Electromagnetic Fields of Moving Point Sources in the Vacuum

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Abstract--**The electromagnetic fields of point sources with time varying charges moving in the vacuum are derived using the Liénard-Wiechert potentials. The properties of the propagation velocities and the Doppler effect are discussed based on their far fields. The results show that the velocity of the electromagnetic waves and the velocity of the sources cannot be added like vectors; the velocity of electromagnetic waves of moving sources are anisotropic in the vacuum; the transverse Doppler shift is intrinsically included in the fields of the moving sources and is not a pure relativity effect caused by time dilation. Since the fields are rigorous solutions of the Maxwell's equations, the findings can help us to abort the long-standing misinterpretations concerning about the classic mechanics and the classic electromagnetic theory. Although it may violate the theory of the special relativity, we show mathematically that, when the sources move faster than the light in the vacuum, the electromagnetic barriers and the electromagnetic shock waves can be clearly predicted using the exact solutions. Since they cannot be detected by observers in the region outside their shock wave zones, an intuitive and reasonable hypothesis can be made that the superluminal sources may be considered as a kind of electromagnetic blackholes.**

I. Introduction

Maxwell's theory is the foundation for handling all electromagnetic problems. In the coordinate system $O(\mathbf{r},t)$, the Maxwell's equations in the vacuum can be expressed by [1-3]

$$\begin{cases} \nabla\times\mathbf{E}(\mathbf{r},t)=-\dfrac{\partial}{\partial t}\mathbf{B}(\mathbf{r},t) \\ \nabla\times\mathbf{H}(\mathbf{r},t)=\dfrac{\partial\mathbf{D}(\mathbf{r},t)}{\partial t}+\mathbf{J}(\mathbf{r},t) \\ \nabla\cdot\mathbf{B}(\mathbf{r},t)=0 \\ \nabla\cdot\mathbf{D}(\mathbf{r},t)=\rho(\mathbf{r},t) \end{cases} \tag{1}$$

where $\mathbf{E}(\mathbf{r},t)$ is the electric field intensity, $\mathbf{H}(\mathbf{r},t)$ is the magnetic field intensity, $\mathbf{D}(\mathbf{r},t)=\varepsilon_0\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)=\mu_0\mathbf{H}(\mathbf{r},t)$ are respectively the corresponding flux densities. ε_0 and μ_0 are respectively the permittivity and the permeability of the vacuum. The fields are generated by the charge density $\rho(\mathbf{r},t)$ and the related current density $\mathbf{J}(\mathbf{r},t)$. In the vacuum, the current density is caused by the motion of the charge density, i.e.,

$$\mathbf{J}(\mathbf{r},t)=\rho(\mathbf{r},t)\mathbf{v}(\mathbf{r},t) \tag{2}$$

$\mathbf{v}(\mathbf{r},t)$ is the velocity of $\rho(\mathbf{r},t)$. In this paper, we consider non-relativistic electromagnetic problems, and ignore the constraints from the theory of special relativity (SR). In the analysis, we are supposed to stay in the same coordinate system $O(\mathbf{r},t)$ and do not care about the observers in other inertial frames.

In the Maxwell's equations, ε_0 and μ_0 are constants in the vacuum, hence, $c_0=1\big/\sqrt{\mu_0\varepsilon_0}$ is also a constant. However, it is obvious that the Maxwell's theory does not state that the velocity of the electromagnetic waves in the vacuum is always c_0. Therefore, we temporarily forget the relativity rule that all objects cannot move faster than the light in the vacuum, and consider that all causal solutions to the Maxwell's equations are physically reasonable.

The electromagnetic fields generated by point charges in the vacuum are very useful because they usually have explicit expressions that can be used to illustrate the main characteristics of the electromagnetic fields. Liénard-Wiechert potentials [1, 4, 5] are widely used in analyzing the electromagnetic fields of a moving charge. They are derived by Liénard in 1898 and Wiechert in 1900. The same techniques can be applied for deriving the fields generated by moving sources with time-varying charges, like the Hertzian dipole [3, 6, 7, 8]. However, most of the published results for moving

Hertzian dipoles are relativistic ones that were used to check the relativistic behaviors. In this paper, the rigorous solutions of a moving point source with time-varying charge and that of a moving Hertzian dipole are derived based on the Liénard-Wiechert potentials. The behaviors of the far fields are analyzed and illustrated with figures. In particular, the exact relationship between the wave velocity and the velocity of the sources are provided, based on which the anisotropic property of the wave velocity and the Doppler effect are demonstrated. These findings are very important because they have broken the three long-standing misinterpretations concerning about the classic physics. Statements can be found in most of the related textbooks and Journal papers that, according to the classic physics, the light velocity is isotropic in the vacuum; the wave velocity and the velocity of the source can be added like vectors; the transverse Doppler shift is a pure relativistic effect due to the time dilation. We show that all these statements are not true. Furthermore, without the velocity limit imposed by SR, when the sources move faster than the light in the vacuum, the electromagnetic barriers and electromagnetic shock waves can be clearly predicted from the exact solutions.

