

An unconditionally stable semi-analytical Lagrangian stream temperature model


An unconditionally stable semi-analytical Lagrangian stream temperature model
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Context

Stream temperature is a critical ecological indicator and plays an important role in river fish habitats and breeding. Typically, stream temperature is simulated using traditional models or discrete Eulerian energy balance models.

Physically-based stream temperature models often only approximate lateral heat diffusion or Eulerian energy approach for simulating the advection, diffusion and energy balance. Such methods are unconditionally stable and time integration increments often have and space step Δx .

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Derivation

Here, we solve for the changing water volume $V_w(t)$ of a parcel of water as it moves through a stream reach. An energy balance on the parcel yields:

$$\frac{dV_w}{dt} + V_w \left(\frac{dV_w}{V_w dt} + \beta \frac{dh_w}{dt} \right) = S(t)$$

where A_w is the reach surface area, V_w is the reach water volume, and the remaining terms are unconditionally defined from the energy balance equation in the following list under leave LSE.

$$\frac{dh_w}{dt} + \beta h_w = S(t)$$

where $S(t)$ includes all of the external forcing terms (radiation, ET, an energy source, groundwater temperature) and energy generated or consumed over a model time step Δt . Then LSE can be solved for a function $h_w(t) = h_w(t_0) e^{-\beta(t-t_0)} + \int_{t_0}^t S(\tau) e^{-\beta(t-\tau)} d\tau$, where $h_w(t_0)$ is the known temperature of the parcel at the reach, yielding the solution of the water temperature as a function of the following variables: $h_w(t_0)$ and mass fraction $S(t)$, expressed discretely for time step Δt .

$$h_w(t) = h_w(t_0) e^{-\beta(t-t_0)} + \sum_{i=1}^n \frac{S_i \Delta t}{\beta} (1 - e^{-\beta(t-t_0 - \Delta t)}) e^{-\beta(t-t_0 - \Delta t)}$$

Assuming that flow dispersion in the channel is defined via a convection approach, the traditional Eulerian energy coupling can be implemented at discrete points with Eulerian input flows through the reach, yielding a final formula for energy output from the reach.

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Results

The semi-analytical approach here was applied within the stream temperature within the 2015 for "Riparian and Pastoral watersheds in southern British Columbia, Canada" a subset of the 1992-2012 model (here, 2012) was used to simulate flow, groundwater temperature, and elevation coefficients were calculated. A final time step of $\Delta t = 0.1$ was used. Figure 1 depicts modelled vs. observed stream temperatures in various sites, as compared to mean air temperatures in the basin.



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Conclusions

A new unconditionally stable approach for simulating stream reach temperatures in distributed hydrological models has been developed, tested, and demonstrated to successfully simulate daily stream temperature fluctuations at the sites. The novel methodology has been incorporated into the River hydrological modelling framework as part of a larger initiative to simulate the complete hydrologic energy balance.

Because of the Lagrangian nature of the solution, a more sophisticated mathematical approach was needed to track individual flow energy fluxes and include the energy balance change. This approach is reported in the following manuscript.

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CONTEXT

Stream temperature is a critical ecosystem indicator and plays an important role in river ice formation and breakup. Typically, stream temperature is simulated either using statistical models or discrete Eulerian energy balance models.

Physically-based stream temperature models often rely upon a grid-based finite difference or finite volume approach for simulating the advection-dominated in-stream energy balance. Such methods are conditionally stable and thus introduce constraints upon time and space step (e.g., Courant and Peclet constraints).

GOALS

Here, a novel semi-analytical technique for simulating advection, latent and sensible heat transfer, heat generation from friction, and hyporheic exchange with groundwater is introduced. It assumes a discrete convolution (i.e., transfer function or unit hydrograph) method is used for routing flows through the reach. The energy balance is applied to discrete parcels of water that travel along the reach exchanging energy with their surroundings; the parcel-based energy balance is solved exactly.

The method is unconditionally stable, runs at the native time step of the hydrological model, and requires no spatial discretization below the reach scale.

The stream temperature model, as implemented within the Raven hydrological modelling framework, is demonstrated at several test catchments in the North American Rocky Mountains.

