

Spatial coexistence of coinvasive communities with mutualism and competition

Naven Narayanan^{1*}, Peter Lutz¹, Allison K. Shaw¹

¹. Department of Ecology, Evolution, Behavior, University of Minnesota
Twin Cities, Saint Paul, MN, 55108, USA ; * Corresponding author

Contact for corresponding author: venka210@umn.edu

Keywords: Mutualism, dispersal, coinvasion, coexistence, integro-difference
equations, mutualism dependence, competition, range expansion

Article type: Letter.

Open Research Statement: The data that support the findings of this study
are openly available in Zenodo at <https://doi.org/10.5281/zenodo.8357360>

Running title: Coexistence during coinvasions

Number of references: 78; Number of figures: 5; Number of tables: 1;

Words in abstract: 150; Words in main text: 3941;

Statement of authorship: NN and AKS conceived study, PL and NN conducted simulations and plotted results under supervision of AKS, NN wrote the first draft with significant contributions from PL, and all authors contributed substantially to revisions.

Abstract

All interactions between multiple species invading together (coinvasion) must be
3 accounted for to predict species coexistence patterns across space. Mutualisms,
particularly, are known to influence species' population dynamics and their in-
vasive ability (e.g. mycorrhizal fungi with partner plants). Yet, while modelling
6 coinvasion, their role in mediating coexistence is overlooked. We build a spatial
model of coinvasion of two competing plant species with a shared fungal mu-
tualist to study how mutualism and competition shape the spatial coexistence
9 of competitors. We find that mutualist presence engenders regional (spatial)
coexistence between competitors even when local coexistence between them is
impossible. Further, increasing mutualist dispersal results in sharp transitions
12 in competitor coexistence outcomes. Finally, differences in mutualist partner
dependence and competitive ability interact to produce a variety of local and
regional coexistence outcomes. Thus, incorporating mutualism dependence and
15 dispersal behaviour into coinvasion models offers new insights into spatial coex-
istence and invasive species distributions.

Introduction

18 Understanding the drivers of species coexistence, both local and regional (i.e.
spatial) is essential to understand the biogeography of communities and the
distribution of its constituent species. Traditional biotic drivers of local and
21 spatial coexistence proposed consist interactions with natural enemies or herbi-
vores, life-history, and competition-colonisation tradeoffs among others (Tilman,
1982; MacArthur, 1970; Chesson, 1994; Janzen, 1970; Amarasekare, 2003; Fa-
24 gan et al., 2005). Biotic interactions shaping spatial coexistence, in particular,
has often been limited to pairwise competition or predator-prey dynamics where
competition between invading species leads to either local coexistence or species
27 replacement and the predator behaviour (generalism or specialism) can gener-
ate range overlaps or limits of the species (Case and Taper, 2000; Holt, 1984;
Hochberg and Ives, 1999; Lutscher, 2019). However, more recently, the role of
30 positive interactions has also been shown to shape the local coexistence between
competing species.

Mutualisms are ubiquitous ecological interactions between two or more species
33 that reciprocally increase each others' growth rates and thus their respective
abundances (Vandermeer and Boucher, 1978; Bronstein, 2015). The degree to
which species attribute their growth to partner benefits is termed as 'mutualism
36 dependence' (Douglas and Smith, 1989; Ollerton, 2006; Janos, 2007). Mutual-
ism dependence often evolves to maximise usage of partner benefits but tradeoffs
can lead to trait loss resulting in reduced growth of a species when the partner
39 is absent (Siefert et al., 2019; Chomicki et al., 2020; Visser et al., 2010; Ellers
et al., 2012). Thus, differences in mutualism dependence could lead to competi-
tors obtaining different magnitudes of benefits from a shared mutualist partner.
42 Differential benefits obtained by competitors from a shared mutualists has been
shown to stabilise the local coexistence of competitors (Bever et al., 1997; Bever,

1999; Umbanhowar and McCann, 2005; Heijden et al., 2008).

45 In addition to influencing local coexistence by shaping competitor popula-
tion growth, mutualisms also shape the spatial dynamics of competing species.
Across a variety of taxa, the coinvasion of two or more species arise through mu-
48 tualistic interactions resulting in repeated, succesful invasions in a phenomenon
termed ‘invasional meltdown’ (Simberloff and Von Holle, 1999). This hypothe-
sis has been borne out in several empirical examples including legume-rhizobial
51 mutualisms and plant-fungal mutualisms (Lopez et al., 2021; Simonsen et al.,
2017; Nuñez et al., 2009). For instance, the invasion and range expansion of
pine species in parts of South America and New Zealand are known to proceed
54 only in the presence of their mutualist ectomycorrhizal fungi (EM) (A. Dickie
and Reich, 2005; Dickie et al., 2017; Traveset and Richardson, 2014; Richardson
et al., 2000). Indeed, increased dependence on their fungal partner is known to
57 increase the invasive abilities of a tree species (Moyano et al., 2020, 2021). This
dependence-invasion correlation can arise when more dependent species mor-
phologically adapt to produce smaller seeds or lower seed wing loading (among
60 other adaptations broadly termed Long Distance Dispersal (LDD) Syndrome)
that disperse further even at the cost of reduced competitive ability (Groom,
2010; Greene and Johnson, 1993). Indeed, it has been shown that mycorrhizal
63 plants are more likely to have structures enabling LDD than non-mycorrhizal
plants (Correia et al., 2018; Vargas et al., 2012; Zhang et al., 2019). Differen-
tial dispersal due to variation in LDD traits could shape invasion dynamics of
66 competitors through priority effects and putatively spatial coexistence through
reduced spatial niche overlap (Fukami, 2015; Hess et al., 2019; Vannette and
Fukami, 2014; Ploughe et al., 2020). Yet, we do not completely understand
69 the conditions under which mutualisms shape local and regional (or spatial)
coexistence of co-invading competitor species.

Here, we build a spatially explicit mathematical model of two coin-
72 competitors (for e.g. congeneric plant species) with a shared mutualist partner
(for e.g. EM fungi) to understand the conditions under which the competitors
coexists locally, regionally, or both. We model the role of mutualistic dependence
75 on competitor population dynamics and on their dispersal ability and study their
interplay in driving coexistence outcomes. We also consider scenarios where the
shared mutualist disperses faster or slower than the competitors to identify how
78 relative dispersal abilities of the three interacting species ultimately drive the
coexistence and spatial distribution of the community.

Methods

81 Model

We build a system agnostic model of competition and mutualism, drawing in-
piration from plant-fungal and legume-rhizobial models of mutualisms. We
84 consider two very similar competing species (e.g. congeneric plant species) such
that they have similar resource requirements, habitat requirements, and share
the same mutualist partner species (rhizobia or EM fungi).

87 We model our system using a set of Integro-Difference Equations (IDEs) that
incorporate growth, species interactions, and dispersal of species. The growth
and dispersal phases repeat in a cycle, over discrete time steps across continuous
90 one-dimensional space. Growth occurs from t to $t + T$, followed by dispersal at
 $t + 1$, where t is a given year or generation. ‘ T ’ is some period of time less than
a year (or generation) i.e. $0 < T < 1$. Growth is modelled by coupled Ordinary
93 Differential Equations (ODEs) for the three species while the dispersal of each
species is governed by their own dispersal kernel (Naven Narayanan and Shaw,
2023 (in press)). We track the speed at which species expand into new regions

96 in space along with how their populations densities are spatially distributed.

In our model species F_1 and F_2 are the focal plant species competitors and P is the partner mutualist of both F_1 and F_2 . F_1 and F_2 differ both in their
 99 dependence on P (and thus receive different magnitudes of mutualistic benefits) and in competitive ability. We arbitrarily assume, without loss of generality, that F_1 depends more on P than F_2 (see Figure 1). The general functional form
 102 of our IDEs are as follows:

$$P_{t+1}(x) = \int_{-\infty}^{\infty} k_P(x-y)M_P(P_t(y), F_{1,t}(y), F_{2,t}(y))dy \quad (1a)$$

$$F_{1,t+1}(x) = \int_{-\infty}^{\infty} k_{F_1}(x-y)M_{F_1}(P_t(y), F_{1,t}(y), F_{2,t}(y))dy \quad (1b)$$

$$F_{2,t+1}(x) = \int_{-\infty}^{\infty} k_{F_2}(x-y)M_{F_2}(P_t(y), F_{1,t}(y), F_{2,t}(y))dy \quad (1c)$$

where k_i represents the dispersal kernel of species 'i' ($i = P, F_1, F_2$), x and y are spatial coordinates after and before dispersal respectively, and M_i is the
 105 nonlinear growth function describing the growth of species 'i' at a point in space.