II. **Fields of Moving Point Sources**

Assume that a particle with harmonic charge $\rho_0 \cos\omega_0 t_1$ moves along a trajectory $\mathbf{x}(t_1)$, as shown in Fig. 1. ω_0 is the oscillating angular frequency of the charge. At the time t_1, the velocity is $\mathbf{v}(t_1)=\dot{\mathbf{x}}(t_1)$, and the acceleration is $\mathbf{a}(t_1)=\dot{\mathbf{v}}(t_1)$. For the sake of brevity, we use the dot on top of a vector to denote the derivation with respect to time t_1. Denote $\mathbf{R}=\mathbf{r}-\mathbf{x}(t_1)$ as the radius vector from the source position to the observation point, and $R=|\mathbf{r}-\mathbf{x}(t_1)|$. The fields generated by the charge at time t_1 propagate to the observation point $\mathbf{r}$ with a time delay of $t-t_1$. In this paper, we generally use $(\mathbf{r},t)$ for the fields and potentials, and $(\mathbf{r}_1,t_1)$ for the sources if not specified otherwise.

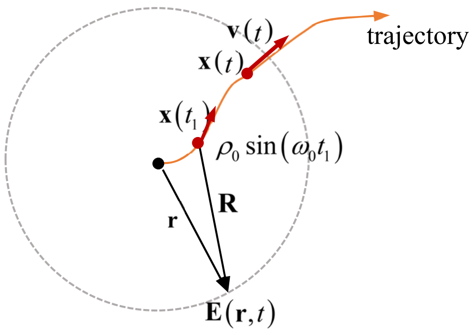


Fig.1. Trajectory of a moving point source with time-varying charge.

The charge density and the corresponding current density can be expressed by

$$\rho(\mathbf{r}_1,t_1)=\rho_0\delta(\mathbf{r}_1-\mathbf{x})\cos(\omega_0 t_1) \tag{3}$$

$$\mathbf{J}(\mathbf{r}_1,t_1)=\rho_0\mathbf{v}\delta(\mathbf{r}_1-\mathbf{x})\cos(\omega_0 t_1) \tag{4}$$

where (t_1) in $\mathbf{v}(t_1)$ and $\mathbf{x}(t_1)$ is suspended for the sake of simplicity. Following exactly the same way in deriving the Liénard-Wiechert potentials for a moving charge [4, 5], we can derive the Liénard-Wiechert potentials for the moving point source with harmonic time varying charges,

$$\phi_{har}(\mathbf{r},t)=\frac{\rho_0}{4\pi\varepsilon_0}\frac{1}{R(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta})}\cos(\omega_0 t_1) \tag{5}$$

$$\mathbf{A}_{har}(\mathbf{r},t)=\frac{\rho_0}{4\pi\varepsilon_0}\frac{\boldsymbol{\beta}}{c_0 R(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta})}\cos(\omega_0 t_1) \tag{6}$$

From the potentials, we can derive the fields of the moving source as [4, 5]

$$\mathbf{E}_{har}(\mathbf{r},t) = -\frac{\partial \mathbf{A}_{har}(\mathbf{r},t)}{\partial t} - \nabla \phi_{har}(\mathbf{r},t)$$

$$= \frac{\rho_0}{4\pi\varepsilon_0}\left\{ \begin{array}{l} \left[\frac{(1-\boldsymbol{\beta}\cdot\boldsymbol{\beta})(\hat{\mathbf{n}}-\boldsymbol{\beta})}{R^2(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta})^3} + \frac{\hat{\mathbf{n}}\times\left[(\hat{\mathbf{n}}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right]}{c_0 R(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta})^3}\right]\cos(\omega_0 t_1) \\ -\frac{\hat{\mathbf{n}}-\boldsymbol{\beta}}{c_0 R(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta})^2}\omega_0 \sin(\omega_0 t_1) \end{array} \right\} \tag{7}$$

$$\mathbf{B}_{har}(\mathbf{r},t) = \frac{1}{c_0}\hat{\mathbf{n}}\times\mathbf{E}_{har}(\mathbf{r},t) \tag{8}$$

In the expressions, we denote $\boldsymbol{\beta} = \mathbf{v}/c_0$, $\dot{\boldsymbol{\beta}} = \dot{\mathbf{v}}/c_0$, $\hat{\mathbf{n}} = \mathbf{R}/R$, and c_0 is the light velocity in the vacuum. According to the solutions of the Maxwell's equations, the field at $(\mathbf{r},t)$ generated by the pulse source at $\mathbf{x}(t_1)$ travels with velocity c_0. Hence, we have

$$R = c_0(t-t_1) = |\mathbf{r}-\mathbf{x}(t_1)| \tag{9}$$

When $\omega_0 = 0$, it is straightforward to check that (7) and (8) are simplified to the fields of the moving point source with a constant charge.

