

# Reconciling Surface Deflections From Simulations of Global Mantle Convection

Conor P. B. O'Malley<sup>1,2</sup>, Gareth G. Roberts<sup>1</sup>, James Panton<sup>3</sup>, Fred D. Richards<sup>1</sup>, J. Huw Davies<sup>3</sup>, Victoria M. Fernandes<sup>1,4</sup>, Sia Ghelichkhan<sup>5</sup>

<sup>1</sup>Department of Earth Science & Engineering, Imperial College London, London SW7 2BP, UK

<sup>2</sup>now at Cathie Group, 2-4 Hanover Square, Newcastle upon Tyne, NE1 3NP, UK

<sup>3</sup>School of Earth & Environmental Sciences, University of Cardiff, Park Place, Cardiff CF10 3AT, UK

<sup>4</sup>now at Section 4.6 Geomorphology, GFZ Potsdam, Telegrafenberg, 14473 Potsdam, Germany

<sup>5</sup>Research School of Earth Sciences, Australian National University, 142 Mills Road, Acton, ACT 0200,

Australia

## Key Points:

- Numeric and analytic predictions of surface deflections from mantle convection simulations are compared.
- Impact of gravitation, excising shallow structure, boundary conditions, and different viscosity and density distributions are quantified.
- Calculated effective contributions to surface deflection emphasize dominance of upper mantle structure.

---

Corresponding author: Conor O'Malley; Gareth Roberts, [c.omalley1@msn.com](mailto:c.omalley1@msn.com);  
[gareth.roberts@imperial.ac.uk](mailto:gareth.roberts@imperial.ac.uk)

**Abstract**

The modern state of the mantle and its evolution on geological timescales is of widespread importance for the Earth sciences. For instance, it is generally agreed that mantle flow is manifest in topographic and drainage network evolution, glacio-eustasy and in the distribution of sediments. There now exists a variety of theoretical approaches to predict histories of mantle convection and its impact on surface deflections. A general goal is to make use of observed deflections to identify Earth-like simulations and constrain the history of mantle convection. Several important insights into the role of radial and non-radial viscosity variations, gravitation, and the importance of shallow structure already exist. Here we seek to bring those insights into a single framework to elucidate the relative importance of popular modelling choices on predicted instantaneous vertical surface deflections. We start by comparing results from numeric and analytic approaches to solving the equations of motion that are ostensibly parameterised to be as-similar-as-possible. Resultant deflections can vary by  $\sim 10\%$ , increasing to  $\sim 25\%$  when viscosity is temperature-dependent. Including self-gravitation and gravitational potential of the deflected surface are relatively small sources of discrepancy. However, spherical harmonic correlations between model predictions decrease dramatically with the excision of shallow structure to increasing depths, and when radial viscosity structure is modified. The results emphasise sensitivity of instantaneous surface deflections to density and viscosity anomalies in the upper mantle. They reinforce the view that a detailed understanding of lithospheric structure is crucial for relating mantle convective history to observations of vertical motions at Earth's surface.

**Plain Language Summary**

Flow of rock within Earth's interior plays a crucial role in evolving the planet. It moves heat and chemicals from deep depths to the surface, for instance. It also moves the lithosphere—the Earth's outer rocky shell—which in turn impacts processes including mountain building, sea-level change, formation of volcanoes, river network evolution, and natural resource distribution. Consequently, we wish to understand the present state, and history, of flowing rock within Earth's interior. Observations exist to address this problem, and mathematics and computing tools can also be used to predict histories of flow and their impact on Earth's surface. We explore how assumptions incorporated into such numeric models can affect calculations of the vertical deflection of Earth's surface. Predictions from different models are compared, with a view to identifying crucial modelling components. Surface sensitivity to deep flow is assessed, demonstrating how surface observations can enlighten flow histories.

**1 Introduction**

Mantle convection plays a crucial role in Earth's evolution (e.g., Hager & Clayton, 1989; Parsons & Daly, 1983; Pekeris, 1935). It is well understood, for instance, that flow in the mantle is fundamental in the transfer of heat and chemicals from the deep Earth to the surface, in driving horizontal and vertical lithospheric motions (thus tectonic processes), and in magnetism via interactions with the core (e.g., Biggin et al., 2012; Davies et al., 2023; Foley & Fischer, 2017; Hoggard et al., 2016; Holdt et al., 2022; Pekeris, 1935). In turn, many processes operating at or close to Earth's surface are impacted, including glacio-eustasy, magmatism, climate, sediment routing, natural resource distribution, drainage network evolution, and development of biodiversity (e.g., Bahadori et al., 2022; Ball et al., 2021; Braun, 2010; Chang & Liu, 2021; Hazzard et al., 2022; O'Malley et al., 2021; Salles et al., 2017; Stanley et al., 2021). Clearly, understanding the physical and chemical evolution of the mantle has broad implications. An important goal is to determine contributions to surface processes from the modern mantle and its history during, say, the last 100 million years.

68 Residual oceanic age-depth measurements, potential field data, seismic tomographic  
69 models and melting histories of young mafic rocks are providing increasingly coherent  
70 observational insights into the modern and recent state of the mantle (e.g., Ball et al.,  
71 2022; Davies et al., 2023; Fichtner et al., 2009, 2013; Fichtner & Villaseñor, 2015; French  
72 & Romanowicz, 2015; Hoggard et al., 2016; Holdt et al., 2022; Kaula, 1963; Lekić & Fis-  
73 cher, 2014; Priestley & McKenzie, 2013; F. D. Richards et al., 2021). Stratigraphic and  
74 geomorphic observations as well as magmatic histories provide clues about the history  
75 of mantle convection on geologic timescales (e.g., Al-Hajri et al., 2009; Czarnota et al.,  
76 2013; Flament et al., 2015; Fernandes et al., 2019; Fernandes & Roberts, 2021; Galloway  
77 et al., 2011; Gunnell & Burke, 2008; Gurnis et al., 2000; Hoggard et al., 2021; Lambeck  
78 et al., 1998; Morris et al., 2020; O’Malley et al., 2021; Stanley et al., 2021). Despite these  
79 advances, observations providing information about the history of mantle convection are  
80 sparse in places, especially within continental interiors and back in time (see e.g., Hog-  
81 gard et al., 2021). Moreover, disentangling contributions from crustal, lithospheric and  
82 sub-lithospheric processes to surface deflections remains challenging and controversial  
83 (see e.g. Hoggard et al., 2021; Wang et al., 2022).

84 Theoretical approaches that retrodict histories of mantle convection can, in prin-  
85 ciple, be used to fill in spatio-temporal gaps in the observational record and disentangle  
86 contributions to surface observables from different geologic processes (e.g., Baumgardner,  
87 1985; Bunge & Baumgardner, 1995; Davies et al., 2013; Flament et al., 2015; Ghe-  
88 lichkhan et al., 2021; Hager et al., 1985; Moucha & Forte, 2011; Steinberger & Antret-  
89 ter, 2006). Increasingly realistic geodynamic simulations incorporating, for instance, plate  
90 motions, gravitation and deflection of gravitational potential fields, complex rheologies,  
91 viscosity laws that can include temperature, pressure, composition, grain size and strain  
92 rate dependence, and assimilation of seismic tomographic information into flow solutions,  
93 result in a diverse array of retrodicted flow histories. Mineralogical phase changes, com-  
94 pressibility, different surface and core-mantle boundary slip conditions (e.g., no-slip, free-  
95 slip), chemical and thermal buoyancy, and plate motion constraints on mantle structure  
96 can also generate diverse predictions of mantle convection and resultant surface deflec-  
97 tions (e.g., Baumgardner, 1985; Bunge et al., 2002, 2003; Corrieu et al., 1995; Cramer  
98 et al., 2012; Dannberg et al., 2017; “Topographic asymmetry of the South Atlantic from  
99 global models of mantle flow and lithospheric stretching”, 2014; Forte, 2007; Ghosh &  
100 Holt, 2012; Glišović & Forte, 2016; Hager & Clayton, 1989; Heister et al., 2017; Liu &  
101 Gurnis, 2008; Panasyuk et al., 1996; Ribe, 2007; Ricard, 2007; Tackley et al., 1993; Zhong  
102 et al., 2008; Zhou et al., 2018). Aside from the fundamental choice of governing equa-  
103 tions and parameterizations underpinning simulations, mathematical and computational  
104 approaches to solve the equations of motion generate different predictions of surface de-  
105 flections. These approaches sit within two broad families: numeric simulations (e.g., AS-  
106 PECT, CitcomS, TERRA; Bangerth et al., 2023; Baumgardner, 1985; Zhong et al., 2000),  
107 and propagator-matrix-based, quasi-analytic techniques, that can be solved in two or three  
108 dimensions, and importantly for our purposes, spherically and spectrally (e.g., Colli et  
109 al., 2016; Hager & O’Connell, 1979; Parsons & Daly, 1983).

110 A challenge then is to establish whether observed surface deflections can be used  
111 to discriminate between theoretical predictions of mantle convection, and, in turn, iden-  
112 tify models that generate realistic and testable retrodictions. In this study we are prin-  
113 cipally concerned with establishing similarities and sensitivities of predicted instanta-  
114 neous vertical surface deflections. We focus on vertical motions for two reasons. First,  
115 inventories of measurements of uplift and subsidence—on timescales of mantle convection—  
116 now exist for most continents and could be compared to predictions from global simu-  
117 lations in future work (e.g. Fernandes & Roberts, 2021, and references therein). Secondly,  
118 many geodynamic simulations incorporate horizontal motions of the lithosphere, which  
119 limits their use as a comparator.

From an observational perspective, it would be useful to establish rules-of-thumb that quantify sensitivity of surface deflections to choices made when predicting them. Many such rules are already well known from analytic and numeric solutions of the equations of motion (e.g., Colli et al., 2016; Hager & O’Connell, 1979; Holdt et al., 2022; Lees et al., 2020; Parsons & Daly, 1983). For instance, a suite of benchmark studies exist that compare predictions from numeric mantle convection simulations with analytic solutions (see e.g., Bauer et al., 2019; Kramer et al., 2021; Zhong et al., 2008, and references therein). Those papers tend to focus on establishing the fidelity of numeric models. In contrast, our goals are to, first, understand how calculated deflections are impacted by the choice of methodology used to solve the equations of motion and, secondly, to establish sensitivities to popular assumptions incorporated into simulations. We want to know the extent to which an improved fit between predictions and observations reflects a more Earth-like density and viscosity structure versus modelling choices. Our thesis is that performing all tests in a self-consistent framework, as we do in this study, provides a straightforward way to collate insights into the sensitivities of predicted surface deflections and to simplify the comparison of predictions from different suites of models.

### 1.1 Our Approach and Paper Structure

We start by exploring the consequences of solving the equations of motion numerically, using the TERRA software, and analytically, using Ghelichkhan et al. (2021)’s propagator matrix algorithms (see Figure 1 & Supporting Information). We make use of the flexibility of numeric approaches by incorporating a variety of assumptions and parameterizations that are not amenable to analytic attack (e.g., temperature-dependent viscosity). All numeric simulations presented in this paper were driven by the plate motion history of Merdith et al. (2021, see Figures 1g–h and S1). The models have a resolution of 60 km at their surface (see Supporting Information for details of model setup and execution). We note that they do not directly assimilate information about the mantle from tomographic models. Ensuring that numeric simulations are accurate and stable means that computational burden is often considerable and hence systematic exploration of parameter space remains challenging. In contrast, analytic approaches can yield calculated surface deflections that are (mathematically) accurate, whilst including features such as radial gravitation, with much less computational cost. Consequently, we make use of propagator matrix techniques to explore parameter space, examine benchmarks, and reproduce results. We establish the sensitivity of solutions to different parameterizations and approaches to solving the equations of motion.

There are at least two important considerations when solving the equations of motion analytically. First, solutions are only known to exist in the spherical harmonic domain for fluid bodies with radial viscosity (i.e., no lateral variability in viscosity). Second, generating solutions in the spherical harmonic domain places practical limits on spatial resolution of solutions. Consider that the number of spherical harmonic coefficients per degree =  $2l+1$ , where  $l$  is degree, so for a given maximum degree  $L$ , there are  $(L+1)^2$  coefficients in total. For instance, when  $L = 50$  there are 2,601 coefficients for each model. Consider also that spatial resolution increases approximately with the reciprocal of  $l$  (see Section 2.2). Incorporating full resolution output from the numeric models (60 km at the surface) would require  $L \approx 880$ , with 776,161 coefficients, which is computationally cumbersome. Furthermore, observational constraints on mantle-related surface deflection are unlikely to be finer than the flexural wavelength of the overlying lithosphere,  $l \approx 50$  (e.g., Holdt et al., 2022). With these limitations in mind, we compared surface deflections predicted using different approaches at the same resolution up to  $l = 50$  (see Supporting Information and Section 2.3).

Most of the tests in this paper compare surface deflections calculated using the entirety of the model domains (i.e., from CMB to the surface). This approach simplifies like-for-like comparisons of model predictions and comparisons to increasingly complex

scenarios. However, amplitudes of calculated deflections will of course not reflect estimated amplitudes of dynamic topography. This approach purposefully avoids isolating passive or plate-driven surface deflections and sub-plate support from numeric simulations or analytic solutions. Since the central focus of this work is merely on quantifying contrasts that arise from choices made when simulating mantle convection, we wish to avoid incorporating additional modifications where possible. In subsequent tests we examine the consequences of simply removing shallow structure, a widely used approach for estimating dynamic support from simulations (see e.g., Flament et al., 2013; Flament, 2018; Wang et al., 2022).

With this framework in place we generate, compare and contrast predicted surface deflections. The first suite of tests are purposefully simple, e.g., incompressible, constant gravitational acceleration (no self-gravitation or radial variation in gravitation) and have radial viscosity independent of temperature. Results are compared to estimates of sub-plate support from oceanic age-depth residuals with a view to quantifying corrections necessary to convert predicted instantaneous surface deflections into estimates of sub-plate support. We then systematically examine the impact of incorporating radial variations in gravitational acceleration, contribution to flow from deflection of the gravitational potential field, removal of shallow density structure, choice of surface and CMB slip conditions, inclusion of temperature-dependent viscosity, and amplification or reduction of viscosity and density anomalies in the upper and lower mantle (Section 4; Tables 1–2). A closed-loop modelling strategy is explored in which predicted surface deflections from these relatively complex models are compared to results from simpler reference models. Finally, a methodology for assessing effective contributions to surface topography from mantle anomalies is presented.

## 2 Numeric and Analytic Calculations of Surface Deflection

The Supporting Information document summarises the formulations of Stokes’ equations that are solved, model parameter values used and the numeric approach to calculating mantle convection using the TERRA finite-element software. Here, we move straight to explaining how those simulations are used to calculate radial stresses,  $\sigma_{rr}$ , thence vertical deflections,  $h$ , at Earth’s surface (Figure 1). A methodology for representing model predictions in the spherical harmonic domain is then described. We then examine analytic solutions obtained using propagator matrix techniques.