Growth Functions

We model growth of the three species at a given point in the landscape using
 108 a set of Ordinary Differential Equations (ODEs). We model competition between F_1 and F_2 using classic Lotka-Volterra dynamics with linear functional responses. Mutualisms between P and F_1 (or F_2) are modelled as bi-directional
 111 consumer-resource interactions sensu (Holland and DeAngelis, 2009, 2010). The benefits obtained are modelled as a saturating function with partner density which captures physiological limits to uptake or handling of resources. Such

114 saturation has been observed in fig-fig wasp, ant-treehopper, ant-aphid mutu-
 alisms (Wright, 1989; Addicott, 1981; Morales, 2000; Bronstein, 2001). In the
 absence of mutualists and competitors, a single species grows logistically to a
 117 carrying capacity. The equations are given as

$$\frac{dP}{dt} = P \left[(r_P + \left(\frac{\alpha_{PF_1} F_1}{h_P + F_1} + \frac{\alpha_{PF_2} F_2}{h_P + F_2} \right) - d_P P \right] \quad (2a)$$

$$\frac{dF_1}{dt} = F_1 \left[(1 - \delta_{F_1}) r_{F_1} + \delta_{F_1} \left(\frac{\alpha_{F_1 P} P}{h_{F_1} + P} \right) - d_{F_1} F_1 - \tau_{12} F_2 \right] \quad (2b)$$

$$\frac{dF_2}{dt} = F_2 \left[(1 - \delta_{F_2}) r_{F_2} + \delta_{F_2} \left(\frac{\alpha_{F_2 P} P}{h_{F_2} + P} \right) - d_{F_2} F_2 - \tau_{21} F_1 \right] \quad (2c)$$

and are integrated from time ‘t’ to ‘t+T’ to obtain M_P, M_{F_1}, M_{F_2} respectively.
 δ_i represents the mutualist partner dependence of species F_i . The total growth
 120 of a species (say F_1) is the sum of its own intrinsic growth and partner benefits
 weighted by its degree of dependence (δ_{F_1}) resulting in some tradeoff between
 intrinsic growth versus benefit uptake from partner. Such tradeoffs between re-
 123 production and root architecture have been observed in invasive forbs with dif-
 ferent degrees of dependence on their mycorrhizal fungal partner (Seifert et al.,
 2009). r_i is the intrinsic birth rates of species ‘i’, α_{ij} is the maximal benefit re-
 126 ceived from species ‘j’ by species ‘i’, h_i is the half-saturation constant of species
 ‘i’, d_i s are the species’ intrinsic death rates, and τ_{ij} is the competitive effect on
 species ‘i’ by species ‘j’. In our model, we do not explicitly model mutualism
 129 costs. We assume that benefits exchanged are ‘net benefits’ and thus the costs
 of mutualism (i.e. resource acquisition of carbon, phosphorous, or nitrogen) are
 embedded in the expression itself.

132 Dispersal Kernel

Species dispersal is governed by their dispersal kernel, a probability density function describing the probability of an individual dispersing to and establish-
 135 ing at a location ‘x’ given it started at another location ‘y’. We assume that each species has a Gaussian dispersal kernel given as

$$k_P(x - y) = \frac{1}{\sqrt{2\pi\sigma_P^2}} e^{-\frac{(x-y)^2}{2\sigma_P^2}} \quad (3a)$$

$$k_{F_i}(x - y) = \frac{1}{\sqrt{2\pi\delta_{F_i}\sigma_i^2}} e^{-\frac{(x-y)^2}{2\delta_{F_i}\sigma_i^2}} \quad (3b)$$

where σ_P^2 is the variance of P and $\sigma_{F_i}^2$ s are the variances of the dispersal
 138 kernel of the competitors. While implementing dependence-dispersal tradeoffs in our simulations, the kernels of the competitors are asymmetric. When these tradeoffs are not implemented, we ignore their δ_{F_i} values and set them to 1 thus
 141 making the kernels symmetric.

Simulations

To simulate the spatial dynamics of the model, we initialize a one-dimensional
 144 landscape. All simulations begin with very low population densities for each species ($P = F_1 = F_2 = 0.1$) at the center of the landscape with symmetric coinvasion in both directions. Each species’ growth phase occurs for an arbitrary T time steps; changing this number does not qualitatively affect results.
 147 Dispersal then occurs thus completing a single iteration of the simulation. We ran each simulation for 500 iterations by which time steady state was reached,
 150 which we defined as when all three species reached a constant range expansion speed at the edge and the population density of species behind their range edge reached equilibrium. We determined range expansion speed by subtracting

153 the range edge location for the previous iteration from the one for the current
 iteration. We also defined a metric for range overlap ρ which allowed us to de-
 termine what type of coexistence arose between the the competitors. We define
 156 ρ as $\frac{R_{F_1} - R_{F_2}}{R_{F_1} \cup R_{F_2}}$ where R_{F_1} and R_{F_2} are the ranges of the F_1 and F_2 respectively.
 The numerator describes the difference in the range size between the two species
 while the denominator depicts the size of space where either F_1 or F_2 is present.
 159 Based on ρ 's value, we can identify what sort of coexistence outcome can be
 expected between the competitors (see Appendix S1: Table S1 for expected
 ranges of ρ for different coexistence outcomes). Table 1 includes the parameter
 162 values used in all simulations.

Scenarios

We ran four sets of simulations: Set 1 - We compare the coexistence outcomes
 165 between competitors in cases where a mutualist coinvaes with them versus is
 absent from the landscape. Here we assume that the dispersal kernels of all
 species are equal. Set 2 - To understand how increasing dispersal ability of
 168 P altered coexistence of competitors under different competitive abilities (i.e.
 strong, intermediate, and weak competition coefficients (τ_{ij})), we swept over
 $\sigma_P^2 \in [0.001, 0.75]$ with step size 0.001 and calculated their range overlap ρ .
 171 Set 3 - Next, for a given dispersal ability (variance in dispersal kernel i.e. σ_P^2)
 of P, we ask how differences in competitive ability influence the spatial coexis-
 tence outcomes of F_1 and F_2 when P spreads a) slower than, b) equal to, or c)
 174 faster than F_1 and F_2 while the competitors have equal dispersal abilities (i.e.
 $\sigma_{F_1} = \sigma_{F_2}$). These kernel asymmetries between mutualist partners are often
 observed in tree-fungal mutualisms where tree seeds are often wind-dispersed
 177 $\sim 100 - 200\text{m}$ whereas fungal spores typically disperse in the order of $\sim 1 - 10\text{m}$
 (Galante et al., 2011; Peay et al., 2012; Greene and Johnson, 1993) Set 4 - We

next incorporate dependence-dispersal tradeoffs which generates asymmetry in
180 the dispersal ability of the competitors themselves to study its interplay with P's
dispersal ability in determining the spatial coexistence of F_1 and F_2 . We fixed
values of dependence $(\delta_{F_1}, \delta_{F_2}) = (0.6, 0.4)$ on P. The difference in dependence
183 modified the dispersal kernels of the competitors and more dependent competi-
tors disperse further (see Equation 3). Our results are robust to changes in the
values of dependence (which vary from 0 to 1). In this set of simulations, the
186 dispersal kernels of all three species were different from one another. We ran all
simulations across a wide range of competitive abilities of F_1 and F_2 (see Table
1 for values chosen).

189 Outcomes

We classified the coexistence outcome of each simulation when a steady state
was reached. If the population of F_2 was zero across all points in the landscape,
192 we defined this as F_1 *dominance or win* i.e. competitive exclusion of F_2 and
vice versa for F_1 . When the fraction of the total combined species range jointly
occupied by both F_1 and F_2 was > 0.95 we defined it as *local coexistence*. In
195 instances where F_1 or F_2 excluded the other over a fraction (≥ 0.05) of the total
occupied range with local coexistence over the remaining range, we termed this
as *local coexistence with F_1 or F_2 dominance*. Finally, when we found F_1 and F_2
198 each exclusively occupying separate portions of the total occupied range due to
either exclusion of differential dispersal abilities, we defined this as *regional or
spatial coexistence* (See Figure 2 for illustrations of each coexistence outcome).