When the source moves uniformly, $\mathbf{x}(t_1) = \mathbf{v}t_1$ and $\dot{\boldsymbol{\beta}}(t_1) = 0$. Denote $\boldsymbol{\beta} = \beta\hat{\boldsymbol{\beta}}$, $\mathbf{r} = r\hat{\mathbf{r}}$. $\hat{\boldsymbol{\beta}}$ and $\hat{\mathbf{r}}$ are the corresponding unit vectors, and $\hat{\boldsymbol{\beta}}\cdot\hat{\mathbf{r}} = \cos\theta$. Solving t_1 from (9) yields

$$t_1 = \frac{c_0^2 t - c_0\beta r\cos\theta - c_0\sqrt{(1-\beta^2\sin^2\theta)r^2 - 2c_0 t\beta\cos\theta r + \beta^2 c_0^2 t^2}}{c_0^2(1-\beta^2)} \tag{10}$$

Note that t_1 must be real, and $t_1 \le t$ must hold according to the causality principle. For a uniformly moving source, we can check that there is at most one solution for t_1 that satisfies these conditions if $\beta \le 1$. In other words, the fields at the position $\mathbf{r}$ and the time instant t comes only from the source at the time instant t_1 when it moves to the position $\mathbf{v}t_1$. We may consider the field generated at a point $(\mathbf{r}_1, t_1)$ as a pulse of electromagnetic fields. The fields generated at each time instant t_1 will propagate with constant velocity c_0. However, a continuously moving point source generates continuous electromagnetic fields when it moves on. All the field pulses by the moving point source are superposed to form the continuous field distributions in the vacuum. However, we can see that the behaviors of the composed fields may become quite different from that of a single electromagnetic pulse: both the wave velocity and the frequency of the composed fields will change.

At places far away from the sources, $r \gg vt$ and $R \gg vt$, we have

$$\sqrt{(1-\beta^2\sin^2\theta)r^2 - 2c_0 t\beta\cos\theta r + \beta^2 c_0^2 t^2} \approx \sqrt{1-\beta^2\sin^2\theta}\, r - \frac{c_0\beta\cos\theta}{\sqrt{1-\beta^2\sin^2\theta}} t$$

Consequently,

$$t_1 \approx \left(\frac{1}{1-\beta^2} + \frac{\beta}{1-\beta^2}\frac{\cos\theta}{\sqrt{1-\beta^2\sin^2\theta}}\right)t - \frac{\beta\cos\theta + \sqrt{1-\beta^2\sin^2\theta}}{c(1-\beta^2)} r \tag{11}$$

Substituting (11) into (7) and discarding the term with respect to $(1/R^2)$, we obtain the approximate expression for the electric far field

$$\mathbf{E}_{har}(\mathbf{r},t) \approx -\frac{\omega_0 \rho_0}{4\pi\varepsilon_0 c_0}\frac{\hat{\mathbf{n}}-\boldsymbol{\beta}}{R(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta})^2}\sin\left[\omega(t-r/c)\right] \tag{12}$$

where the velocity c and the angular frequency ω of the composed far field are, respectively,

$$c = \frac{c_0}{\sqrt{1-\beta^2 \sin^2\theta}} \tag{13}$$

$$\omega = \frac{\omega_0}{1-\beta^2}\left(1+\frac{\beta\cos\theta}{\sqrt{1-\beta^2\sin^2\theta}}\right) \tag{14}$$

Obviously, (12) represents a spherical wave. The wave velocity c and the angular frequency ω of the composed far fields are both functions of three variables: the velocity β of the source, the oscillating frequency ω_0 of the source, and the angle θ between the propagation direction of the fields and the moving direction of the source.

A. Wave velocity

The wave velocity of the electromagnetic far fields is dependent on the propagation direction. It is obvious that $c \geq c_0$ for all θ. The velocity reaches its maximum of $c_0/\sqrt{1-\beta^2}$ at $\theta = \pm\pi/2$. It is exactly c_0 at $\theta = 0$ or π, in which the observer is on the path of the source. The wave velocity of the far field is isotropic only in the case that $\beta = 0$, i.e., the source is motionless or moves within a bounded region. To state that the wave velocity in the vacuum is constant according to the classic physics rules is a misinterpretation.

A typical plot of the velocity is shown in Fig.2. When $\beta = 0.1$, the largest wave velocity is only $1.005c_0$ by (13). It would be $1.1c_0$ if the two velocities are added directly. Equation (13) clearly shows that the propagation velocity of the electromagnetic waves and the moving velocity of the sources do not satisfy the vector addition rule. To state that the two velocities are added like vectors according to the classic physics rules is another misinterpretation.