### 2.1 Deflections calculated using radial stresses from numeric simulation

Following Parsons and Daly (1983), surface deformation is estimated from numeric simulations of mantle convection by making use of the requirement that normal stress is continuous across the upper boundary of the solid Earth (see also McKenzie, 1977; Ricard, 2015). In other words, radial stresses generated by the solid Earth are required to be balanced by stresses generated by the overlying (oceanic or atmospheric) fluid. There are three contributions to normal stress at this boundary from the mantle: hydrostatic stress that would exist even in the absence of convection, dynamic stress arising from convection, and viscous stress which opposes fluid motion (see Equations 2–6 in Supporting Information). To satisfy the continuity condition, these stresses must be balanced by those generated by the water (or air) column atop this boundary. If the pressure from the overlying column is hydrostatic, the resultant condition is

$$\rho_w g_s h = \rho_m g_s h + \sigma_{rr}, \quad (1)$$

where  $\sigma_{rr}$  incorporates deviatoric viscous stresses generated by mantle convection and dynamic pressure ( $\sigma_{rr} = \tau_{rr} - p$ ), obtained by solving Equation 2 in Supporting Information. In practice, since values for this term are obtained by subtracting radial litho-

219 static stress from the total stress, values of  $\sigma_{rr}$  integrate to zero globally.  $g_s$  is gravita-  
 220 tional acceleration at Earth’s surface,  $\rho_m$  is the mean density for the surficial layer, and  
 221  $\rho_w$  is the density of the overlying fluid (see Table S1). Note that we do not impose ad-  
 222 ditional oceanic plate cooling, e.g., due to hydrothermal circulation at ridges. Cooling  
 223 and subsequent subsidence, as well as passive return flow at ridges, arise naturally from  
 224 solution of the governing equations laid out in Section 2 of Supporting Information.

225 Surface deflection arising in response to predicted convective flow,  $h$ , is approxi-  
 226 mated by rearranging Equation 1,

$$h \approx -\frac{\sigma_{rr}}{(\rho_m - \rho_w)g_s}. \quad (2)$$

227 Deflections are estimated from radial stresses at times of interest (e.g., the present-  
 228 day) by re-running one time-step of the TERRA model. During that time-step, a free-  
 229 slip boundary condition, for which analytic approximations for surface deflection exist,  
 230 is imposed instead of the plate-slip condition prescribed during the main model run rou-  
 231 tine (see Section 2.3; Ricard, 2015). The numeric models themselves apply a quasi-rigid  
 232 condition at the surface, whereby flow is driven by estimates of real plate velocities (from  
 233 Meredith et al., 2021), and so the surface layers behave as a series of rigid, laterally mo-  
 234 bile plates rather than a single rigid shell. We assess the accuracy of modifying bound-  
 235 ary conditions in this way by converting calculated deflections into the spherical harmonic  
 236 domain and comparing them to predictions generated using the analytic propagator ma-  
 237 trix approach. The consistent boundary flux (CBF) method provides an alternative means  
 238 to accurately calculate normal stresses (Zhong et al., 1993). Previous benchmarking with  
 239 TERRA has shown mean errors of a few percent or less for surface deflection predictions  
 240 at low harmonic degrees,  $l \leq 16$  (Davies et al., 2013).

## 241 2.2 Surface Deflections Calculated in the Spherical Harmonic Domain

242 Transforming stress, or surface deflections, calculated using numeric approaches into  
 243 the frequency domain provides straightforward means of comparing results to analytic  
 244 solutions and of quantifying spectral power, i.e., the magnitude of contribution to the  
 245 total signal from different wavelengths. Since the models that we investigate are global  
 246 in scope, we do so using spherical harmonics. The methodology for calculating spheri-  
 247 cal harmonics and the definition of power adopted in this study are included as Sup-  
 248 porting Information. Figure 2 shows an example of surface stresses calculated using the  
 249 TERRA code, their spherical harmonic representation, calculated surface deflections in  
 250 the spherical harmonic domain, and associated statistics and power spectra.

251 Using the total power per degree convention, Hoggard et al. (2016) derived a rule-  
 252 of-thumb for estimating the power spectrum of dynamic topography (see their Support-  
 253 ing Information),  $P_l^{DT}$ , using Kaula (1963)’s approximation for the long-wavelength grav-  
 254 ity field of Earth as a function of  $l$ :

$$P_l^{DT} \approx \left(\frac{GM}{ZR^2}\right)^2 \left(\frac{2}{l} - \frac{3}{l^2} + \frac{1}{l^4}\right), \quad (3)$$

255 where  $G$  is the gravitational constant,  $M = 5.97 \times 10^{24}$  kg is the mass of the Earth,  
 256  $R \approx 6370$  km is Earth’s radius. The value of low-degree admittance,  $Z$ , between grav-  
 257 ity and topography varies as a function of viscosity, as well as the depth and wavelength  
 258 of internal density anomalies (Colli et al., 2016). Hoggard et al. (2016) found that as-  
 259 suming an average value of  $Z = 12$  mGal km<sup>-1</sup> provides a reasonable approximation  
 260 of observed residual topographic trends, thus we make use of that value in the remain-  
 261 der of the paper. Finally, it is useful to note that Jeans (1923) related spherical harmonic  
 262 degree to wavelength  $\lambda$ , which at Earth’s surface can be approximated via  $\lambda \approx 2\pi R/\sqrt{l(l+1)}$ .

263

### 2.3 Surface Deflections Calculated Analytically

264

265

266

267

268

269

270

271

272

273

274

275

276

277

278

The second methodology used to calculate surface deflection in response to mantle convection is the analytic propagator matrix technique (e.g., Craig & McKenzie, 1987; Gantmacher, 1959; Ghelichkhan et al., 2021; Parsons & Daly, 1983; M. A. Richards & Hager, 1984). The approach we take stems from the work of Hager and O’Connell (1981) who used Green’s functions to solve the equations of motion in the spherical harmonic domain. Those solutions are used to generate sensitivity kernels that straightforwardly relate, for example, density or temperature anomalies in the mantle to surface deflections. The kernels are generated in the frequency domain, and constructed such that surface deflection sensitivity to mantle (e.g., density) anomalies is calculated as a function of depth (or radius) and wavenumber. A global spherical harmonic implementation introduced by Hager et al. (1985) has been extended to include compressibility, the effect of warping of the gravitational potential by subsurface density distributions, and radial gravity variations calculated using radial mean density values (Corrieu et al., 1995; Forte & Peltier, 1991; Hager & O’Connell, 1981; M. A. Richards & Hager, 1984; Thoraval et al., 1994).

279

280

In this study, following Ghelichkhan et al. (2021), surface deflection for each spherical harmonic coefficient,  $h_{lm}$ , is calculated in the spectral domain such that

$$h_{lm} = \frac{1}{(\rho_m - \rho_w)} \int_{R_{\text{CMB}}}^R A_l \delta\rho_{lm}(r) \cdot dr. \quad (4)$$

281

282

283

284

Products of the sensitivity kernel,  $A_l$ , and density anomalies,  $\delta\rho_{lm}$ , of spherical harmonic degree,  $l$ , and order,  $m$ , are integrated with respect to radius,  $r$ , between the core-mantle boundary and Earth’s surface radii,  $R_{\text{CMB}}$  and  $R$ , respectively. The sensitivity kernel is given by

$$A_l = - \left( \frac{\eta_0}{Rg_R} \right) \left( u_1 + \frac{\rho_w}{\rho_0} u_3 \right), \quad (5)$$

285

286

where  $u_n(r)$  represents a set of poloidal variables, which are posed for solution of the set of simultaneous equations by matrix manipulation, such that

$$u(r) = [y_1\eta_0 \quad y_2\eta_0\Lambda \quad (y_3 + \bar{\rho}(r)y_5)r \quad y_4r\Lambda \quad y_5r\rho_0\Lambda \quad y_6r^2\rho_0]^T, \quad (6)$$

287

288

289

290

291

292

293

294

295

where  $\Lambda = \sqrt{l(l+1)}$ , and  $y_1$  to  $y_6$  represent the spherical harmonic coefficients of radial velocity  $v_r$ , lateral velocity  $v_{\theta,\phi}$ , radial stress  $\sigma_{rr}$ , lateral stress  $\sigma_{r\theta,\phi}$ , gravitational potential  $V$ , and gravitational potential gradient  $\partial V/\partial r$ , respectively (Hager & Clayton, 1989; Panasyuk et al., 1996).  $\bar{\rho}$  is the layer mean ( $l=0$ ) density. The kernel  $A_l$  includes both  $u_1$  and  $u_3$ , two terms in the matrix solution to the governing equations that affect surface topography by directly exerting stress on the surface boundary ( $u_1$ ), and by changing the gravitational potential at the surface ( $u_3$ ). The functional forms of calculated sensitivity kernels depend on chosen radial viscosity profiles and boundary conditions (e.g., free-slip or no-slip; Parsons & Daly, 1983).

296

### 3 Spatial and Spectral Comparison of Model Predictions

297

298

299

To quantify impacts of modelling assumptions and approaches used to solve the equations of motion we compare calculated surface deflections using the following metrics.

300

### 3.1 Euclidean Comparisons of Amplitudes

301  
302

First, we calculate root-mean-squared difference,  $\chi$ , between predicted surface deflections in the spatial domain,

$$\chi = \sqrt{\frac{1}{N} \sum_{n=1}^N w_{\phi} (h_n^a - h_n^b)^2}, \quad (7)$$

303  
304  
305  
306  
307  
308  
309

where  $h_n^a$  and  $h_n^b$  are predicted surface deflections from the two models being compared.  $N$  = number of points in the  $1 \times 1^\circ$  gridded maps being compared (e.g., Figure 3b;  $N = 65341$ ). The prefactor  $w_{\phi}$  is proportional to  $\cos \phi$ , where  $\phi$  is latitude, and is included to correct biases in cell size with latitude; mean  $w_{\phi} = 1$ . This metric is closely associated with the mean vertical distance ( $L^2$ -norm distance) between predicted and reference surfaces, i.e.,  $\Delta \bar{h} = 1/N \sum_{n=1}^N w_{\phi} |h_n^a - h_n^b|$ . These metrics are sensitive to differences in amplitudes and locations of surface deflections.

310

### 3.2 Spectral Correlation Coefficients

311  
312  
313  
314

Second, we use `pyshtools v4.10` to compute correlation coefficients,  $r_l$ , between predicted surface deflections in the spectral domain (Wieczorek & Meschede, 2018). Correlation coefficients as a function of degree  $l$ , adapted from Forte (2007), are calculated such that

$$r_l = \frac{\sum f_1^* f_2}{\sqrt{\sum f_1^* f_1} \sqrt{\sum f_2^* f_2}}, \quad \text{where} \quad \sum = \sum_{m=-l}^{+l}, \quad (8)$$

315  
316  
317  
318  
319  
320  
321

$f_1$  and  $f_2$  are the spherical harmonic coefficients of the two fields (i.e., surface deflections) being compared, which vary as a function of order,  $m$ , and  $l$ ;  $f = f_l^m$ . \* indicates complex conjugation (see also Becker & Boschi, 2002; O'Connell, 1971). This metric is sensitive to the difference between predicted and reference surface deflection signals in the frequency domain, but not to their amplitudes. To summarize spectral similarity between models concisely, we later refer to the mean value of  $r_l$  over every degree (0–50) as  $\bar{r}_l$ . We refer to the standard deviation of  $r_l$  across degrees as  $s_r$ .

322

### 3.3 Comparing Calculated Power Spectra

323  
324

Lastly, to estimate closeness of fit between power spectra of surface deflections predicted in this study and independent estimates, we calculate

$$\chi_p = \sqrt{\frac{1}{L} \sum_{l=1}^L (\log_{10} P_l - \log_{10} P_l^K)^2} + \sqrt{\frac{1}{L} \sum_{l=1}^L (\log_{10} P_l - \log_{10} P_l^H)^2}, \quad (9)$$

325  
326  
327  
328  
329

where  $L$  = number of spherical harmonic degrees being compared ( $L = 50$ ).  $P_l$  = power of predicted surface deflections generated in this study at degrees  $1 \leq l \leq L$  (Equation 11 in Supporting Information).  $P_l^K$  = power of surface deflections estimated using Kaula's law (Equation 3).  $P_l^H$  = power of residual oceanic age-depth measurements from Holdt et al. (2022).

330

## 4 Model Parameterizations

331  
332

The models examined in this paper are summarised in Table 1. In terms of assumptions tested there are two families of models, those with viscosity independent of tem-

**Table 1. Summary of mantle convection simulations.** Column labeled ‘Method’ indicates surface deflections calculated using either ‘*Numeric*’ (i.e., from surface normal stresses calculated using TERRA) or ‘*Analytic*’ (i.e., propagator matrix) approaches; ‘*Mixed*’ indicates spherical harmonic fitting of surface stresses calculated using numeric code, enabling comparison with solutions to propagator matrix code.  $\eta(r)$  indicates models with radial viscosity (e.g. independent of temperature; Models 1–10).  $\eta(r, T)$  indicates models with temperature-dependent (therefore laterally varying) viscosity (Models 11–20); note that analytic Models 12–20 incorporate radial viscosity calculated using mean radial viscosity from Model 11a. † indicates with respect to Model 12. See Table 1, Section 4 and figures referred to in column 5 for details.

Model	Method	Viscosity	Parameterizations	Figures
1a	Numeric	$\eta(r)$	Full resolution numeric model	1g-h, 2a-c, S1-2
1b	Mixed	$\eta(r)$	Spherical harmonic fit to 1a	2d-i
2	Analytic	$\eta(r)$	Propagator matrix solutions	3, S3
3	Analytic	$\eta(r)$	Radial gravitation, $g(r)$	4a-c, S4
4	Analytic	$\eta(r)$	Gravitational potential terms	4d-e, S5
5	Analytic	$\eta(r)$	Removing upper 50 km of mantle	5a-b, S7a-d
6	Analytic	$\eta(r)$	Removing upper 100 km of mantle	5c-d, S7e-h
7	Analytic	$\eta(r)$	Removing upper 200 km of mantle	5e-f, S7i-l
8	Analytic	$\eta(r)$	No-slip surface, free CMB	6a-d
9	Analytic	$\eta(r)$	Free surface, no-slip CMB	6e-h
10	Analytic	$\eta(r)$	No-slip surface, no-slip CMB	6i-l
11a	Numeric	$\eta(r, T)$	Full resolution numeric model	S8-S10, S12a-c
11b	Mixed	$\eta(r, T)$	Spherical harmonic fit to 11a	7, S8-10, S12d-g
12	Analytic	$\eta(r)$	Mean radial $\eta(r, T)$ from Model 11a	7, S11, S12h-k
13	Analytic	$\eta(r)$	Decrease† radial upper mantle $\eta$	8a-b, S13a-d
14	Analytic	$\eta(r)$	Increase† radial upper mantle $\eta$	8c-d, S13e-h
15	Analytic	$\eta(r)$	Increase† radial upper mantle $\eta$	8e-f, S13i-l
16	Analytic	$\eta(r)$	Constant radial $\eta$	8g-h, S13m-p
17	Analytic	$\eta(r)$	Upper mantle densities $\times 2^\dagger$	8i, S14a-c
18	Analytic	$\eta(r)$	Upper mantle densities $\times 1/2^\dagger$	8j, S14d-f
19	Analytic	$\eta(r)$	Lower mantle densities $\times 2^\dagger$	8k, S14g-i
20	Analytic	$\eta(r)$	Lower mantle densities $\times 1/2^\dagger$	8l, S14j-l

333 perature (Models 1–10), and those with temperature-dependent viscosity (Models 11–  
 334 20). We note that Models 12-20 incorporate mean radial viscosity from the numeric Model  
 335 11a in which viscosity depends on temperature. The two approaches used to solve the  
 336 equations of motion are annotated ‘*Numeric*’ and ‘*Analytic*’ in Table 1, which refers to  
 337 solutions from the TERRA and propagator matrix code, respectively. Viscosities and den-  
 338 sities calculated using TERRA were used as input for the propagator matrix code and  
 339 thus used to generate analytic estimates of surface deflection. Since analytic solutions  
 340 are obtained by spherical harmonic expansion, surface deflections from TERRA were fit  
 341 using spherical harmonics before predicted deflections were compared (annotated ‘*Mixed*’  
 342 in Table 1; Section 2.2). We compare predicted deflections that arise from flow across  
 343 entire model domains, i.e., from the CMB to the surface. Parameterizations of these mod-  
 344 els and resultant surface deflections are discussed in the following sections, with sum-  
 345 mary statistics given in Table 2.

