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Supporting Information for

**Advancements in Planetary Unstructured Equivalent Source Inversion and Current Circulation Modeling Technology for Earth's Magnetic Field**

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**Introduction**

The supplementary materials for this study include comprehensive texts (Texts S1 to S3) that detail the forward modeling procedure, the division of the inversion mesh (as illustrated in Figures S1 and S2), and the development of the magnetization current source (shown in Figures S3). Additionally, Dataset S1, comprising the SWARM data utilized in our inversion example, is provided. To further aid in visualizing our findings, we have included a series of videos (Movies S1-S3) that display the 3D models of the current in detail. These videos are in .MP4 format and can be conveniently accessed using standard media players.

**Text S1:** *Quadratic Finite Element Discrete Solve Scheme*

For each tetrahedron element, there 10 nodes on the 4 vertices and the 6 middle point of edges, respectively, as shown in Figure S1. We use the discretized forms (Eq. S1) for potential field () with the domain is divided to tetrahedral elements (). The based function  ()represent the potential field in each discretization elements subdomain .

In each tetrahedron element of our model, there are 10 nodes: these include nodes at the 4 vertices and nodes at the 6 midpoints of the edges, as depicted in Figure S1. The discretized formulation for the potential field () is utilized, outlined in Eq. S1, where the domain is segmented into tetrahedral elements (). The basis functions, denoted as  (), represent the potential field within the subdomains  of each discretized element.

 (S1)

The governing equation, as outlined in Eq. (3), is solvable using the Galerkin method within Cartesian coordinates, a process which is detailed in Eq. (S2).

 (S2)

Subsequently, the linear equation  with the accompanying terms defined as: :

 (S3a)

 (S3b)

where  and .

**Text S2:** *Data Processing*

A total of 35,768 three-component magnetic vector data points were obtained from the International Geomagnetic Reference Field (IGRF-13) model. These data points are uniformly distributed over a spherical surface, defined in geocentric coordinates with the Earth's center as the origin. The x-axis aligns with the Greenwich meridian and equator, while the z-axis points north, referencing the World Geodetic System 1984 (WGS84) spheroid. Initially, the magnetic vectors from IGRF are based in a North-East-Down (NED) reference frame, within a spherical coordinate system. For the purposes of our inverse method, which operates in a geocentric coordinate system, these vectors are transformed from the local NED coordinates to Geocentric Earth-Centered, Earth-Fixed (ECEF) Cartesian coordinates.

Our study employs a 3-D cubic inverse grid, spanning 40,000 km in each dimension, which encompasses a spherical core model with a radius of 3,479 km, positioned at the origin of the Cartesian coordinate system. This sphere, representing the Earth’s core, is uniformly divided to form the inverse model grid. The boundary of this spherical model lies 2,900 km beneath the Earth's surface. To accommodate the central, finely-detailed core model, the unstructured grid is supplemented with coarser tetrahedrons in the surrounding space. The region encompassing the observational data is further refined for accuracy. Overall, the grid comprises 2,337,505 tetrahedral cells, with the core model itself containing 3,133,643 nodes, as illustrated in Fig. S2.

**Text S3:** *Magnetization current model*

The 3-D magnetic flux inverse method we have implemented is anchored in a partial differential equation framework, uniquely defined within the Cartesian coordinate system. This approach contrasts with the conventional integral equation framework typically employed in a spherical coordinate system. For our method, the initial value of the model is set at 0.001 A/m². The regularization parameter β, crucial for the inversion process, is consistently set at 1E2 across all examples. It is observed that setting β to a larger value can hinder convergence, but it results in a more sharply defined magnetization or current inverted model. According to classical electromagnetic theory, the magnetic field , generated by a free electric current , can equivalently originate from an electric current . Both phenomena conform to Ampère's law, which states:

 (S4)

Moreover, these two types of electric currents,  and , are known to exhibit an explicit and defined relationship:

 (S5)

The Earth's core is assumed to be a nonlinear medium. Consequently, the equivalent susceptibilities  cannot be directly probed or estimated, making it impractical to calculate the real free electric current  using the established equation. Therefore, the equivalent current circulation model in this study serves as an approximate representation of the actual internal source.

In our analysis, we estimate equivalent currents and magnetic fields across the entire 3D space. However, for clarity in our visual representations, only select portions of these are depicted in the figures, chosen at specific intervals. Displaying all data points would result in an overly dense visual where key features are obscured by the sheer number of lines filling the 3D space. This point is demonstrated in Fig. S4, where we show an example with a greater number of electric current vectors drawn without excessive interval. Similarly, for reasons of visual clarity, the electric gyre currents in 3D space are not projected onto a 2D map, as such a representation would not adequately convey the intricate spatial relationships.

**Figure S1:**



Figure S1 A quadratic tetrahedral element for magnetization model.