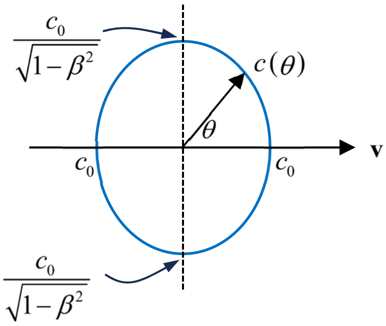


Fig. 2. The velocity of a uniformly moving source.

B. Doppler effect

The angular frequency ω of the far fields is also dependent on the propagation direction. This is the Doppler effect. We denote the normalized angular frequency as

$$s(\theta) = \frac{\omega}{\omega_0} = \frac{1}{1-\beta^2} + \frac{\beta}{1-\beta^2}\frac{\cos\theta}{\sqrt{1-\beta^2\sin^2\theta}} \tag{15}$$

The relative Doppler shift is $s(\theta)-1$. The Doppler shift is zero at the angle θ_d that satisfies

$$\cos\theta_d = -\frac{\beta}{\sqrt{1+\beta^2}} \tag{16}$$

For slowly moving sources, β is small. $s(\theta)$ can be expanded to the power series with respect to β,

$$s(\theta) \approx 1 + \beta\cos\theta + \beta^2 + \cdots \tag{17}$$

We can check that $s(0) = 1+\beta/(1-\beta) \approx 1+\beta$ and $s(\pi) = 1+\beta/(1+\beta) \approx 1-\beta$. They agree with the classic mechanics results and the results based on SR to the first order of β.

Particularly, (14) shows that transverse Doppler effect clearly exists at $\theta = \pm\pi/2$,

$$s\left(\pm\frac{\pi}{2}\right)=\frac{1}{1-\beta^2}\approx 1+\beta^2+\cdots \tag{18}$$

The Doppler effect is explicitly included in the wave solutions of the Maxwell's equations at places far away from the sources. It is caused by the superposition of the fields radiated by the moving source at different times. It is directly solved in the laboratory frame without necessity to apply any invariant principles related to coordinate transformations. We have to note that the coefficient of the second-order Doppler shift is different from that predicted using SR, which is (-0.5) [4].

Conventionally, the transverse doppler effect is considered as a pure relativistic effect that is directly related to time dilation [4, 9]. It has been used as a strong support to the theory of special relativity. Now we have shown that this is once again a misinterpretation concerning about the classic physics. The transverse Doppler effect is definitely included in the exact solution (14) obtained using the principles of the classical physics.

C. Field patterns

The point source with harmonic charge is the simplest example for illustrating the radiation property. It can be treated as an imaginary source because it is not a constant and may violate the conventional charge conservation law. However, this is not fatal because we can combine two of them with opposite charges together to form a Hertzian dipole, the charge in which is conserved. Although the expressions for the wave velocity (13) and the frequency (14) are derived from a uniformly moving point source with harmonic time-varying charge, they can be used for predicting the propagation properties of the far fields of general sources in uniform motions. For general sources, the patterns of their fields may have much more complex directivities. However, the propagation velocity and the Doppler effect of their far fields are the same and can be respectively described by (13) and (14).

As an example, we have derived the far fields of a moving Hertzian dipole. It is composed of two anti-phase harmonic charges with small distance l, as shown in Fig.3. The electric momentum is expressed by $\rho_0 l\cos(\omega_0 t_1)\hat{\mathbf{p}}$, where $\hat{\mathbf{p}}$ is the polarization unit vector. The fields of the uniformly moving Hertzian dipole can be derived using exactly the same techniques in deriving the fields of the single harmonic charge.

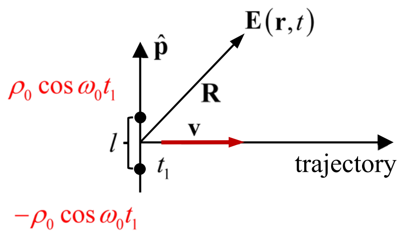


Fig. 3. Trajectory of a moving Hertzian dipole. $\mathbf{r}$ is not depicted.

