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Supporting Information for

A Comprehensive Criterion for Threshold of Motion of Bioclastic Sediments under Steady Unidirectional Flow

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Text S1

The threshold of sediment motion is determined as the critical condition under which a bed surface materials initiate to move under the unidirectional flow. However, there is no universally accepted criteria for defining the initiation of the sediment motion, leading to discrepancies in previous datasets. In this study, we adopted the quantitative criteria ε proposed by Yalin (1972) to determine the threshold of motion of coral particles based on the video images recorded in the experiment. The dimensionless parameter ε is defined as,

$$\varepsilon = \left(\frac{N}{A\Delta t}\right)\left[\frac{\rho d^5}{(\rho_\mathrm{p} - \rho)g}\right]^{1/2} \tag{S1}$$

where N is the number of grains detached from the bed in the time Δt, over the given bed surface area, A, g is the gravitational acceleration, and d is the characteristic grain diameter of the sediment. The value of ε was suggested to be small, on the order of 10^{-6}. Then, the value of N for each sample can be estimated, which in turn determines the threshold condition of the initiation of sediment motion in the experiments. In this study, Δt =120 sec, A =0.0266 m^2, and $\varepsilon \geq 10^{-6}$ was applied to determine the sediment threshold of motion, and d has been taken as the nominal diameter d_n of each sample.

Text S2

In the context of flows over rough beds, conventional Reynolds equations or Reynolds averaged Navier-Stokes equations are inadequate due to the spatial heterogeneity observed in the near-bed region (Padhi et al., 2018). To address this complexity, the time averaging is conceptually supplemented by the area averaging in the layer parallel to the mean bed surface, which is called a double-averaging method (DAM) (Nikora et al., 2007). With DAM, the local instantaneous horizontal and vertical velocities (u, w) are separated into a time-averaged velocity quantity $(\bar{u}, \bar{w})$ and a turbulent fluctuation (u', w') by Reynolds decomposition. Then, the horizontally varying time-averaged velocities $(\bar{u}, \bar{w})$ are decomposed as a double-averaged (DA) velocity $(\langle\bar{u}\rangle, \langle\bar{w}\rangle)$, averaged over horizontal space, and a horizontal fluctuation of the time-averaged velocity $(\tilde{u}, \tilde{w})$. In this study, the DAM was employed to analyze the flow field over the coral debris bed.

The near-bed turbulent boundary layer governs the particle motion at the bed. The bed shear velocity, u_*, can be calculated from the double-averaged (DA) velocity profile within the boundary layer based on the law of the wall. The structure of the boundary layer depends on the hydraulic regime of the flow, which is determined by the shear Reynolds number $R_*=u_* k_s/\nu$, where k_s is the Nikuradse roughness length and ν is kinetic viscosity of the fluid. The law of the wall is defined as follows,

$$\frac{\langle\bar{u}\rangle}{u_*} = \frac{1}{\kappa}\ln\left(\frac{z}{z_0}\right) \tag{S2}$$

where $\langle\bar{u}\rangle$ is the DA streamwise horizontal velocity, κ is the von Kármán constant (=0.41), z is the elevation from the origin $z = 0$ determined at the gravel crest on the bed surface, and z_0 is the level where $\langle\bar{u}\rangle = 0$. With the measured $\langle\bar{u}\rangle$ and corresponding z within the logarithmic layer, the values of u_* and z_0 are obtained by the linear regression line of $\langle\bar{u}\rangle$ and $\ln(z)$ using Equation S2.

Text S3

Alcerreca et al. (2013) proposed a simple equation for the computation of particle settling velocity based on the dataset of 1557 calcareous sand grains, which is expressed as,

$$\frac{w_s d}{\nu} = \left(\sqrt{22 + 1.13 d_*^2} - 4.67\right)^{1.5} \tag{S3}$$

where $d_* = [(\rho_s - \rho)g/\rho\nu^2]^{1/3} d$, and d is taken as d_n in his study.