346

347

## 4.1 Models with Viscosity Independent of Temperature

348

### 4.1.1 Reference models

349

350

351

352

353

354

355

356

357

Models 1 and 2 are the simplest explored in this paper. They were designed to be as similar as possible, with a view to quantifying differences and similarities arising solely from the choice of numeric or analytic methodology used to solve equations of motion and to calculate surface deflections. Model 1 was parameterized with the radial viscosity structure shown in Figure 2c. Radial viscosity used in other geodynamic studies are shown alongside for comparison (Ghelichkhan et al., 2021; Mitrovica & Forte, 2004; Steinberger & Calderwood, 2006). Figure 2d shows spherical harmonic expansion of the surface stress field predicted by Model 1 at 0 Ma (cf. Figure 2a). We call this result Model 1b. The original, full-resolution, numerical result is referred to as Model 1a.

358

359

360

361

362

363

364

365

366

Model 2 is the analytic model parameterized to be as similar as possible to Model 1. Its sensitivity kernel, generated assuming water loading ( $\rho_w = 1030 \text{ kg /m}^3$ ), free-slip surface and CMB boundary conditions, and the radial viscosity profile shown in Figure 2c, is shown in Figure 3a. Values of the other parameters used to generate these kernels are stated in Table S1. Similar to many previous studies, the kernel indicates that surface deflections will be especially sensitive (across all degrees incorporated,  $l \leq 50$ ) to density anomalies in the upper mantle (Parsons & Daly, 1983; Hager & Clayton, 1989; Ghelichkhan et al., 2021). Models 1 and 2 are used as points of reference for other more complex models explored in the remainder of this paper.

367

### 4.1.2 Gravitation

368

369

370

371

372

373

We start by incorporating more complex parameterizations of gravitation. The analytic Model 3 was parameterized in the same way as Model 2 with the addition of radial gravitation (following Hager & Clayton, 1989; Panasyuk et al., 1996, see Equation 5). The solid curve in Figure 4b shows the radial gravity function used to calculate surface deflections. It was generated using the density distribution produced by (the numerical) Model 1a (see Figure S1) by calculating

$$g(r) = \frac{4\pi G}{r^2} \left[ \int_{R_{\text{CMB}}}^r \bar{\rho}(r') r'^2 dr' \right] + F_{\text{core}}, \quad (10)$$

374

375

376

377

where  $\bar{\rho}(r)$  is layer mean density and  $F$  is a factor chosen to account for core mass, and such that  $g = 9.8 \text{ m s}^{-2}$  at the surface. This formulation is derived from Gauss's law assuming spherically symmetric density, combined with Newton's law of universal gravitation (Turcotte & Schubert, 2002).

378

379

380

381

382

383

384

385

The analytic Model 4 incorporates stress perturbations induced by deflections of the gravitational potential field. This model assumes  $g = 10 \text{ m s}^{-2}$  everywhere, even within the deflected surface layer, as was the case for Models 1–2. Following Hager and Clayton (1989) and Panasyuk et al. (1996), when solving for surface deflection using propagator matrices, the effect on flow of perturbation of gravitational potential is included via the  $u_3$  term in Equation 6 (see also Ribe, 2007; Ricard, 2015). Sensitivity kernels for Models 3 and 4 are shown in Figure S6. TERRA simulations do not include this component in flow calculations (see Supporting Information).

386

### 4.1.3 Discarding Shallow Structure

387

388

The uppermost few hundred kilometers of geodynamic simulations are often not included in predictions of surface deflections (see e.g. Flament et al., 2013; Flament, 2018;

389 Davies et al., 2019, and references therein). To quantify the impact of discarding shal-  
 390 low structure on our calculations, we examine differences in calculated deflections in the  
 391 spatial and spherical harmonic domains. We present three tests, resulting in Models 5,  
 392 6 and 7, where structure shallower than 50, 100 and 200 km is removed from Model 2.

#### 393 **4.1.4 Changing Boundary Conditions**

394 Up to now, we have only considered instantaneous analytic and numeric solutions  
 395 for surface deflection where both the surface and CMB have free-slip conditions imposed  
 396 (i.e., vertical component of flow velocity  $\mathbf{u}_r = 0$ , horizontal components are allowed to  
 397 freely vary). No gradient/Neumann constraint (e.g., on  $\partial\mathbf{u}/\partial z$ ) is imposed. This con-  
 398 dition is generally deemed appropriate for the surface of the convecting mantle, and CMB,  
 399 since at both boundaries, cohesion within convecting mantle is thought to be much stronger  
 400 than adhesion to the boundary. Analytic solutions for sensitivity kernels for propaga-  
 401 tor matrices also exist for no-slip Dirichlet boundary conditions, where horizontal com-  
 402 ponents of  $\mathbf{u} = 0$ , which may be more appropriate when the Earth's lithosphere is im-  
 403 plicitly included in mantle convection models, as is the case here (Parsons & Daly, 1983;  
 404 Thoraval & Richards, 1997). Therefore, we test the effect of changing the surface bound-  
 405 ary condition to no-slip on predicted surface deflections (Model 8). Although there is lit-  
 406 tle reason to believe the adhesion of the CMB would be strong, for completeness, we test  
 407 scenarios in which no- and free-slip conditions are assumed for the CMB and the sur-  
 408 face, respectively (Model 9), and both have no-slip conditions (Model 10).

#### 409 **4.2 Models with Temperature-Dependent Viscosity**

410 We investigate the impact of including the temperature dependence of viscosity,  
 411  $\eta(r, T)$ , on predicted global mantle flow in numeric models, and on subsequent estimates  
 412 of surface deflection. We do so by first generating the numeric Model 11, which is iden-  
 413 tical to Model 1 in terms of all boundary conditions, initialization, and physical param-  
 414 eters, except for the fact that viscosity depends on temperature in the manner described  
 415 by Equation 7 in Supporting Information.

416 The radial distribution of viscosity, but not its absolute value, plays a crucial role  
 417 in determining sensitivity of instantaneous solutions for surface deflections to density (and  
 418 thermal) anomalies in the mantle (e.g., Parsons & Daly, 1983; Hager, 1984). Consequently,  
 419 to assess sensitivity of surface deflections to arbitrary changes to radial viscosity,  $\eta(r)$ ,  
 420 we performed a suite of analytic tests. Since the analytic approaches require viscosity  
 421 to only vary as a function of radius, we first test the impact of inserting layer-mean vis-  
 422 cosity from the present-day 3D temperature-dependent viscosity structure predicted by  
 423 numeric Model 11 (Figure S8). This parameterization is used to generate (the analytic)  
 424 Model 12. The sensitivity kernel for Model 12 is shown in Figure S11a.

425 We stress that in Models 3–10 analytic instantaneous solutions for surface deflec-  
 426 tion, with adjusted parameters and boundary conditions, were simply compared with Model  
 427 2; no new numeric models were generated using TERRA. In contrast, the additional tests  
 428 examined here correspond to a new TERRA model (Model 11) in which temperature de-  
 429 pendence of viscosity affects mantle flow across the entire run time.

430 The sensitivity of surface deflections to arbitrary modification of upper and lower  
 431 mantle viscosity and densities were then examined. Mean upper and lower mantle (ra-  
 432 dial) temperature-dependent viscosity was decreased or increased by an order of mag-  
 433 nitude from that used to generate Model 12 (see solid black curve in Figure 8). The re-  
 434 sultant impact on calculated surface deflections (Models 13–16) was quantified by compar-  
 435 ison with results generated using reference Model 12 (Figure S11). Figures 8i–l and  
 436 S14 show the amplitudes of density anomalies in the upper and lower mantle that were  
 437 systematically increased or decreased to generate Models 17–20. Similar to the tests shown

438 in Figures 8a–h and S13, densities are amplified relative to Model 12. Radial viscosity  
 439 is constant for each of these tests (black curve in Figure 8a; i.e., same as that used to  
 440 generate Model 12).

## 441 5 Results

### 442 5.1 Models with Viscosity Independent of Temperature

#### 443 5.1.1 Reference Models: Comparing Numeric and Analytic Solutions

444 We first compare solutions generated from numeric Model 1a, with its spherical har-  
 445 monic representation (Model 1b), and analytic Model 2, which were designed to be as  
 446 similar as possible. Figure 1g–h shows calculated densities that arise in Model 1a at 0  
 447 and 100 Ma (see Figure S1 for extended results). The history of plate motions used to  
 448 drive these models is also indicated on these figures. The resultant normal stresses,  $\sigma_{rr}$ ,  
 449 calculated at the surface of Model 1, and associated statistics are shown in Figure 2a–  
 450 b. By convention, positive stresses imply compression and hence downward surface de-  
 451 flection, which could be manifest as lithospheric drawdown, i.e., subsidence. Prominent  
 452 regions of positive stress anomalies in this model include locations atop imposed colli-  
 453 sion zones, where subduction naturally results, e.g., along the Pacific margin of South  
 454 America. Negative stresses imply dilation and hence positive lithospheric support (i.e.,  
 455 surface uplift). Figure 2a shows dilatational stresses beneath Southern Africa, for exam-  
 456 ple, and along mid-oceanic ridges in the Indian and Atlantic Oceans.

457 Surface stresses calculated by fitting radial stresses from Model 1a with a global  
 458 spherical harmonic interpolation up to maximum degree  $l = 50$ , i.e., minimum wave-  
 459 length of  $\approx 800$  km, is shown in Figure 2d–e. The resultant power spectrum in terms  
 460 of total power at each degree is shown in Figure 2f. Aside from the lack of structure at  
 461 degree 0, amplitudes decrease steadily with increasing degree (i.e., decreasing wavelength)  
 462 and can be approximated by red noise. The spherical harmonic representation of deflec-  
 463 tions calculated by converting stress using Equation 2, assuming water loading, are shown  
 464 in Figures 2g and S2. A comparison of calculated power spectra, expected surface de-  
 465 flection from Kaula’s rule (Equation 3), and spectra generated from observed residual  
 466 ocean age-depth measurements is also included in Figures 2 and S2 (Kaula, 1963; Hog-  
 467 gard et al., 2016; Holdt et al., 2022). For completeness, surface deflections calculated as-  
 468 suming air loading are shown in Figure S2f–j.

469 Surface deflections predicted by Model 2 and its associated sensitivity kernel are  
 470 shown in Figure 3a–b. An expanded set of results including sensitivity kernels for wa-  
 471 ter and air loading, and histograms of deflection and associate power spectra are included  
 472 in Figure S3.

473 Deflections predicted from these numeric and analytic models are visually similar  
 474 (cf. Figures 2g & 3b). Absolute differences in amplitudes are greatest close to subduc-  
 475 tion zones (e.g., in South America and Asia; Figure 3c). The differences are broadly nor-  
 476 mally distributed and centred on 0 (Figure 3d). The spherical harmonic correlation be-  
 477 tween these models is high (close to 1 for all degrees; cf. Forte, 2007, Figure 3e). The  
 478 ratios between surface deflection values in these predictions indicate that analytic solu-  
 479 tions tend to be damped compared to numeric solutions. This result is emphasized by  
 480 the histogram shown in Figure 3g. Multiplying amplitudes of deflections from the prop-  
 481 agator matrix solutions by a factor of 1.1 brings them in-line with the numeric solutions.  
 482 These results indicate that the propagator matrix approach dampens solutions by  $\approx 10\%$ .  
 483 We note that power spectral slopes between Model 1b and 2 are similar (cf. Figures 2i  
 484 and S3d). These and all other results are discussed in Section 6.

### 485 **5.1.2 Incorporating Self-Gravitation and Gravitational Potential of the** 486 **Deflected Surface**

487 Differences in deflections predicted by Model 2, which assumes constant  $g = 10$   
 488  $\text{m s}^{-2}$  across all radii, and Model 3, which incorporates self-consistently calculated  
 489 radial gravitation, are shown in Figure 4a and 4c. Deviations in predicted instantaneous  
 490 deflections are  $\sim 10\%$  of maximum amplitudes predicted by Model 2 (see Table 2). Note  
 491 that, for the viscosity structure used in these models, changing  $g$  in this way impacts sen-  
 492 sitivity kernels most at low degrees  $l \lesssim 10$  in the mid-mantle (see Figures 2c, 3a and  
 493 S6).

494 We suggest that the broadly hemispherical differences in calculated deflections arise  
 495 from three contributing factors. First, deviations in  $g$  between the two models are great-  
 496 est in the mid-mantle, which, secondly, results in subtly different sensitivity kernels (see  
 497 Figure S6). In general, surface deflection sensitivity to mid-mantle structure is highest  
 498 at low degrees ( $l = 1-3$ ), and is almost negligible at higher degrees compared to contri-  
 499 butions from the near-surface. Thus it seems likely that differences between these ker-  
 500 nels would be manifest in low-degree (e.g. hemispherical) differences in surface deflec-  
 501 tions. Third, in the final timestep, which is used to calculate deflections, there occurs  
 502 a greater proportion of negative and positive deflections in the northern and southern  
 503 hemispheres, respectively.

504 We note that incorporating radially varying gravitation into numeric simulations,  
 505 which is not trivial, might materially impact calculated mantle flow fields and hence pre-  
 506 dictions of surface deflections. Our results are consistent with the rule of thumb outlined  
 507 in Section 7.02.2.5.2 of Ricard (2015), whereby magnitudes of differences incurred by in-  
 508 clusion of full self-gravitation, i.e.,  $g(\theta, \phi, r)$ , decay as a function of spherical harmonic  
 509 degree, proportionately to  $3/(2l + 1)$ .

510 As expected, induced differences in surface displacement predictions are much lower  
 511 in magnitude when gravitational potential of the deflected surface is included compared  
 512 to when radial gravitation is incorporated (cf. Figure 4a and 4d). We note that they are  
 513 of the same order of magnitude as the geoid height anomalies predicted by these mod-  
 514 els. The mean Euclidean distance between the two predicted surfaces in Models 2 and  
 515 4 is only  $\sim 110$  m (compared to maximum amplitudes  $> 8$  km), and the spherical har-  
 516 monic correlation is very high across all degrees (see Table 2). Similar to the results for  
 517 Model 3, the differences are concentrated at low spherical harmonic degree  $l$ . We stress  
 518 that this test investigates the effect of the  $u_3$  term on instantaneous solution for surface  
 519 deflection (Equation 5). It cannot be ruled out from this test that inclusion of the ef-  
 520 fect of gravitational potential field perturbation would result in greater differences across  
 521 the entire model run time of a numeric model, although it is unlikely (Zhong et al., 2008).

### 522 **5.1.3 Excising Shallow Structure**

523 As expected from examination of surface deflection sensitivity kernels (e.g., Fig-  
 524 ure 3a), removal of shallow structure (Models 4–6) results in significantly reduced am-  
 525 plitudes of surface deflections (Figure 5). Doing so results in amplitudes of power spec-  
 526 tra that more closely align with independent estimates (Figure 5b, f, j). The reduction  
 527 in differences is largely due to the fact that the reference Model 2 has surface deflections  
 528 that are much larger than independent estimates of dynamic topographic power across  
 529 all degrees. We note that power spectral slopes for predicted surface deflection from Model  
 530 2 are close to those generated from Kaula’s rule, and observed oceanic residual depths  
 531 (Figures 2i, S2 and S3). Removing shallow structure steepens spectral slopes (i.e., re-  
 532 duces power at high degrees) beyond those expected from theoretical considerations (Kaula’s  
 533 rule) or observed from oceanic residual depths, akin to results from other work that ex-  
 534 cised shallow structure (e.g., Flament et al., 2013; Moucha et al., 2008; Steinberger, 2007).  
 535 This result is emphasized by calculated spectral coherence,  $r_l$ , between deflections with

536 and without shallow structure removed (cf. Figure 5b, d, f). While degree 1 and 2 struc-  
 537 ture remains coherent, coherence across degrees  $\gtrsim 20$  decreases from  $\sim 0.9$  to as low  
 538 as 0.5, which are the largest discrepancies between any models examined in this study  
 539 (Figure S7).