We here consider the case that the dipole moves in a path perpendicular to its polarization direction, $\mathbf{v}\cdot\hat{\mathbf{p}}=0$, as shown in Fig.3. The scalar potential of the two charges is of the same form as (5),

$$\phi_\pm(\mathbf{r},t)=\pm\frac{\rho_0}{4\pi\varepsilon_0}\frac{1}{R_\pm(1-\hat{\mathbf{n}}_\pm\cdot\boldsymbol{\beta})}\cos(\omega_0 t_{1\pm}) \tag{19}$$

where $R_\pm$, $\hat{\mathbf{n}}_\pm$, and $t_{1\pm}$ are respectively related to the charge $\pm\rho_0\cos\omega_0 t_1$. The total scalar potential is then obtained by

$$\phi(\mathbf{r},t)=l\lim_{l\to 0}\left[\frac{\phi_+(\mathbf{r},t)+\phi_-(\mathbf{r},t)}{l}\right] \tag{20}$$

We let $l\to 0$ but keep $\rho_0 l$ constant. By applying the vector identities in deriving (7) [5], we get the final scalar potential for the moving Hertzian dipole,

$$\phi(\mathbf{r},t)=\frac{\rho_0 l}{2\pi\varepsilon_0}\left\{\frac{(1-\boldsymbol{\beta}\cdot\boldsymbol{\beta})(\hat{\mathbf{n}}\cdot\hat{\mathbf{p}})}{R^2(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta})^3}\cos(\omega_0 t_1)-\frac{\hat{\mathbf{n}}\cdot\hat{\mathbf{p}}}{c_0 R(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta})^2}\omega_0\sin(\omega_0 t_1)\right\} \tag{21}$$

The vector potential for the Hertzian dipole is derived in a similar way,

$$\mathbf{A}(\mathbf{r},t)=\frac{\rho_0 l}{2\pi\varepsilon_0}\left\{\frac{(1-\boldsymbol{\beta}\cdot\boldsymbol{\beta})(\hat{\mathbf{n}}\cdot\hat{\mathbf{p}})\boldsymbol{\beta}}{Rc_0^2(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta})^3}\cos(\omega_0 t_1)-\frac{(\hat{\mathbf{n}}\cdot\hat{\mathbf{p}})\boldsymbol{\beta}+(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta})\hat{\mathbf{p}}}{c_0^2 R(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta})^2}\omega_0\sin(\omega_0 t_1)\right\} \tag{22}$$

Note that the vector potential includes the contributions not only from the two moving charges but also from the moving dipole momentum. The fields can be derived from them rigorously with the same formulae as those in [4, 5]. Keeping the main terms that include the first order of $(1/R)$, the far fields of the uniformly moving Hertzian dipole can be approximately expressed by

$$\mathbf{E}_{dip}(\mathbf{r},t)=\frac{\omega_0^2 E_0}{R(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta})^2}\left[\hat{\mathbf{p}}-\frac{\hat{\mathbf{n}}\cdot\hat{\mathbf{p}}}{1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta}}(\hat{\mathbf{n}}-\boldsymbol{\beta})\right]\cos(\omega_0 t_1)+\frac{\omega_0 E_0(1-\boldsymbol{\beta}\cdot\boldsymbol{\beta})(\hat{\mathbf{n}}\cdot\hat{\mathbf{p}})}{R(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta})^4}\boldsymbol{\beta}\sin(\omega_0 t_1) \tag{23}$$

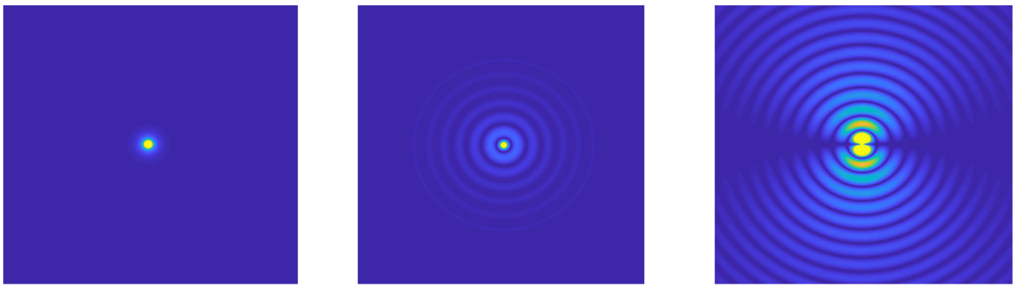
$$\mathbf{B}_{dip}(\mathbf{r},t)\approx\frac{1}{c_0}\hat{\mathbf{n}}\times\mathbf{E}_{dip}(\mathbf{r},t) \tag{24}$$

where $E_0=\rho_0 l/(2\pi\varepsilon_0 c_0^2)$ is a source-related constant. The propagation property is exactly the same as the uniformly moving single harmonic charge, except that there is a directivity factor related to the polarization of the dipole. For $\mathbf{v}=0$, we can check that (23) turns into

$$\mathbf{E}_{dip}(\mathbf{r},t)\approx\frac{\omega_0^2 E_0}{R}(\hat{\mathbf{p}}-\hat{\mathbf{r}}\hat{\mathbf{r}}\cdot\hat{\mathbf{p}})\cos(\omega_0 t_1)$$

which is the far electric field of a fixed Hertzian dipole.