201 Results

Coinvasion of shared mutualist with competitors promotes their regional coexistence

204 We first ran simulations (Set 1) with coinvasion of F_1 and F_2 in the absence of P. We found that for all possible combinations of competition coefficients (τ_{12}, τ_{21}) pairs, species F_2 always excluded F_1 across the landscape resulting in
207 its complete extinction. Due to F_1 's increased dependence of partner relative to F_2 , its growth and competitive effect on F_2 was low leading to its extinction. We then ran these coinvasion simulations in the presence of spreading
210 P. We found that P's presence altered coexistence outcomes (Figure 3). When $\tau_{12} \gg \tau_{21}$, we find similar to earlier that F_2 locally and spatially excludes F_1 . On the other hand, when $\tau_{12} < \tau_{21}$, F_1 wins and excludes F_2 across its range.
213 Here, F_1 receives more benefits from P than F_2 and this, combined with its better competitive ability excludes F_2 . When both species are weak competitors $\tau_{12} \approx \tau_{21} \in [0, 0.07]$, locally coexistence due to P providing unequal buffers to
216 each competitor's growth. Finally, when $\tau_{12} \gtrsim \tau_{21}$, we observe regional coexistence where F_2 always outcompetes F_1 in regions of common occurrence but F_1 spreads faster across space thereby partially escaping competition from F_2 . P
219 provides benefits which increases F_1 's growth resulting in greater dispersal and increased speed. Thus, mutualisms promote both local and regional coexistence between competitors in scenarios where exclusion might have occurred in their
222 absence.

Increasing mutualist dispersal leads to qualitatively different coexistence outcomes in different competitive environments

Next, we relaxed the assumption that all species had the same dispersal kernel and ran simulations to identify how mutualist dispersal ability could shape spatial coexistence outcomes (Set 2). Differences in dispersal ability between mutualistic partners of different taxonomic groups (for e.g. plant and fungi) have been well documented and can vary to multiple orders of magnitude (Galante et al., 2011; Peay et al., 2012). We modified the variance of P's dispersal kernel (σ_P^2) from 0.001 (extremely low dispersal relative to competitors) to 0.099 (extremely high dispersal ability) and studied its effect on spatial coexistence of the competitors by measuring the overlap of their ranges. We found that when competitive abilities of both species were intermediate or high, increasing mutualist dispersal resulted in sharp transitions between outcomes from exclusion of more dependent species (F_1) to less dependent species (F_2) with a narrow region of intermediate values of σ_P^2 resulting in regional coexistence (Figure 4).

On the other hand, when F_1 and F_2 were weak competitors, there was a slight increase in ρ with increasing σ_P^2 but neither competitor excluded the other. Rather, increasing σ_P^2 simply altered which species dominated at the edge of the co-invading community's range but maintained local coexistence at core of their ranges. Determining whether competition led to regional coexistence or local coexistence with one dominant species at the range edge required comparing the observed ρ for these simulations with expected ρ s from either of the aforementioned coexistence outcomes (Appendix S2: Figure S1).

Differences in competitive ability result in different coexistence outcomes of the competitors with symmetric kernels

249 In our previous result, we found that spatial coexistence was influenced by P's dispersal as well as overall strength of competition between F_1 and F_2 . Here, we relaxed the assumption that F_1 and F_2 were equivalent competitors 252 and explored how differences in competitive ability between F_1 and F_2 could influence coexistence outcomes (Figure 5 a-c). We chose to study these outcomes for three different values of dispersal for P (i.e. $\sigma_P^2 = 0.01, 0.05, 0.075$) (Set 3).

255 We found that results greatly varied based on F_1 and F_2 's relative competitive ability. When P's dispersal was slower than the competitors ($\sigma_P^2 < \sigma_{F_1}^2 = \sigma_{F_2}^2$), there were three possible outcomes. When $\tau_{12} > \tau_{21}$, F_2 excluded 258 F_1 across space due to its lower dependence on P for growth and its relatively equal (if not greater) competitive ability to F_1 . When competition was weak i.e. ($\tau_{12} \approx \tau_{21} \in [0, 0.07]$), F_2 locally coexisted with F_1 yet excluded it at F_2 's range 261 edge. However, when $\tau_{12} < \tau_{21}$, we find that both species regionally coexist. In these simulations, F_2 spreads faster than F_1 due to lesser dependence on the slow spreading P but get excluded behind their range front by F_1 . Here, F_2 is 264 not a strong competitor and F_1 growth is buffered by P.

When P's dispersal is equal to the competitors ($\sigma_P^2 = \sigma_{F_1}^2 = \sigma_{F_2}^2$), we see a shift in coexistence. While we still see F_2 excluding F_1 when $\tau_{12} \gg \tau_{21}$, 267 we also see regions where F_1 excludes F_2 when $\tau_{12} \ll \tau_{21}$. When competition is very low, both species locally coexist across their ranges while for slightly higher values of τ_{21} , local coexistence with F_1 dominance at its range edge is 270 observed. The zone of regional coexistence shrinks to only occupy regions where $\tau_{12} \gtrsim \tau_{21}$ or $\tau_{12} \lesssim \tau_{21}$. Interestingly, these are qualitatively different forms of regional coexistence. In the former, F_1 is present on the edge of the invasion 273 but F_2 outcompetes F_1 behind the expansion front while the reverse pattern is

observed in the latter case.

When P's dispersal is greater the competitors ($\sigma_P^2 > \sigma_{F_1}^2 = \sigma_{F_2}^2$), there was
276 another abrupt shift in the coexistence outcomes. When $\tau_{12} < \tau_{21}$, as opposed
to regional coexistence (seen above), we find F_1 excluding F_2 . Although we find
cases where local and regional coexistence arise, we do not find scenarios where
279 F_2 excludes F_1 .

Dispersal-Dependence tradeoffs increase zone of regional coexistence of the competitors

282 Finally, we incorporate asymmetries in the dispersal kernels of the competitors
that arise from adaptations correlated with degree of dependence on P and
perform simulations for different dispersal abilities of P (Set 4). Here, too
285 we see an abrupt transition in coexistence outcomes based on P's dispersal
ability (Figure 5 d-f). When P's dispersal was slower than the competitors
($\sigma_P^2 < \sigma_{F_2}^2 < \sigma_{F_1}^2$), we see either local coexistence with F_2 dominance, F_1
288 exclusion, or regional coexistence similar to the case when the competitors'
dispersal kernels were similar (Figure 5a). However, when P's dispersal ability
was intermediate ($\sigma_{F_1}^2 > \sigma_P^2 > \sigma_{F_2}^2$), there is a shift in coexistence outcomes
291 where F_1 excludes F_2 when $\tau_{12} < \tau_{21}$ but the competitors regionally coexist
when F_2 is the better competitor with local coexistence (with F_1 dominance)
occurring when both τ_{12} and τ_{21} were low. This pattern was recapitulated even
294 as we increased P's dispersal ability i.e. ($\sigma_P^2 > \sigma_{F_1}^2 > \sigma_{F_2}^2$). Thus, we find
that differential competitor dispersal creates a sharper transition between the
coexistence patterns that are generated with increasing σ_P^2 .

297 Discussion

There is growing theoretical and empirical evidence of the role mutualisms have in shaping the outcomes of local coexistence between competitors as well as
300 shaping the invasion and range expansion speeds of participating species (Umbanhowar and McCann, 2005; Siefert et al., 2019; Lewis et al., 2002; Nuñez et al., 2009; Lopez et al., 2021). However, we have yet to completely understand how
303 these two phenomena interact to shape the spatial coexistence of competing, dispersing species. In this paper, we sought to understand how the mutualisms shaped the coexistence of two coinvasive competitor species.

