The Limits to Equity in Water Allocation Under Scarcity

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Key Points:

- A categorization of widely implemented water allocation mechanisms into three canonical types based on the adequacy distribution.
- Evaluation of the performance limits for the canonical and arbitrary allocation mechanisms in terms of equity and reliability.
- Validation of the theoretical performance limits through Monte-Carlo simulations generated for a selected agricultural area.

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Abstract

Equitable water allocation in real-world irrigation systems is hampered by supply fluctuations, posing a significant challenge to the goal of promoting fairness among consumers. In this paper, we concern ourselves with the limits of equity achievable for any water allocation scheme across the entire spectrum of water supply conditions. In the process, we develop a typology of canonical water allocation mechanisms that categorizes mechanisms w.r.t. the distribution of fulfilled demand across the users. Adopting specific notions of supply reliability and distribution equity, we derive the theoretical performance limits for all canonical mechanisms and extend the analysis to arbitrary allocation mechanisms. We show that for any value of supply reliability, the best possible equity is realized by mechanisms that uniformly distribute water among users, whereas the worst possible equity is associated with mechanisms that prioritize the demand of some users before allocating water to others. We also show that any intermediate equity level can be realized by adjusting the initial entitlements prior to allocating water to fulfill demands, in an approach we categorize as hybrid allocation. We parameterize the performance boundaries for such allocation schemes based on the fraction of supply allocated to initial entitlements. We discuss how this parameter can serve as a policy tool to balance the goals of equitable water access with other system-level objectives. In the end, we complement the analytical results with numerical simulations of a selected agricultural district from a real-world irrigation system and speculate about the application of our study to large-scale hierarchical systems.

1 Introduction

Inadequate water management is recognized as a primary instigator of water scarcity, among other factors such as population growth, rapid industrialization, economic growth, and environmental pollution, to name a few (Dolan et al., 2021). Water scarcity is projected to increase even further in the future due to climate change effects (UN, 2018). There is also increasing evidence that these effects do not impact all sections of society in the same manner, with marginalized and poor communities being disproportionately affected by water scarcity (Organization et al., 2019; He et al., 2021). Therefore, adequate management approaches that effectively deal with water shortages are not only critical for supporting resource conservation efforts but also for protecting vulnerable segments of society and upholding social justice and equity in the consuming population.

Reducing water demand through an increase in consumption efficiency is often seen as an appealing solution to deal with water shortages (Hatfield & Dold, 2019; Ward & Pulido-Velazquez, 2008). Agriculture, being the largest consumer of freshwater has been the primary target of such interventions. These interventions however are observed to fall victim to various externalities and rebound effects, whereby water demand adjusts in a manner that offsets the original decrease in consumption (Grafton et al., 2018). Moreover, efficient consumption technologies alone are also inadequate in compensating for supply fluctuations driven by climate change-induced environmental shocks. Since these technologies remain largely inaccessible to vulnerable segments of society, especially in developing countries (Deichmann et al., 2016), the disparity in climate risk exposure becomes even more pronounced. Therefore, despite remarkable progress in water-use efficiency, there is a distinct need to devise specific approaches to promote a fair sharing of such risks among the consuming population. Among such approaches, water allocation mechanisms may ensure a fair distribution of risk by implementing systematic procedures that prioritize equitable access to water resources among different users (Hassan et al., 2023). In this paper, we explore the potential boundaries of allocation mechanisms in achieving both equitable water access and other system-level targets of interest.

Water allocation mechanisms form an integral part of agricultural activity, and are often designed to support key economic objectives (Liang et al., 2020). These objectives may include maximization of agricultural yield, resource conservation, satisfaction of demand, and cost minimization (Dinar et al., 1997; Speed et al., 2013). Many allocation mechanisms prioritize consumers in accordance with these objectives and allocate water to them in order of their priority (Li et al., 2020; Hassan et al., 2021). In some other systems, these priorities may be fixed beforehand on the basis of seniority, birthright, or other historical claims (Ali Shah et al., 2022). Many allocation schemes currently practiced in large-scale irrigation systems, prioritize users according to these, or related criteria (we discuss some representative systems in the following sections). However, multiple studies report instances where allocation systems designed for maximizing economic performance fail to ensure fairness and equity in water distribution (Bell et al., 2015; Muhammad et al., 2016). This failure arises from the fact that while optimizing their prescribed objectives, these systems inadvertently neglect the distributional impacts on marginalized or disadvantaged segments of society. In such systems, the very criteria that enhance productivity and economic output undermine considerations of social justice and fairness. Hence, even within allocation mechanisms, tension persists between the objectives of maximizing productivity and promoting social justice. While legacy allocation mechanisms from the age of the Industrial Revolution (such as those practiced in many large-scale irrigation systems), are designed for equity, they are dominantly supply-based, without the flexibility to adjust allocation during run-time to enhance the productivity of water that is already scarce to begin with (Anwar et al., 2016). Therefore, there is a need to explicitly incorporate equity-based design within priority-based allocation frameworks to balance productivity and equity goals. Hybrid allocation schemes like these are currently operational in various real-world irrigation systems, and specific examples are elaborated upon in the subsequent sections.

In this paper, we concern ourselves with the limits of equity that can be achieved across an extensive spectrum of water allocation schemes. As mentioned previously, these limits are sensitive to the availability of water at each particular time. In practice, various methods of assessing equity are employed such as the coefficient of variation (Siddiqi et al., 2018; D. J. Molden & Gates, 1990), the Gini coefficient (Anwar & Ul Haq, 2013), and other statistical measures that report the variation of a distribution. However, less thought is given to the limits of performance that can be obtained w.r.t. these statistical metrics and the entire range of the metrics is often assumed to be in play. Here, we assert that the performance space of any allocation mechanism is fundamentally constrained by limits inherent to the mechanism itself. These limits, if understood correctly, can provide valuable insights into the potential of the individual mechanisms to achieve equitable distribution and perform well w.r.t. measures related to fair resource allocation. Furthermore, if the sensitivity of these performance limits to variability in the resource supply is quantified beforehand, it can help predict the performance of the allocation mechanism when subjected to climate shocks and other perturbations to prevailing environmental conditions. Thus, the evaluation of any allocation mechanism should not be restricted to single measures of equity alone, but must also be coupled with measures of reliability in supply.

In what follows, we conduct a novel analysis of the performance limits of water allocation mechanisms in a coupled equity-reliability space. We begin by introducing a typology of canonical water allocation mechanisms that categorize all possible mechanisms w.r.t. the resulting distribution of demand fulfillment among individual consumers. Uniform distribution, resulting from equity-inspired allocation schemes lies at one end of the spectrum, whereas prioritized distribution, resulting from efficiency-inspired allocation schemes lies at the other end. Hybrid mechanisms that combine aspects of both uniform and priority-based mechanisms lie in between. After defining system-level metrics of supply reliability, and distribution equity, we rigorously derive the theoretical performance limits of all canonical allocation mechanisms w.r.t. these metrics. We discuss the equity-reliability space in detail and comment on feasible operating regions within this space. We see that for the entire range of supply reliability, the best performance w.r.t. equity is obtained when allocation is purely uniform, whereas the worst performance is obtained when allocation is purely priority-based. Any operating point in-between may be realized by varying the proportion of initial entitlements in a hybrid allocation scheme. The theoretical results are complemented by numerical simulations of a selected agricultural district in the Central Punjab region of Pakistan. In the end, we speculate about the application of this study to large-scale irrigation systems of hierarchical structure where satisfactory performance must be obtained at multiple levels within the hierarchy. We conclude with a discussion of limitations and directions for future research.

2 Water allocation mechanisms

Here, we characterize allocation mechanisms based on the adequacy distribution of individual users. Adequacy is the widely used metric for performance evaluation of irrigation water allocations (Kharrou et al., 2013; D. J. Molden & Gates, 1990). It is defined as the fraction of water demand that is fulfilled. We categorize the widely used water allocation mechanisms into three canonical types: priority-based allocation, uniform allocation, and hybrid allocation. Priority-based and uniform allocation can be viewed as special cases of hybrid allocation. The canonical types of water allocation mechanisms, their adequacy distributions, and real-world examples of countries where these canonical allocation mechanisms are applied at different locations and sectors are described in Table 1. It is important to note that these canonical allocation types do not cover the entire range of possible allocation mechanisms. Therefore, we separately define the arbitrary allocation mechanism with any possible adequacy distribution. Below, we describe the types of water allocation mechanisms in detail.

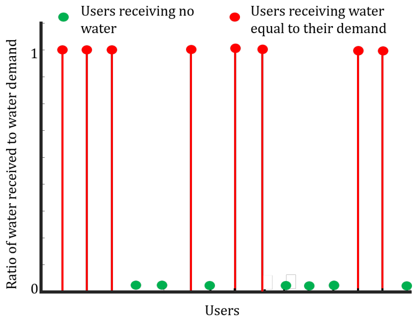
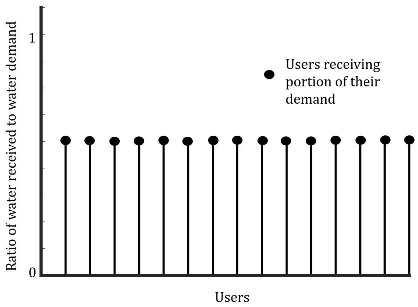
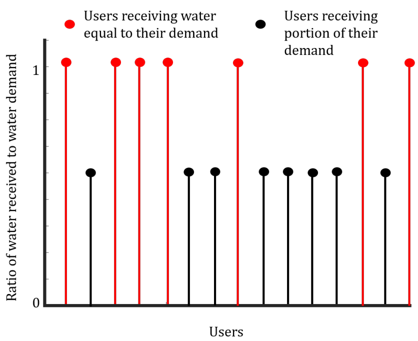
2.1 Priority-based allocation

In such type of allocation mechanisms, some users are prioritized over others. In priority-based allocation, some users receive water exactly equal to their demand, one user's demand is partially fulfilled and remaining users receive no water at all. The water allocation principle of "first in time, first in right" used in Western United States is one example of the priority-based allocation (Savenije & Van der Zaag, 2000). Many demand-based allocation mechanisms fall into the category of priority-based allocation. These demand-based allocation mechanisms prioritize users according to their water demand. For example, in a demand-based allocation mechanism described by Hassan et al. (2021), users bid for water by providing information of water demand and willingness to pay the water authority. Next, the water authority arranges the competing users' bids so that users with the highest bid receive water with the first preference and so on. The scale of such allocation mechanisms remains low due to lack of demand information availability (Nakasone & Torero, 2016). Various regions in different other countries use priority-based allocation mechanisms to fulfill demand of users e.g., Kazakhstan (Karatayev et al., 2017), Iran (RazaviToosi & Samani, 2019), and Australia (Gómez-Limón et al., 2020).

2.2 Uniform allocation

In this type of allocation, adequacy level of all users is same. One example of uniform allocation is supply-based allocation. Supply-based allocation mechanisms do not take into account the actual water demand of the users. Water delivered in excess of the demand is usually wasted in the form of run-off. Similarly, any deficit in supply is borne by the crop as water stress. Suppose the irrigation delivery schedule determined by the authority does not align with a farmer's crop water requirements. In that case, the farmer either has to invest in an expensive alternative such as pumped groundwater or face a low yield. For example, in Indus basin irrigation system, water is allocated at the farm level using a supply-based fixed rotational schedule scheme, known as *warabandi*, institutionalized under British colonial rule (Anwar & Ahmad, 2020). Under this supply-based

Table 1. The types of canonical water allocation mechanisms, their adequacy distributions, and real-world examples of countries where allocation mechanisms are applied at different locations and sectors.