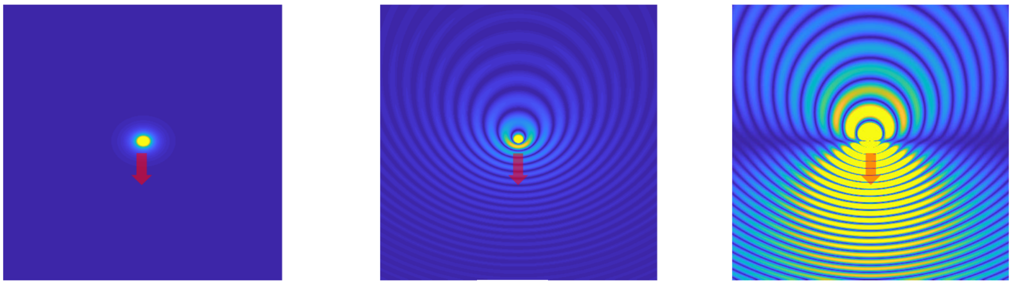
We will use three types of sources to illustrate the properties of the fields at $\beta=0$, 0.5, and 0.9. The electric fields for the time invariant charge, harmonic time varying charge are calculated with (7). The fields of the Hertzian dipole are calculated with (23), but only keep the term with respect to ω_0^2 since we focus on high frequency cases. For motionless sources, the results are shown in Fig.4. The static charge simply generates a static field, as shown in Fig.4(a). For the sources with time varying charges, the wave velocity is c_0 in all directions, as shown in Fig.4(b) and (c).



(a) (b) (c)

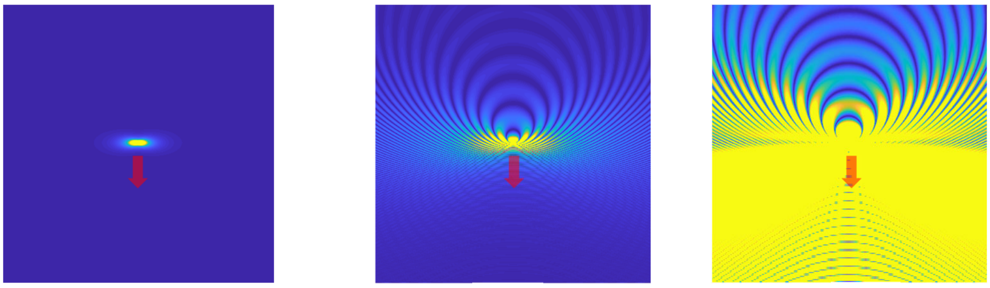
Fig.4. The field patterns of the motionless sources. (a) Static charge. (b) Harmonic charge. (c) Hertzian dipole.

The electric fields of the moving sources are shown in Fig.5 and Fig.6, in which red arrows indicate the moving directions. When the static charge moves, it generates a time-varying field in the space with a spectrum dependent on the moving velocity. Its field patterns contract along the moving direction. For moving sources with time varying charges, the anisotropic property of the wave velocity and the Doppler effect are clearly demonstrated in their fields.



(a) (b) (c)

Fig.5. The field patterns of the moving sources at $\beta = 0.5$. (a) Time invariant charge. (b) Harmonic time varying charge. (c) Hertzian dipole.

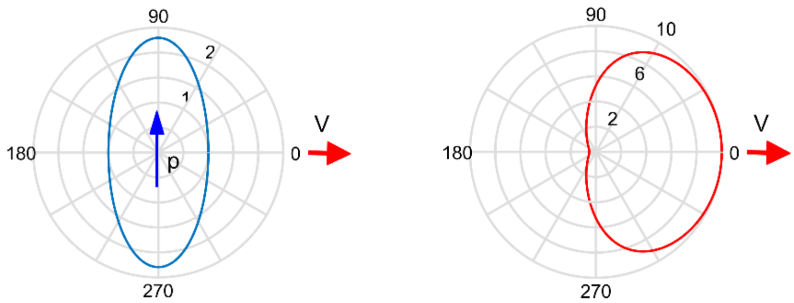


(a) (b) (c)

Fig.6. The field patterns of the moving sources at $\beta = 0.9$. (a) Time invariant charge. (b) Harmonic time varying charge. (c) Hertzian dipole.

The radiation directivity of the Hertzian dipole is obviously different from that of the single moving harmonic charge. When the source velocity is close to c_0, very strong radiations with very high frequencies can be observed in the moving direction of the Hertzian dipole, while radiations with lower frequencies can be observed in the opposite direction of the dipole.