**Figure S2:**

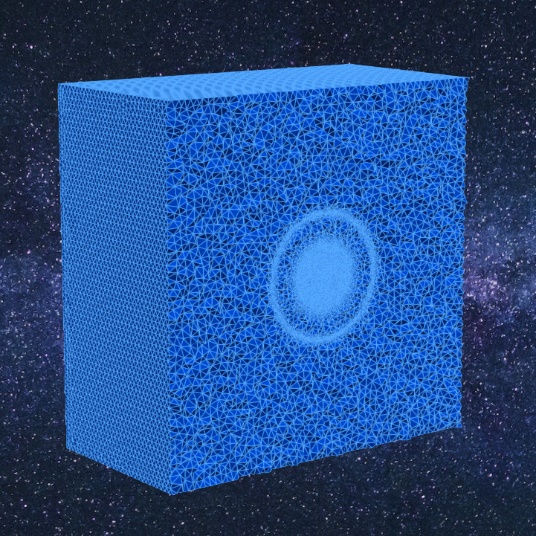
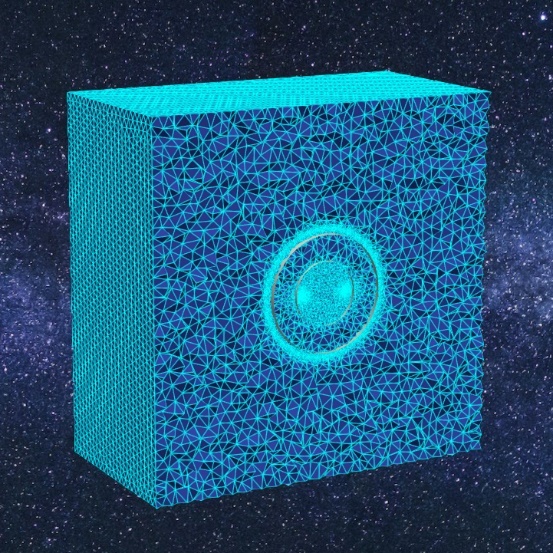


Figure S2: Meshes Used in Forward Modeling and Inversion.

a. Forward Modeling Mesh for the Synthetic Model featuring a Current Circular Ring. This figure shows the large circular ring representing the refined mesh around observation points, essential for accurately capturing the model's dynamics. b. Inversion Mesh for the Examples Inversion. The central circular region in this figure indicates the fine homogeneous inversion are. The radius of the inversion region sphere is set at 3,479 km, corresponding to the spherical model of the Earth's core.

**Figure S3:**

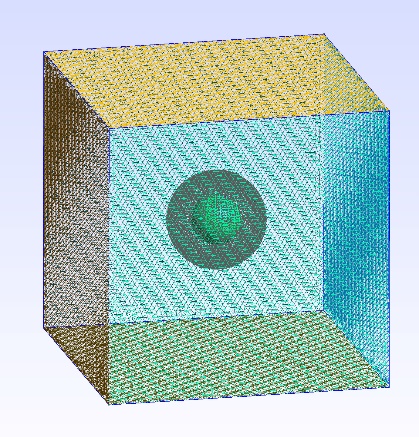
 

Figure S3: Observation Point Locations in the Swarm Satellite Dataset.

a. Distribution of Observation Points within the Inversion Mesh. This part of the figure displays how observation points from the Swarm dataset are positioned within the 3-D cubic inversion mesh, providing a comprehensive view of their spatial arrangement. b. Detailed View of the Observation Locations (marked as black dots). This zoomed-in view offers a closer look at the specific positioning of the Swarm dataset's observation points within the inversion mesh. The green sphere in this figure represents the boundary of the inversion region, giving context to the placement of the observation points.

**Figure S4:**

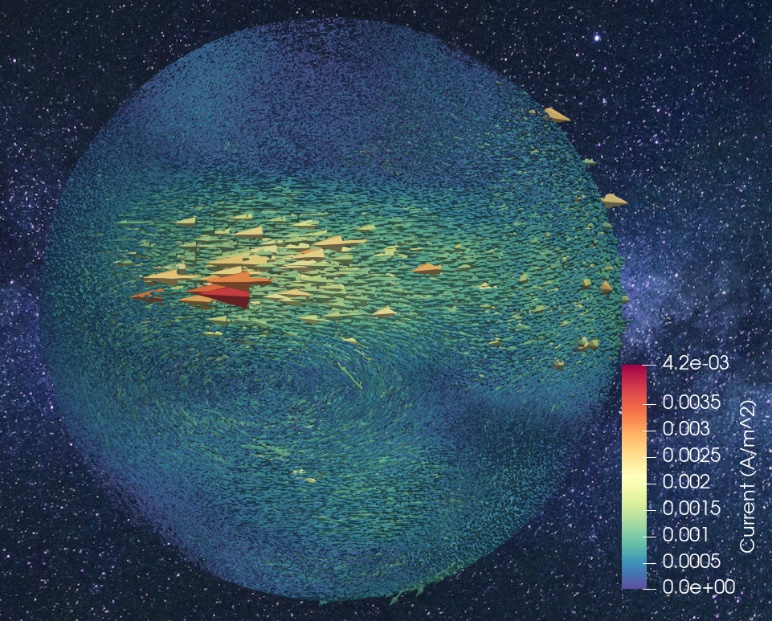


Figure S4: Inverted Current from SWARM Dataset.

This figure presents the inverted current model derived from the Swarm dataset, displayed without applying intervals to the data points. The comprehensive visualization captures the full array of current data, providing an unfiltered representation of the inverted current patterns.