Allocation Mechanism Types	**Adequacy distributions**	**Real-world examples**
Priority based allocation: In this type of mechanism some users are prioritise over others, e.g., the western United States water allocation principle of "first in time, first in right" (Bruns et al., 2005).		Western United States (Savenije & Van der Zaag, 2000), Kazakhstan (Karatayev et al., 2017), Iran (RazaviToosi & Samani, 2019), and Australia (Gómez-Limón et al., 2020).
Uniform allocation: In this type of allocation, the adequacy level of all users is same, e.g., In Pakistan, a round-robin mechanism is being used to distribute equal amount of water among users (Ali Shah et al., 2022).		Pakistan (Hassan et al., 2021), India (Khepar et al., 2000), Netherlands, and Europe (M. Van Rijswick et al., 2012; H. Van Rijswick, 2015).
Hybrid allocation (Priority-based allocation with water rights): In this type of distribution, all users receive a base amount of water and some users receive additional water depending upon their demand, e.g., water entitlements in most of the water markets provide base amount of water to users (Bajaj et al., 2022)		Chile (Donoso et al., 2021), Australia (Ann Wheeler & Garrick, 2020), and California, United State (Arellano-Gonzalez et al., 2021).

scheme, during a rotation cycle, each user receives a fixed amount of water in fixed turns depending upon her farm land size (Ireson, 2019). This is in contrast to demand-based water allocation schemes, where the water received by farmers depends upon their demand (Xiao et al., 2016). Various regions and countries in the world use supply side water allocation mechanisms to ensure equitable supply of water amoung users, e.g., Pakistan, Mexico, India, Netherlands, Europe, Chile, and Australia (Hassan et al., 2021; Khepar et al., 2000; Delorit et al., 2019; Gleick, 2003; M. Van Rijswick et al., 2012; H. Van Rijswick, 2015).

2.3 Hybrid allocation

In such type of distributions, all users receive a base amount of water equivalent to their entitlement or water right, and some users receive additional water depending on their demand using priority-based allocation approach. For example, in water markets, water entitlements or initial water rights provide a base amount of water to users and users with high water demand buy additional water (Bajaj et al., 2022). Hybrid allocation mechanisms are commonly implemented in two ways. In one case, all users first receive their entitled water then additional water is allocated to certain priority users. The prioritized users are defined based on their socio-economic vulnerability or other similar factors. In second case, demand fulfilment of prioritized users is ensured first. The remaining water is then distributed equally among the other users (Wheeler, 2014). Hybrid allocations have recently gained popularity in many regions of the world, e.g., California, United States (Arellano-Gonzalez et al., 2021), Australia (Ann Wheeler & Garrick, 2020), and Chile (Donoso et al., 2021).

2.4 Arbitrary allocation

In arbitrary allocation, we consider the possibility that the canonical allocation mechanisms may not represent all real-world allocation mechanisms. Therefore, we separately study mechanisms in which adequacy distributions may take any form and call them arbitrary allocation mechanisms. These cover the canonical allocation mechanisms and any other allocation mechanism.

The statistics for the allocation distribution arising from the canonical allocation mechanisms are more easily analyzed than arbitrary allocation mechanisms. Thus, in this paper we first assess the adequacy, reliability and equity of the canonical allocation mechanisms. The analysis is then extended towards arbitrary allocation mechanisms.

3 Metrics for Performance Assessment

The performance of an allocation scheme can be gauged by how well adequate and reliable water supply is provided equitably to users (Fan et al., 2018; D. J. Molden & Gates, 1990). In an irrigation system, if farms do not receive adequate water, crops may go under stress. Furthermore, the reliable water supply gives farmers the ability to predict and plan to obtain maximum agricultural yield. Moreover, a successful irrigation system requires fairness among water users (De Loë & Bjornlund, 2008). Thus, measuring the current performance of allocation mechanisms is necessary, so actions can be taken to improve the performance of the allocation mechanism. Performance assessment is regarded as an effective method to reduce the gap between the actual and projected performance of allocation schemes (Kazbekov et al., 2009).

For performance assessment, different performance measures are used to analyze water allocation schemes. Performance assessment of the water allocation schemes is performed for various reasons, such as comparing the performance of one allocation mechanism with another (D. Molden et al., 2007; Cordery et al., 2009), evaluating trends of the water users (Moreno-Pérez & Roldán-Cañas, 2013), and diagnostic evaluation (Shakir et al., 2010). The performance assessment measures of allocation schemes are classified into internal and external assessment measures. Most allocation schemes' performance evaluation research focuses on external assessment indicators, mainly water-use efficiency, economic efficiency, environmental footprints, and agricultural yield (Xu et al., 2011; Hollanders et al., 2005). However, few studies have evaluated internal performance assessment indicators, such as adequacy, reliability, and equity, due to lack of information about water demand and supply.

The performance measures of adequacy, reliability, and equity are considered when assessing allocation mechanisms from the water supply and demand management perspective. Adequacy is a measure of supply to demand, one of the main targets of an allocation system. Adequacy is used to calculate reliability and equity. Reliability is defined as the temporal stability of the water supply; it is the degree to which the allocation mechanism satisfies the expectations of the users. Equity is the notion of fairness among users. Equity and reliability depend on water supply, water management policies, legal structure, regulations, and infrastructure for water deliveries (Klümper et al., 2017). The definition of sustainable water availability for water users also has an embedded notion of reliability, and equity (Siddiqi et al., 2018). Reliability and equity vary depending on the adequacy distribution encountered as a result of the selected allocation mechanism. Thus, in this research, we use adequacy, reliability, and equity to gauge the performance of different allocation mechanisms.

Equity and reliability depend on a number of factors that include surface water supply, water entitlements, allocation policy, access and management of infrastructure and regulations for water supply (Klümper et al., 2017). Equity in surface water allocation is achieved when the amount of water provided to each user is proportional to her water demand. Reliability represents consistency in surface water supply to users. The reliability of surface water supply is very crucial for users like farmers, as it enables them to optimally use and predict water availability that can help improve crop production.

We suppose there are N users in the water allocation and distribution network. A single user is denoted by $n \in \{1, 2, 3, ..., N\}$. A complete cropping season consists of a total of K allocation cycles. The kth allocation cycle is $((k-1)T, kT]$, where $k \in \{0, 1, 2, 3, ..., K\}$ and the fixed time interval T denotes the duration between successive water allocation decisions.

3.1 Adequacy

We define the adequacy $\alpha_n(k)$ as a ratio of amount of water received to the water demand. The ratio $\alpha_n(k)$ for the user n in the kth decision cycle is as follows

$$\alpha_n(k) = \min\left(1, \frac{\text{water received}}{\text{water demand}}\right). \tag{1}$$

The above ratio is such that $\alpha_n(k) = 0$ when no water is received by a user, $0 < \alpha_n(k) < 1$ when water received by a user is less than her demand, and $\alpha_n(k) = 1$ when water delivered is equivalent to the water demand. If a user receives water greater than her water demand, the extra water received is of no use. Therefore, we assume that the maximum adequacy value is 1. An adequacy distribution $\alpha(k)$ for an allocation cycle is a vector of adequacy values for each user in that allocation cycle interval, i.e., $\alpha(k) = [\alpha_1(k), \alpha_2(k), \alpha_3(k), ..., \alpha_N(k)]$.

3.2 Interval reliability

Interval reliability $r(k)$ is the average water received by all users in the kth decision cycle. This metric measures the range in which water demands are met for all users during the kth decision cycle. If the water demand of all N users is fully met during the kth decision cycle of a cropping season, then $r(k) = 1$. Interval reliability is calculated as follows

$$r(k) = \frac{\sum_{n=1}^{N} \alpha_n(k)}{N}. \tag{2}$$

The maximum value for interval reliability is attained when all users receive water equal to their demand, i.e., $\alpha_1(k) = \alpha_2(k) = \alpha_3(k) = ... = \alpha_N(k) = 1$. Now, replacing these $\alpha_n(k)$ values in (2) gives us the value 1 for the interval reliability. It implies that interval reliability cannot get value more than 1 when water is being distributed using any allocation scheme.

The minimum value of the interval reliability is achieved when no one receives water, i.e., $\alpha_n(k) = 0 \; \forall \, n$. In this case interval reliability would be zero.

So, the interval reliability always remains in the interval $[0, 1]$. It implies that the interval reliability cannot get value less than zero or greater than 1.

3.3 Interval Equity

Interval equity $e(k)$ is the coefficient of variation (Brown, 1998) of adequacy ($\alpha_n(k)$) across the users in a water allocation network during the kth allocation decision cycle. This concept is derived from the idea that geographical variation in performance can be used to assess equity (D. J. Molden & Gates, 1990; Kaghazchi et al., 2021). When the variation across users in a kth decision cycle is low, then interval equity $e(k)$ is low. Water allocation is absolutely equitable when $e(k) = 0$. Interval equity quantifies the variation in water allocation across different users, and it evaluates whether different users in a water allocation and distribution network are getting equal water supply or not. For example, if the water received by all users is 90% of their water demand in a kth decision cycle, then perfect interval equity is achieved. In contrast, the interval equity value is higher if half users are receiving 40% of their water demand while the rest are receiving 80% of their demand. Interval equity is calculated as follows

$$e(k) = \frac{\sigma(\alpha(k))}{\mu(\alpha(k))}, \tag{3}$$

where $\mu(\alpha(k))$ and $\sigma(\alpha(k))$ are the mean and standard deviation respectively and are defined as follows

$$\begin{aligned} \mu(\alpha(k)) &= \frac{1}{N} \sum_{n=1}^{N} \alpha_n(k), \\ \sigma(\alpha(k)) &= \sqrt{\frac{1}{N-1} \sum_{n=1}^{N} (\alpha_n(k) - \mu(\alpha_n(k)))^2}. \end{aligned} \tag{4}$$

The maximum value for interval equity $e(k)$ is attained when $N-1$ users have $\alpha_n(k) = 0$ and only one user has non-zero α value, i.e., $\alpha_N(k) = a$, where $0 < a \leq 1$, (Cox, 2010). Now, replacing these $\alpha_n(k)$ values in (3) give us a maximum interval equity value of $\sqrt{N}$. It shows that interval equity cannot get value more than $\sqrt{N}$ when water is being distributed using the priority based allocation.

The minimum value of the interval equity is achieved when all users receive same amount of water, i.e., $\alpha_n(k) = a \; \forall \, n$. In this case $\mu(\alpha_n(k)) = a$ and $\sigma(\alpha_n(k)) = 0$ $\forall \, n$, therefore interval equity would be zero.

Therefore, the interval equity always remains in the interval $[0, \sqrt{N}]$, where N is the total number of users. It implies that interval equity cannot get a value less than zero and greater than $\sqrt{N}$.

3.4 Reliability

Reliability R is define as the mean of interval reliability $r(k)$ across all allocation cycles. Reliability is calculated as follows

$$R = \frac{\sum_{k=1}^{K} r(k)}{K}. \tag{5}$$

The reliability range is same as the interval reliability range $[0, 1]$, since reliability is simply the mean of the interval reliability.

3.5 Equity

Equity E is defined as the mean of interval equity $e(k)$ across all allocation cycles. Equity is calculated as follows

$$E = \mu(e(k)) = \frac{\sum_{k=1}^{K} e(k)}{K}. \tag{6}$$

The equity range is same as interval equity range $[0, \sqrt{N}]$ since equity is simply the mean of interval equity.

3.6 The Equity-Reliability Triangle

In previous subsections, ranges of the performance metrics are given in general when water is allocated to users through any water allocation mechanism. In reality, these metrics are related to each other, i.e., they influence each others ranges. Intuitively, this also makes sense, because reliability and equity depend on the same adequacy distributions, therefore there exists some co-relation between these two quantities. In next sections, we will prove that the equity-reliability space is as shown in Fig. 1 when water is allocated using any allocation mechanism. The shaded region in Fig. 1 shows the range of equity values for each reliability value that is achievable when water is allocated through any mechanism.

In case of water abundance, when all users receive water equal to their demand, the perfect equity-reliability point is achieved in the equity-reliability space, i.e., $R = 1$ and $E = 0$, as shown in Fig. 1. Ideally, one wants to operate an allocation mechanism at perfect equity-reliability point. However, under scarcity the worst-case performance degrades as shown in Fig. 1. In case of scarcity, the reliability decreases which can be distributed among users in many ways. The best case occurs if scarcity is distributed among users equally and the reliability-equity point occurs on reliability axis. The worst case occurs when scarcity is distributed disproportionately towards only few users. It is important to note that since the reliability is average of interval reliability across multiple allocation cycles, therefore it is possible to perform even worse than that of a single allocation cycle.

4 Performance limits

In this section, we first derive the performance limits for equity and reliability of each canonical allocation mechanism. We present the detailed mathematical calculations, discuss the placement of each mechanism in the equity-reliability space, and identify how allocation policy can be tuned to traverse this space. In the end, we show that the bounds are valid for adequacy distributions arising from any possible allocation mechanism, not just the three canonical mechanisms for which the bounds are calculated.