For $\beta = 0.9$, the wave velocity is shown in Fig.7(a). It is exactly c_0, at $\theta = 0, \pi$, and is about $2.3c_0$ at $\theta = \pm\pi/2$. The normalized frequency is plotted in Fig. 7(b). In this case, $\theta_d = 132°$ and $228°$. The red-shift area is much smaller than the blue-shift area.



(a) (b)

Fig. 7. Moving Hertzian dipole for $\beta = 0.9$. (a) Wave velocity. (b) Normalized angular frequency.

The normalized electric field is plotted in Fig.8(a). The radiation pattern is quite different from that of the motionless Hertzian dipole shown in Fig.8(b). Obviously, these results show that both the radiation pattern and the signal frequency of an antenna implemented on a fast-moving platform may suffer sever distortions and need to be carefully compensated.

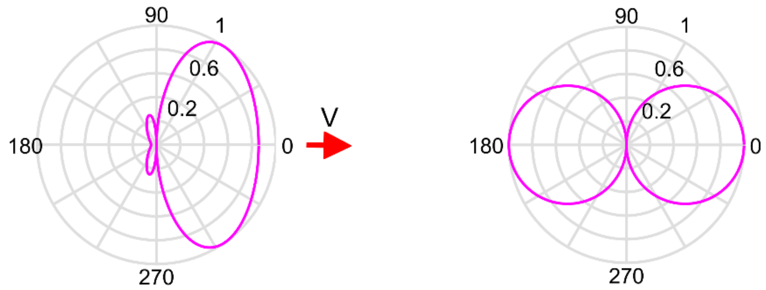


Fig. 8. Radiation patterns of Hertzian dipole. (a) Moving at $\beta = 0.9$. (b) Motionless.

III. **Cherenkov Radiation and Electromagnetic Shock Waves**

As we can see, the fields we obtained are the exact solutions to the Maxwell's equations. Since the Maxwell's theory itself does not put any restrictions on the velocity of the electromagnetic waves, the formulae (7)-(14) are valid for all β , including $\beta \geq 1$. Therefore, we temporarily put aside the velocity limitation and analyze the properties of the causal solutions to the Maxwell's equations when the sources move faster than light in the vacuum. However, we must observe the causality principle that the fields cannot be generated by the sources in the future. Explicitly, it is required that t_1 should be real and $t_1 \leq t$.

Without loss of generality, we choose the position of the source at time t as the origin of the coordinate for the sake of simplicity, that is, we choose $t = 0$. In order to guarantee that t_1 is real for $t = 0$, we derive from (10) that the range of the angle should satisfy

$$\beta^2 \sin^2 \theta \leq 1 \tag{25}$$

To meet the causality condition $t_1 \leq 0$, we check from (10) that

$$\cos\theta \leq 0 \tag{26}$$

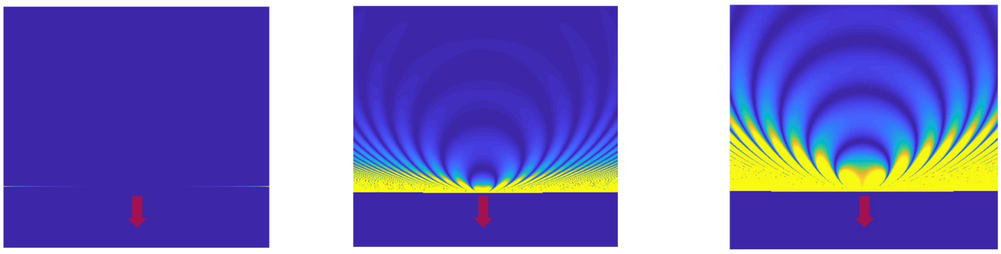
Denote the critical angle as

$$\theta_c = \pi - \sin^{-1}(1/\beta) \tag{27}$$

It is obvious that the electromagnetic fields only exist in the conical region of $\theta_c \leq \theta \leq 2\pi - \theta_c$. In particular, at the edge of the conical zone, $(1 - \hat{\mathbf{n}} \cdot \boldsymbol{\beta}) \to 0$, the amplitude of the fields tends to become infinitely large for point sources. For general sources, the amplitude is finite but may be unusually large. The electromagnetic fields at the edge form a kind of electromagnetic barrier of very large amplitude.

For a moving source with a constant charge, Cherenkov radiation occurs when $\beta \geq 1$ [10]. It is considered that Cherenkov radiation cannot occur in the vacuum since sources cannot move faster than the light in the vacuum. However, it can occur in a medium because sources can move faster than the light in that medium. Cherenkov radiation was discovered in 1934 in the experiment of bombarding the water with γ -rays. Bluish light was clearly observed in the experiment.

When the velocity of the sources with harmonic time varying charges increase to c_0 , then $\beta = 1$, and $\theta_c = \pi/2$, the wave fronts of the electromagnetic waves are in the same plane of the source. The source crosses the electromagnetic barrier [4, 10], similar to that of the sonic barrier that supersonic planes may encounter when they cross the sound speed. The patterns of the electric fields are shown in Fig.9. It is reasonable to predict that the electromagnetic shock waves may be observed in media.