#### 540 **5.1.4 Adjusting Boundary Conditions**

541 Figure 6a, e and i show predicted sensitivity kernels as a function of depth and de-  
 542 gree, for no-slip/free-slip, free-slip/no-slip and no-slip/no-slip boundaries respectively,  
 543 where the first condition is the surface slip condition, and the second the CMB slip con-  
 544 dition. Differences to the sensitivity kernel for Model 2 (free-slip/free-slip; Figure 3a) are  
 545 shown in panels b, f and j. Those panels, and panels c, g and k, demonstrate that when  
 546 the surface boundary condition is ‘no-slip’, there is decreased sensitivity to short wave-  
 547 length shallow structure, and increased sensitivity to long-wavelength (low degree) struc-  
 548 ture across all depths. Figure 6d, h and l reveal that induced misfit in the spatial do-  
 549 main is impacted to a greater degree than in tests of gravitation (Models 3 & 4), but not  
 550 necessarily more severely than for removal of, say the upper 200 km of density structure  
 551 from surface deflection calculations. Spectral correlation with Model 2 is most severely  
 552 impacted when both surface and CMB boundaries are no-slip, which is probably phys-  
 553 ically unrealistic (Model 7; see Table 2; Section 4.1.4).

### 554 **5.2 Adjusting Viscosity and Density Anomaly Amplitudes**

#### 555 **5.2.1 Temperature-Dependent Viscosity**

556 Slices through the three-dimensional viscosity and density structure of Model 11,  
 557 which incorporated temperature-dependent viscosity, are shown in Figure 1a, c and e.  
 558 Density anomalies in the models parameterized with temperature-dependent viscosity  
 559 are more localised (‘sharper’) than in the models with viscosity independent of temper-  
 560 ature (e.g., Model 1; see Figures 7 & S8–S10). This result is unsurprising since tempera-  
 561 ture-dependent viscosity provides stronger mechanical contrasts between cooler subducting  
 562 regions and surrounding asthenosphere (cf. Figure 1g–h & S9; Zhong et al., 2000). Nonethe-  
 563 less, power spectra of calculated surface deflections are very similar (cf. Figure S10j &  
 564 2i). This result emphasises the relatively small impact incorporating temperature-dependent  
 565 viscosity has on surface deflections compared to, say, excising shallow structure.

566 Calculated power spectra from the analytic Model 12, which was generated using  
 567 layer-mean (radial) viscosity from Model 11a, reinforces this view (cf. Figure S3a–d &  
 568 Figure S11a–d). Similar to the results obtained for models without temperature-dependent  
 569 viscosity (Figure 3), deflections calculated analytically are damped relative to numeric  
 570 solutions (see Figure 7f). The best fit amplification factor to align propagator matrix  
 571 and numeric solutions is 1.24 (24%). The effect is greater than that seen when compar-  
 572 ing Models 1b and 2 because of increased short wavelength structure in Model 11 (see  
 573 also Zhong et al., 2000). Nonetheless, spherical harmonic correlations,  $r_l$ , are  $> 0.75$  for  
 574 all degrees examined ( $l \leq 50$ ), and  $> 0.85$  for most degrees. Cell-to-cell differences in  
 575 surface deflections are broadly normally distributed and centred on zero (Figure 7d).

576 A summary of comparisons between models with and without temperature-dependent  
 577 viscosity is shown in Figure S12. Discrepancies in cell-to-cell deflections are broadly nor-  
 578 mally distributed and centred on zero, clustering along the 1:1 relationship with max-  
 579 imum  $\chi = 1.51$  for full resolution (numeric) models (Figure S12b–c; see Table 2). Un-  
 580 surprisingly, spherical harmonic fits and analytic results have tighter normal distribu-  
 581 tions and lower  $\chi$  values. Correlation coefficients are  $> 0.75$  for nearly all degrees in all  
 582 comparisons.

### 5.2.2 Sensitivity to Upper/Lower Mantle Viscosity and Density Anomalies

In order to explore the consequences of modified viscosity and density on calculated deflections we also systematically increased and decreased contrasts in the upper and lower mantle (Models 13–20) with respect to Model 12. Figure 8 summarises the results, which include decreasing upper mantle viscosity by an order of magnitude and show the impact of using increasingly simple radial viscosity in analytic calculations. Calculated sensitivity kernels for the adjusted viscosity profiles demonstrate that decreasing upper mantle viscosity reduces sensitivity of surface deflections to long-wavelength density structure, especially in the lower mantle (Figures S13 & 8d, f, h). Models 13–16 have broad similarities with the reference Model 12 even when  $\eta(r)$  is drastically varied: average  $\chi$  misfit = 0.17–0.38 km, and  $r_l > 0.97$  across all degrees. These results emphasize that the viscosity adjustments we examined exert a relatively minor control on the amplitudes of instantaneous surface deflection (Table 2, see, e.g., Ghosh et al., 2010; Moucha et al., 2007; Lu et al., 2020).

In contrast, increasing (Model 17) or decreasing (Model 18) upper mantle densities is much more impactful on amplitudes of calculated surface deflections (see Figure S11, and S14). For instance, increasing or decreasing upper mantle densities by a factor of two (relative to Model 12) results in  $\chi$  values of 0.97 and 0.48, respectively. Modifying lower mantle densities has a much smaller impact on amplitudes of deflection (Models 18 & 19). Spherical harmonic correlation between models is approximately as good as for the radial viscosity tests (Models 13–16), which is to be expected since we do not vary locations of density anomalies here, only their amplitudes, and  $r_l$  is insensitive to amplitudes of the two results being compared. Significant is the fact that mean vertical differences between Models 17–20 and 12 (i.e.,  $\chi$  and  $\Delta\bar{h}$ ) are higher than those calculated for Models 13–16 (in which viscosity is varied; see Table 2).

These results emphasize the relative sensitivity of instantaneous surface deflections to upper mantle density anomalies compared to, say, radial viscosity or lower mantle densities. Even quite large uncertainties in lower mantle density anomalies have relatively little impact on instantaneous surface deflections. These results reinforce the view that accounting for shallow (e.g., lithospheric and asthenospheric) densities is crucial when estimating surface deflection, and dynamic topography, from mantle convection simulations (e.g., Colli et al., 2016; Flament et al., 2013; Holdt et al., 2022; Wang et al., 2022).

616

## 6 Discussion

### 6.1 Similarities of Analytic and Numeric Solutions

In this paper we compare numeric and analytic predictions of instantaneous surface deflections generated by mantle convection simulations. First, we simply compared predictions from numeric and analytic approaches parameterised to be as similar as possible. In this test, the models were purposefully simple: viscosity is radial, models are incompressible, and they do not include self-gravitation, or radial variation in  $g$ . Numeric solutions were transformed into the frequency (spherical harmonic) domain so that they could be compared with analytic solutions, and so that power spectra could be directly compared at appropriate scales. The results show that, for as-similar-as-possible parameterizations, amplitudes of analytic solutions are  $\approx 10\%$  lower than numeric solutions (Figure 3). If the numeric model incorporates temperature-dependent viscosity, this discrepancy increases to 25% (Figure 7). We interpret these results in two ways. First, once armed with viscosity and density fields, numeric and analytic approaches broadly yield

630

**Table 2. Inter-model comparison of predicted surface deflections.** Models being compared are summarised in Table 1. Metrics: root-mean-squared difference ( $\chi$ , km), mean Euclidean ( $L^2$ -norm) difference in predicted deflection ( $\Delta\bar{h}$ , km), and mean spherical harmonic correlation between models ( $\bar{r}_l$ ). Standard deviation of  $r_l$  distribution across degrees ( $s_r$ ) is also stated: note that  $r_l \leq 1$ . All spherical harmonic representations of output from numeric code and generated by the propagator matrix code are expanded up to maximum degree,  $l = 50$ . See body text, figures referred to in column 6, and Table 1 for details.

Models	$\chi$	$\Delta\bar{h}$	$\bar{r}_l$	$s_r$	Figures
1b & 2	0.95	0.69	0.97	0.02	3
2 & 3	0.57	0.47	0.99	$4 \times 10^{-4}$	4
2 & 4	0.13	0.11	0.99	$2 \times 10^{-5}$	4
2 & 5	0.67	0.48	0.93	0.04	5a-b
2 & 6	1.03	0.74	0.87	0.06	5c-d
2 & 7	1.57	1.12	0.63	0.15	5e-f
2 & 8	1.26	1.04	0.99	$1 \times 10^{-3}$	6a-d
2 & 9	1.09	0.97	0.99	0.04	6e-h
2 & 10	1.00	0.74	0.96	0.28	6i-l
1a & 11a	1.51	1.04	—	—	S12a-c
1b & 11b	1.44	0.98	0.79	0.26	S12d-g
11b & 12	1.20	0.80	0.95	0.02	7
2 & 12	0.92	0.64	0.85	0.27	S12h-k
12 & 13	0.31	0.20	0.99	$9 \times 10^{-3}$	8a-b, S13a-d
12 & 14	0.17	0.10	0.99	$3 \times 10^{-3}$	8c-d, S13e-h
12 & 15	0.32	0.20	0.98	0.01	8e-f, S13i-l
12 & 16	0.38	0.23	0.98	0.01	8g-h, S13m-p
12 & 17	0.97	0.64	0.98	$7 \times 10^{-3}$	8i, S14a-c
12 & 18	0.48	0.32	0.98	$6 \times 10^{-3}$	8j, S14d-f
12 & 19	0.43	0.29	0.99	$3 \times 10^{-3}$	8k, S14g-i
12 & 20	0.22	0.14	0.99	$1 \times 10^{-3}$	8l, S14j-l

631 similar estimates of surface deflections. Second, the relatively damped analytic solutions  
632 are a consequence of smoothing steps in the propagator matrix approach.

633 The smoothness of analytic solutions, and subsequent damping of topographic amplitudes,  
634 is perhaps surprising, given the fact that they are being compared with numeric  
635 models expanded into the spherical harmonic domain to the same maximum degree,  $l =$   
636  $50$ . However, the surface stresses used to generate Model 1a have full horizontal reso-  
637 lution ( $\approx 45$  km) across depths, and *only* the surface layer is smoothed by spherical har-  
638 monic fitting, to generate Model 1b. Therefore, Model 1b inherently contains some con-  
639 tribution from degrees  $\geq 50$ , in the sense that finer-resolution density structure at depth  
640 could affect longer-wavelength flow nearer the surface. In contrast, to generate the an-  
641 alytic solution (Model 2), the density structure of each layer of the model is smoothed,  
642 by expansion to maximum  $l = 50$ , *before* integration of their contributions to surface  
643 deflection. The analytic solution would provide a better match to stress estimates from  
644 numeric models if such estimates were calculated using density structure smoothed to  
645 the same maximum  $l$  across all depths, which is currently challenging (see Section 1.1).

646 Nonetheless, the similarity of results indicates that the relatively low-cost propa-  
647 gator matrix approach can be used to explore consequences of including additional model  
648 complexity. A systematic sweep of parameters, including radial gravitation (Figure 4a-  
649 c) and gravitational potential field effects (Figure 4d-e) indicates that their effects on sur-  
650 face deflection are relatively modest. A useful rule of thumb is that self-gravitation per-  
651 turbs instantaneous surface deflections by  $O(1-10)\%$  when compared to models with con-  
652 stant gravitational acceleration, and even less difference is observed at high degree (e.g.,  
653 Ricard, 2015, their Section 7.02.2.5.2). Incorporating the effect of deflections of gravi-  
654 tational potential field on flow has a modest impact on amplitudes of surface deflections  
655 at degrees 1–2, but overall it contributes even less than radial variation in  $g$  to surface  
656 deflections across the scales of interest. We note that incorporating full 3-D self-gravitation  
657 into numeric simulations is currently challenging. Nonetheless, establishing its impact  
658 on the flow field over time, and resultant impact on surface deflections, may be impor-  
659 tant future work.

## 660 **6.2 Importance of Viscosity and Shallow Density Anomalies for Isolating** 661 **Dynamic Support**

662 Figure 8 demonstrates that even quite large (order of magnitude) variations in vis-  
663 cosity do not have much impact on instantaneous surface deflections when compared to,  
664 say, modified upper mantle density anomalies, which appears to agree with the results  
665 of Davies et al. (2019) (see also Flament, 2019; Steinberger et al., 2019). Assuming no-  
666 slip boundary conditions at Earth’s surface may be appropriate for driving near-surface  
667 (lithospheric) flow throughout the main model run time, but it less clear whether no-  
668 or free-slip boundary conditions are most appropriate for calculating instantaneous dy-  
669 namic topography (see, e.g., Forte & Peltier, 1994; Thoraval & Richards, 1997). Nonethe-  
670 less, all calculated sensitivity kernels in this study indicate that shallow density anoma-  
671 lies make significant contributions to surface topography regardless of viscosity profile  
672 or boundary conditions chosen (e.g., Figure 3a; see also Colli et al., 2016; Parsons & Daly,  
673 1983).

674 It is well known that disentangling contributions to Earth’s surface topography from  
675 mantle convection, lithospheric isostasy and flexure is important but not trivial (see, e.g.,  
676 Davies et al., 2019; Cao & Liu, 2021; Fernandes & Roberts, 2021; Hoggard et al., 2021;  
677 Steinberger, 2016; Stephenson et al., 2021; Zhou & Liu, 2019; Wang et al., 2022). Pre-  
678 vious studies simulating mantle convection have addressed this issue by discarding den-  
679 sity anomalies in radial shells shallower than specified depths before calculating surface  
680 stresses (e.g., Spasojevic & Gurnis, 2012; Flament et al., 2013; Molnar et al., 2015). Sim-  
681 ilarly, analytic approaches have isolated contributions from the convecting mantle by only

682 incorporating information from deep shells (e.g., Colli et al., 2018). This method has the  
 683 advantage of effectively removing the effect of lithospheric cooling through time from sur-  
 684 face deflection estimates. It also avoids the need to incorporate, say, realistic crustal or  
 685 depleted lithospheric layers within the viscous flow parameterization. However, uncer-  
 686 tain oceanic and continental lithospheric thicknesses mean that choosing appropriate cut-  
 687 off depths is not simple.

688 Out of all the tests performed in this study, removing shallow structure resulted  
 689 in the largest impact on predicted surface deflections. It modifies amplitudes of deflec-  
 690 tions, locations of uplift and subsidence, and degrees over which they are resolved, and  
 691 hence modifies power spectral scalings (Table 2, Figure 5). Making quantitative predic-  
 692 tions of dynamic topography from such an approach is fraught for at least two reasons.  
 693 First, if the chosen depth is shallower than the lithosphere-asthenosphere boundary in  
 694 places, plate and sub-plate contributions to topography will be entangled. Second, dis-  
 695 carding deeper layers to ensure that all plate contribution is definitely avoided means  
 696 that some contributions from asthenospheric flow will be missed. Thus, such a step is  
 697 unlikely to be desirable if mantle flow models are to be used to understand, say, litho-  
 698 spheric vertical motions, or vice versa (see e.g., Figure 3a; Davies et al., 2019; Hoggard  
 699 et al., 2016). Given the calculated sensitivity kernels, excising layers in the upper few  
 700 100 km is likely to result in predictions of surface deflections that are especially inaccu-  
 701 rate at short wavelengths, i.e., high spherical harmonic degree. An alternative approach,  
 702 which may be fruitful future work, is removal of structure based on appropriately cali-  
 703 brated plate models, or globally averaged age-dependent density trends (e.g., F. D. Richards  
 704 et al., 2020, 2023).