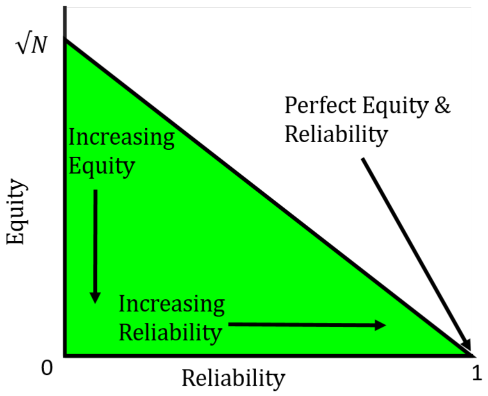


Figure 1. For any arbitrary allocation mechanism, shaded region shows the possible range of equity for each value of reliability.

4.1 Priority-based allocation

In priority-based allocation, the water authority makes a priority list and allocates water to the first user in the priority list. Next, if there is still water left, a user with second priority gets water, and so on. Therefore, when water is allocated through a priority-based allocation mechanism, some users' demand gets fulfilled, one user's demand gets partially fulfilled and the remaining users get no water. Hence, in a network of N users, fulfilled population count is i, one user's demand is partially fulfilled, and unfulfilled population count is $N-i-1$. For priority-based allocation, we assume that first i users' in the set $\{1, 2, 3.....N\}$ are the users with fulfilled demand, the $i+1$th user in the set is the user with partially fulfilled demand and $N-i-1$ users with unfulfilled demand are the users from $i+2$ to N user in the same set $\{1, 2, 3.....N\}$. The adequacy level of the i users with fulfilled demand, the one user with partially fulfilled demand, and the $N-i$ users with unfulfilled demand is one, $0 \leq b(k) < 1$, and zero, respectively. Graphical demonstration of the priority-based allocation is shown in Fig. 4a. Next, to find reliability and equity performance limits, we first need interval reliability and interval equity when i users' demand is fulfilled.

4.1.1 Relationship between interval equity and interval reliability

We first calculate the interval reliability when water is allocated through priority-based allocation. The interval reliability $r(k)$, when the fulfilled population count is i, is calculated by placing $\alpha_1(k) = \alpha_2(k) = ... = \alpha_{i-1} = \alpha_i = 1$, $\alpha_{i+1}(k) = b(k)$, and $\alpha_{i+2}(k) = \alpha_{i+3}(k) = ... = \alpha_{N-1} = \alpha_N = 0$ in (2) which gives us

$$r(k) = \frac{i + b(k)}{N}. \tag{7}$$

To find the equity performance limits we need interval equity when the fulfilled population count is i.

Lemma 4.1. In priority-based allocation, for N users the interval equity is given as,

$$e(k) = \frac{1}{r(k)} \sqrt{\frac{\lfloor Nr(k) \rfloor + (Nr(k) - \lfloor Nr(k) \rfloor)^2 - Nr(k)^2}{(N-1)}}, \tag{8}$$

where the interval reliability is $r(k)$ and $\lfloor . \rfloor$ is the floor function.

Proof. For proof see Appendix A.1. ■

For the different number of users in a network, the relationship between interval reliability and interval equity is graphically demonstrated in Fig. 2.

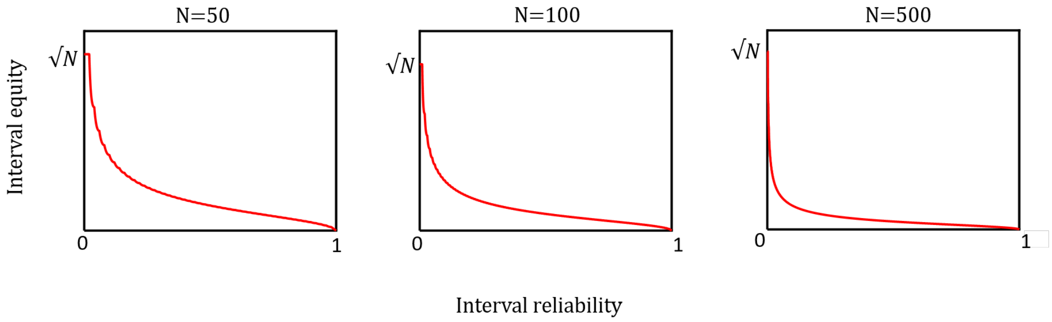


Figure 2. Graphical representation of relationship between interval equity and interval reliability when water is allocated using priority-based allocation.

Next, we use the interval reliability and the interval equity value when the fulfilled population count is i to measure the relationship between reliability and equity for the priority-based allocation. Reliability and equity are calculated over the total K allocation cycles, while interval reliability and interval equity are calculated for a single allocation cycle $k \in \{0, 1, 2, ...K\}$. To find the reliability and equity relationship, we measure the fulfilled population count i and interval reliability for each allocation cycle of the season.

4.1.2 Relationship between equity and reliability

To find the relationship between reliability and equity, we find the equity performance limits for each reliability value from the complete reliability range $[0, 1]$. A reliability value R is equal to the mean of interval reliability across all allocation cycles i.e., $R = \frac{\sum_{k=1}^{K} r(k)}{K} = \frac{1}{K}\left(r(1) + r(2) + ... + r(K-1) + r(K)\right)$, where $0 \leq r(k) \leq 1$. Therefore, to find the maximum and minimum equity value for each reliability value R, we can maximize and minimize following optimization problem for the complete reliability R range $[0, 1]$,

$$\max_{\{r(1),r(2),....,r(K)\}} / \min \frac{1}{K}\left(\sum_{k=1}^{K} \frac{1}{r(k)} \sqrt{\frac{\lfloor Nr(k) \rfloor + (Nr(k) - \lfloor Nr(k) \rfloor)^2 - Nr(k)^2}{(N-1)}}\right),$$
$$\text{Subject to}$$
$$\frac{1}{K}\left(r(1) + r(2) + ... + r(K-1) + r(K)\right) = R,$$
$$0 < r(k) \leq 1. \tag{P1}$$

The objective function of optimization Problem P1 is non-convex and thus the Problem is NP-hard (Hochba, 1997). We can relax problem P1 by replacing $\lfloor Nr(k) \rfloor$ with $Nr(k)$. This relaxation implies $b(k) = 0$, i.e., users are either receiving water equal to their demand or receiving no water at all. Hence, a user is either part of the fulfilled population with count i or the unfulfilled population with count $N - i$. This also implies the following relations

$$r(k) = r_i = \frac{i}{N}, \tag{9}$$

and

$$e(k) = e_i = \sqrt{\frac{N(1 - r(k))}{r(k)(N-1)}} = \sqrt{\frac{N(N-i)}{i(N-1)}}, \tag{10}$$

where $i \in \{1, 2, 3, ..., N\}$. After substitution, the relaxation of Problem P1 can be rewritten as follows

$$\begin{gathered}
\max_{\{r(1),r(2),....,r(K)\}} / \min \frac{1}{K}\left(\sum_{k=1}^{K}\sqrt{\frac{N(1-r(k))}{r(k)(N-1)}}\right), \\
\text{Subject to} \\
\frac{1}{K}\left(r(1) + r(2) + ... + r(K-1) + r(K)\right) = R, \\
0 < r(k) \leq 1.
\end{gathered} \tag{P2}$$

Even though the Problem P2 is nonconvex. We can transform this problem into a liner optimization problem. Since there are K decision cycles and in each decision cycle k the interval reliability $r(k)$ and interval equity $e(k)$ can only take on values r_i and e_i given in (9) and (10) respectively. For this purpose, we rewrite equity E and reliability R in terms of interval equity e_i and interval reliability r_i respectively. The equity E can be written as the weighted sum of interval reliability e_i as follows

$$E = \sum_{i=1}^{N} w_i e_i, \tag{11}$$

where w_i represents the friction of the total allocation cycles K in which interval equity is e_i. The sum of weights is equal to 1, i.e., $w_1 + w_2 + ... + w_{i-1} + w_i + w_{i+1} + ... + w_N = 1$. All weights $w_1, w_2, ..., w_N$ are also non negative, i.e., $w_i \geq 0$. We can similarly write reliability R as the weighted sum of interval reliability r_i i.e., $R = \sum_{i=1}^{N} w_i r_i = r_1 w_1 + r_2 w_2 + ... + r_{N-1} w_{N-1} + r_N w_N$. We add this reliability constraint to find the equity value for each specific reliability value R from the reliability range $[0, 1]$. So in order to find the relationship between reliability and equity, we have to maximize and minimize following optimization problem which is a relaxed version of the optimization problem P1,

$$\begin{gathered}
\max_{\{w_1,w_2,...,w_N\}} / \min \sum_{i=1}^{N} w_i e_i, \\
\text{Subject to} \\
w_1 + w_2 + ... + w_{N-1} + w_N = 1, \\
r_1 w_1 + r_2 w_2 + ... + r_i w_i + ... + r_{N-1} w_{N-1} + r_N w_N = R, \\
0 < w_i \leq 1, \\
0 \leq R \leq 1.
\end{gathered} \tag{P3}$$

where r_i, e_i are constant and N is a system parameter representing the total number of users. Fig. 3 shows the equity range for each value of reliability by solving P3 numerically. We see for a very large number of users, the lower bound on equity converges to zero. Comparing Fig. 3 with fig. 2 it can be seen that for the same x-axis value the lower bound on equity is same as the interval equity value. It is important to note that since the equity is average of interval equity across multiple allocation cycles, therefore it is possible to perform even worse than that of a single allocation cycle.

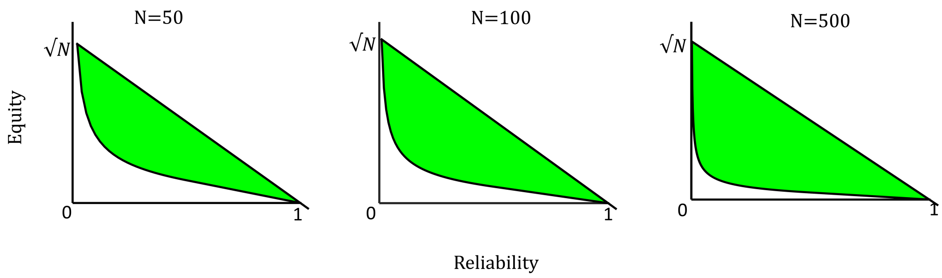


Figure 3. For different numbers of users N, the shaded region in each figure shows the equity range for each value of the reliability when water is allocated using the priority-based allocation.

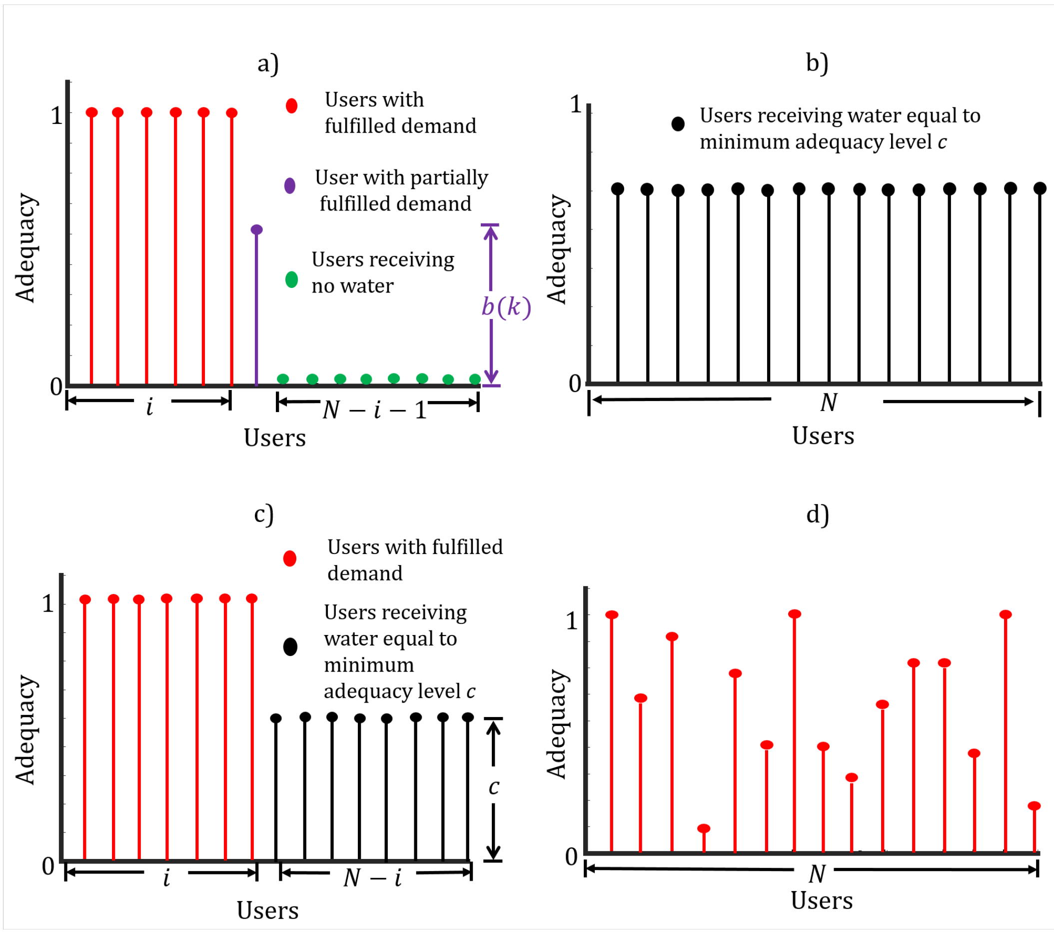


Figure 4. Graphical representation of the three canonical allocations and arbitrary allocation. a) priority-based allocation, b)the uniform allocation, c) hybrid allocation, d) arbitrary allocation.