DERIVATION

Here, we solve for the changing volumetric enthalpy h_w [MJ/m³] of a parcel of water as it moves through a stream reach. An energy balance on the parcel yields:

$$V \frac{dh_w}{dt} = k^* A_s (T_{air} - T) + R^{inc} A_s - R_{LW}^{og} A_s - ET \rho_w \lambda_v A_s + Q_f A_s + q_{mix} A_b c_w \rho_w (T_{GW} - T)$$

change in
enthalpy

convective
(sensible) exchange
with atmosphere

incoming
radiation

outgoing
longwave
radiation

evaporative
cooling

friction

hyporheic
exchange

where A_s is the reach surface area, V is the reach water volume, and the remaining terms are not explicitly defined here for brevity. This expression can be converted to the following first order linear ODE:

$$\frac{dh_w}{dt} + \beta h_w = S(t)$$

where $S(t)$ includes all of the external forcing terms (radiation, ET, air temperature, groundwater temperature), each presumed constant over a model time step, Δt . This ODE can be solved for a duration $t_R = i\Delta t$, where t_R is the discrete travel time of the parcel in the reach, yielding the enthalpy of the outflowing water as a function of the inflowing enthalpy, $h_{in}(t)$, and source history, $S(t)$, expressed discretely for time step n :

$$h_{out}^n = h_{in}^{n-i} e^{-\beta i \Delta t} + \sum_{j=1}^i \frac{S^{n-i+j}}{\beta} (1 - e^{-\beta \Delta t}) e^{-\beta(i-j)\Delta t}$$

Assuming that flow dissipation in the channel is solved via a convolution approach (e.g., via analytical diffusive wave routing), we can track all parcels with distinct travel times through the reach, yielding a final formula for energy output from the reach

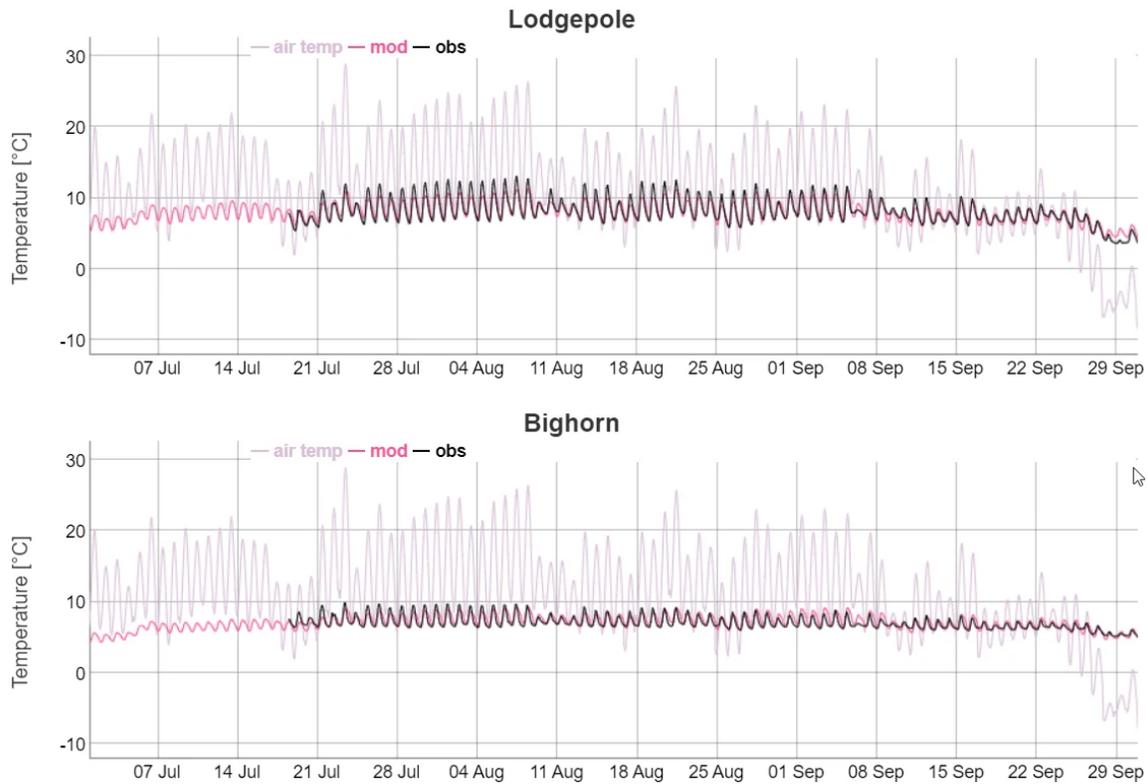
$$(Qh_{out})^n = \sum_{i=0}^{N_U} \left[(Qh_{in})^{n-i} e^{-\beta i \Delta t} + Q^{n-i} \sum_{j=1}^i \frac{S^{n-i+j}}{\beta} (1 - e^{-\beta \Delta t}) e^{-\beta(i-j)\Delta t} \right]$$

where N_U is the number of streamtubes with different travel times. The resultant flow-weighted enthalpy is readily converted to temperature via a simple relation.