### 705 6.3 Assessing ‘Effective’ Contributions to Instantaneous Deflections

706 The results emphasise the importance of considering sensitivities of instantaneous  
 707 vertical surface deflections to the location and scale of flow in the mantle. Taking inspi-  
 708 ration from Hager and O’Connell (1981) and Parsons and Daly (1983), we calculate the  
 709 net contributions from density anomaly structure to deflections, as a function of radius,  
 710 latitude and longitude across all spherical harmonic degrees considered (i.e.,  $l = 1$  to  
 711 50). Contributions to deflections from densities at particular radii  $r$ , across all spheri-  
 712 cal harmonic degrees and orders, for each latitude and longitude,  $(\theta, \phi)$ , are calculated  
 713 such that

$$h_e(\theta, \phi, r) = \sum_{l=1}^L \sum_{m=-l}^{m=l} [Y_{lm}(\theta, \phi) \cdot \delta\rho_{lm}(r) \cdot A_l(r) \cdot \Delta r], \quad (11)$$

714 where  $\Delta r$  is the radial width of the spherical shell included in the calculation ( $\approx 45$  km  
 715 for all shells from the surface to the CMB; see Supporting Information) and  $Y_{lm}$  are spheri-  
 716 cal harmonic coefficients. Mean density anomalies,  $\delta\rho_{lm}$ , within each shell at each lati-  
 717 tude and longitude, and sensitivities  $A_l$  at the top of each shell are used to calculate  $h_e$   
 718 (see Section 2.3). Contributions at specific locations to surface deflections as a function  
 719 of latitude and longitude, and spherical shell depth are shown in Figure 9 for Model 12,  
 720 for  $1 \leq l \leq 50$ . Results for lower maximum degrees are shown in Supporting Infor-  
 721 mation. Panels a-d show slices through effective density in the upper (at 45, 135, 360  
 722 km) and lower mantle (1445 km). A  $180^\circ$  cross-section showing effective densities from  
 723 the core-mantle-boundary to the surface beneath the Pacific to the Indian Ocean encom-  
 724 passing South America and southern Africa (the same transect as shown in Figure 1)  
 725 is shown in panel e. This figure again emphasizes the contribution of density anomalies  
 726 in the upper mantle to surface deflections, and the risks associated with discarding shal-  
 727 low structure when predicting dynamic topography.

728

## 6.4 Summary and Future Work

729

730

731

732

733

734

735

736

Encouragingly, although predicted instantaneous surface deflections are sensitive to different parameterizations, broadly coherent patterns emerge in all models tested. Moreover, calculated deflections are relatively insensitive to the methodologies used to solve the equations of motion. For instance, incorporation of gravitational potential of deflected surfaces, self-gravitation and viscosity anomalies each generate subtly different surface deflections. Choosing to solve the equations of motion analytically or numerically changes calculated deflections by  $< 25\%$ , even when temperature-dependent viscosity is included throughout the duration of a simulation.

737

738

739

740

741

742

743

744

745

In contrast, removal of shallow structure produces much larger discrepancies between predicted deflections. For instance, surface deflections calculated using the entire modelling domain (core-mantle boundary to surface) have spectral slopes consistent with those of oceanic age-depth residuals, however amplitudes are over-predicted by 1–2 orders of magnitude. In contrast, by not including the shallowest 200 km, calculated power spectra more closely match observed amplitudes, especially at spherical harmonic degrees  $> 10$  (Figure 5). However, the spectral slopes of predicted deflections are redder than for the oceanic residuals, which implies that a different approach to removing the contribution of lithospheric structure is required.

746

747

748

749

750

751

752

753

An obvious necessary next step for accurately predicting modern dynamic support from mantle convection simulations is to incorporate accurate information about lithospheric structure from, for instance, tomographic models (e.g., Priestley & McKenzie, 2013; F. D. Richards et al., 2020). Another useful next step is to establish sensitivity of surface deflections to time-dependent parameters that impact predicted flow histories, including plate motions. The results in this paper indicate that comparing predicted and observed surface deflections, combined with knowledge of lithospheric structure, could be used to identify optimal models.

754

755

756

757

758

759

760

761

762

763

764

Finally, the body of geologic and geomorphologic observations that could be used to test predicted histories of surface deflections from mantle convection simulations has grown substantially in the last decade (e.g., uplift and subsidence histories; Section 1; see, e.g., Hoggard et al., 2021, and references therein). A suite of other geologic and geophysical observables are also predicted by, or can be derived from, such simulations (e.g., mantle temperatures, heat flux, geoid, seismic velocities, true polar wander). Using them alongside histories of surface deflections to identify optimal simulations is an obvious avenue for future work (e.g., Ball et al., 2021; Lau et al., 2017; Panton et al., 2023; F. D. Richards et al., 2023). Using such data and the methodologies explored in this paper may be a fruitful way of identifying optimal simulations from the considerable inventory that already exists.

765

## 7 Conclusions

766

767

768

769

770

771

772

773

774

775

776

777

This study is concerned with quantifying sensitivities and uncertainties of Earth’s surface deflections that arise in simulations of mantle convection. Calculated sensitivities of instantaneous deflection of Earth’s surface to mantle density structure emphasise the importance of accurate mapping of the upper mantle. Surface deflections are somewhat sensitive to the distribution of viscosity throughout the mantle, but especially to the locations and scales of density anomalies in the upper mantle. The largest discrepancies between predicted deflections seen in this study are generated when upper mantle structure is excised or altered. Doing so changes both the amplitude and distribution of calculated deflections, modifying their power spectral slopes. These results emphasise the importance of incorporating accurate models of lithospheric structure into calculation of sub-plate support of topography, and also the need to accurately determine plate contributions to topography. In contrast, the choice of methodology to es-

778 timate surface deflections—analytic or numeric—or boundary conditions are relatively  
 779 small sources of uncertainty. Similarly, assumed gravitational profiles and temperature  
 780 dependence of viscosity are relatively minor contributors to uncertainty given reason-  
 781 able, Earth-like, parameterizations. Nonetheless, these parameterizations may impact  
 782 surface deflections through their role in determining how upper mantle flow evolves through  
 783 geologic time. A fruitful next step could be to use the approaches developed in this pa-  
 784 per, in combination with careful isolation of plate cooling signatures from surface deflec-  
 785 tion predictions, to test mantle convection simulations using the existing and growing  
 786 body of geologic, geomorphologic and geophysical observations.

## 787 Open Research Section

788 TERRA models are archived [here]. The propagator matrix code is archived [here].  
 789 Parameterization files are archived [here]. [TO ED: this section will be completed upon  
 790 final submission, when confirmation of the precise models published is obtained after re-  
 791 view.]

## 792 Acknowledgments

793 We thank A. Biggin, H. Brown, C. Davies, A. Ferreira, M. Holdt, P. Japsen, P. Koele-  
 794 meijer, F. McNab, R. Myhill and J. Ward for helpful discussion. We also thank N. Fla-  
 795 ment and an anonymous reviewer for helping us to clarify our thesis. C.O., J.P. and V.M.F.  
 796 were supported by NERC Grant NE/T01684/1.

## 797 References

- 798 Al-Hajri, Y., White, N., & Fishwick, S. (2009). Scales of transient convective sup-  
 799 port beneath Africa. *Geology*, *37*(10), 883–886. doi: 10.1130/G25703A.1
- 800 Bahadori, A., Holt, W., & Feng, R. e. a. (2022). Coupled influence of tectonics,  
 801 climate, and surface processes on landscape evolution in southwestern North  
 802 America. *Nat Commun*, *13*(4437). doi: 10.1038/s41467-022-31903-2
- 803 Ball, P. W., Duvernay, T., & Davies, D. R. (2022). A coupled geochemical-  
 804 geodynamic approach for predicting mantle melting in space and time. *Geo-*  
 805 *chemistry, Geophysics, Geosystems*, *23*, 1–31. doi: 10.1029/2022gc010421
- 806 Ball, P. W., White, N. J., MacLennan, J., & Stephenson, S. N. (2021). Global Influ-  
 807 ence of Mantle Temperature and Plate Thickness on Intraplate Volcanism. *Na-*  
 808 *nature Communications*, *12*(2045), 1–13. doi: 10.1038/s41467-021-22323-9
- 809 Bangerth, W., Dannberg, J., Fraters, M., Gassmoeller, R., Glerum, A., Heister,  
 810 T., . . . Naliboff, J. (2023). ASPECT v2.5.0 [Computer software manual].  
 811 Retrieved 2023-08-04, from <https://zenodo.org/record/3924604> doi:  
 812 10.5281/zenodo.8200213
- 813 Bauer, S., Huber, M., Ghelichkhan, S., Mohr, M., Rude, U., & Wohlmuth, B.  
 814 (2019). Large-scale simulation of mantle convection based on a new  
 815 matrix-free approach. *Journal of Computational Science*, *31*, 60–76. Re-  
 816 trieved from <https://doi.org/10.1016/j.jocs.2018.12.006> doi:  
 817 10.1016/j.jocs.2018.12.006
- 818 Baumgardner, J. R. (1985). Three-dimensional treatment of convective flow in the  
 819 Earth’s mantle. *Journal of Statistical Physics*, *39*(5-6), 501–511. doi: 10.1007/  
 820 BF01008348
- 821 Becker, T. W., & Boschi, L. (2002). A comparison of tomographic and geodynamic  
 822 mantle models. *Geochemistry, Geophysics, Geosystems*, *3*(1), 1–48. doi: 10  
 823 .1029/2001GC000168
- 824 Biggin, A. J., Steinberger, B., Aubert, J., Suttie, N., Holme, R., Torsvik, T. H., . . .  
 825 Van Hinsbergen, D. J. (2012). Possible links between long-term geomagnetic  
 826 variations and whole-mantle convection processes. *Nature Geoscience*, *5*(8),

- 827 526–533. doi: 10.1038/ngeo1521
- 828 Braun, J. (2010). The many surface expressions of mantle dynamics. *Nature Geo-*  
829 *science*, 3(12), 825–833. doi: 10.1038/ngeo1020
- 830 Bunge, H.-P., & Baumgardner, J. R. (1995). Mantle convection modeling on par-  
831 allel virtual machines. *Computers in Physics*, 9(2), 207–215. doi: 10.1063/1  
832 .168525
- 833 Bunge, H.-P., Hagelberg, C. R., & Travis, B. J. (2003). Mantle circulation models  
834 with variational data assimilation: inferring past mantle flow and structure  
835 from plate motion histories and seismic tomography. *Geophysical Journal*  
836 *International*, 152, 280–301. doi: 10.1046/j.1365-246X.2003.01823
- 837 Bunge, H.-P., Richards, M. A., & Baumgardner, J. R. (2002). Mantle-circulation  
838 models with sequential data assimilation: Inferring present-day mantle struc-  
839 ture from plate-motion histories. *Philosophical Transactions of the Royal*  
840 *Society A: Mathematical, Physical and Engineering Sciences*, 360(1800), 2545–  
841 2567. doi: 10.1098/rsta.2002.1080
- 842 Cao, Z., & Liu, L. (2021). Origin of Three-Dimensional Crustal Stress Over  
843 the Conterminous United States. *Journal of Geophysical Research: Solid*  
844 *Earth*, 126(11), e2021JB022137. (e2021JB022137 2021JB022137) doi:  
845 <https://doi.org/10.1029/2021JB022137>
- 846 Chang, C., & Liu, L. (2021). Investigating the formation of the Cretaceous Western  
847 Interior Seaway using landscape evolution simulations. *GSA Bulletin*, 133(1-  
848 2), 347–361. doi: 10.1130/B35653.1
- 849 Colli, L., Ghelichkhan, S., & Bunge, H.-P. (2016). On the ratio of dynamic topogra-  
850 phy and gravity anomalies in a dynamic Earth. *Geophysical Research Letters*,  
851 43, 2510–2516. doi: 10.1002/2016GL067929
- 852 Colli, L., Ghelichkhan, S., Bunge, H.-P., & Oeser, J. (2018). Retrodictions of Mid  
853 Paleogene mantle flow and dynamic topography in the Atlantic region from  
854 compressible high resolution adjoint mantle convection models: Sensitivity to  
855 deep mantle viscosity and tomographic input model. *Gondwana Research*, 53,  
856 252–272. doi: 10.1016/j.gr.2017.04.027
- 857 Corrieu, V., Thoraval, C., & Ricard, Y. (1995). Mantle dynamics and geoid Green  
858 functions. *Geophysical Journal International*, 120(2), 516–523. doi: 10.1111/  
859 j.1365-246X.1995.tb01835.x
- 860 Craig, C. H., & McKenzie, D. (1987). Surface deformation, gravity and the  
861 geoid from a three-dimensional convection model at low Rayleigh num-  
862 bers. *Earth and Planetary Science Letters*, 83, 123–136. doi: 10.1016/  
863 0012-821X(87)90056-2
- 864 Crameri, F., Schmeling, H., Golabek, G. J., Duretz, T., Orendt, R., Buiter, S. J.,  
865 ... Tackley, P. J. (2012). A comparison of numerical surface topogra-  
866 phy calculations in geodynamic modelling: An evaluation of the ‘sticky  
867 air’ method. *Geophysical Journal International*, 189(1), 38–54. doi:  
868 10.1111/j.1365-246X.2012.05388.x
- 869 Czarnota, K., Hoggard, M. J., White, N., & Winterbourne, J. (2013). Spa-  
870 tial and temporal patterns of Cenozoic dynamic topography around Aus-  
871 tralia. *Geochemistry, Geophysics, Geosystems*, 14(3), 634–658. doi:  
872 10.1029/2012GC004392
- 873 Dannberg, J., Eilon, Z., Faul, U., Gassmüller, R., Moulik, P., & Myhill, R. (2017).  
874 The importance of grain size to mantle dynamics and seismological obser-  
875 vations. *Geochemistry, Geophysics, Geosystems*, 18(8), 3034–3061. doi:  
876 10.1002/2017GC006944
- 877 Davies, D. R., Davies, J. H., Bollada, P. C., Hassan, O., Morgan, K., & Nithiarasu,  
878 P. (2013). A hierarchical mesh refinement technique for global 3-D spher-  
879 ical mantle convection modelling. *Geoscientific Model Development*, 6(4),  
880 1095–1107. doi: 10.5194/gmd-6-1095-2013
- 881 Davies, D. R., Ghelichkhan, S., Hoggard, M. J., Valentine, A. P., & Richards, F. D.