4.2 Uniform allocation

In case of uniform allocation, the adequacy level of all users is same. Therefore, the interval reliability for uniform allocation is equal to the adequacy level of an allocation cycle because the interval reliability is the simple mean of the adequacy of all users in an allocation cycle. If adequacy level for all users in an allocation cycle is equal to $a(k)$, the interval reliability is calculated by placing $\alpha_1(k) = \alpha_2(k) = ... = \alpha_{N-1} = \alpha_N = a(k)$ in (2), which give us interval reliability equal to the adequacy level $a(k)$. The graphical demonstration of the uniform allocation is shown in Fig. 4b.

The interval equity, when the adequacy level in an allocation cycle is $a(k)$ for all users, can be calculated by placing $\alpha_1(k) = \alpha_2(k) = ... = \alpha_{N-1} = \alpha_N = a(k)$ in (4) which gives us the mean equal to the adequacy level $a(k)$, and the standard deviation equal to zero. In this case, the interval equity value would be zero.

To find the reliability and equity, we use the above calculated interval reliability and interval equity values for uniform allocation. The reliability is simply the mean of the interval reliability a(k), i.e., $R = \frac{\sum_{k=1}^{K} a(k)}{K} = a(k)$.

Thus, for uniform allocation, the equity always remains zero because the interval equity is zero during all allocation cycles.

4.3 Hybrid allocation

In hybrid allocation, a user is either receiving water equal to their demand or a minimum base amount of water equal to their entitlement. Therefore, when water is allocated through hybrid allocation, a user is either part of the population count i with fulfilled demand or population count $N-i$ with partially fulfilled demand. For hybrid allocation, we assume that first i users in the set $\{1, 2, 3, ..., N\}$ are the users with fulfilled demand, and remaining $N-i$ users in the set are the users with partially fulfilled demand. The adequacy level of the i users with fulfilled demand and $N-i$ users with partially fulfilled demand is one and c respectively, where $0 \leq c \leq 1$ is called minimum adequacy level. The graphical demonstration of the hybrid allocation is shown in Fig. 4c. The minimum adequacy level c is decided at the beginning of the season and remains constant throughout the season because initial water rights or entitlements are same once decided. Next, to find reliability and equity performance limits we need interval reliability and interval equity when the fulfilled population count is i.

4.3.1 Relationship between interval equity and interval reliability

We first calculate the interval reliability when water is allocated through hybrid allocation. The interval reliability $r(k)$ when the fulfilled population count is i, is calculated by substituting $\alpha_1(k) = \alpha_2(k) = ... = \alpha_{i-1} = \alpha_i = 1$, and $\alpha_{i+1}(k) = \alpha_{i+2}(k) = ... = \alpha_{N-1} = \alpha_N = c$ in (2) which gives us

$$r(k) = r_i = \frac{i + (N-i)c}{N}. \tag{12}$$

To find the equity performance limits we need interval equity when the fulfilled population count is i.

Lemma 4.2. In hybrid allocation, for N users the interval equity $e(k)$ is given as,

$$e(k) = e_i = \frac{1-c}{i + (N-i)c}\sqrt{\frac{N.i.(N-i)}{N-1}} \tag{13}$$

where i represents the number of users with fulfilled demand and c represents the adequacy level of the population with partially-fulfilled demand.

Proof. For proof see Appendix A.2. ∎

We know from (12) that $Nr(k) = i + (N-i)c$ where $i \in \{1, 2, 3, ..., N\}$ and $0 \leq$ 1. Therefore for a value of interval reliability $r(k)$, there are multiple possible values of

i and c. Hence, for a value of interval reliability $r(k)$. there is a range of interval equity $e(k)$ given in Lemma Appendix A.2.

Lemma 4.3. In hybrid allocation, for N users the range of interval equity $e(k)$ is given as,

$$\left[0, \quad \frac{1-r(k)}{r(k)}\sqrt{\frac{N.\lfloor Nr(k)\rfloor}{(N-1)(N-\lfloor Nr(k)\rfloor)}}\right], \tag{14}$$

where the interval reliability is r(k) and $\lfloor . \rfloor$ is the floor function.

Proof. for proof see Appendix A.3. ∎

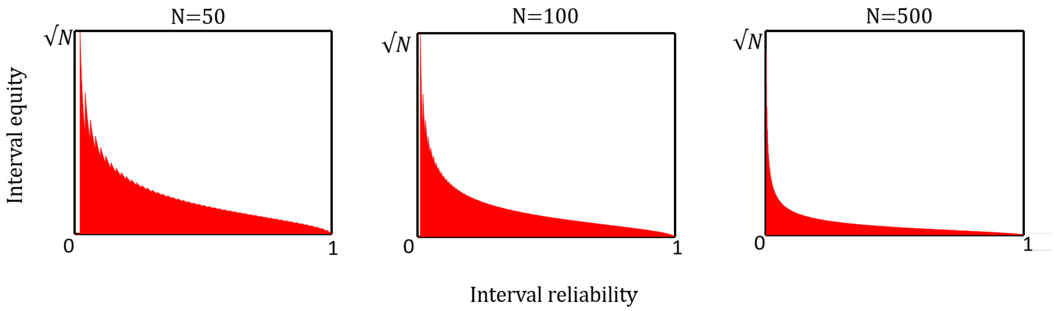


Figure 5. Graphical representation of interval equity range with respect to interval reliability when water is allocated using hybrid allocation.

For different number of users in a network, the range of interval equity with respect to interval reliability is graphically demonstrated in Fig. 5. At the small interval reliability values the upper range of interval equity is high.

Now, we use the interval reliability and the interval equity value when the fulfilled population count is i to calculate the relationship between reliability and equity for the hybrid allocation. Reliability and equity are calculated over the total of k allocation cycles while interval reliability and interval equity are calculated for a single allocation cycle $k \in \{0, 1, 2, ...K\}$. The adequacy for partially-filled users throughout the season is c. The equity and reliability are not independent. The equity value changes as the reliability value changes, and vice versa. Next, we describe the relationship between reliability and equity when water is allocated using the hybrid allocation.

4.3.2 Relationship between equity and reliability

To find the relationship between reliability and equity, we find the equity performance limits. We know from Lemma Appendix A.3 that for all values of interval reliability the lower limit on interval equity is zero. Thus, the lower bound on equity is always zero due to the possibility of all population having adequacy equal to the minimum adequacy level c. In this case, the reliability would be $0 \leq c \leq 1$ and equity would be zero. We find the upper limit on equity for each reliability value from the complete reliability range $[0, 1]$. A reliability value R is equal to the mean of interval reliability across all allocation cycles i.e., $R = \frac{\sum_{k=1}^{K} r(k)}{K} = \frac{1}{K}\left(r(1) + r(2) + ... + r(K-1) + r(K)\right)$, where $0 \leq r(k) \leq 1$. Since there are K decision cycles and in each decision cycle k the interval reliability $r(k)$ and interval equity $e(k)$ can only take on values r_i and e_i given in (12) and (A5) respectively. For this purpose, we rewrite equity E and reliability R

in terms of interval equity e_i and interval reliability r_i respectively. The equity E can be written as the weighted sum of interval reliability e_i as follows

$$E = \sum_{i=1}^{N} w_i e_i, \tag{15}$$

where w_i represents the friction of the total allocation cycles K in which interval equity is e_i. The sum of weights is equal to 1, i.e., $w_1 + w_2 + ... + w_{i-1} + w_i + w_{i+1} + ... + w_N = 1$. All weights $w_1, w_2, ..., w_N$ are also non negative, i.e., $w_i \geq 0$. We can similarly write reliability R as the weighted sum of interval reliability r_i i.e., $R = \sum_{i=1}^{N} w_i r_i = r_1 w_1 + r_2 w_2 + ... + r_{N-1} w_{N-1} + r_N w_N$. We add this reliability constraint to find the equity value for each specific reliability value R from the reliability range $[0, 1]$. Therefore, to find the upper limit on equity for each reliability value R, we can maximize following optimization problem for the complete reliability R range $[0, 1]$,

$$\begin{gathered}
\max_{\{w_1, w_2, ..., w_N\}} \sum_{i=1}^{N} w_i e_i, \\
\text{Subject to} \\
w_1 + w_2 + ... + w_{N-1} + w_N = 1, \\
r_1 w_1 + r_2 w_2 + ... + r_i w_i + ... + r_{N-1} w_{N-1} + r_N w_N = R, \\
0 < w_i \leq 1, \\
0 \leq R \leq 1.
\end{gathered} \tag{P4}$$

where r_i, e_i are constant and N is a system parameter representing the total number of users. Fig. 6 shows the upper bound on equity for each value of reliability and different values of minimum adequacy level c by solving P4 numerically. Priority-based allocation is spacial case of hybrid allocation when $c = 0$. The worse case performance of equity improves as the minimum adequacy level c of the users with partially-fulfilled demand increases. As the adequacy level c increases the domain of reliability change, e.g., when $c = 50\%$ the possible reliability range is $(0.50, 1]$. In hybrid allocation, the equity is zero when reliability is less than $\frac{1+(N-1)c}{N}$ for single allocation cycle because in this case water supply is not enough to fulfill atleast one user demand and all users get water equal to the minimum adequacy level c where $c = \frac{R}{N}$. For example, in Fig. 6, for $c = 0$, the equity value is zero for reliability range $[0, \frac{1}{N})$ and non zero at reliability $\frac{1}{N}$.

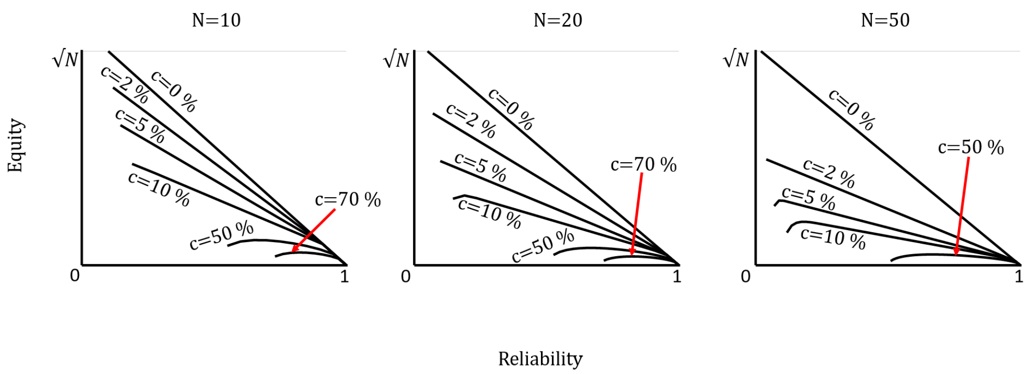


Figure 6. For different numbers of users N and different values of minimum adequacy level c, upper limit on equity for the complete range of reliability values.

4.4 Arbitrary allocation

In arbitrary allocation, the adequacy distributions of users may take any form including the adequacy distributions of the three canonical allocation mechanisms. The graphical demonstration of arbitrary allocation is shown in Fig. 4d. Next, we present the relationship between reliability and equity for allocation mechanism with any arbitrary adequacy distribution. For this purpose we first present the bound on interval equity.