Text S4

Dietrich (1982) presented the fourth-order polynomial fitted equation of the relationship between d_* and w_{*0} for spherical particles, expressed as

$$\begin{aligned}\log W_{*0} = &-3.76715 + 1.92944(\log D_*) - 0.09815(\log D_*)^{2.0} - 0.00575(\log D_*)^{3.0} \\ &+ 0.00056(\log D_*)^{4.0}\end{aligned} \tag{S4}$$

where $W_{*0} = w_{*0}^3$ and $D_* = d_*^3$. Equation S4 is valid within the range of $0.05 \le D_* \le 5 \times 10^9$.

Figure S1

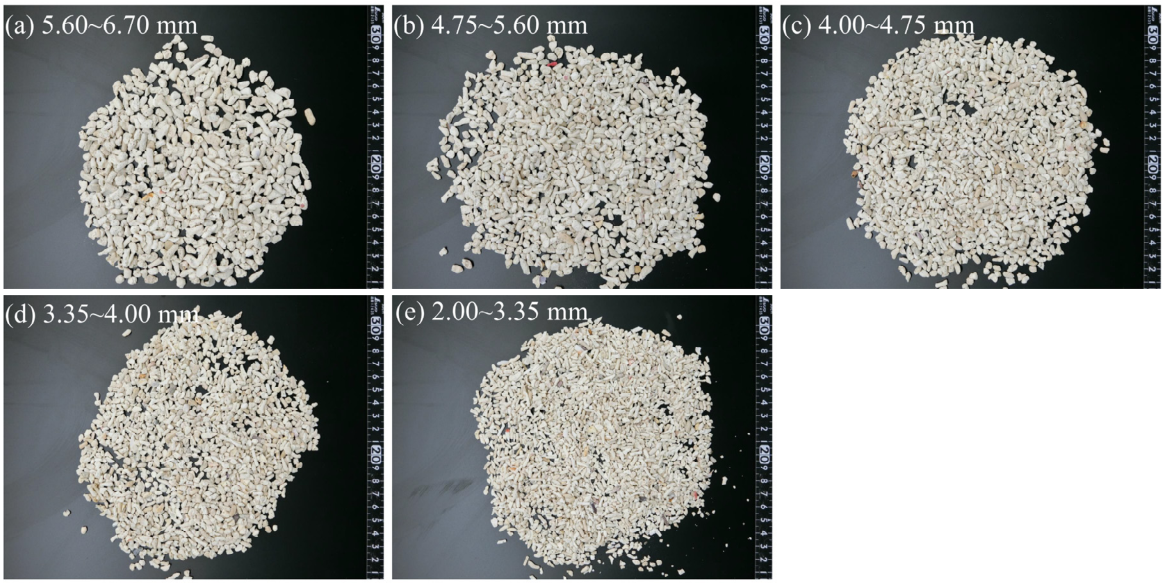

Figure S1. Photographs of the five coral debris fractions sorted through sieving.