306 Our model goes beyond past theory to consider several species invading together in line with growing biological evidence of coinvasion seen across a number of aquatic and terrestrial taxa (Johnson et al., 2009; Preston et al.,
309 2012; Kuebbing and Nuñez, 2015; Jackson, 2015; Carrasco et al., 2018). When these coinvasive species are mutualists, the invasion meltdown hypothesis posits that species invasion is accelerated. Successful invasions can then result in
312 downstream ecosystem impacts on native community structure and interactions (Prior et al., 2015; Christian, 2001; Rudgers and Clay, 2008). However, multi-species coinvasions often include other biotic interactions, which are often negative
315 (Kuebbing and Nuñez, 2015; Jackson, 2015). By incorporating these additional negative interactions in our model, we find first that variation in mutualism traits i.e. dependence engenders both local coexistence (by buffering
318 against competitive exclusion of weaker species at a location in space) as well as regional coexistence (by providing benefits which in turn increase growth and range expansion speeds of the more dependent species). We also find that competition acts alongside species spread resulting in the constriction of the range
321 size of the poorer competitor. In the absence of a mutualist, invasion into a region where the better competitor is present is not possible. Competition is

324 a common mechanism invoked to generate species range borders (Yodzis, 2013;
 Fowler and Levin, 1984; Case and Taper, 2000). However, unlike past models
 with only antagonistic interactions, co-invasion with the mutualist and regional
 327 coexistence by the weaker competitor if it is a more dependent species. This
 is due to differential benefits obtained by the competitors as well as mutualism
 mediated seed trait evolution (modelled here as dispersal-dependence tradeoffs).

330 We next find that the dispersal ability of the mutualist influences the range
 of possible coexistence outcomes between the competitors. We find that increas-
 ing dispersal ability of P results in a transition from the more dependent to the
 333 less dependent species getting excluded when competition between F_1 and F_2
 is roughly equivalent and interspecific reduction of growth rates are high. On
 the other hand, when competition is weak ($\tau_{12}, \tau_{21} < 0.05$), increased dispersal
 336 of P does not change patterns of coexistence but alters which species dominates
 at the range edge (Figure 4). Greater dispersal of P leads to mutualists being
 present at high densities at a new habitat prior to either competitor reaching it.
 339 We also observe that differences in competitive ability between F_1 and F_2 also
 lead to different coexistence patterns for a fixed value of P's dispersal ability.
 Thus, we find that mutualism, competition, and dispersal interact to generate
 342 three qualitatively different types of coexistence outcomes based on whether
 the mutualist is faster, slower, or similar in dispersal ability to the competi-
 tors (Figure 5). We also find that incorporating dispersal-dependence tradeoffs,
 345 sharpens the transition between the coexistence patterns that are generated by
 P's dispersal ability (Figure 5d and 5e).

The absence or reduced density of mutualist partners has been shown to to
 348 slow down and hinder species range expansions particularly in plant-microbial in-
 teractions (Wilkinson, 1998; Stanton-Geddes and Anderson, 2011; Nuñez et al.,
 2009). However, in the presence of a competitor, our model predicts that more

351 dependent species are not just slowed down (due to lower intrinsic growth rates)
 but more likely to go extinct during the invasion process (Figure 3). This is in
 contrast to scenarios where the mutualist is a far better disperser than the
 354 competitors, where the more dependent species (F_1) benefits more than its
 competitor in expanding into ranges its partner is present in. Here, the less
 dependent species is more likely to be outcompeted even in cases where it is a
 357 better competitor (i.e. $\tau_{12} > \tau_{21}$). This is because, F_1 spreads faster and grows
 to higher densities before F_2 can invade these regions. Evidence for more depen-
 dent plant species showing greater invasive ability has been shown in pine-EM
 360 fungal mutualisms (Moyano et al., 2021, 2020). Hence, F_2 is excluded through
 priority effects mediated by F_1 's significantly higher densities in regions F_2 is
 newly invading to. Our model also predicts that the set of coexistence (or ex-
 363 clusion) outcomes is more diverse when the dispersal ability of the mutualist
 is equal to the competitors (Figure 5). Here, the mutualist is present at low
 densities (similar to the competitors) at their range front and all three species
 366 are in the transient phase of their growth. This transience allows for several
 equilibria to be accessed based on the species' relative competitive abilities and
 mutualist dependence.

369 Our model could also be interpreted in the context of mutualism trait evo-
 lution during invasion. By considering F_1 and F_1 as two variants (within a
 species) with different degrees of investment into mutualism traits, our results
 372 identify conditions during coinvasion in which mutualism should be selected
 for (i.e. F_1 dominance across space). On one hand, selection could favour
 increasing dependence on mutualism during invasion if these interactions pro-
 375 vide F_1 a fitness advantage over the less dependent (F_2) conspecific invading
 with it or being present in the native community (Rodríguez-Echeverría et al.,
 2009; Rodríguez-Echeverría et al., 2012). On the other hand, selection for lesser

378 mutualism dependence could arise particularly in the presence of competitors,
reduced mutualist availability in novel habitats or life-history tradeoffs (Shelby
et al., 2016; terHorst et al., 2018; Seifert et al., 2009). Indeed, evolution during
381 invasion could lead to mutualism breakdowns between plant and symbiont and
result in more exploitative symbiont phenotypes (Wendlandt et al., 2021).

Although not presented explicitly, our model implicitly predicts outcomes
384 of invader-native plant competition in the presence of a common mutualist.
Such competitive interactions are observed globally with outcomes of these in-
teractions potentially modifying future communities through soil legacy effects
387 (Reinhart and Callaway, 2006; Fahey and Flory, 2022; Van der Putten et al.,
2010). In our model, scenarios where $\sigma_P^2 > \sigma_{F_1}^2 > \sigma_{F_2}^2$ are equivalent to F_2
invading a native plant-fungal mutualism and outcomes of local and regional
390 coexistence should proceed similar results presented in Figure 5c (or 5e).

There are several possible extensions to our model one of which is relax-
ing the assumption that the landscape across which species spread is homoge-
393 neous. This implies our model does not generate coexistence patterns that arise
due spatial heterogeneity such as spatial storage effects (Chesson, 2000; Ama-
rasekare, 2003). Abiotic variation across space is also an important generator
396 of dispersing species' range limits. In our model, range limits are generated
purely through competition, a commonly observed biotic factor generating lim-
its (Fowler and Levin, 1984; Case and Taper, 2000; Case et al., 2005). Further,
399 mutualisms can themselves be context dependent and vary across space in how
much benefits are exchanged between species (Cunning and Baker, 2014; Drew
and King, 2022; Chamberlain et al., 2014). We aim to extend our work to focus
402 on the effects of spatial variation in mutualistic benefit exchange which can then
shape coexistence outcomes of invasive species.

In conclusion, we show that extending models of co-invasion to include mul-

405 tiple biotic interactions can give rise to an array of local and regional coexis-
tence outcomes through both population dynamic effects and modification of
dispersal abilities. Particularly, even in the absence of stable equilibria between
408 competitors locally, we find conditions where coexistence between competitors
occurs globally due to unequal acquisition of mutualist benefits and dependence-
dispersal tradeoffs. We hope the theoretical framework presented here fosters
411 further research into the role of mutualism as an important biotic driver of local
and regional coexistence of co-invading communities.

Acknowledgements

414 We thank members of the Shaw lab for helpful discussions and the Minnesota
Supercomputing Institute (MSI) for their resources. This manuscript is a result
on work supported by the National Science Foundation under Grant No. DEB-
417 2109965.