Lemma 4.4. In arbitrary allocation, for N users the interval equity range is given as

$$\left[0, \quad \frac{1}{r(k)}\sqrt{\frac{\lfloor Nr(k)\rfloor + (Nr(k) - \lfloor Nr(k)\rfloor)^2 - Nr(k)^2}{(N-1)}}\right], \tag{16}$$

where the interval reliability is $r(k)$ and $\lfloor . \rfloor$ is the floor function.

Proof. For proof see Appendix A.4. ■

For different number of users in a network, the range of interval equity with respect to reliability is graphically demonstrated in Fig. 7. At the small interval reliability values the interval equity values are high. Next, we use the upper and lower limits of interval equity range to find out the equity range for each possible reliability value for arbitrary allocation. The lower bound on equity is zero for the complete range of reliabil-

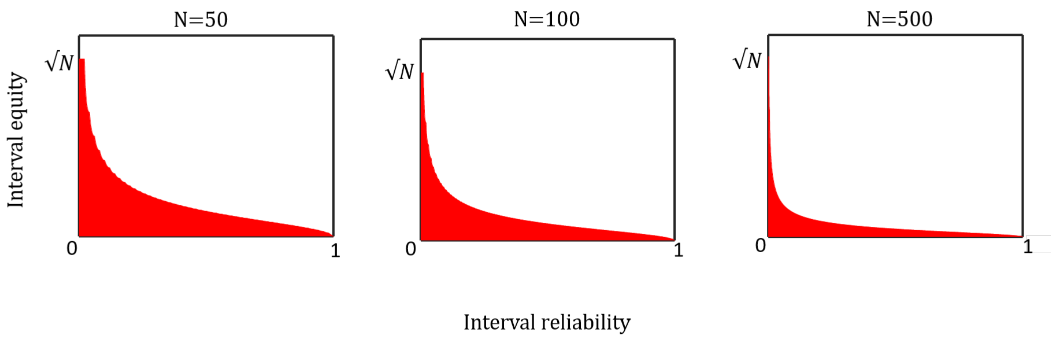


Figure 7. Graphical representation of interval equity range with respect to interval reliability when water is allocated using arbitrary allocation.

ity, because equity is average of interval equity and interval equity lower bound is zero for the complete range of interval reliability as shown in Lemma Appendix A.4. For arbitrary allocation, the upper bound on interval equity range is same as the interval equity value for priority-based allocation. Hence, for arbitrary allocation, upper bound on equity is same as the upper bound in equity for the priority-based allocation. The upper bound on equity for priority-based allocation is derived in Section 4.1.2. The range of equity for each value of reliability is shown in Fig. 1.

5 Simulations

In this section, we first present the simulation setup to evaluate reliability and equity performance limits when water is allocated using a particular priority-based allocation mechanism and a hybrid allocation mechanism. The setup is constructed from data gathered from a real-world irrigation district in Central Punjab in Pakistan. Then we present simulations for arbitrary allocation. We develop Monte-Carlo simulations to analyze equity and reliability over the realistic range of model parameters for priority-based, hybrid, and arbitrary allocation.

5.1 Simulation setup

The selected study area is located in the Moza Joyia area in the Okara district of the Punjab province, Pakistan. The 91093 / L watercourse irrigates the case study area. There are 24 farmers who own a portion of the land in the case study region, i.e., $N = 24$. Data collected at the case study site is from July 2018 to October 2018. We use data for a khareef cropping season, from July 2018 to October 2018, which is 17 weeks, that is, $K = 17$.

We carry out Monte-Carlo simulations by changing the parameter values over a range that mimics the real-world context. We need data on water demand and water supply in the study area to carry out the simulations. The maximum surface water supply for the farmers in the study region watercourse is 1.6×10^6 cusec, and available surface water supply range is $[0, 1.6 \times 10^6]$ cusec. The water demand of a farm is calculated using deep percolation constant, reference evapotranspiration, crop coefficient, precipitation, and soil moisture level. Details about measuring ranges of these parameters in the study region are provided by Hassan et al. (2021). The dimensionless deep percolation constant range is $[0.5 \times 10^{-4}, 2 \times 10^{-4}]$. The reference evapotranspiration and crop coefficient are randomly chosen from the range provided by Ullah et al. (2001) for the study region. The precipitation data set for the study region is taken from the NASA data portal (NASA, 2021). The given 21 samples are used to randomly select daily precipitation for the study region. For each farm, the initial soil moisture level is selected from an interval $[10\%, 25\%]$ using a uniform distribution.

5.2 Priority-based allocation

We validate the reliability and equity performance limits derived in Section 4.1 of priority-based allocation by presenting Monte-Carlo simulations for a particular priority-based allocation mechanism describe in details by Hassan et al. (2021). In this priority-based allocation mechanism, each user is interested in securing a shared resource, i.e., surface water. The shared resource is managed by a central planner. The complete season is divided into time interval of fixed duration. At the beginning of each time interval, the central planner asks for bids from the users. Users' bids consist of information on water demand and their valuation of water. Then, based on the demand, valuation, and availability of the shared resource, the central planner sorts the bids in descending order and starts fulfilling the demand from the top. Details of this mechanism are provided in Hassan et al. (2021). This mechanism of resource allocation falls under the category of priority-based allocation, as some users are receiving water equal to their demand and remaining users do not receive water.

Fig. 8a represents the equity vs. reliability relationship for Monte-Carlo simulations when water is allocated using the demand-based allocation mechanism described above. We have reiterated the simulations for 1×10^4 time instances. There are 24 users in the network, i.e., $N = 24$. equity for the range of reliability should remain within the upper and lower bounds of the priority-based allocation. For the Monte-Carlo simulations, all the equity and reliability values stay within these limits as shown in Fig. 8a, which validates the upper and lower bounds derived earlier.

5.3 Hybrid allocation

Next, we validate the reliability and equity performance limits of hybrid allocation by presenting Monte-Carlo simulations for a particular hybrid allocation mechanism. This

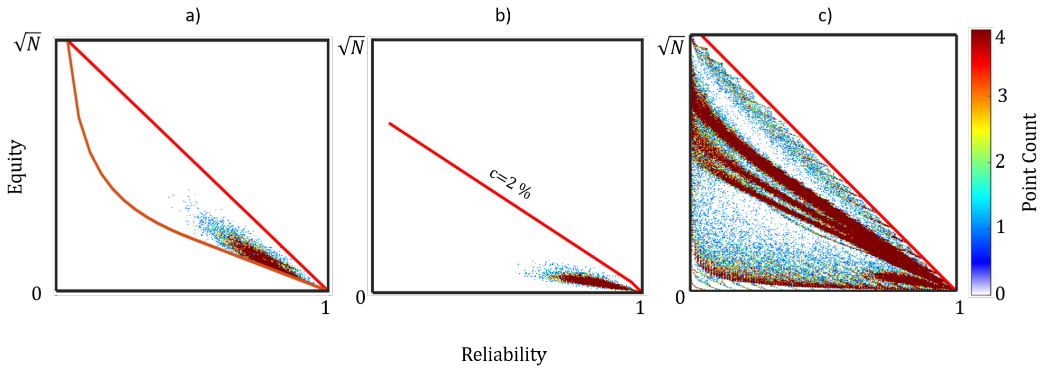


Figure 8. The equity vs. reliability relationship for Monte-Carlo simulations: a) when water is allocated using a demand-based allocation mechanism described in Hassan et al. (2021), which falls under the category of priority-based allocation, b) when water is allocated using uniform allocation, c) when water is allocated using arbitrary allocation. The plane is discretized into a grid and individual points lie at the center of each cell. The color of the points represents the number of times a simulated scenario fell within that cell.

hybrid allocation consists of two steps: surface water allocation based on initial water rights and demand-based surface water reallocation and distribution based on a priority-based mechanism. First, the central planner allocates water based on initial water rights. We call the initial water rights the minimum adequacy level. Next, the central planner distributes the remaining water using the priority-based allocation mechanism described in Section 5.2. This mechanism of allocating resources falls under the category of hybrid allocation, as some users receive resources equal to their demand and remaining users receive resources equal to the minimum adequacy level.

Fig. 8b represents the equity vs. reliability relationship for Monte-Carlo simulations when water is allocated using a particular hybrid allocation mechanism described above. We have reiterated the simulations for 1×10^4 times. For hybrid allocation, when there are 24 users in a network, i.e., $N = 24$, and minimum adequacy level is 2 %, i.e., $c = 0.20$. Then, the equity for the range of reliability should remain below the upper bound of the hybrid allocation. In this case, the lower bound on equity is zero for the complete range of reliability. For the Monte-Carlo simulations, all the equity and reliability values remain within the upper and lower bounds. Hence, the simulations validate the performance limits of the hybrid allocation derived previously.

5.4 Arbitrary allocation

We validate the reliability and equity performance limits of arbitrary allocation mechanism by presenting Monte-Carlo simulations. Fig. 8c represents the equity vs. reliability relationship for Monte-Carlo simulations when water is allocated using arbitrary allocation. In Monte-Carlo simulations, we assume there are 24 users, i.e., $N = 24$ in the network, and the season consists of 17 allocation cycles, i.e., $K = 17$. We have repeated the simulations for 0.206×10^6 times. The adequacy level for each user during each allocation cycle is generated using the beta distribution with a mean value between 0 and 1. Fig. 8c shows that the equity and reliability value for each simulated point remains within the upper and lower limit of the equity and reliability, which implies that these bounds are valid for any allocation mechanisms. In Fig. 8, there are some regions with zero point count and some regions with multiple point count. This is mainly because of the selected scarcity range for each simulation.

6 Towards Large-scale Irrigation Systems

In this paper, we have conducted a comprehensive analysis of the performance limits for water allocation mechanisms w.r.t. a coupled equity-reliability space. The analysis conducted applies to the allocation of water at a single level, i.e., a single allocation scheme that directly receives water from the supply and distributes it to the farmers without the action of any intermediary decision-making bodies. While our study encompasses all allocation systems of this kind, it is essential to recognize that the assumption of a single-level hierarchy being valid in practical systems holds true only up to a certain scale. This assumption is rarely applicable to large-scale irrigation systems.

Figure 9 illustrates the hierarchical structure of large-scale irrigation systems, featuring representative irrigation districts in Pakistan and China [1]. The process of resource allocation in these system is complex and consists of multiple spatial and temporal scales. For example, in the Indus Basin Irrigation System (IBIS) in Pakistan (Shah et al., 2016), primary canals draw water from large reservoirs or headworks located on main rivers. The water drawn in these canals is subject to the decision of a national-level authority that is responsible for determining the water share of each province. Each province is further subdivided into administrative units referred to as circles, also known as zones. The provincial-level authorities are responsible for managing water allocation among these circles. These circles are then divided into smaller divisions, with secondary canals serving as conduits for water distribution among these divisions. A circle-level authority assumes the responsibility of allocating water resources among the various divisions. Subsequently, divisions are further segmented into irrigation districts. At this level, divisional authorities make decisions regarding the allocation of water among these districts. Within each district, decisions pertaining to water distribution are made by the district irrigation authority, and the actual distribution of water is facilitated through tertiary canals. On farms, water is accessed directly from these tertiary canals by utilizing watercourses. At the farm level, the agriculture department assumes responsibility for the equitable distribution of water among individual farmers. A similar hierarchical structure is commonly observed in other large-scale irrigation systems worldwide.

The analysis of our paper is concerned with only a single level of hierarchy. In order to apply the analysis to a large-scale hierarchical irrigation system, the framework will need to be extended. In such an application, the individual decision-making blocks are situated in a cascaded manner, so that the decisions made at each stage are propagated down to the subsequent stage until the farm-level decisions are implemented. Since the allocation decisions themselves are made in accordance with the water demand, information on the demand travels in a feedback manner from the farm level in a stage-wise sequence up to the highest decision-making level. While the performance analysis presented in this paper can be applied in isolation to each level, the framework will need to be extended so that feedback from each individual level is effectively incorporated in order to evaluate the performance of the entire irrigation system as a whole.

Figure 10 depicts our vision of the application of our performance evaluation framework to a large-scale hierarchical irrigation system. The figure shows the cascaded decision-making levels and the propagation of demands as feedback in the hierarchy. Under the scope of this study, we have considered the evaluation of allocations made over the entire season as a batch process. Since the decisions are typically made over smaller intervals[2] an opportunity is lost to compensate for any degradation in performance due to unforeseen circumstances at run time such as climate shocks and other associated disturbances. If the performance evaluation can be reformulated as an iterative process, the

[1] These systems are depicted here as representative examples. Such large-scale hierarchical irrigation systems exist in many other parts of the world as well (Plusquellec, 2009).