Figure S2

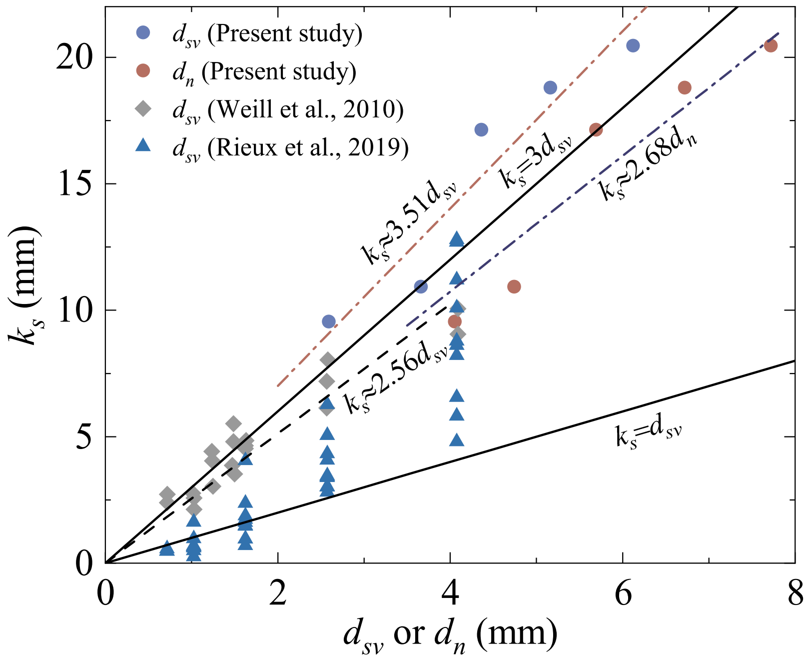

Figure S2. The Nikuradse roughness length k_s as a function of sieve diameter d_{sv} or nominal diameter d_n for the five coral debris beds. In this graph, the correlations between k_s and d_{sv} reported by Weill et al. (2010) and Rieux et al. (2019) are also plotted.

Figure S3

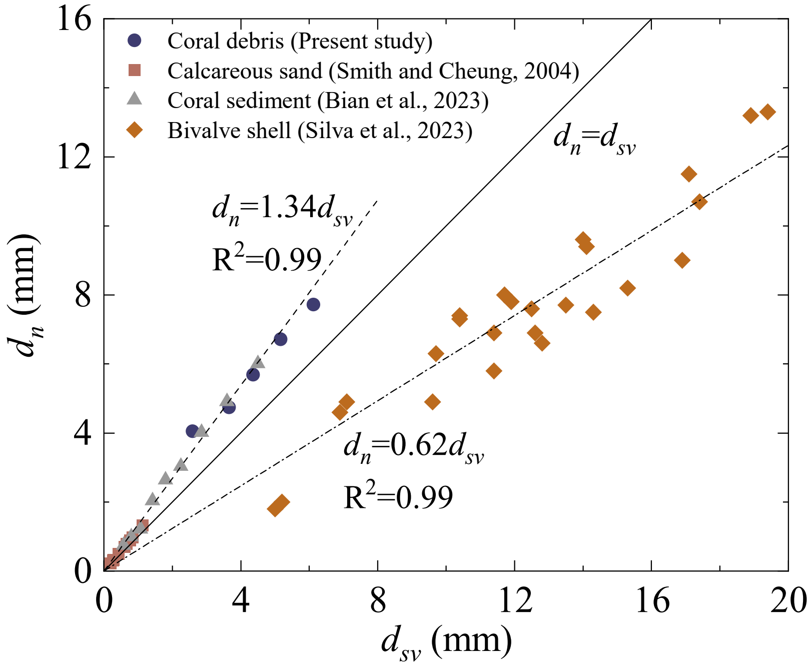

Figure S3. Correlation between the nominal diameter d_n and the sieve dimeter d_{sv} of investigated coral particles, calcareous sand (Smith and Cheung, 2004), coral sediment (Bian et al., 2023), and bivalve shells (Silva et al., 2023).

Figure S4

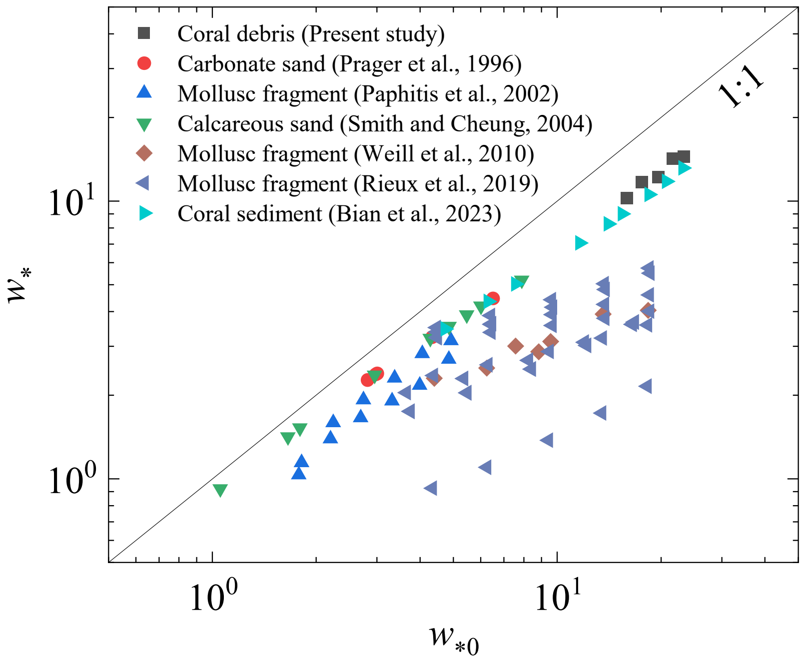

Figure S4. Correlation between w_* and the dimensionless settling velocity of a volume-equivalent sphere w_{*0} for considered bioclastic grains in this study.

