchers in Science, Medicine and Engineering, as well as for those working on the development of fluid mechanics. Study of the phenomenon of the phase difference ϕ , thermal radiation R , Hartman number M , thermal slip γ and viscosity parameter ε influence and operations is also used to improve the conditions of peristaltic motion.

Appendix

$$A = M^2 \times \cos^2(\alpha), \quad B = \frac{\text{Re}}{Fr} \times \sin(\zeta), \quad k = \frac{-\beta}{1+R},$$

$$c_1 = k(h_1 - y) \times (-h_2 + y) + k(h_1 - h_2)\gamma, \quad c_2 = \frac{2(h_1 - y + \gamma)}{(h_1 - h_2 + 2\gamma)},$$

$$c_3 = S_r \times S_c, \quad c_4 = 2 - c_3 k \times [(h_1 - h_2) \times (h_1 - y) \times (h_2 - y)],$$

$$c_5 = 2 \times (h_1 - h_2),$$

$$a_1 = \frac{e^{\frac{-y\varepsilon}{\varepsilon y - 1}}}{12(e^{\frac{h_1\varepsilon}{\varepsilon y - 1}} - e^{\frac{h_2\varepsilon}{\varepsilon y - 1}}) \times (h_1 - h_2 + 2\gamma) \times (h_1 - h_2)\varepsilon^3},$$

$$a_2 = -e^{\frac{(h_2 + y)\varepsilon}{\varepsilon y - 1}} (h_2 - y) \times [2 + (-2h_1 + h_2 + y)\varepsilon - 2\varepsilon y],$$

$$a_3 = e^{\frac{(h_1 + h_2)\varepsilon}{\varepsilon y - 1}} (h_1 - h_2) \times [-2 + h_1\varepsilon - h_2\varepsilon + 2\varepsilon y],$$

$$a_4 = -e^{\frac{(h_1 + y)\varepsilon}{\varepsilon y - 1}} (h_1 - y) \times [-2 + h_1\varepsilon - y\varepsilon + 2\varepsilon y],$$

$$a_5 = e^{\frac{(h_1 + h_2)\varepsilon}{\varepsilon y - 1}} (h_1 - h_2) \times [(h_1 - h_2)^2 \varepsilon^2 - 12(\varepsilon y - 1)^2],$$

$$a_6 = e^{\frac{(h_2 + y)\varepsilon}{\varepsilon y - 1}} (h_2 - y) \times [-12 + \varepsilon \times (6h_1 - 6y + (h_2 - y) \times (-3h_1 + h_2 + 2y)\varepsilon) \\ + 24\varepsilon y + 6(-h_1 + y)\varepsilon^2 y - 12\varepsilon y^2],$$

$$a_7 = -e^{\frac{(h_1 + y)\varepsilon}{\varepsilon y - 1}} (h_1 - y) \times [-12 + \varepsilon \times (6(h_2 - y) + (h_1 - y) \times (h_1 - 3h_2 + 2y)\varepsilon) \\ + 24\varepsilon y + 6(-h_2 + y)\varepsilon^2 y - 12\varepsilon y^2],$$

$$a_8 = Br(h_1 - h_2 + 2\gamma) \times [6\varepsilon(a_2 + a_3 + a_4) + c_3(h_1 - h_2)k \times (a_5 + a_6 + a_7)],$$

$$a_9 = Gr \times [-12\varepsilon + (h_1 - h_2 + 2\gamma) \times (-12k + (6 + (h_1 - h_2)k(h_1 - h_2 + 2\gamma))\varepsilon^2) \\ + 12(2k(h_1 - h_2 + 2\gamma) + \varepsilon)\varepsilon y - 12k(h_1 - h_2 + 2\gamma)\varepsilon y^2],$$