[2] For instance in the IBIS, the allocations at the farm level are made according to a weekly schedule.

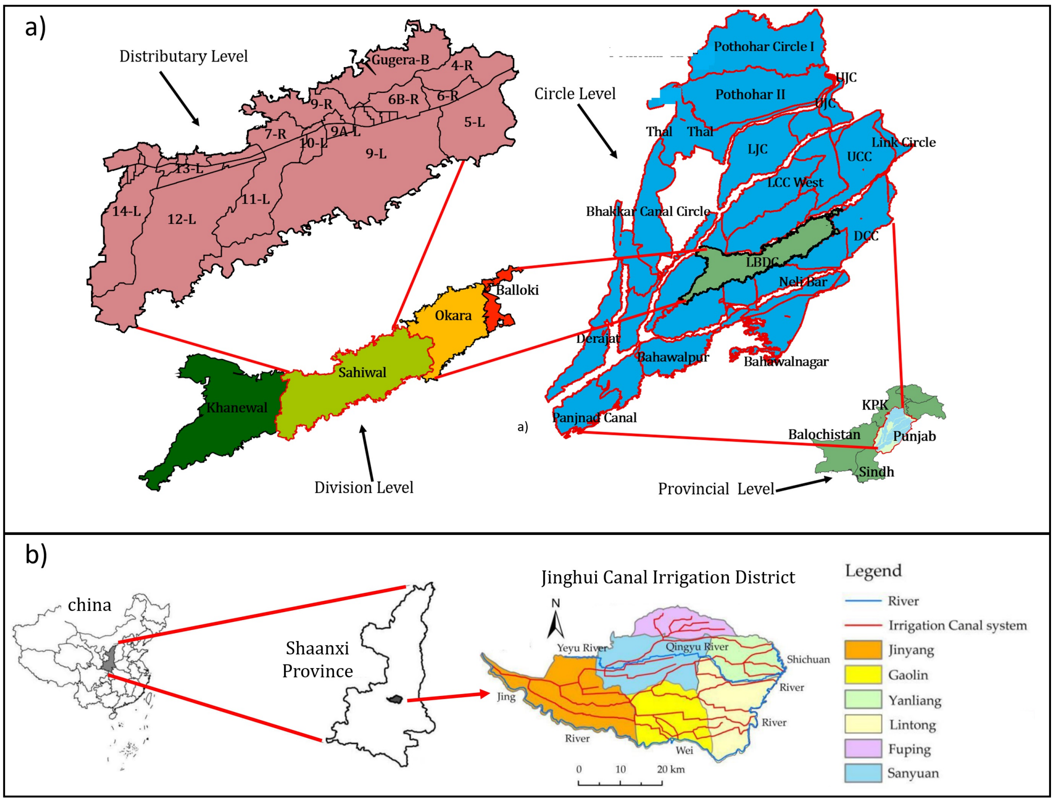


Figure 9. a) A demonstrative diagram of a large-scale irrigation system in Pakistan. b) A demonstrative diagram of a large-scale irrigation system in China.

allocations can be adjusted according to the current performance of the system in order to salvage some of the potential performance of the system over the duration of the entire season. This presents a compelling avenue for future research.

7 Discussion and Conclusion

Although equity and reliability values are widely used to inform policymakers, their performance limits and the relation between these assessment indicators are not commonly used in practice, to the best of our knowledge. The performance limits calculated in this paper can enable policy makers and irrigation system planners to gauge the overall performance of any selected allocation mechanism. To assess an allocation mechanism's performance, the irrigation planners can measure the equity and see where these values fall on the equity-reliability triangle. If the equity vs. reliability point is close to the zero equity axis in the equity-reliability triangle in Fig. 1, the allocation mechanism is working well. However, if the equity vs. reliability point is located towards the worst performance bound, then the irrigation planners may seek to modify the allocation scheme.

Theoretically, as per the analysis in this paper, the best way to allocate water is through uniform allocation since the equity value is zero for all reliability values in this case. Practically however, uniform allocation is difficult to implement since the irrigation planner needs to devise policy to achieve multiple objectives in water allocation, from ensuring water rights and environmental flows to attaining tough economic goals. To meet these economic, environmental, and social objectives, policymakers allocate water using priority-based allocation in many parts of the world. But the worst-case performance

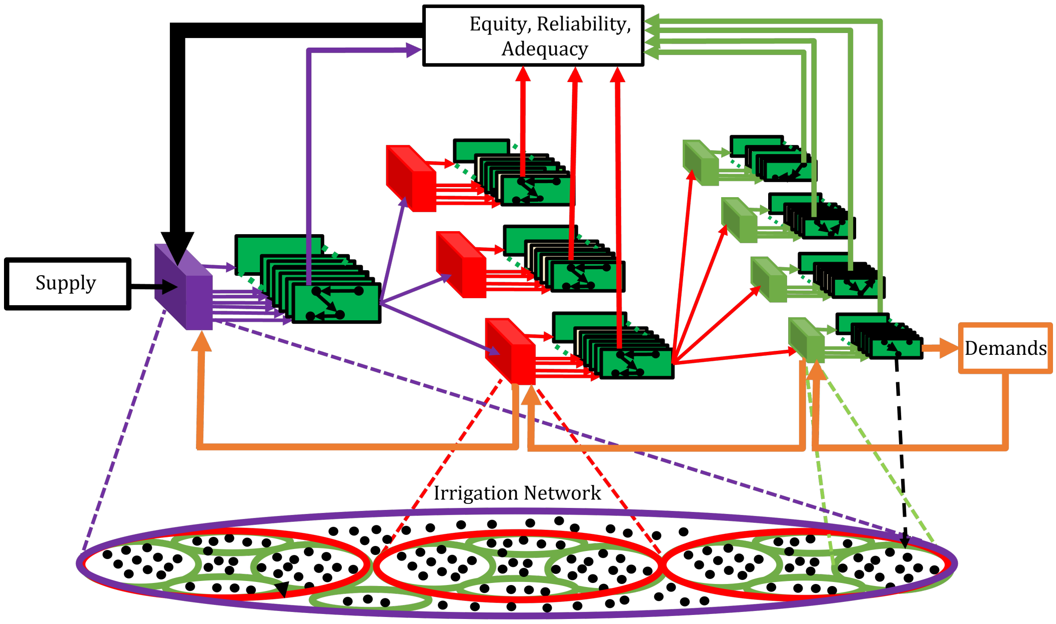


Figure 10. A block-diagram representing an imaginative framework for performance assessment of large-scale irrigation systems.

of the priority-based allocation is low compared to the hybrid allocation. As shown in Fig. 6, the upper bound of equity for priority-based allocation is represented by the $c = 0$ line. As the policymaker adjusts the value of c the worst-case performance of the hybrid allocation improves compared to the priority-based allocation. Thus, using hybrid allocation, policymakers may tune the parameter c to meet the different economic, environmental, and social objectives while simultaneously performing well in terms of equity. In the past, researchers have conducted case studies and developed allocation mechanisms to improve performance by using hybrid allocation mechanism approach, i.e., by adjusting the minimum adequacy level c. For example Hipel et al. (2013) and Wang et al. (2003) presented an allocation mechanism which in first step allocate water entitlements and in second step reallocate and distribute water for social welfare maximization. In the context of our framework, the allocation mechanism presented by Hipel et al. (2013) and Wang et al. (2003) adjust minimum adequacy level c to improve fairness among farmers.

Fig. 1 shows the equity. vs reliability space when water is allocated using any water allocation mechanism. Policymakers may seek to improve the performance of their allocation mechanism when the operating point is not close to the perfect equity vs. reliability point. Priority-based allocation can not guarantee any desired equity because the irrigation planner has to prioritize some users over others. This is also true for uniform allocation because irrigation planner is already allocating water equitably and there is no degree of freedom to move the operating point in the equity-reliability space. Therefore, priority-based and uniform allocation do not give freedom to improve performance by simply tuning some parameter during the allocation process. However, in hybrid allocation, the irrigation planners may control the minimum adequacy level c. As we can see in Fig. 6, as the value of c increases, the worst-case performance improves. Hence, policymakers can tune the parameter c to improve performance if their operating point is not close to the perfect equity-reliability point.

It is important to note that while we have taken equity as a representative metric for fairness or justice among users, there also exist multiple other notions of fairness (Francez, 2012). For example, Benthamite Utilitarianism theory of fairness maximizes the overall welfare of all users (Mulgan, 2014; Viner, 1949), and Rawlsian theory of fairness suggests providing maximum benefit to the neglected part of the society(Kittay, 2018; Sen, 1976). In this particular study, we have used the Egalitarian principle of fairness, which states that all users are equal (Cohen, 2021; Woodburn, 1982). Nonetheless, one can view this work as a blueprint for other fairness notions as well.

Even with the above understanding of there not being any universal definition of fairness, we also wish to highlight that the coefficient of variation is only one measure of equity among multiple others that exist. Indeed many researchers in the irrigation sector assert that the coefficient of variation can be used as a primary summary statistic to measure equity (Siddiqi et al., 2018; D. J. Molden & Gates, 1990). In many other studies however, researchers employ the Gini coefficient to measure equity (Anwar & Ul Haq, 2013), and this coefficient is also widely applied in the irrigation system of Pakistan (Shah et al., 2016) and the distribution of other economic resources (Bell et al., 2016). For many practical cases, the coefficient of variation and Gini coefficient are positively correlated, implying that the general conclusions of our analysis should remain the same for either indicator. However it has been observed that for quantities that are relatively precise (such as the adequacy of irrigation water) the coefficient of variation is more sensitive to individuals in the right tail of the frequency distribution (also called outliers). Therefore, the coefficient of variation may often be recommended over the Gini coefficient if a measure of relative precision is selected to assess inequality (Bendel et al., 1989).

To know performance of an irrigation system over time, we need equity-reliability as a phase space over which trajectories are analyzed. Although we have not looked at irrigation systems that evolve over seasons, the framework is perfect to look at how a system would evolve over time, whether it has stalled in performance, improving or deteriorating. This type of information can be really useful for practitioners and policy-makers.

Finally, the scope of the proposed framework is not only limited to water allocation mechanisms. The framework applies to any resource allocation problem. Far example, in COVID-19 pandemic allocation of resource in health care (Silva et al., 2020), resource allocation in cloud computing (Abid et al., 2020), power allocation in communication technologies (Jayakumar et al., 2021), and scheduling in smart grids (Nair et al., 2018). We have presented this framework as a guideline for equitable allocation of resources in practice, with the hope that it will stimulate further research to add economic, social, and environmental dimensions to the physical distribution of water.

8 Open Research

The data on which this article is based are available in Hassan et al. (2021) and Hassan et al. (2023).

Appendix A

Lemma Appendix A.1. In priority-based allocation, for N users the interval equity is given as,

$$e(k) = \frac{1}{r(k)} \sqrt{\frac{\lfloor Nr(k) \rfloor + (Nr(k) - \lfloor Nr(k) \rfloor)^2 - Nr(k)^2}{(N-1)}}, \tag{A1}$$

where the interval reliability is $r(k)$ and $\lfloor . \rfloor$ is the floor function.