Table S1. Experimental parameters at the entrainment threshold.

Sample	d_{sv} (mm)	d_n (mm)	A_{mp} (mm^2)	d_s/d_i	d_i/d_l	S_f	w_s (m/s)	$u_{*\mathrm{cr}}$ (×10^{-2} m/s)	Re_*	R_*	k_s (mm)	θ_{cr} (×10^{-2})	Λ_{c}
PS1	6.12	7.72	79.04	0.73	0.43	0.47	0.296	6.65	513.41	1360.8	20.46	6.67	0.22
PS2	5.16	6.72	58.11	0.76	0.43	0.49	0.291	5.67	380.83	1065.4	18.80	5.56	0.19
PS3	4.36	5.69	42.35	0.69	0.52	0.48	0.250	5.20	295.99	891.7	17.14	5.54	0.21
PS4	3.66	4.74	29.45	0.67	0.55	0.48	0.240	4.86	230.33	531.4	10.94	5.80	0.20
PS5	2.59	4.05	21.27	0.68	0.53	0.49	0.210	4.48	181.45	427.9	9.56	5.76	0.21

Table S2. Data compilation of threshold of motion of bioclastic sediments under steady current obtained from preceding studies.

Data source	Abbreviation	Number of data	Materials	Solid density (g/cm^3)	Used grain diameter	Shape factor	Settling velocity
Prager et al. (1996)	PR	5	Carbonate sand	2.50~2.73	d_{sv}	Not given	Not given
Paphitis et al. (2002)	PA	12	Mollusc fragment	2.72~2.80	d_{sv} and d_q	Not given	Given
Smith & Cheung (2004)	SC	9	Calcareous sand	2.60	d_{sv} and d_n	Not given	Not given
Weill et al. (2010)	WE	8	Mollusc fragment	2.60~2.71	d_{sv} and d_q	Not given	Given
Rieux et al. (2019)	RI	50	Mollusc fragment	2.01~2.80	d_{sv}	Not given	Given
Bian et al. (2023)	BI	45	Coral sediment	2.82	d_{sv} and d_n	S_f	Not given

*d_q is the equivalent settling diameter.

Table S3. Regression statistics and parameters for different scenarios. where $\boldsymbol{N}$ denotes the number of data used for regression, ARE is the absolute relative error, and $\boldsymbol{k}$ is the angular coefficient indicating the slope of the fitted line, with $\boldsymbol{k} = \mathbf{1}$ denoting perfect agreement.

Data source	N	Scenarios			a	R^2	k	ARE (%)
		φ	d for calculating Λ_{c0}	F				
PS	5	0.42	d_n	1	—	0.997	0.778	22.11
PS	5	0.42	d_n	S_f	-0.344	0.997	1.002	3.67
PS	5	0.42	d_n	w_*/w_{*0}	-0.568	0.998	1.003	3.39
PS, SC	14	0.42 (PS), 0 (SC)	d_{sv}	1	—	0.995	0.778	18.21
PS, SC	14	0.42 (PS), 0 (SC)	d_n	1	—	0.993	0.799	14.46

PS, SC	14	0.42 (PS), 0 (SC)	d_n	w_*/w_{*0}	-0.560	0.995	1.012	9.37
PS, PR, PA, SC, WE, RI	89	0.42 (PS), 0 (others)	d_{sv}	1	—	0.862	0.554	35.64
PS, PR, PA, SC, WE, RI	89	0.42 (PS), 0 (others)	d_{sv}	w_*/w_{*0}	-0.589	0.972	0.935	14.81