Bibliography

References

- 420 A. Dickie, I., and P. B. Reich. 2005. Ectomycorrhizal fungal communities at
forest edges. *Journal of Ecology* 93:244–255. Publisher: John Wiley & Sons,
Ltd.
- 423 Addicott, J. F. 1981. Stability properties of 2-species models of mutualism:
Simulation studies. *Oecologia* 49:42–49.
- Amarasekare, P. 2003. Competitive coexistence in spatially structured

- 426 environments: a synthesis. *Ecology Letters* 6:1109–1122. .eprint:
<https://onlinelibrary.wiley.com/doi/pdf/10.1046/j.1461-0248.2003.00530.x>.
- Bever, J. 1999. Dynamics within mutualism and the maintenance of diversity:
 429 inference from a model of interguild frequency dependence. *Ecology Letters*
 2:52–61. .eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.1046/j.1461-0248.1999.21050.x>.
- 432 Bever, J. D., K. M. Westover, and J. Antonovics. 1997. Incorporating the Soil
 Community into Plant Population Dynamics: The Utility of the Feedback Ap-
 proach. *Journal of Ecology* 85:561–573. Publisher: [Wiley, British Ecological
 435 Society].
- Bronstein, J. L. 2001. The Costs of Mutualism. *azoo* 41:825–839. Publisher:
 The Society for Integrative and Comparative Biology.
- 438 Bronstein, J. L., ed. 2015. *Mutualism*. 1st ed. Oxford University Press, Oxford,
 United Kingdom.
- Carrasco, D., G. A. Desurmont, D. Laplanche, M. Proffit, R. Gols, P. G. Becher,
 441 M. C. Larsson, T. C. J. Turlings, and P. Anderson. 2018. With or without
 you: Effects of the concurrent range expansion of an herbivore and its natural
 enemy on native species interactions. *Global Change Biology* 24:631–643.
 444 .eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.1111/gcb.13836>.
- Case, T. J., R. D. Holt, M. A. McPeck, and T. H. Keitt. 2005. The community
 context of species' borders: ecological and evolutionary perspectives. *Oikos*
 447 108:28–46. .eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.0030-1299.2005.13148.x>.
- Case, T. J., and M. L. Taper. 2000. *Interspecific Competition*, Environmental

- 450 Gradients, Gene Flow, and the Coevolution of Species' Borders. *The American Naturalist* 155:583–605. Publisher: The University of Chicago Press.
- Chamberlain, S. A., J. L. Bronstein, and J. A. Rudgers. 2014. How context
453 dependent are species interactions? *Ecol Lett* 17:881–890.
- Chesson, P. 1994. Multispecies Competition in Variable Environments. *Theoretical Population Biology* 45:227–276.
- 456 ———. 2000. General Theory of Competitive Coexistence in Spatially-Varying Environments. *Theoretical Population Biology* 58:211–237.
- Chomicki, G., E. T. Kiers, and S. S. Renner. 2020. The Evolution of Mutualistic
459 Dependence. *Annual Review of Ecology, Evolution, and Systematics* 51:409–432. *eprint*: <https://doi.org/10.1146/annurev-ecolsys-110218-024629>.
- Christian, C. E. 2001. Consequences of a biological invasion reveal the importance
462 of mutualism for plant communities. *Nature* 413:635–639.
- Correia, M., R. Heleno, P. Vargas, and S. Rodríguez-Echeverría. 2018. Should
I stay or should I go? Mycorrhizal plants are more likely to invest in long-
465 distance seed dispersal than non-mycorrhizal plants. *Ecology Letters* 21:683–691. *eprint*: <https://onlinelibrary.wiley.com/doi/pdf/10.1111/ele.12936>.
- Cunning, R., and A. C. Baker. 2014. Not just who, but how many: the importance
468 of partner abundance in reef coral symbioses. *Frontiers in Microbiology* 5.
- Dickie, I. A., J. L. Bufford, R. C. Cobb, M.-L. Desprez-Loustau, G. Grelet, P. E.
471 Hulme, J. Klironomos, A. Makiola, M. A. Nuñez, A. Pringle, P. H. Thrall, S. G. Tourtellot, L. Waller, and N. M. Williams. 2017. The emerging science of linked plant–fungal invasions. *New Phytologist* 215:1314–1332. *eprint*:
474 <https://onlinelibrary.wiley.com/doi/pdf/10.1111/nph.14657>.

- Douglas, A. E., and D. C. Smith. 1989. Are endosymbioses mutualistic? *Trends in Ecology & Evolution* 4:350–352.
- 477 Drew, G. C., and K. C. King. 2022. More or Less? The Effect of Symbiont
Density in Protective Mutualisms. *The American Naturalist* 199:443–454.
Publisher: The University of Chicago Press.
- 480 Ellers, J., E. Toby Kiers, C. R. Currie, B. R. McDonald,
and B. Visser. 2012. Ecological interactions drive evolution-
ary loss of traits. *Ecology Letters* 15:1071–1082. _eprint:
483 <https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1461-0248.2012.01830.x>.
- Fagan, W. F., M. Lewis, M. G. Neubert, C. Aumann, J. L. Apple, and J. G.
Bishop. 2005. When Can Herbivores Slow or Reverse the Spread of an Invas-
486 ing Plant? A Test Case from Mount St. Helens. *The American Naturalist*
166:669–685. Publisher: The University of Chicago Press.
- Fahey, C., and S. L. Flory. 2022. Soil microbes alter competition between native
489 and invasive plants. *Journal of Ecology* 110:404–414.
- Fowler, N. L., and D. A. Levin. 1984. Ecological Constraints on the Estab-
lishment of a Novel Polyploid in Competition with Its Diploid Progenitor.
492 *The American Naturalist* 124:703–711. Publisher: The University of Chicago
Press.
- Fukami, T. 2015. Historical contingency in community assembly: integrating
495 niches, species pools, and priority effects. *Annual review of ecology, evolution,
and systematics* 46:1–23.
- Galante, T. E., T. R. Horton, and D. P. Swaney. 2011. 95% of basidiospores fall
498 within 1 m of the cap: a field-and modeling-based study. *Mycologia* 103:1175–
1183. Publisher: Taylor & Francis _eprint: <https://doi.org/10.3852/10-388>.

- Greene, D. F., and E. A. Johnson. 1993. Seed Mass and Dispersal Capacity in
501 Wind-Dispersed Diaspores. *Oikos* 67:69.
- Groom, P. 2010. Implications of terminal velocity and wing loading on hakea
(proteaceae) seed dispersal. *Journal of the Royal Society of Western Australia*
504 93:175.
- Heijden, M. G. A. V. D., R. D. Bardgett, and N. M. V. Straalen. 2008.
The unseen majority: soil microbes as drivers of plant diversity and pro-
507 ductivity in terrestrial ecosystems. *Ecology Letters* 11:296–310. _eprint:
<https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1461-0248.2007.01139.x>.
- Hess, M. C., F. Mesléard, and E. Buisson. 2019. Priority effects: Emerging prin-
510 ciples for invasive plant species management. *Ecological Engineering* 127:48–
57.
- Hochberg, M. E., and A. R. Ives. 1999. Can natural ene-
513 mies enforce geographical range limits? *Ecography* 22:268–
276. _eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1600-0587.1999.tb00502.x>.
- Holland, J. N., and D. L. DeAngelis. 2009. Consumer-resource
theory predicts dynamic transitions between outcomes of inter-
specific interactions. *Ecology Letters* 12:1357–1366. _eprint:
519 <https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1461-0248.2009.01390.x>.
- . 2010. A consumer-resource approach to the density-dependent popu-
lation dynamics of mutualism. *Ecology* 91:1286–1295. Place: Hoboken Pub-
522 lisher: Wiley WOS:000277867600006.
- Holt, R. D. 1984. Spatial Heterogeneity, Indirect Interactions, and the Coexis-

- tence of Prey Species. *The American Naturalist* 124:377–406. Publisher: The
 525 University of Chicago Press.
- Jackson, M. C. 2015. Interactions among multiple invasive animals. *Ecology*
 96:2035–2041. _eprint: [https://onlinelibrary.wiley.com/doi/pdf/10.1890/15-](https://onlinelibrary.wiley.com/doi/pdf/10.1890/15-0171.1)
 528 0171.1.
- Janos, D. P. 2007. Plant responsiveness to mycorrhizas differs from dependence
 upon mycorrhizas. *Mycorrhiza* 17:75–91.
- 531 Janzen, D. H. 1970. Herbivores and the Number of Tree Species in Tropical
 Forests. *The American Naturalist* 104:501–528. Publisher: The University of
 Chicago Press.
- 534 Johnson, P. T. J., J. D. Olden, C. T. Solomon, and M. J. Vander Zanden. 2009.
 Interactions among invaders: community and ecosystem effects of multiple
 invasive species in an experimental aquatic system. *Oecologia* 159:161–170.
- 537 Kuebbing, S. E., and M. A. Nuñez. 2015. Negative, neutral, and posi-
 tive interactions among nonnative plants: patterns, processes, and man-
 agement implications. *Global Change Biology* 21:926–934. _eprint:
 540 <https://onlinelibrary.wiley.com/doi/pdf/10.1111/gcb.12711>.
- Lewis, M. A., B. Li, and H. F. Weinberger. 2002. Spreading speed and linear
 determinacy for two-species competition models. *J Math Biol* 45:219–233.
- 543 Lopez, Z. C., M. L. Friesen, E. Von Wettberg, L. New, and S. Porter. 2021.
 Microbial mutualist distribution limits spread of the invasive legume *Medicago*
polymorpha. *Biol Invasions* 23:843–856.
- 546 Lutscher, F. 2019. Integrodifference equations in spatial ecology. Springer.
- MacArthur, R. 1970. Species packing and competitive equilibrium for many
 species. *Theoretical Population Biology* 1:1–11.