Proof. In priority-based allocation, when fulfilled population count is i, then, $\alpha_1(k) = \alpha_2(k) = ... = \alpha_{i-1} = \alpha_i = 1$, $\alpha_{i+1}(k) = b(k)$, and $\alpha_{i+1}(k) = \alpha_{i+2}(k) = ... = \alpha_{N-1} = \alpha_N = 0$. Now, substituting $\alpha_n(k)$ values in (4) give us the standard deviation

$$\sigma(\alpha(k)) = \sqrt{\frac{i}{N-1}\left(1 - \frac{i+b(k)}{N}\right)^2 + \frac{1}{N-1}\left(b(k) - \frac{i+b(k)}{N}\right)^2 + \frac{(N-i-1)}{N-1}\left(\frac{(i+b(k))}{N}\right)^2},$$

and the mean

$$\mu(\alpha(k)) = \frac{i+b(k)}{N}. \tag{A2}$$

The interval equity value while using mean and standard deviation value from (A2) is as follows

$$e(k) = \frac{1}{i+b(k)} \sqrt{\frac{Ni(N-i) + b(k)^2 N(N-1) - 2Nb(k)i}{(N-1)}}, \tag{A3}$$

We know from (7) that $b(k) = Nr(k) - i$, where the fulfilled population count i is equal to floor of $Nr(k)$, i.e., $i = \lfloor Nr(k) \rfloor$. Now substituting $b(k) = Nr(k) - i$ in (A3) gives us

$$e(k) = \frac{1}{r(k)} \sqrt{\frac{\lfloor Nr(k) \rfloor + (Nr(k) - \lfloor Nr(k) \rfloor)^2 - Nr(k)^2}{(N-1)}}. \tag{A4}$$

Hence, the statement is true when interval reliability is $r(k)$. ■

Lemma Appendix A.2. In hybrid allocation, for N users the interval equity $e(k)$ is given as,

$$e(k) = e_i = \frac{1-c}{i+(N-i)c}\sqrt{\frac{N.i.(N-i)}{N-1}} \tag{A5}$$

where i represents the number of users with fulfilled demand and c represents the adequacy level of the population with partially-fulfilled demand.

Proof. In hybrid allocation, when fulfilled population count is i, and partially fulfilled population count is $N-i$, then, $\alpha_1(k) = \alpha_2(k) = ... = \alpha_{i-1} = \alpha_i = 1$, and $\alpha_{i+1}(k) = \alpha_{i+2}(k) = ... = \alpha_{N-1} = \alpha_N = c$. Now, substituting $\alpha_n(k)$ values in (4) give us the mean

$$\mu(\alpha(k)) = \frac{i+(N-i)c}{N},$$

and the standard deviation

$$\begin{aligned} \sigma(\alpha(k)) &= \sqrt{\frac{i}{N-1}\left(1-\frac{i+(N-i)c}{N}\right)^2 + \frac{(N-i)}{N-1}\left(c-\frac{i+(N-i)c}{N}\right)^2}, \\ &= \frac{1-c}{N}\sqrt{\frac{N.i.(N-i)}{N-1}}. \end{aligned} \tag{A6}$$

The interval equity value while using mean and standard deviation value from (A6) is as follows

$$e(k) = \frac{1-c}{i+(N-i)c}\sqrt{\frac{N.i.(N-i)}{N-1}}. \tag{A7}$$

Hence, the statement is true. ■

Lemma Appendix A.3. In hybrid allocation, for N users the range of interval equity $e(k)$ is given as,

$$\left[0,\ \frac{1-r(k)}{r(k)}\sqrt{\frac{N.\lfloor Nr(k)\rfloor}{(N-1)(N-\lfloor Nr(k)\rfloor)}}\right], \tag{A8}$$

where the interval reliability is r(k) and $\lfloor . \rfloor$ is the floor function.

Proof. The best possible value of interval equity $e(k)$ is achieved, when no user demand is fulfilled, i.e., $i = 0$ and all N users' demand is partially fulfilled. This implies that the adequacy level of all user would be equal to $c = r(k)$. Now, substituting $i = 0$ and $c = r(k)$ in (A9) give us the interval equity $e(k) = 0$. Hence, the lower bound on interval equity for all values of interval reliability would be zero.

The worst possible value of interval equity $e(k)$ is achieved, when the population count with fulfilled demand is $i = \lfloor Nr(k)\rfloor$ and population count with partially fulfilled demand is $N-\lfloor Nr(k)\rfloor$. A proof of the statement similar to this one is given in the next section for arbitrary allocation. This implies that the adequacy level of the population count with fulfilled demand $\lfloor Nr(k)\rfloor$ and population count with partially fulfilled demand $N-\lfloor Nr(k)\rfloor$ is one and $c = \frac{Nr(k)-\lfloor Nr(k)\rfloor}{N-\lfloor Nr(k)\rfloor}$ respectively. Now, substituting $i = \lfloor Nr(k)\rfloor$ and $c = \frac{Nr(k)-\lfloor Nr(k)\rfloor}{N-\lfloor Nr(k)\rfloor}$ in (A9) give us the interval equity

$$e(k) = \frac{1-r(k)}{r(k)}\sqrt{\frac{N.\lfloor Nr(k)\rfloor}{(N-1)(N-\lfloor Nr(k)\rfloor)}}. \tag{A9}$$

Hence, the statement is true for N number of users and the interval reliability is $r(k)$.

■

Lemma Appendix A.4. In arbitrary allocation, for N users the interval equity range is given as

$$\left[0, \quad \frac{1}{r(k)}\sqrt{\frac{\lfloor Nr(k)\rfloor + (Nr(k)-\lfloor Nr(k)\rfloor)^2 - Nr(k)^2}{(N-1)}}\right], \tag{A10}$$

where the interval reliability is $r(k)$ and $\lfloor . \rfloor$ is the floor function.

Proof. The lower bound on interval equity is zero because one of the possible arbitrary allocation is when adequacy level of all users is same. The arbitrary allocation with the same adequacy level for all users can arise for any interval reliability value $r(k)$. In this case, the fulfilled population count $i = 0$ and adequacy level $c = \frac{r(k)}{N}$ for all the users, which always results in zero interval equity. Thus, the lower limit on interval equity is zero for complete range of interval reliability.

To find the upper bound on interval equity, we know from Lemma Appendix A.1 that interval equity when interval reliability is $r(k)$ and water is allocated using the priority-based allocation is given by $\frac{1}{r(k)}\sqrt{\frac{\lfloor Nr(k)\rfloor+(Nr(k)-\lfloor Nr(k)\rfloor)^2-Nr(k)^2}{(N-1)}}$. In this case, the adequacy levels of the fulfilled population count i is 1, i.e., $\alpha_1(k) = \alpha_2(k) = ... = \alpha_{i-1} = \alpha_i = 1$, one user's demand is partially fulfilled, i.e., $\alpha_{i+1} = b(k)$ and adequacy level of the unfulfilled population count $N - i - 1$ is zero, i.e., $\alpha_{i+2}(k) = ... = \alpha_{N-1} = \alpha_N = 0$. For simplicity, we assume that $b(k) = 0$, i.e., $\alpha_{i+1} = 0$.

To show that the interval equity of this allocation is always greater than the interval equity of any arbitrary allocation, we first show that we can transform any arbitrary adequacy distribution of constant interval reliability $r(k)$ from this adequacy distribution and then show that this transformation always results in decrease in interval equity. To construct any adequacy distribution form this adequacy distribution, we shuffle adequacy Δs from user $j \in \{1, 2, 3, ...i\}$ to user $m \in \{i+1, i+2, ...N\}$, such that $\alpha_j(k) \geq \alpha_m(k)$, $\alpha_j(k) = 1$, and $0 \leq \Delta s \leq \alpha_j(k) - \alpha_m(k)$. After Δs shuffle in adequacy, the adequacy distribution would be as follows, $\alpha_1(k) = \alpha_2(k) = ... = \alpha_{j-1}(k) = 1$, $\alpha_j(k) = 1 - \Delta s$, $\alpha_{j+1}(k) = \alpha_{j+2}(k) = ... = \alpha_{i-1} = \alpha_i = 1$, $\alpha_{i+1}(k) = \alpha_{i+2}(k) = ... = \alpha_{m-1} = 0$, $\alpha_m(k) = \Delta s$, and $\alpha_{m+1}(k) = \alpha_{m+2}(k) = ... = \alpha_N = 0$. We continue shuffling adequacy Δs from j to m till we get the desired arbitrary adequacy distribution.

Next, if we prove that Δs shuffle in an arbitrary adequacy distribution constructed above always result in decrease in interval equity, it can be inferred that the interval equity would always be less than the interval equity of the adequacy distribution from which we constructed that arbitrary distribution. To prove that, we assume that the adequacy of a user $n \in \{1, 2, 3, ...N\}$ is $\alpha_n(k)$. Now replacing $\alpha_n(k)$ values in (4) give us the mean,

$$\mu(\alpha_n(k)) = \frac{\sum_{n=1}^{N} \alpha_n(k)}{N} = \frac{\alpha}{N},$$

and the standard deviation

$$\sigma(\alpha_n(k)) = \sqrt{\frac{1}{N-1}\sum_{n=1}^{N}\left(\alpha_n(k) - \frac{\alpha}{N}\right)^2}. \tag{A11}$$

Interval equity value while using mean and standard deviation value from (A11) is as follows

$$e(k) = \frac{N}{\alpha}\sqrt{\frac{1}{N-1}\sum_{n=1}^{N}\left(\alpha_n(k) - \frac{\alpha}{N}\right)^2}. \tag{A12}$$

Now, if we shuffle adequacy Δs from user j to users m where $j, m \in \{1, 2, 3, ...N\}$, such that $\alpha_j(k) \geq \alpha_m(k)$ and $0 \leq \Delta s \leq \alpha_j(k) - \alpha_m(k)$. After shuffle in adequacy, replacing the new values $\alpha_n(k)$ in (4) give us the mean

$$\mu(\alpha_n(k)) = \frac{\sum_{n=1}^{N} \alpha_n(k) + \Delta s - \Delta s}{N} = \frac{\alpha}{N},$$

and the standard deviation

$$\sigma(\alpha_n(k)) = \sqrt{\frac{1}{N-1}\left(\sum_{n=1}^{N-2}\left(\alpha_n(k)-\frac{\alpha}{N}\right)^2 + \left(\alpha_j(k)-\Delta s-\frac{\alpha}{N}\right)^2 + \left(\alpha_m(k)+\Delta s-\frac{\alpha}{N}\right)^2\right)}. \tag{A13}$$

Interval equity value while using mean and standard deviation value from (A13) is as follows,

$$e(k) = \frac{N}{\alpha}\sqrt{\frac{1}{N-1}\sum_{n=1}^{N}\left(\alpha_n(k)-\frac{\alpha}{N}\right)^2 + \frac{2}{N-1}\Delta s\left(\Delta s - (\alpha_j(k)-\alpha_m(k))\right)}. \tag{A14}$$

Since, $0 \leq \Delta s \leq \alpha_j(k) - \alpha_m(k)$, therefore the term $\frac{2}{N-1}\Delta s(\Delta s - (\alpha_j(k) - \alpha_m(k)))$ would always be negative. Hence, the interval equity for this new shuffled arbitrary adequacy distribution would always be less than the interval equity (A14) of the arbitrary adequacy distribution without shuffling. Therefore, we can infer that the interval equity of the allocation mechanism with any arbitrary adequacy level is less than the interval equity $\frac{1}{r(k)}\sqrt{\frac{\lfloor Nr(k)\rfloor + (Nr(k)-\lfloor Nr(k)\rfloor)^2 - Nr(k)^2}{(N-1)}}$. Hence, the statement is true. ■

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References

Abid, A., Manzoor, M. F., Farooq, M. S., Farooq, U., & Hussain, M. (2020). Challenges and issues of resource allocation techniques in cloud computing. *KSII Transactions on Internet and Information Systems (TIIS)*, *14*(7), 2815–2839.

Ali Shah, M. A., Bell, A. R., & Anwar, A. (2022). Enabling volumetric flow measurement in the indus basin irrigation scheme: perceptions and conflict reduction. *Water Resources Research*, *58*(1), e2020WR029464.

Ann Wheeler, S., & Garrick, D. E. (2020). A tale of two water markets in australia: lessons for understanding participation in formal water markets. *Oxford Review of Economic Policy*, *36*(1), 132–153.

Anwar, A. A., & Ahmad, W. (2020). Precision surface irrigation with conjunctive water use. *Sustainable Water Resources Management*, *6*(5), 1–17.

Anwar, A. A., Ahmad, W., Bhatti, M. T., & Ul Haq, Z. (2016). The potential of precision surface irrigation in the indus basin irrigation system. *Irrigation science*, *34*(5), 379–396.

Anwar, A. A., & Ul Haq, Z. (2013). An old–new measure of canal water inequity. *Water international*, *38*(5), 536–551.

Arellano-Gonzalez, J., AghaKouchak, A., Levy, M. C., Qin, Y., Burney, J., Davis, S. J., & Moore, F. C. (2021). The adaptive benefits of agricultural water markets in california. *Environmental Research Letters*, *16*(4), 044036.

Bajaj, A., Singh, S., & Nayak, D. (2022). Impact of water markets on equity and efficiency in irrigation water use: a systematic review and meta-analysis. *Agricultural Water Management*, *259*, 107182.