- 882 (2023). Observations and Models of Dynamic Topography: Current Status and  
 883 Future Directions. In J. Duarte (Ed.), *Dynamics of plate tectonics and mantle*  
 884 *convection* (pp. 223–269). Elsevier. doi: 10.1016/B978-0-323-85733-8.00017-2
- 885 Davies, D. R., Valentine, A. P., Kramer, S. C., Rawlinson, N., Hoggard, M. J.,  
 886 Eakin, C. M., & Wilson, C. R. (2019). Earth’s multi-scale topographic  
 887 response to global mantle flow. *Nature Geoscience*, *12*, 845–850. doi:  
 888 10.1038/s41561-019-0441-4
- 889 Fernandes, V. M., & Roberts, G. G. (2021). Cretaceous to Recent net continental  
 890 uplift from paleobiological data: Insights into sub-plate support. *GSA Bulletin*,  
 891 *133*, 1–20. doi: 10.1130/b35739.1
- 892 Fernandes, V. M., Roberts, G. G., White, N., & Whittaker, A. C. (2019).  
 893 Continental-Scale Landscape Evolution: A History of North American To-  
 894 pography. *Journal of Geophysical Research: Earth Surface*, *124*, 1–34. doi:  
 895 10.1029/2018jf004979
- 896 Fichtner, A., Kennett, B. L. N., Igel, H., & Bunge, H.-P. (2009). Full seismic wave-  
 897 form tomography for upper-mantle structure in the Australasian region using  
 898 adjoint methods. *Geophysical Journal International*, *179*(3), 1703–1725. doi:  
 899 10.1111/j.1365-246X.2009.04368.x
- 900 Fichtner, A., Trampert, J., Cupillard, P., Saygin, E., Taymaz, T., Capdeville, Y., &  
 901 Villaseñor, A. (2013). Multiscale full waveform inversion. *Geophysical Journal*  
 902 *International*, *194*, 534–556. doi: 10.1093/gji/ggt118
- 903 Fichtner, A., & Villaseñor, A. (2015). Crust and upper mantle of the western  
 904 Mediterranean - Constraints from full-waveform inversion. *Earth and Plane-*  
 905 *tary Science Letters*, *428*, 52–62. doi: 10.1016/j.epsl.2015.07.038
- 906 Flament, N. (2018). Present-day dynamic topography and lower-mantle structure  
 907 from palaeogeographically constrained mantle flow models. *Geophysical Jour-*  
 908 *nal International*, *216*(3), 2158–2182. doi: 10.1093/gji/ggy526
- 909 Flament, N. (2019). Present-day dynamic topography and lower-mantle structure  
 910 from palaeogeographically constrained mantle flow models. *Geophysical Jour-*  
 911 *nal International*, *216*(3), 2158–2182. doi: 10.1093/gji/ggy526
- 912 Flament, N., Gurnis, M., & Muller, R. D. (2013). A review of observations and mod-  
 913 els of dynamic topography. *Lithosphere*, *5*(2), 189–210. doi: 10.1130/L245.1
- 914 Flament, N., Gurnis, M., Müller, R. D., Bower, D. J., & Husson, L. (2015). Influ-  
 915 ence of subduction history on South American topography. *Earth and Plane-*  
 916 *tary Science Letters*, *430*, 9–18. doi: 10.1016/j.epsl.2015.08.006
- 917 Foley, S. F., & Fischer, T. P. (2017). An essential role for continental rifts and litho-  
 918 sphere in the deep carbon cycle. *Nature Geoscience*, *10*(12), 897–902. doi: 10  
 919 .1038/s41561-017-0002-7
- 920 Forte, A. M. (2007). Constraints on Seismic Models from Other Disciplines -  
 921 Implications for Mantle Dynamics and Composition. In B. Romanowicz &  
 922 A. Dziewonski (Eds.), *Seismology and the structure of the earth* (pp. 805–858).  
 923 Elsevier B.V. doi: 10.1016/B978-044452748-6.00027-4
- 924 Forte, A. M., & Peltier, R. (1991). Viscous Flow Models of Global Geophysical  
 925 Observables 1. Forward Problems. *Journal of Geophysical Research*, *96*(B12),  
 926 20131–20159. doi: 10.1029/91JB01709
- 927 Forte, A. M., & Peltier, W. R. (1994). The Kinematics and Dynamics of Poloidal-  
 928 Toroidal Coupling in Mantle Flow: The Importance of Surface Plates and  
 929 Lateral Viscosity Variations. *Advances in Geophysics*, *36*, 1–119. doi:  
 930 10.1016/S0065-2687(08)60537-3
- 931 French, S. W., & Romanowicz, B. (2015). Broad plumes rooted at the base of the  
 932 Earth’s mantle beneath major hotspots. *Nature*, *525*(7567), 95–99. doi: 10  
 933 .1038/nature14876
- 934 Galloway, W. E., Whiteaker, T. L., & Ganey-Curry, P. (2011). History of Ceno-  
 935 zoic North American drainage basin evolution, sediment yield, and accu-  
 936 mulation in the Gulf of Mexico basin. *Geosphere*, *7*(4), 938–973. doi:

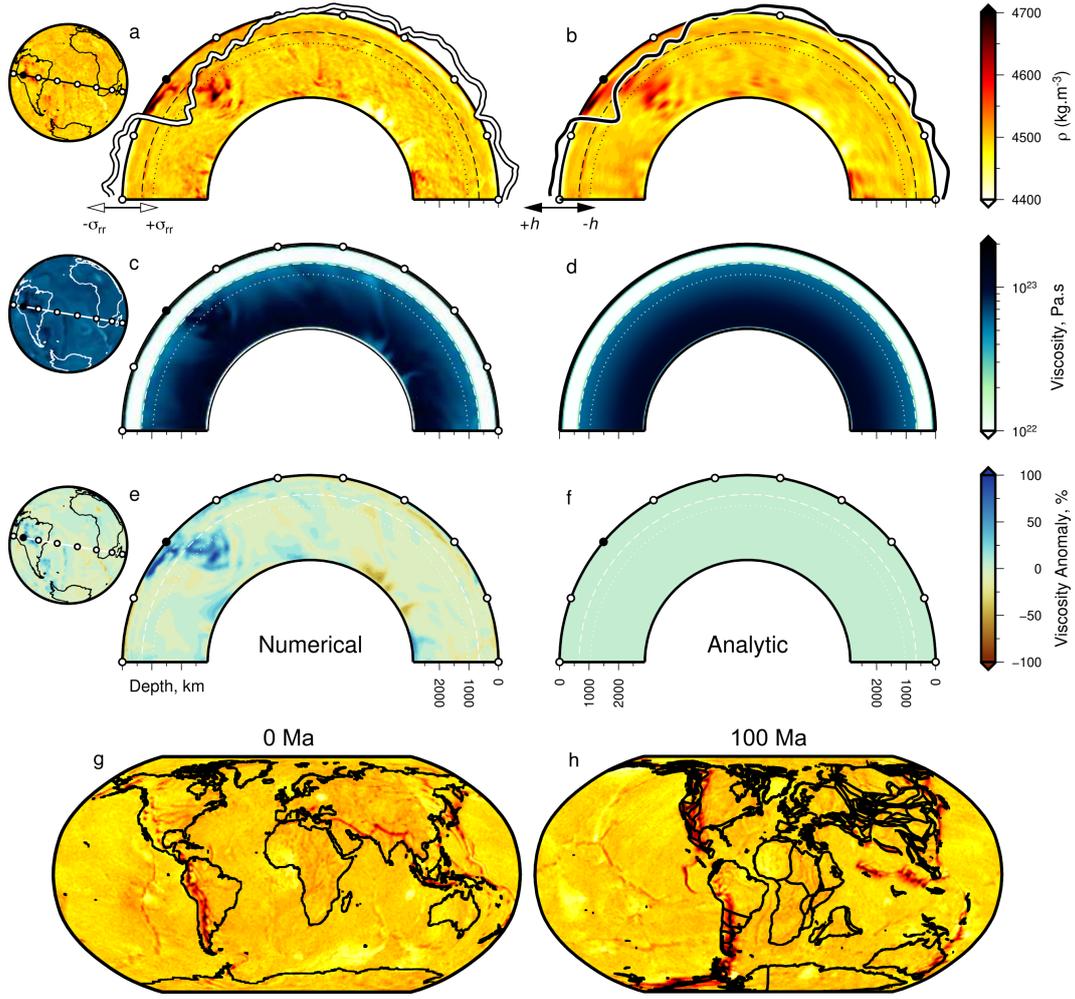
- 937 10.1130/GES00647.1  
938 Gantmacher, F. R. (1959). *The Theory of Matrices*. New York: Chelsea Publishing  
939 Company.
- 940 Ghelichkhan, S., Bunge, H.-P., & Oeser, J. (2021). Global mantle flow retrod-  
941 ictions for the early Cenozoic using an adjoint method: Evolving dynamic  
942 topographies, deep mantle structures, flow trajectories and sublithospheric  
943 stresses. *Geophysical Journal International*, *226*(2), 1432–1460. doi:  
944 10.1093/gji/ggab108
- 945 Ghosh, A., Becker, T. W., & Zhong, S. J. (2010). Effects of lateral viscosity vari-  
946 ations on the geoid. *Geophysical Research Letters*, *37*(1), 2–7. doi: 10.1029/  
947 2009GL040426
- 948 Ghosh, A., & Holt, W. E. (2012). Plate Motions and Stresses from Global Dynamic  
949 Models. *Science*, *335*(6070), 838–843. doi: 10.1126/science.1214209
- 950 Glišović, P., & Forte, A. M. (2016). A new back-and-forth iterative method for  
951 time-reversed convection modeling: Implications for the Cenozoic evolution of  
952 3-D structure and dynamics of the mantle. *Journal of Geophysical Research:*  
953 *Solid Earth*, *121*(6), 4067–4084. doi: 10.1002/2016JB012841
- 954 Gunnell, Y., & Burke, K. (2008). The African Erosion Surface: A Continental-Scale  
955 Synthesis of Geomorphology, Tectonics, and Environmental Change over the  
956 Past 180 Million Years. *Memoir of the Geological Society of America*, *201*,  
957 1–66. doi: 10.1130/2008.1201
- 958 Gurnis, M., Mitrovica, J. X., Ritsema, J., & Van Heijst, H.-J. (2000). Constraining  
959 mantle density structure using geological evidence of surface uplift rates: The  
960 case of the African Superplume. *Geochemistry, Geophysics, Geosystems*, *1*(7),  
961 1–35. doi: 10.1029/1999GC000035
- 962 Hager, B. H. (1984). Subducted Slabs and the Geoid: Constraints on Mantle Rheol-  
963 ogy and Flow. *Journal of Geophysical Research*, *89*(B7), 6003–6015.
- 964 Hager, B. H., & Clayton, R. W. (1989). Constraints on the Structure of Man-  
965 tle Convection Using Seismic Observations, Flow Models, and the Geoid. In  
966 W. R. Peltier (Ed.), *Mantle convection: Plate tectonics and global dynamics*  
967 (pp. 657–763). New York: Gordon and Breach Science Publishers.
- 968 Hager, B. H., Clayton, R. W., Richards, M. A., Comer, R. P., & Dziewonski, A. M.  
969 (1985). Lower mantle heterogeneity, dynamic topography and the geoid. *Nat-*  
970 *ure*, *313*, 541–545. doi: 10.1038/314752a0
- 971 Hager, B. H., & O’Connell, R. J. (1979). Kinematic Models of Large-Scale Flow in  
972 the Earth’s Mantle. *Journal of Geophysical Research*, *84*(B3), 1031–1048.
- 973 Hager, B. H., & O’Connell, R. J. (1981). A Simple Global Model of Plate Dynamics  
974 and Mantle Convection. *Journal of Geophysical Research*, *86*(B6), 4843–4867.  
975 doi: 10.1029/JB086iB06p04843
- 976 Hazzard, J. A. N., Richards, F. D., Goes, S. D. B., & Roberts, G. G. (2022). Prob-  
977 abilistic Assessment of Antarctic Thermomechanical Structure: Impacts on Ice  
978 Sheet Stability. *EarthArXiv*. doi: 10.31223/X5C35R
- 979 Heister, T., Dannberg, J., Gassmüller, R., & Bangerth, W. (2017). High accuracy  
980 mantle convection simulation through modern numerical methods – II: realistic  
981 models and problems. *Geophysical Journal International*, *210*(2), 833–851.  
982 doi: 10.1093/gji/ggx195
- 983 Hoggard, M. J., Austermann, J., Randel, C., & Stephenson, S. (2021). Observa-  
984 tional Estimates of Dynamic Topography Through Space and Time. In *Man-*  
985 *tle convection and surface expressions* (pp. 371–411). AGU. doi: 10.1002/  
986 9781119528609.ch15
- 987 Hoggard, M. J., White, N., & Al-Attar, D. (2016). Global dynamic topography  
988 observations reveal limited influence of large-scale mantle flow. *Nature Geo-*  
989 *science*, *9*(May), 1–8. doi: 10.1038/ngeo2709
- 990 Holdt, M. C., White, N. J., Stephenson, S. N., & Conway-Jones, B. W. (2022).  
991 Densely Sampled Global Dynamic Topographic Observations and Their Signifi-

- cance. *Journal of Geophysical Research: Solid Earth*, *127*, 1–32.
- 992  
993 Jeans, J. H. (1923). The Propagation of Earthquake Waves. *Proceedings of the Royal*  
994 *Society of London A*, *102*(718), 554–574.
- 995 Kaula, W. M. (1963). Determination of the Earth’s Gravitational Field. *Reviews of*  
996 *Geophysics*, *1*(4), 507–551.
- 997 Kramer, S. C., Davies, D. R., & Wilson, C. R. (2021). Analytical solutions for man-  
998 tle flow in cylindrical and spherical shells. *Geoscientific Model Development*,  
999 *14*(4), 1899–1919. doi: 10.5194/gmd-14-1899-2021
- 1000 Lambeck, K., Smither, C., & Johnston, P. (1998). Sea-level change, glacial rebound  
1001 and mantle viscosity for northern Europe. *Geophysical Journal International*,  
1002 *134*, 102–144. doi: 10.1046/j.1365-246X.1998.00541.x
- 1003 Lau, H. C. P., Mitrovica, J. X., Davis, J. L., Tromp, J., Yang, H.-Y., & Al-Attar, D.  
1004 (2017). Tidal tomography constrains Earth’s deep-mantle buoyancy. *Nature*,  
1005 *551*(7680), 321–326. doi: 10.1038/nature24452
- 1006 Lees, M. E., Rudge, J. F., & McKenzie, D. (2020). Gravity, topography, and melt  
1007 generation rates from simple 3D models of mantle convection. *Geochemistry,*  
1008 *Geophysics, Geosystems*, *21*, 1–29. doi: 10.1029/2019gc008809
- 1009 Lekić, V., & Fischer, K. M. (2014). Contrasting lithospheric signatures across the  
1010 western United States revealed by Sp receiver functions. *Earth and Planetary*  
1011 *Science Letters*, *402*, 90–98. doi: 10.1016/j.epsl.2013.11.026
- 1012 Liu, L., & Gurnis, M. (2008). Simultaneous inversion of mantle properties and initial  
1013 conditions using an adjoint of mantle convection. *Journal of Geophysical Re-*  
1014 *search: Solid Earth*, *113*(B8), 1–17. doi: 10.1029/2008jb005594
- 1015 Lu, C., Forte, A. M., Simmons, N. A., Grand, S. P., Kagan, M. N., Lai, H., & Gar-  
1016 nero, E. J. (2020). The Sensitivity of Joint Inversions of Seismic and Geo-  
1017 dynamic Data to Mantle Viscosity. *Geochemistry, Geophysics, Geosystems*,  
1018 *21*(4), 1–29. doi: 10.1029/2019gc008648
- 1019 McKenzie, D. (1977). Surface deformation, gravity anomalies and convection. *Geo-*  
1020 *physical Journal of the Royal Astronomical Society*, *48*, 211–238. doi: 10.1111/  
1021 j.1365-246X.1977.tb01297.x
- 1022 Merdith, A. S., Williams, S. E., Collins, A. S., Tetley, M. G., Mulder, J. A., Blades,  
1023 M. L., ... Müller, R. D. (2021). Extending full-plate tectonic models into deep  
1024 time: Linking the Neoproterozoic and the Phanerozoic. *Earth-Science Reviews*,  
1025 *214*(103477), 1–44. doi: 10.1016/j.earscirev.2020.103477
- 1026 Mitrovica, J. X., & Forte, A. M. (2004). A new inference of mantle viscosity based  
1027 upon joint inversion of convection and glacial isostatic adjustment data. *Earth*  
1028 *and Planetary Science Letters*, *225*(1-2), 177–189. doi: 10.1016/j.epsl.2004.06  
1029 .005
- 1030 Molnar, P., England, P. C., & Jones, C. H. (2015). Mantle dynamics, isostasy, and  
1031 the support of high terrain. *Journal of Geophysical Research: Solid Earth*,  
1032 *120*(3), 1932–1957. doi: 10.1002/2014JB011724
- 1033 Morris, M., Fernandes, V. M., & Roberts, G. G. (2020). Extricating dynamic to-  
1034 pography from subsidence patterns: Examples from Eastern North America’s  
1035 passive margin. *Earth and Planetary Science Letters*, *530*(115840), 1–13. doi:  
1036 10.1016/j.epsl.2019.115840
- 1037 Moucha, R., & Forte, A. M. (2011). Changes in African topography driven by man-  
1038 tle convection: supplementary information. *Nature Geoscience*, *4*(10), 707–712.  
1039 doi: 10.1038/ngeo1235
- 1040 Moucha, R., Forte, A. M., Mitrovica, J. X., & Daradich, A. (2007). Lateral vari-  
1041 ations in mantle rheology: Implications for convection related surface observ-  
1042 ables and inferred viscosity models. *Geophysical Journal International*, *169*(1),  
1043 113–135. doi: 10.1111/j.1365-246X.2006.03225.x
- 1044 Moucha, R., Forte, A. M., Mitrovica, J. X., Rowley, D. B., Quéré, S., Simmons,  
1045 N. A., & Grand, S. P. (2008). Dynamic topography and long-term sea-level  
1046 variations: There is no such thing as a stable continental platform. *Earth and*