- 549 Morales, M. A. 2000. Mechanisms and Density Dependence of
Benefit in an Ant–Membracid Mutualism. *Ecology* 81:482–
489. _eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.1890/0012-9658%282000%29081%5B0482%3AMADDOB%5D2.0.CO%3B2>.
552
- Moyano, J., M. A. Rodriguez-Cabal, and M. A. Nuñez. 2021. Invasive trees
rely more on mycorrhizas, countering the ideal-weed hypothesis. *Ecology*
555 102:e03330.
- Moyano, J., M. A. Rodriguez-Cabal, and M. A. Nuñez. 2020. Highly invasive
tree species are more dependent on mutualisms. *Ecology* 101:e02997. _eprint:
558 <https://onlinelibrary.wiley.com/doi/pdf/10.1002/ecy.2997>.
- Nuñez, M., T. Horton, and D. Simberloff. 2009. Lack of belowground mutualisms
hinders Pinaceae invasions. *Ecology* 90:2352–9.
- 561 Ollerton, J. 2006. Biological barter”: patterns of specialization compared across
different mutualisms. *Plant-pollinator interactions: from specialization to
generalization*. University of Chicago Press, Chicago pages 411–435.
- 564 Peay, K. G., M. G. Schubert, N. H. Nguyen, and T. D. Bruns. 2012. Mea-
suring ectomycorrhizal fungal dispersal: macroecological patterns driven
by microscopic propagules. *Molecular Ecology* 21:4122–4136. _eprint:
567 <https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1365-294X.2012.05666.x>.
- Ploughe, L. W., C. N. Carlyle, and L. H. Fraser. 2020. Priority effects: How
the order of arrival of an invasive grass, *bromus tectorum*, alters productivity
570 and plant community structure when grown with native grass species. *Ecology
and Evolution* 10:13173–13181.
- Preston, D. L., J. S. Henderson, and P. T. J. Johnson. 2012. Commu-
573 nity ecology of invasions: direct and indirect effects of multiple inva-

- sive species on aquatic communities. *Ecology* 93:1254–1261. [_eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1890/11-1821.1](https://onlinelibrary.wiley.com/doi/pdf/10.1890/11-1821.1).
- 576 Prior, K. M., J. M. Robinson, S. A. Meadley Dunphy, and M. E. Frederickson.
2015. Mutualism between co-introduced species facilitates invasion and alters
plant community structure. *Proceedings of the Royal Society B: Biological*
579 *Sciences* 282:20142846. Publisher: Royal Society.
- Reinhart, K. O., and R. M. Callaway. 2006. Soil biota and invasive plants. *New*
Phytologist 170:445–457.
- 582 Richardson, D. M., N. Allsopp, C. M. D’antonio, S. J. Milton, and M. Rejmánek.
2000. Plant invasions — the role of mutualisms. *Biological Reviews*
75:65–93. [_eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1469-](https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1469-185X.1999.tb00041.x)
585 [185X.1999.tb00041.x](https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1469-185X.1999.tb00041.x).
- Rodríguez-Echeverría, S., S. Fajardo, B. Ruiz-Díez, and M. Fernández-Pascual.
2012. Differential effectiveness of novel and old legume–rhizobia mutualisms:
588 implications for invasion by exotic legumes. *Oecologia* 170:253–261.
- Rodríguez-Echeverría, S., J. A. Crisóstomo, C. Nabais, and H. Freitas. 2009.
Belowground mutualists and the invasive ability of *Acacia longifolia* in coastal
591 dunes of Portugal. *Biol Invasions* 11:651–661.
- Rudgers, J. A., and K. Clay. 2008. An invasive plant–fungal mutualism reduces
arthropod diversity. *Ecology Letters* 11:831–840.
- 594 Seifert, E. K., J. D. Bever, and J. L. Maron. 2009. Evidence for the evolution
of reduced mycorrhizal dependence during plant invasion. *Ecology* 90:1055–
1062.
- 597 Shelby, N., R. P. Duncan, W. H. van der Putten, K. J. McGinn, C. Weser,

- and P. E. Hulme. 2016. Plant mutualisms with rhizosphere microbiota in introduced versus native ranges. *Journal of Ecology* 104:1259–1270.
- 600 Siefert, A., K. W. Zillig, M. L. Friesen, and S. Y. Strauss. 2019. Mutualists Stabilize the Coexistence of Congeneric Legumes. *The American Naturalist* 193:200–212. Publisher: The University of Chicago Press.
- 603 Simberloff, D., and B. Von Holle. 1999. Positive Interactions of Nonindigenous Species: Invasional Meltdown? *Biological Invasions* 1:21–32.
- Simonsen, A. K., R. Dinnage, L. G. Barrett, S. M. Prober, and P. H. Thrall.
- 606 2017. Symbiosis limits establishment of legumes outside their native range at a global scale. *Nature Communications* 8:1–9. Publisher: Nature Publishing Group.
- 609 Stanton-Geddes, J., and C. G. Anderson. 2011. Does a facultative mutualism limit species range expansion? *Oecologia* 167:149–155.
- terHorst, C. P., C. Wirth, and J. A. Lau. 2018. Genetic variation in mutualistic
- 612 and antagonistic interactions in an invasive legume. *Oecologia* 188:159–171.
- Tilman, D. 1982. *Resource Competition and Community Structure*. (MPB-17), Volume 17. Princeton University Press.
- 615 Traveset, A., and D. M. Richardson. 2014. Mutualistic Interactions and Biological Invasions. *Annu. Rev. Ecol. Evol. Syst.* 45:89–113. Publisher: Annual Reviews.
- 618 Umbanhowar, J., and K. McCann. 2005. Simple rules for the coexistence and competitive dominance of plants mediated by mycorrhizal fungi. *Ecology Letters* 8:247–252. _eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1461-0248.2004.00714.x>.
- 621

- Van der Putten, W. H., M. Macel, and M. E. Visser. 2010. Predicting species
distribution and abundance responses to climate change: why it is essential
624 to include biotic interactions across trophic levels. *Philosophical Transactions
of the Royal Society B: Biological Sciences* 365:2025–2034. Publisher: Royal
Society.
- 627 Vandermeer, J. H., and D. H. Boucher. 1978. Varieties of mutualistic interaction
in population models. *Journal of Theoretical Biology* 74:549–558.
- Vannette, R. L., and T. Fukami. 2014. Historical contingency in species inter-
630 actions: towards niche-based predictions. *Ecology letters* 17:115–124.
- Vargas, P., R. Heleno, A. Traveset, and M. Nogales. 2012. Colonization
of the Galápagos Islands by plants with no specific syndromes for long-
633 distance dispersal: a new perspective. *Ecography* 35:33–43. _eprint:
<https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1600-0587.2011.06980.x>.
- Visser, B., C. Le Lann, F. J. den Blanken, J. A. Harvey, J. J. M. van Alphen,
636 and J. Ellers. 2010. Loss of lipid synthesis as an evolutionary consequence
of a parasitic lifestyle. *Proceedings of the National Academy of Sciences*
107:8677–8682. Publisher: Proceedings of the National Academy of Sciences.
- 639 Wendlandt, C. E., E. Helliwell, M. Roberts, K. T. Nguyen, M. L. Friesen, E. von
Wettberg, P. Price, J. S. Griffitts, and S. S. Porter. 2021. Decreased coevo-
lutionary potential and increased symbiont fecundity during the biological
642 invasion of a legume-rhizobium mutualism. *Evolution* 75:731–747. _eprint:
<https://onlinelibrary.wiley.com/doi/pdf/10.1111/evo.14164>.
- Wilkinson, D. 1998. Mycorrhizal fungi and Quaternary plant migra-
645 tions. *Global Ecology & Biogeography Letters* 7:137–140. _eprint:
<https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1466-8238.1998.00268.x>.