Bell, A. R., Shah, M. A. A., Anwar, A., & Ringler, C. (2015). What role can information play in improved equity in pakistan's irrigation system? evidence from an experimental game in punjab. *Ecology and Society*, *20*(1).

Bell, A. R., Ward, P. S., & Shah, M. A. A. (2016). Increased water charges improve efficiency and equity in an irrigation system. *Ecology and Society*, *21*(3).

Bendel, R., Higgins, S., Teberg, J., & Pyke, D. (1989). Comparison of skewness coefficient, coefficient of variation, and gini coefficient as inequality measures within populations. *Oecologia*, *78*(3), 394–400.

Brown, C. E. (1998). Coefficient of variation. In *Applied multivariate statistics in geohydrology and related sciences* (pp. 155–157). Springer.

Bruns, B. R., Ringler, C., Meinzen-Dick, R. S., et al. (2005). *Water rights reform: Lessons for institutional design.* Intl Food Policy Res Inst.

Cohen, G. A. (2021). If you're an egalitarian, how come you're so rich? In *If you're an egalitarian, how come you're so rich?* Harvard University Press.

Cordery, I., et al. (2009). Water productivity in a rigid irrigation delivery system. *Water resources management*, *23*(6), 1025–1040.

Cox, N. J. (2010). Speaking stata: The limits of sample skewness and kurtosis. *The Stata Journal*, *10*(3), 482–495.

Deichmann, U., Goyal, A., & Mishra, D. (2016). Will digital technologies transform agriculture in developing countries? *Agricultural Economics*, *47*(S1), 21–33.

De Loë, R., & Bjornlund, H. (2008). *Irrigation and water security: the role of economic instruments and governance* (Vol. 112). WIT Press.

Delorit, J. D., Parker, D. P., & Block, P. J. (2019). An agro-economic approach to framing perennial farm-scale water resources demand management for water rights markets. *Agricultural Water Management*, *218*, 68–81.

Dinar, A., Subramanian, A., et al. (1997). Water pricing experiences. *World Bank Technical Paper*, *386*.

Dolan, F., Lamontagne, J., Link, R., Hejazi, M., Reed, P., & Edmonds, J. (2021). Evaluating the economic impact of water scarcity in a changing world. *Nature communications*, *12*(1), 1–10.

Donoso, G., Barria, P., Chadwick, C., & Rivera, D. (2021). Assessment of water markets in chile. In *Water markets.* Edward Elgar Publishing.

Fan, Y., Gao, Z., Wang, S., Chen, H., & Liu, J. (2018). Evaluation of the water allocation and delivery performance of jiamakou irrigation scheme, shanxi, china. *Water*, *10*(5), 654.

Francez, N. (2012). *Fairness.* Springer Science & Business Media.

Gleick, P. H. (2003). Global freshwater resources: soft-path solutions for the 21st century. *Science*, *302*(5650), 1524–1528.

Gómez-Limón, J. A., Gutiérrez-Martín, C., & Montilla-López, N. M. (2020). Agricultural water allocation under cyclical scarcity: The role of priority water rights. *Water*, *12*(6), 1835.

Grafton, R. Q., Williams, J., Perry, C. J., Molle, F., Ringler, C., Steduto, P., ... others (2018). The paradox of irrigation efficiency. *Science*, *361*(6404), 748–750.

Hassan, W., Manzoor, T., Jaleel, H., & Muhammad, A. (2021). Demand-based water allocation in irrigation systems using mechanism design: A case study from pakistan. *Agricultural Water Management*, *256*, 107075.

Hassan, W., Manzoor, T., & Muhammad, A. (2023). Improving equity in demand-driven irrigation systems through a rights-preserving water allocation mechanism. *Agricultural Water Management*, *287*, 108443.

Hatfield, J. L., & Dold, C. (2019). Water-use efficiency: advances and challenges in a changing climate. *Frontiers in plant science*, *10*, 103.

He, C., Liu, Z., Wu, J., Pan, X., Fang, Z., Li, J., & Bryan, B. A. (2021). Future global urban water scarcity and potential solutions. *Nature Communications*, *12*(1), 4667.

Hipel, K. W., Fang, L., & Wang, L. (2013). Fair water resources allocation with application to the south saskatchewan river basin. *Canadian water resources journal*, *38*(1), 47–60.

Hochba, D. S. (1997). Approximation algorithms for np-hard problems. *ACM Sigact News*, *28*(2), 40–52.

Hollanders, P., Schultz, B., Shaoli, W., & Lingen, C. (2005). Drainage and salinity assessment in the huinong canal irrigation district, ningxia, china. *Irrigation and Drainage: The journal of the International Commission on Irrigation and Drainage*, *54*(2), 155–173.

Ireson, W. R. (2019). Farmer participation in irrigation management: limited roles in pakistan. In *Farmer participation and irrigation organization* (pp. 151–172). CRC Press.

Jayakumar, S., et al. (2021). A review on resource allocation techniques in d2d communication for 5g and b5g technology. *Peer-to-Peer Networking and Applications*, *14*(1), 243–269.

Kaghazchi, A., Shahdany, S. M. H., & Roozbahani, A. (2021). Simulation and evaluation of agricultural water distribution and delivery systems with a hybrid bayesian network model. *Agricultural Water Management*, *245*, 106578.

Karatayev, M., Kapsalyamova, Z., Spankulova, L., Skakova, A., Movkebayeva, G., & Kongyrbay, A. (2017). Priorities and challenges for a sustainable management of water resources in kazakhstan. *Sustainability of Water Quality and Ecology*, *9*, 115–135.

Kazbekov, J., Abdullaev, I., Manthrithilake, H., Qureshi, A., & Jumaboev, K. (2009). Evaluating planning and delivery performance of water user associations (wuas) in osh province, kyrgyzstan. *Agricultural water management*, *96*(8), 1259–1267.

Kharrou, M. H., Le Page, M., Chehbouni, A., Simonneaux, V., Er-Raki, S., Jarlan, L., … Chehbouni, G. (2013). Assessment of equity and adequacy of water delivery in irrigation systems using remote sensing-based indicators in semi-arid region, morocco. *Water resources management*, *27*(13), 4697–4714.

Khepar, S., Gulati, H., Yadav, A., & Brar, T. (2000). A model for equitable distribution of canal water. *Irrigation Science*, *19*(4), 191–197.

Kittay, E. F. (2018). ten human dependency and rawlsian equality. In *Feminists rethink the self* (pp. 219–266). Routledge.

Klümper, F., Herzfeld, T., & Theesfeld, I. (2017). Can water abundance compensate for weak water governance? determining and comparing dimensions of irrigation water security in tajikistan. *Water*, *9*(4), 286.

Li, M., Fu, Q., Singh, V. P., Liu, D., & Gong, X. (2020). Risk-based agricultural water allocation under multiple uncertainties. *Agricultural Water Management*, *233*, 106105.

Liang, Z., Liu, X., Xiong, J., & Xiao, J. (2020). Water allocation and integrative management of precision irrigation: A systematic review. *Water*, *12*(11), 3135.

Molden, D., Burton, M., & Bos, M. (2007). Performance assessment, irrigation service delivery and poverty reduction: benefits of improved system management. *Irrigation and Drainage: The journal of the International Commission on Irrigation and Drainage*, *56*(2-3), 307–320.

Molden, D. J., & Gates, T. K. (1990). Performance measures for evaluation of irrigation-water-delivery systems. *Journal of irrigation and drainage engineering*, *116*(6), 804–823.

Moreno-Pérez, M. F., & Roldán-Cañas, J. (2013). Assessment of irrigation water management in the genil-cabra (córdoba, spain) irrigation district using irrigation indicators. *Agricultural water management*, *120*, 98–106.

Muhammad, A., Haider, B., & Ahmad, Z. (2016). Iot enabled analysis of irrigation rosters in the indus basin irrigation system. *Procedia Engineering*, *154*, 229–

235.
Mulgan, T. (2014). *Understanding utilitarianism.* Routledge.
Nair, A. S., Hossen, T., Campion, M., Selvaraj, D. F., Goveas, N., Kaabouch, N., & Ranganathan, P. (2018). Multi-agent systems for resource allocation and scheduling in a smart grid. *Technology and Economics of Smart Grids and Sustainable Energy*, *3*(1), 1–15.
Nakasone, E., & Torero, M. (2016). A text message away: Icts as a tool to improve food security. *Agricultural Economics*, *47*(S1), 49–59.
NASA. (2021). *last accessed: 23.1.2021.* last accessed: 23.1.2021. Retrieved from `https://power.larc.nasa.gov/data-access-viewer/`
Organization, W. H., et al. (2019). *Progress on household drinking water, sanitation and hygiene 2000-2017: special focus on inequalities.* World Health Organization.
Plusquellec, H. (2009). Modernization of large-scale irrigation systems: is it an achievable objective or a lost cause. *Irrigation and Drainage*, *58*(S1), S104–S120.
RazaviToosi, S. L., & Samani, J. (2019). A fuzzy group decision making framework based on ism-fanp-ftopsis for evaluating watershed management strategies. *Water Resources Management*, *33*(15), 5169–5190.
Savenije, H. H., & Van der Zaag, P. (2000). Conceptual framework for the management of shared river basins; with special reference to the sadc and eu. *Water policy*, *2*(1-2), 9–45.
Sen, A. (1976). Welfare inequalities and rawlsian axiomatics. *Theory and decision*, *7*(4), 243–262.
Shah, M. A. A., Anwar, A. A., Bell, A. R., & Ul Haq, Z. (2016). Equity in a tertiary canal of the indus basin irrigation system (ibis). *Agricultural Water Management*, *178*, 201–214.
Shakir, A., Khan, N., & Qureshi, M. (2010). Canal water management: Case study of upper chenab canal in pakistan. *Irrigation and Drainage: The journal of the International Commission on Irrigation and Drainage*, *59*(1), 76–91.
Siddiqi, A., Wescoat Jr, J. L., & Muhammad, A. (2018). Socio-hydrological assessment of water security in canal irrigation systems: a conjoint quantitative analysis of equity and reliability. *Water Security*, *4*, 44–55.
Silva, K. R. d., Souza, F. G. d., Roquete, F. F., Faria, S. M. d. C., Peixoto, B. C. F., & Vieira, A. (2020). Allocation of resources for health care in covid-19 pandemic times: integrative review. *Revista Brasileira de Enfermagem*, *73*.
Speed, R., Yuanyuan, L., Zhiwei, Z., Le Quesne, T., & Pegram, G. (2013). *Basin water allocation planning: Principles, procedures and approaches for basin allocation planning.* Asian Development Bank, GIWP, UNESCO, and WWF-UK.
Ullah, M. K., Habib, Z., & Muhammad, S. (2001). *Spatial distribution of reference and potential evapotranspiration across the indus basin irrigation systems* (Vol. 24). IWMI.
UN, W. W. A. P. (2018). The united nations world water development report 2018 (united nations educational, scientific and cultural organization, new york, united states). Retrieved from `https://www.unwater.org/publications/world-water-development-report-2018/`
Van Rijswick, H. (2015). Mechanisms for water allocation and water rights in europe and the netherlands-lessons from a general public law perspective. *Journal of Water Law*, *24*(3/4), 141–149.
Van Rijswick, M., van Rijswick, H., & Havekes, H. J. (2012). *European and dutch water law.* UWA Publishing.
Viner, J. (1949). Bentham and js mill: The utilitarian background. *The American economic review*, *39*(2), 360–382.
Wang, L., Fang, L., & Hipel, K. (2003). Water resources allocation: a cooperative game theoretic approach. *Journal of Environmental Informatics*, *2*(2), 11–22.

Ward, F. A., & Pulido-Velazquez, M. (2008). Water conservation in irrigation can increase water use. *Proceedings of the National Academy of Sciences*, *105*(47), 18215–18220.

Wheeler, S. A. (2014). Insights, lessons and benefits from improved regional water security and integration in australia. *Water Resources and Economics*, *8*, 57–78.

Woodburn, J. (1982). Egalitarian societies. *Man*, 431–451.

Xiao, Y., Hipel, K. W., & Fang, L. (2016). Incorporating water demand management into a cooperative water allocation framework. *Water resources management*, *30*(9), 2997–3012.

Xu, X., Huang, G., Qu, Z., & Pereira, L. S. (2011). Using modflow and gis to assess changes in groundwater dynamics in response to water saving measures in irrigation districts of the upper yellow river basin. *Water Resources Management*, *25*(8), 2035–2059.