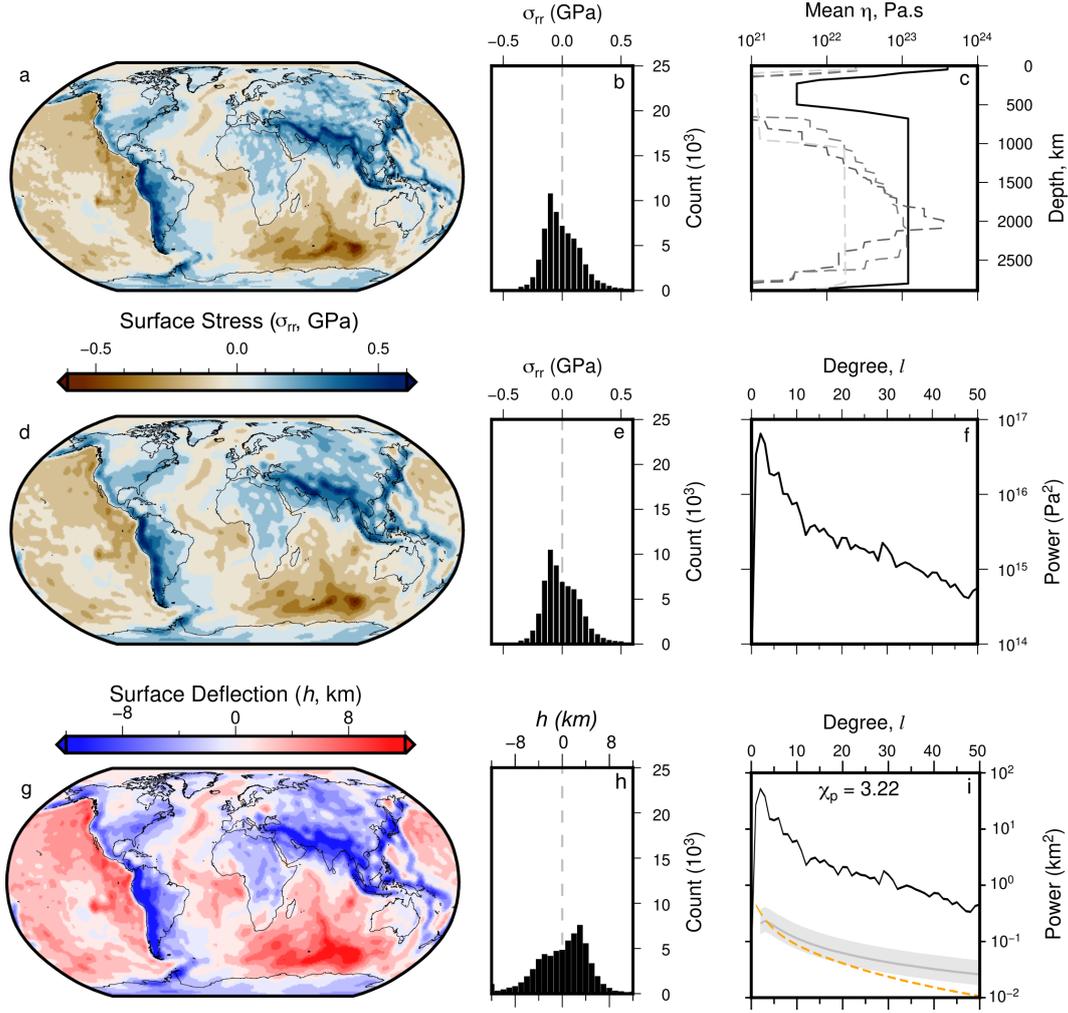
- 1047 *Planetary Science Letters*, 271(1-4), 101–108. doi: 10.1016/j.epsl.2008.03.056
- 1048 O’Connell, R. J. (1971). Pleistocene Glaciation and the Viscosity of the Lower  
1049 Mantle. *Geophysical Journal of the Royal Astronomical Society*, 23(3), 299–  
1050 327. doi: 10.1111/j.1365-246X.1971.tb01823.x
- 1051 O’Malley, C. P. B., White, N. J., Stephenson, S. N., & Roberts, G. G. (2021).  
1052 Large-Scale Tectonic Forcing of the African Landscape. *Journal of Geophysical  
1053 Research: Earth Surface*, 126, 1–37. doi: 10.1029/2021jf006345
- 1054 Panasyuk, S. V., Hager, B. H., & Forte, A. M. (1996). Understanding the effects  
1055 of mantle compressibility on geoid kernels. *Geophysical Journal International*,  
1056 124(1), 121–133. doi: 10.1111/j.1365-246X.1996.tb06357.x
- 1057 Panton, J., Davies, J. H., & Myhill, R. (2023). The Stability of Dense Oceanic  
1058 Crust Near the Core-Mantle Boundary. *Journal of Geophysical Research: Solid  
1059 Earth*, 128, 1–21. doi: 10.1029/2022JB025610
- 1060 Parsons, B., & Daly, S. (1983). The relationship between surface topography, gravity  
1061 anomalies and temperature structure of convection. *Journal of Geophysical Re-  
1062 search*, 88(B2), 1129–1144. doi: 10.1029/JB088iB02p01129
- 1063 Pekeris, C. L. (1935). Thermal Convection in the Interior of the Earth. *Geophysical  
1064 Supplements to the Monthly Notices of the Royal Astronomical Society*, 3(8),  
1065 343–367.
- 1066 Priestley, K., & McKenzie, D. (2013). The relationship between shear wave  
1067 velocity, temperature, attenuation and viscosity in the shallow part of  
1068 the mantle. *Earth and Planetary Science Letters*, 381, 78–91. doi:  
1069 10.1016/j.epsl.2013.08.022
- 1070 Ribe, N. M. (2007). Analytical Approaches to Mantle Dynamics. *Treatise on Geo-  
1071 physics*, 7, 167–226. doi: 10.1016/B978-044452748-6.00117-6
- 1072 Ricard, Y. (2007). Physics of Mantle Convection. *Treatise on Geophysics*, 7, 31–88.
- 1073 Ricard, Y. (2015). Physics of Mantle Convection. In G. Schubert (Ed.), *Treatise on  
1074 geophysics* (pp. 23–71). doi: 10.1016/B978-044452748-6.00115-2
- 1075 Richards, F. D., Hoggard, M. J., Ghelichkhan, S., Koelemeijer, P., & Lau, H. C. P.  
1076 (2021). Geodynamic, geodetic, and seismic constraints favour deflated and  
1077 dense-cored LLVPs. *EarthArXiv*, 1–20. doi: 10.31223/X55601
- 1078 Richards, F. D., Hoggard, M. J., Ghelichkhan, S., Koelemeijer, P., & Lau, H. C. P.  
1079 (2023). Geodynamic, geodetic, and seismic constraints favour deflated and  
1080 dense-cored LLVPs. *Earth and Planetary Science Letters*, 602(117964), 1–13.  
1081 doi: 10.1016/j.epsl.2022.117964
- 1082 Richards, F. D., Hoggard, M. J., White, N., & Ghelichkhan, S. (2020). Quantifying  
1083 the relationship between short-wavelength dynamic topography and thermo-  
1084 mechanical structure of the upper mantle using calibrated parameterization  
1085 of anelasticity. *Journal of Geophysical Research: Solid Earth*, 125, 1–36. doi:  
1086 10.1029/2019JB019062
- 1087 Richards, M. A., & Hager, B. H. (1984). Geoid Anomalies in a Dynamic  
1088 Earth. *Journal of Geophysical Research*, 89(B7), 5987–6002. doi: 10.1029/  
1089 JB089iB07p05987
- 1090 Salles, T., Flament, N., & Müller, D. (2017, jan). Influence of mantle flow on the  
1091 drainage of eastern Australia since the Jurassic Period. *Geochemistry, Geo-  
1092 physics, Geosystems*, 18(1), 280–305. doi: 10.1002/2016GC006617
- 1093 Spasojevic, S., & Gurnis, M. (2012). Sea level and vertical motion of continents from  
1094 dynamic earth models since the Late Cretaceous. *AAPG Bulletin*, 96(11),  
1095 2037–2064. doi: 10.1306/03261211121
- 1096 Stanley, J. R., Braun, J., Baby, G., Guillocheau, F., Robin, C., Flowers, R. M.,  
1097 ... Beucher, R. (2021). Constraining Plateau Uplift in Southern Africa by  
1098 Combining Thermochronology, Sediment Flux, Topography, and Landscape  
1099 Evolution Modeling. *Journal of Geophysical Research: Solid Earth*, 126, 1–34.  
1100 doi: 10.1029/2020JB021243
- 1101 Steinberger, B. (2007). Effects of latent heat release at phase boundaries on flow in

- 1102 the Earth’s mantle, phase boundary topography and dynamic topography at  
 1103 the Earth’s surface. *Physics of the Earth and Planetary Interiors*, 164(1-2),  
 1104 2–20. doi: 10.1016/j.pepi.2007.04.021
- 1105 Steinberger, B. (2016). Topography caused by mantle density variations:  
 1106 Observation-based estimates and models derived from tomography and litho-  
 1107 sphere thickness. *Geophysical Journal International*, 205(1), 604–621. doi:  
 1108 10.1093/gji/ggw040
- 1109 Steinberger, B., & Antretter, M. (2006). Conduit diameter and buoyant rising  
 1110 speed of mantle plumes: Implications for the motion of hot spots and shape  
 1111 of plume conduits. *Geochemistry, Geophysics, Geosystems*, 7(11), 1–25. doi:  
 1112 10.1029/2006GC001409
- 1113 Steinberger, B., & Calderwood, A. R. (2006). Models of large-scale viscous flow  
 1114 in the Earth’s mantle with constraints from mineral physics and surface  
 1115 observations. *Geophysical Journal International*, 167(3), 1461–1481. doi:  
 1116 10.1111/j.1365-246X.2006.03131.x
- 1117 Steinberger, B., Nelson, P. L., Grand, S. P., & Wang, W. (2019). Yellowstone plume  
 1118 conduit tilt caused by large-scale mantle flow. *Geochemistry, Geophysics,*  
 1119 *Geosystems*, 20, 5896–5912. doi: 10.1029/2019gc008490
- 1120 Stephenson, S. N., White, N. J., Carter, A., Seward, D., Ball, P. W., & Klöcking,  
 1121 M. (2021). Cenozoic Dynamic Topography of Madagascar. *Geochemistry,*  
 1122 *Geophysics, Geosystems*, 22, 1–38. doi: 10.1029/2020gc009624
- 1123 Tackley, P. J., Stevenson, D. J., Glatzmaier, G. A., & Schubert, G. (1993). Ef-  
 1124 fects of an endothermic phase transition at 670 km depth on spherical mantle  
 1125 convection. *Nature*, 361, 699–704. doi: 10.1038/361699a0
- 1126 Thoraval, C., Machete, P., & Cazenave, A. (1994). Influence of mantle compressibil-  
 1127 ity and ocean warping on dynamical models of the geoid. *Geophysical Journal*  
 1128 *International*, 117, 566–573. doi: 10.1111/j.1365-246X.1994.tb03954.x
- 1129 Thoraval, C., & Richards, M. A. (1997). The geoid constraint in global geody-  
 1130 namics: Viscosity structure, mantle heterogeneity models and boundary  
 1131 conditions. *Geophysical Journal International*, 131, 1–8. doi: 10.1111/  
 1132 j.1365-246X.1997.tb00591.x
- 1133 Topographic asymmetry of the south atlantic from global models of mantle flow and  
 1134 lithospheric stretching. (2014). *Earth and Planetary Science Letters*, 387,  
 1135 107–119. doi: 10.1016/j.epsl.2013.11.017
- 1136 Turcotte, D. L., & Schubert, G. (2002). *Geodynamics* (Second Edi ed.). Cambridge  
 1137 University Press.
- 1138 Wang, Y., Liu, L., & Zhou, Q. (2022). Topography and gravity reveal denser cra-  
 1139 tonic lithospheric mantle than previously thought. *Geophysical Research Let-*  
 1140 *ters*, 49(1). doi: <https://doi.org/10.1029/2021GL096844>
- 1141 Wieczorek, M. A., & Meschede, M. (2018). SHTools: Tools for Working with Spheri-  
 1142 cal Harmonics. *Geochemistry, Geophysics, Geosystems*, 19, 1–19. doi: 10.1029/  
 1143 2018GC007529
- 1144 Zhong, S., Gurnis, M., & Hulbert, G. (1993). Accurate determination of surface nor-  
 1145 mal stress in viscous flow from a consistent boundary flux method. *Physics of*  
 1146 *the Earth and Planetary Interiors*, 78, 1–8. doi: 10.1016/0031-9201(93)90078  
 1147 -N
- 1148 Zhong, S., McNamara, A., Tan, E., Moresi, L., & Gurnis, M. (2008). A benchmark  
 1149 study on mantle convection in a 3-D spherical shell using CitcomS. *Geochem-*  
 1150 *istry, Geophysics, Geosystems*, 9(10), 1–32. doi: 10.1029/2008GC002048
- 1151 Zhong, S., Zuber, M. T., Moresi, L., & Gurnis, M. (2000). Role of temperature-  
 1152 dependent viscosity and surface plates in spherical shell models of mantle  
 1153 convection. *Journal of Geophysical Research*, 105(B5), 11063–11082.
- 1154 Zhou, Q., & Liu, L. (2019). Topographic evolution of the western United States  
 1155 since the early Miocene. *Earth and Planetary Science Letters*, 514, 1–12. doi:  
 1156 10.1016/j.epsl.2019.02.029