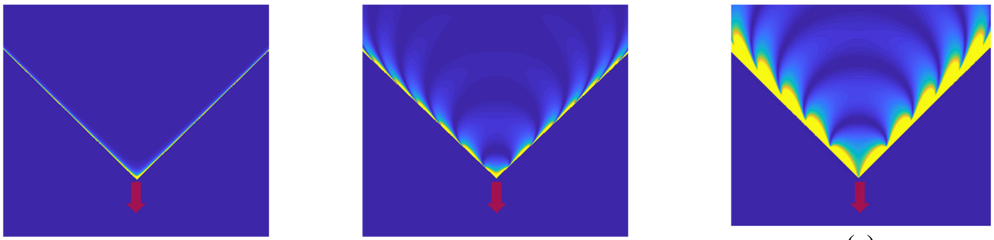


(a) (b) (c)

Fig.9. Electromagnetic barriers generated by moving sources at $\beta = 1$. (a) Time invariant charge. (b) Harmonic time varying charge. (c) Hertzian dipole.

For superluminal sources, $\beta > 1$, $\theta_c > \pi/2$. The wave fronts lag behind the source. The fields are confined in the conical zone and cannot surpass the source. The solutions describe electromagnetic shock waves, just like the sonic shock waves. For point sources, at the edges of the shock waves, the fields tend to be infinitely large and form conical-shaped electromagnetic barriers.

The electric field distributions for the uniformly moving point sources of the three types are shown in Fig.10.



(a) (b) (c)

Fig.10. Electromagnetic fields at $\beta = 1.5$. (a) Cherenkov radiation of a moving time-invariant charge. (b) Shock wave of a moving point source with harmonic charge. (c) Shock wave of a moving Hertzian dipole.

At the electromagnetic barriers, not only the amplitudes of the fields are very large, the frequencies and the velocities of the fields also tend to become very large, as we can check that

$$s(\theta_c) = \frac{\beta^2}{1-\beta^2} + \frac{\beta}{1-\beta^2}\frac{\cos\theta_c}{\sqrt{1-\beta^2\sin^2\theta_c}} \to \infty \tag{28}$$

$$c(\theta_c) = \frac{c_0}{\sqrt{1-\beta^2\sin^2\theta}} \to \infty \tag{29}$$

The superluminal sources radiate high energy rays with velocity much larger than c_0 at the electromagnetic barriers. The observer will experience an electrmagnetic boom when he is swept by the electromagnetic barriers of the superluminal sources. However, for an observer outside the shock wave zone of a superluminal source, the superluminal source is invisible to the observer. We may consider the superluminal source as an electromagnetic blackhole to the observer.

The wave velocity, the normalized frequency, and the amplitude of the fields in the shock wave region in the far region for $\beta = 0.9$ are plotted in Fig.11. As can be seen, the fields in the middle area of the shock wave conical region have the largest red-shift and propagate with velocities slightly larger than c_0.

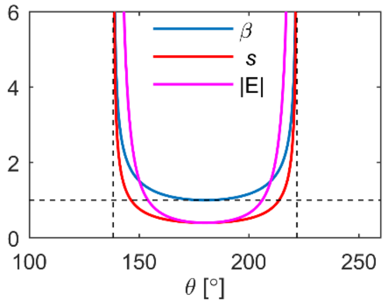


Fig. 11. The velocity, the normalized frequency, and the normalized field amplitude in the shock wave region for $\beta = 0.9$.

IV. Conclusions

It is often stated in textbooks that, according to the rules of the classic physics, the wave velocity and the velocity of the sources are added like vectors; the light velocity in the vacuum is constant in all directions and is independent upon the light sources; the transverse Doppler shift is a pure relativity effect. These are three longstanding misinterpretations concerning about the classic physics. The exact solutions of the electromagnetic fields of uniformly moving point sources in the vacuum reveal that the far fields are spherical waves with velocity depending on the propagation direction. The wave velocity and the moving velocity of the source satisfies $c = c_0 \big/ \sqrt{1-\beta^2 \sin^2\theta}$. This relationship clearly demonstrates that the two velocities cannot be added like vectors under the principles of the classic physics. Moreover, transverse Doppler effect is intrinsically included in the solutions.

Although superluminal sources have not been officially confirmed [11, 12, 13], it does not discourage us from performing mathematical analysis on the causal solutions of the fields of sources moving faster than the light in the vacuum. The electromagnetic shock waves are naturally introduced according to the solutions, together with a reasonable hypothesis that the superluminal sources can be considered as electromagnetic blackholes to observers staying in regions outside the shock wave zone.

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