- Wright, D. H. 1989. A Simple, Stable Model of Mutualism Incorporating Handling Time. *The American Naturalist* 134:664–667. Publisher: The University of Chicago Press.
- Yodzis, P. 2013. Competition for space and the structure of ecological communities, vol. 25. Springer Science & Business Media.
- Zhang, X., Z. Dai, F. Gao, D. Du, and Y. Liu. 2019. The role of long-distance dispersal and mycorrhizas on plant colonisation within mainland Germany. *Flora* 258:151443.

Figures

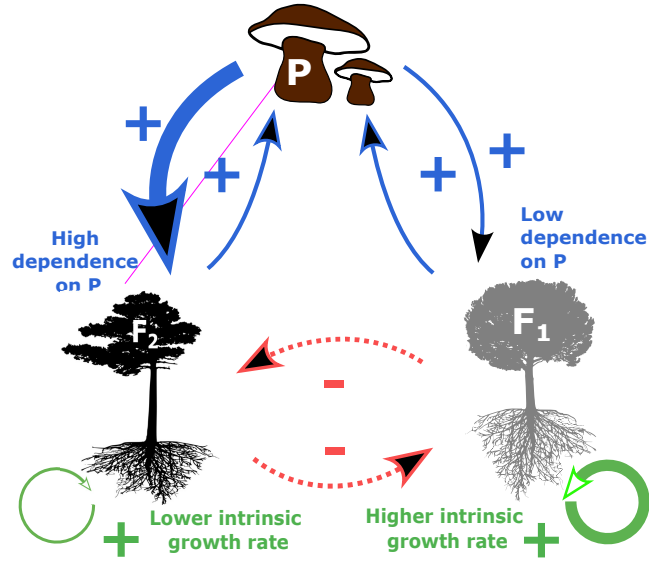


Figure 1: Schematic representation of interactions in the model. Species F_1 and F_2 are the focal competitor species (congeneric plant species) with shared mutualist P (fungal partner) where all species disperse and coinvaade new territory. F_1 has greater dependence on mutualist partner P (in blue) but also lower intrinsic growth rate (in green) than F_2 . Interspecific competition is denoted by dotted red arrows

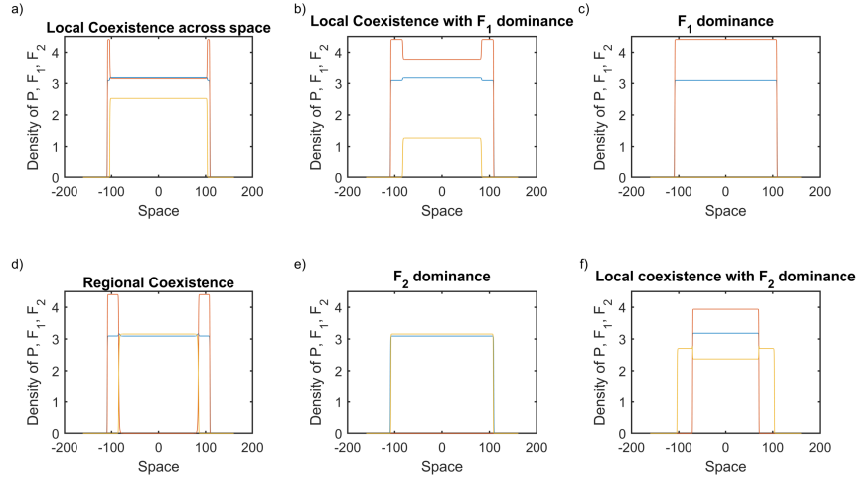


Figure 2: Illustration of different qualitative outcomes observed. All figures are density versus space plots with species P in blue, species F_1 in orange and F_2 in yellow. Subpanel a) shows local coexistence of competitors, b) shows local coexistence with exclusion of F_2 by F_1 at their range edges, c) shows competitive exclusion of F_2 across space, d) shows regional coexistence of both species (but not local coexistence), e) shows local coexistence with F_2 excluding F_1 at the edges, and f) shows F_2 competitively excluding F_1 across all space. Parameter values used for these simulations are: $r_i = 0.3 (i = P, F_1, F_2)$, $\delta_{F_1} = 0.6$, $\delta_{F_2} = 0.4$, $\sigma_{F_1}^2 = \sigma_{F_2}^2 = \sigma_P^2 = 0.05$ ($\sigma_P^2 = 0.02$ for subpanel f), $(\tau_{12}, \tau_{21}) = (0.05, 0.02), (0.05, 0.05), (0.05, 0.15), (0.15, 0.05), (0.3, 0.2), (0.02, 0.02)$

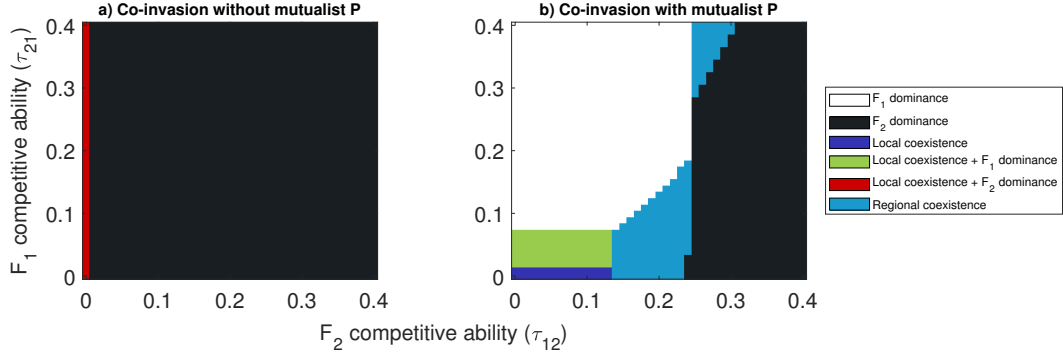


Figure 3: Coexistence of competitors of differing dependence arises in the presence of a co-invading mutualist. Panel a) shows competitive exclusion of the more dependent F_1 without a mutualist while b) shows different possible coexistence outcomes between the competitors in the presence of P for differing relative competitive abilities. Parameters: $r_i = 0.3 (i = P, F_1, F_2)$, $\delta_{F_1} = 0.6$, $\delta_{F_2} = 0.4$, $\sigma_{F_1}^2 = \sigma_{F_2}^2 = \sigma_P^2 = 0.05$ (only for b)