Because this solution is analytical, it is unconditionally stable and can easily simulate edge cases that are exceedingly difficult using discrete methods, such as near-infinite convective exchange coefficient. The only approximation errors are associated with the model time step used to represent the inflow time series, the spatial resolution of the reach network, and the assumption of full mixing at the nodes of the stream network.

RESULTS

The semi-analytical approach here was applied within the Raven hydrological modelling framework (<http://raven.uwaterloo.ca>) (Craig et al., 2020) to simulate observed stream temperatures within the 1936 km² Wigwam and Flathead watersheds in southern British Columbia, Canada. A variant of the HBV-EC model (Jost, 2012) was used to simulate flows. Groundwater temperatures and convection coefficients were calibrated. A fixed time step of $\Delta t=2\text{hrs}$ was used. Figure 1 depicts modelled vs. observed stream temperatures at two sites, as compared to mean air temperatures in the basin:



The semi-analytical model clearly adequately performs as applied to this real-world test case. However, the key benefits are not accuracy of the physical model, but rather the fact that this accuracy is reached without the need for discretization of each reach and the associated time and space step constraints. The model is therefore exceedingly fast, and incurs little computational overhead atop the flow problem.

Because the problem is posed in terms of enthalpy, the model also can provide some information about partially frozen conditions when it may be assumed that ice doesn't impede flow.

CONCLUSIONS

A new unconditionally stable algorithm for simulation of stream reach temperatures in distributed hydrological models has been developed, tested, and demonstrated to successfully simulate sub-daily temperature fluctuations at two sites. The novel methodology has been incorporated into the Raven hydrological modelling framework as part of a larger initiative to simulate the complete landscape energy balance.

Because of the Lagrangian nature of the solution, a rather sophisticated mathematical approach was needed to back-calculate the net energy fluxes and evaluate the energy balance closure. This approach is reported in the forthcoming manuscript.

AUTHOR INFORMATION

Prof. James Craig is the Canadian Research Chair in Hydrological Modelling and Analysis at the University of Waterloo in Ontario, Canada.

Prof. Craig's interest include the development and application of new modelling approaches for addressing difficult environmental/water resources problems in both surface water and groundwater systems. He and his research group are invested in making these tools and techniques accessible and useful to consultants, educators, and regulators.

website <http://www.civil.uwaterloo.ca/jrcraig/> (<http://www.civil.uwaterloo.ca/jrcraig/index.html>)

ABSTRACT

Stream temperature is a critical ecosystem indicator and plays an important role in river ice formation and breakup. Typically, stream temperature is simulated either using statistical models or discrete Eulerian energy balance models. These physically-based models often rely upon a grid-based finite difference or finite volume approach for simulating the advection-dominated in-stream energy balance. Such methods are conditionally stable and thus introduce constraints upon time and space step (e.g., Courant and Peclet constraints). They are also not conceptually consistent with commonly applied convolution-based hydrologic routing approaches, which don't require reach discretization. Here, a novel semi-analytical technique for simulating advection, latent and sensible heat transfer, heat generation from friction, and hyporheic exchange with groundwater is introduced. It assumes a discrete convolution (i.e., transfer function or unit hydrograph) method is used for routing flows through the reach. The energy balance is applied to discrete parcels of water that travel along the reach exchanging energy with their surroundings; the parcel-based energy balance is solved exactly. The method is unconditionally stable, runs at the native time step of the hydrological model, and requires no spatial discretization. It can also easily simulate edge cases that are exceedingly difficult using discrete methods, such as a near-infinite convective exchange coefficient. The only approximation errors are associated with the model time step used to represent the inflow time series, the linearization of the Stefan-Boltzmann equation for longwave radiation flux, and the full mixing assumed at nodes of the stream network.

The stream temperature model, as implemented within the Raven hydrological modelling framework, is demonstrated at several test catchments in the North American Rocky Mountains.

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