$$\begin{aligned}
a_{10} &= -e^{\frac{(h_1+h_2)\varepsilon}{\varepsilon y-1}} (h_1-h_2) \times [-12(A-B+\frac{dp}{dx}) \times (h_1-h_2+2\gamma)\varepsilon^2 + a_9], \\
a_{11} &= h_1^3 k \varepsilon^2 + h_1^2 k \times (-4h_2 + y + 8\gamma)\varepsilon^2, \\
a_{12} &= 2h_2 k \times [6 + \varepsilon(3y + 6\gamma + (y^2 + 3y\gamma - 6\gamma^2)\varepsilon) - 12\varepsilon y - 3(y + 2\gamma)\varepsilon^2 y + 6\varepsilon y^2], \\
a_{13} &= -3h_2^2 k \varepsilon \times [y\varepsilon - 2(-1 + \gamma\varepsilon + \varepsilon y)], \\
a_{14} &= h_1 \times [6\varepsilon^2 + k \times (-12 + \varepsilon(-6y + 3h_2^2\varepsilon - 2(y-3\gamma) \times (y+2\gamma)\varepsilon + \\
&\quad 2h_2(3+y\varepsilon-9\gamma\varepsilon)) + 24\varepsilon y + 6(-h_2+y)\varepsilon^2 y - 12\varepsilon y^2)], \\
a_{15} &= -2 \times [2k\gamma(y^2\varepsilon^2 - 3y\varepsilon(\varepsilon y - 1) + 6(\varepsilon y - 1)^2) + 3\varepsilon(y\varepsilon - 2(-1 + \gamma\varepsilon + \varepsilon y))], \\
a_{16} &= e^{\frac{(h_1+y)\varepsilon}{\varepsilon y-1}} \times [12(h_1-h_2+2\gamma)\varepsilon^2 \times (-(A-B+\frac{dp}{dx}) \times (h_1-\gamma) + \varepsilon) \\
&\quad + Gr(h_1-y) \times (a_{11} + a_{12} + a_{13} + a_{14} + a_{15})], \\
a_{17} &= h_2^3 k \varepsilon^2 + h_2^2 k \times (y-8\gamma)\varepsilon^2 + 3h_1^2 k \varepsilon \times (-2 + h_2\varepsilon - y\varepsilon - 2\gamma\varepsilon + 2\varepsilon y), \\
a_{18} &= -2h_2 \times [-3\varepsilon^2 + k \times (6 + \varepsilon(3y + (y-2\gamma) \times (y+3\gamma)\varepsilon) \\
&\quad - 12\varepsilon y - 3\varepsilon^2 y^2 + 6\varepsilon y^2)], \\
a_{19} &= 2h_1 \times [-6\varepsilon^2 + k \times (6 + \varepsilon(3(h_2+y-2\gamma) + (-2h_2^2 - 3y\gamma - 6\gamma^2 \\
&\quad + h_2(y+9\gamma))\varepsilon) - 12\varepsilon y - 3(h_2+y-2\gamma)\varepsilon^2 y + 6\varepsilon y^2)], \\
a_{20} &= e^{\frac{(h_2+y)\varepsilon}{\varepsilon y-1}} \times [-12(h_1-h_2+2\gamma)\varepsilon^2 \times (-(A-B+\frac{dp}{dx}) \times (h_2-\gamma) + \varepsilon) \\
&\quad + Gr(h_2-y) \times (a_{17} + a_{18} + a_{19} + a_{15})], \\
b_1 &= 12(e^{\frac{h_1\varepsilon}{\varepsilon y-1}} - e^{\frac{h_2\varepsilon}{\varepsilon y-1}}) \times (h_1-h_2+2\gamma)\varepsilon^4, \quad b_2 = e^{\frac{(h_1+h_2)\varepsilon}{2(\varepsilon y-1)}}, \\
b_3 &= -12(h_1-h_2) \times (-h_1+h_2) \times (h_1-h_2+2\gamma)\varepsilon^3 \times \cosh(\frac{(h_1-h_2)\varepsilon}{2(\varepsilon y-1)}), \\
b_4 &= -24(h_1-h_2) \times (h_1-h_2+2\gamma)\varepsilon^2 \times \sinh(\frac{(h_1-h_2)\varepsilon}{2(\varepsilon y-1)}), \\
b_5 &= A \times b_3, \quad b_6 = -B \times b_3, \\
b_7 &= -12k + (6 + (h_1-h_2)k \times (h_1-h_2+6\gamma))\varepsilon^2, \\
b_8 &= 12(2k(h_1-h_2+2\gamma) + \varepsilon)\varepsilon y - 12k(h_1-h_2+2\gamma)\varepsilon y^2, \\
b_9 &= Gr(h_1-h_2) \times (-h_1+h_2)\varepsilon \times \cosh(\frac{(h_1-h_2)\varepsilon}{2(\varepsilon y-1)}),
\end{aligned}$$

$$\begin{aligned}
b_{10} &= c_3(h_1 - h_2)k \times [(-h_1 + h_2)^2 \varepsilon^2 - 12(\varepsilon y - 1)^2], \\
b_{11} &= 6\varepsilon(-2 + h_1\varepsilon - h_2\varepsilon + 2\varepsilon y), \\
b_{12} &= Br(-h_1 + h_2) \times (h_1 - h_2 + 2\gamma)\varepsilon \times \cosh\left(\frac{(h_1 - h_2)\varepsilon}{2(\varepsilon y - 1)}\right) \times [b_{10} + b_{11}], \\
b_{13} &= B \times b_4, \\
b_{14} &= -12c_3(h_1 - h_2)k \times (\varepsilon y - 1)^3 + \varepsilon(12 + (h_1 - h_2)\varepsilon(-6 + h_1\varepsilon - h_2\varepsilon) - 24\varepsilon y + 6(h_1 - h_2)\varepsilon^2 y + 12\varepsilon y^2), \\
b_{15} &= 2Br(h_1 - h_2 + 2\gamma) \times \sinh\left(\frac{(h_1 - h_2)\varepsilon}{2(\varepsilon y - 1)}\right) \times b_{14}, \\
b_{16} &= h_1^2 \varepsilon^2 \times [\varepsilon + 6k\gamma(\varepsilon y - 1)] + h_2^2 \varepsilon^2 \times [\varepsilon + 6k\gamma(\varepsilon y - 1)], \\
b_{17} &= 2h_1[\varepsilon^2(3 + h_2\varepsilon - 3\varepsilon y) + 6k((h_2 - \gamma)\gamma\varepsilon^2 + (\varepsilon y - 1)^2) \times (\varepsilon y - 1)], \\
b_{18} &= 6h_2(\varepsilon y - 1) \times [\varepsilon^2 + 2k(1 + \gamma\varepsilon - \varepsilon y) \times (-1 + \gamma\varepsilon - \varepsilon y)], \\
b_{19} &= -12(\varepsilon y - 1) \times [\varepsilon + 2k\gamma(\varepsilon y - 1)^2 - \varepsilon(\gamma\varepsilon + \varepsilon y)], \\
b_{20} &= 2Gr(h_1 - h_2) \times \sinh\left(\frac{(h_1 - h_2)\varepsilon}{2(\varepsilon y - 1)}\right) \times [b_{16} + b_{17} + b_{18} + b_{19}].
\end{aligned}$$

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