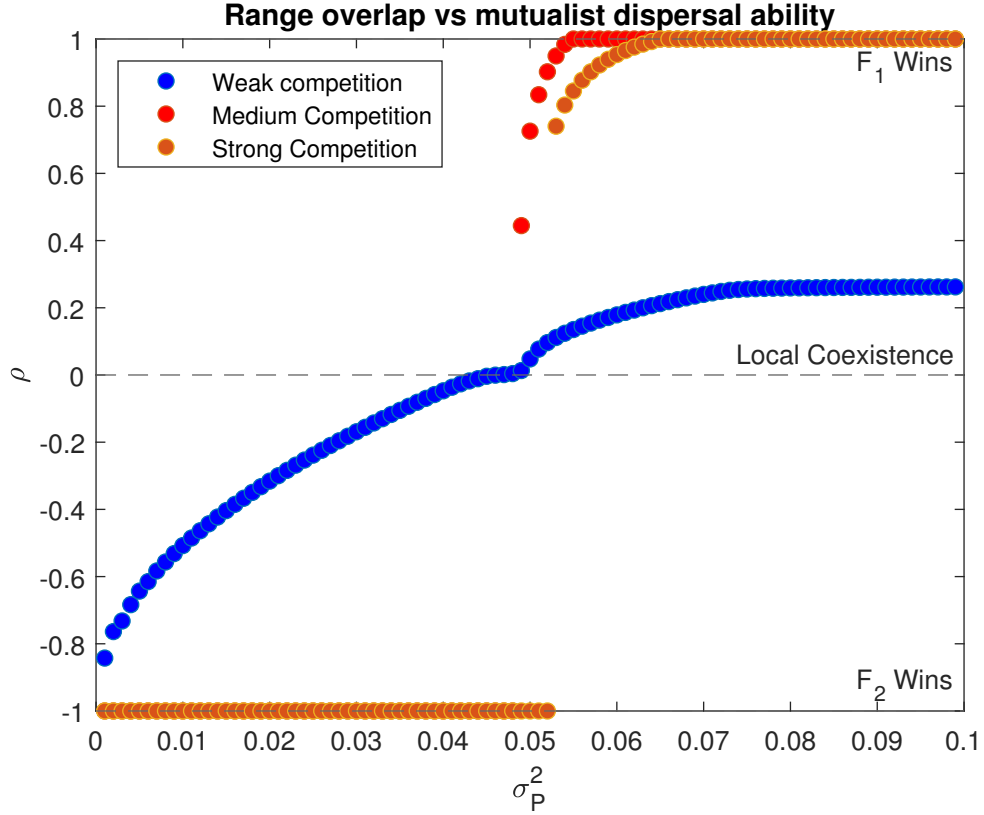


Figure 4: Increasing mutualist dispersal ability alters coexistence type in qualitatively different manners based on strength of competition. When strength of competition is weak (blue), spatial coexistence outcome (denoted by ρ , the range overlap) shifts from F_1 exclusion to local coexistence with F_1 dominance. When the competition strength is intermediate (red) or strong (orange), there is more abrupt shift from F_1 exclusion to F_2 exclusion with regions of regional coexistence for small regions of σ_P^2 . Parameters chosen: $r_i = 0.3 (i = P, F_1, F_2)$, $\delta_{F_1} = 0.6$, $\delta_{F_2} = 0.4$, $\sigma_{F_1}^2 = \sigma_{F_2}^2 = 0.05$ ($\tau_{12}, \tau_{21} = (0.02, 0.02), (0.2, 0.15), (0.37, 0.29)$) for weak, intermediate, and strong competition respectively.

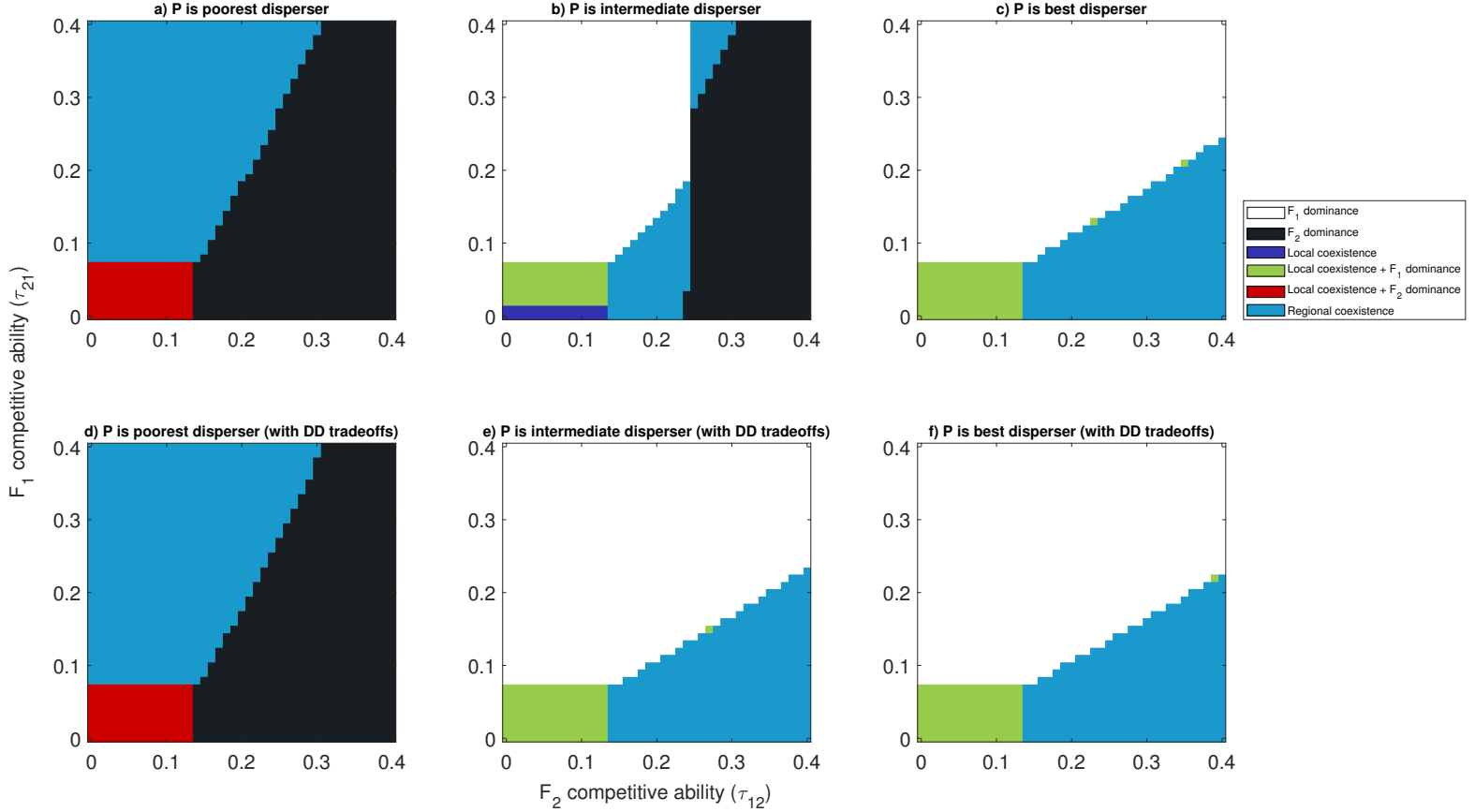


Figure 5: Set of coexistence outcomes shaped by mutualist's dispersal ability and asymmetry in competitors' dispersal kernels arising from dependence-dispersal tradeoffs. In subpanels a), b), and c), we assume both F_1 and F_2 have symmetric dispersal kernels ($\sigma_{F_1}^2 = \sigma_{F_2}^2$) and in d), e), and f) assume F_1 and F_2 have asymmetric dispersal kernels ($\sigma_{F_1}^2 \neq \sigma_{F_2}^2$). We consider P to have lower ($\sigma_P^2 < \sigma_{F_1}^2, \sigma_{F_2}^2$), similar ($\sigma_P^2 \approx \sigma_{F_1}^2, \sigma_{F_2}^2$), and greater ($\sigma_P^2 > \sigma_{F_1}^2, \sigma_{F_2}^2$) dispersal ability than the competitors. When competitors are asymmetric dispersers, only two outcomes are observed. Parameters: $r_i = 0.3 (i = P, F_1, F_2)$, $\delta_{F_1} = 0.6, \delta_{F_2} = 0.4$. For first row: $\sigma_P^2 = (0.01(a), 0.05(b), 0.075(c))$; For second row: $\sigma_{F_1}^2 = 0.03, \sigma_{F_2}^2 = 0.02, \sigma_P^2 = 0.01(a), 0.025(b), 0.075(c)$.

Tables

Table 1: Model parameters and values used in simulations

Symbol	Description	Simulation values
P	Population density of species P	...
F_1	Population density of species F_1	...
F_2	Population density of species F_2	...
τ_{ij}	Competition coefficient of species F_j on F_i ($i, j = 1, 2; i \neq j$)	$0 \leq \tau_{ij} \leq 0.4$
δ_{F_1}	Dependence on mutualism for growth of species F_1	0.6
δ_{F_2}	Dependence on mutualism for growth of species F_2	0.4
r_i	Intrinsic growth rate of species i ($i = P, F_1, F_2$)	0.3
α_{iP}	Mutualist benefits provided by species P to F_i ($i = F_1, \text{ or } F_2$)	0.5
α_{Pj}	Mutualist benefits provided by species F_i to P ($j = F_1, \text{ or } F_2$)	0.01
h_i	Half saturation constant of benefits provided to species i ($i = P, F_1, F_2$)	0.3
d_i	Death rate of species i ($i = P, F_1, F_2$)	0.1
σ_i^2	Variance of dispersal kernel for species i ($i = F_1, F_2$)	0.05
σ_P^2	Variance of dispersal kernel for species P (low, intermediate, high)	(0.01, 0.05, 0.1)