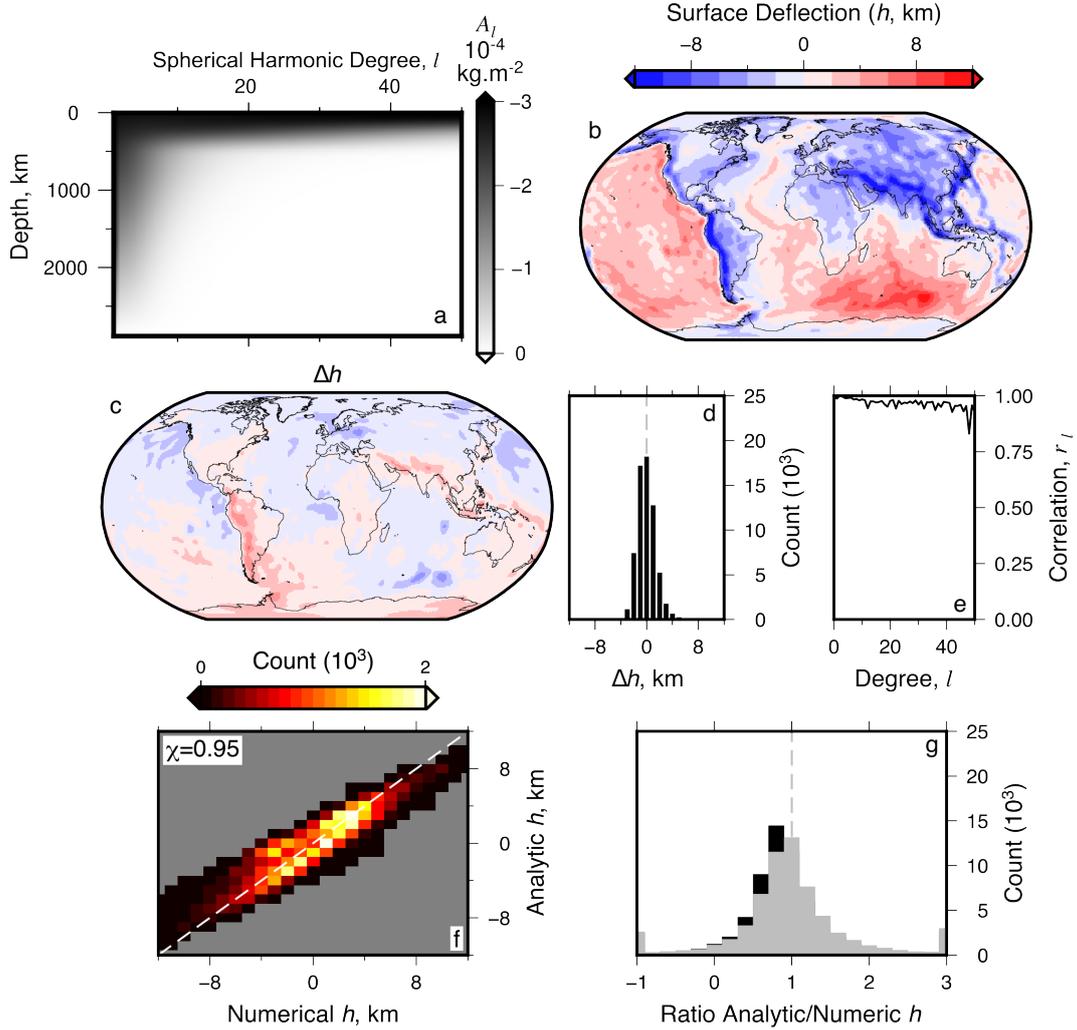
1157 Zhou, Q., Liu, L., & Hu, J. (2018). Western US volcanism due to intruding oceanic  
1158 mantle driven by ancient Farallon slabs. *Nature Geoscience*, *11*, 70–76. doi: 10  
1159 .1038/s41561-017-0035-y



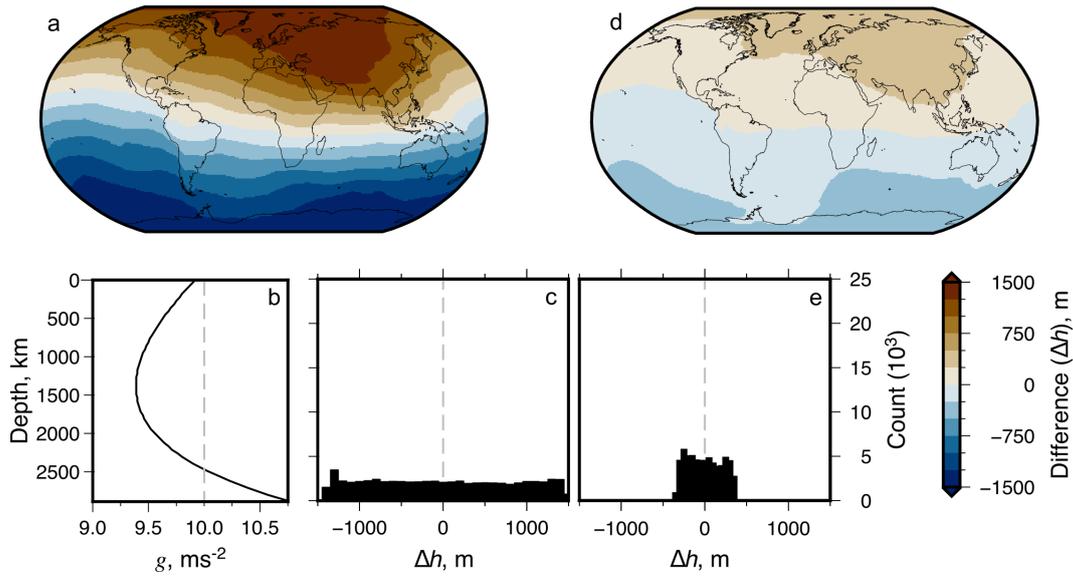
**Figure 1. Examples of mantle densities and viscosity used to calculate stresses and surface deflections numerically and analytically.** (a) Great-circle slice ( $180^\circ$ ) through full-resolution, present-day, density  $\rho$ , predicted by numeric model TERRA with temperature dependent viscosity (Model 11a; see Table 1 and body text); see globe to left for location. White circles =  $20^\circ$  intervals; filled black circle indicates orientation of cross section; dashed line = 660 km depth contour; dotted line = 1038 km depth contour, at which depth  $\rho$  is plotted on globe; white-black curve = numeric prediction of surface normal stress  $\sigma_{rr}$  from Model 11a. (b) As (a) but slice is through spherical harmonic expansion of density structure, to maximum degree  $l = 50$  ( $\lambda \approx 792$  km; Model 11b); black-white curve = surface deflection  $h$ , calculated using (analytic) propagator matrix approach (Model 12). (c) As (a) but for slice through full-resolution viscosity structure of numeric model. (d) As (c) but for mean (radial) viscosity structure, used along with the density structure shown in (b) to generate analytic solution for surface deflection shown by black-white curve atop (b). (e-f) As (c-d) but viscosity is expressed as a percentage anomaly with respect to the layer (radial) mean. (g-h) Predicted densities at 270 km depth at 0 and 100 Ma from numeric model with viscosity independent of temperature (Model 1a). Extended results are shown in Figure S1. Plate motions and paleo-coastlines are from Merdith et al. (2021).



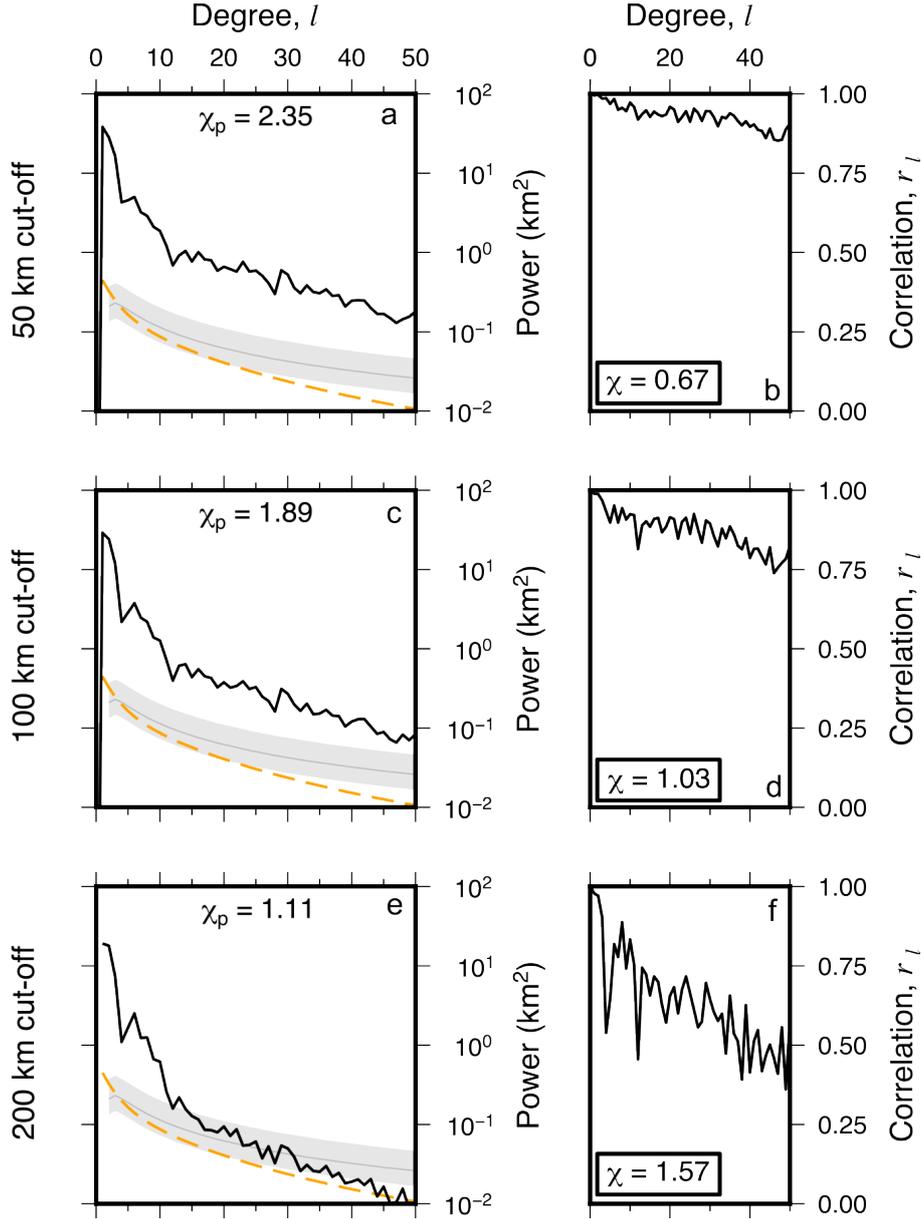
**Figure 2.** Surface stresses and deflections from numeric simulation of mantle convection with spherical harmonic expansion up to degree 50. (a) Predicted present-day surface radial stress,  $\sigma_{rr}$  (Model 1a). (b) Histogram of values shown in (a). (c) Black line = radial viscosity structure used to drive Model 1a and thus produce grid shown in panel (a). Gray dashed lines = alternative viscosity profiles of (from darkest to lightest) Mitrovića and Forte (2004), Steinberger and Calderwood (2006), and  $\mu_1$ ,  $\mu_2$  from Ghelichkhan et al. (2021). (d) Model 1b: Spherical harmonic fit to Model 1a (panel a) up to maximum degree  $l = 50$  (minimum wavelength  $\lambda \approx 792$  km). (e) Histogram of values shown in panel (d). (f) Power spectrum—total power per degree—of stress field shown in panel (d). (g) Spherical harmonic fit to surface deflections (Model 1b; up to degree  $l = 50$ ). (h) Histogram of values shown in panel (g). (i) Black curve = power spectrum of calculated water-loaded surface deflections (panel g); gray line and band = expected dynamic topography from Kaula’s rule using admittance  $Z = 12 \pm 3$  mGal  $\text{km}^{-1}$  (Kaula, 1963). Orange dashed line = expected power spectrum for water-loaded residual topography (from Holdt et al., 2022) via analytic solution of special case of Equation 15.  $\chi_p$  = root-mean-squared difference between calculated (black) and independent (orange & grey) surface deflection power (see Equation 20). All histograms are weighted by latitude to correct to equal-area. Figure S2 shows extended results including air-loaded deflections.



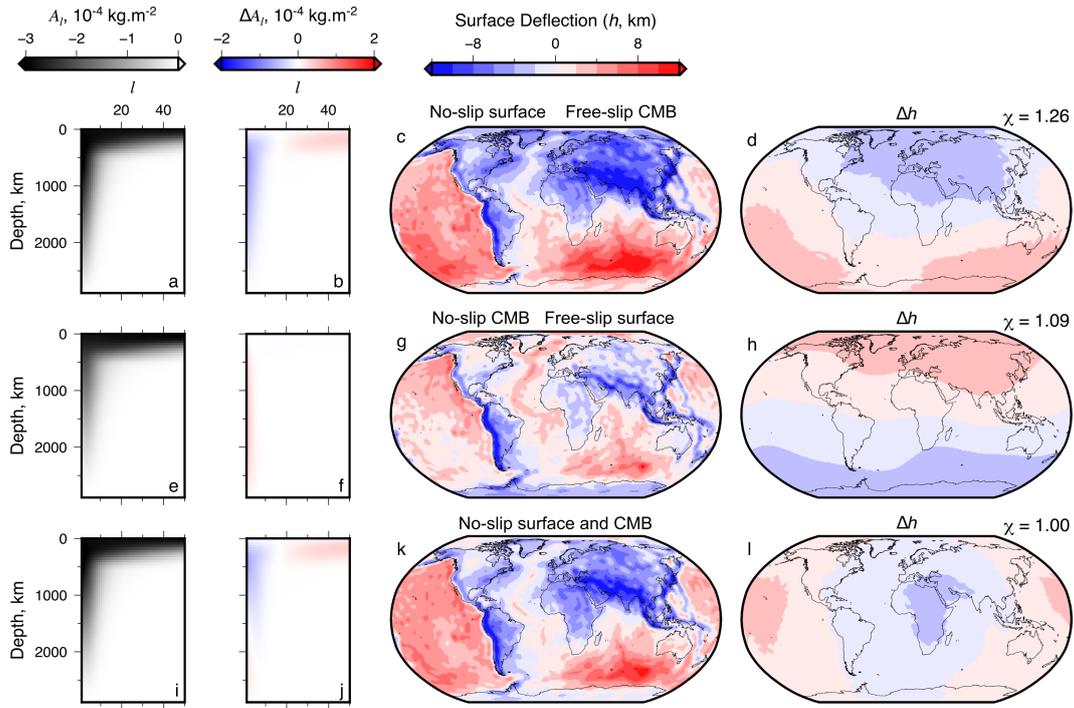
**Figure 3. Comparisons of numeric (Model 1b) and analytic (Model 2) estimation of surface deflections from models with identical parameterization.** (a) Surface deflection sensitivity kernel  $A_l$  as a function of spherical harmonic degree,  $l$ , and depth (Model 2). (b) Propagator matrix (analytic) solution for water-loaded surface deflection calculated using sensitivity kernel shown in panel (a). Figure S3 shows extended results including power spectra and air-loaded deflections. (c) Difference,  $\Delta h$ , of surface deflections in Models 1b and 2. (d) Histogram of difference values shown in (c). (e) Spectral correlation coefficient,  $r_l$ , between Models 1b and 2; Equation 8. (f) Comparison of predicted surface deflections;  $\chi$  = root-mean-squared difference between predictions (Equation 7); gray dashed line = 1:1 ratio. (g) Black bars = histogram of ratios between analytic:numeric solutions for surface deflection as in (f). Gray dashed line = 1 (i.e., identical values). Gray bars = as black bars, but for propagator matrix solution amplitudes scaled up by optimal factor to fit numeric solution (=10%). All histograms are weighted by latitude to correct to equal-area.



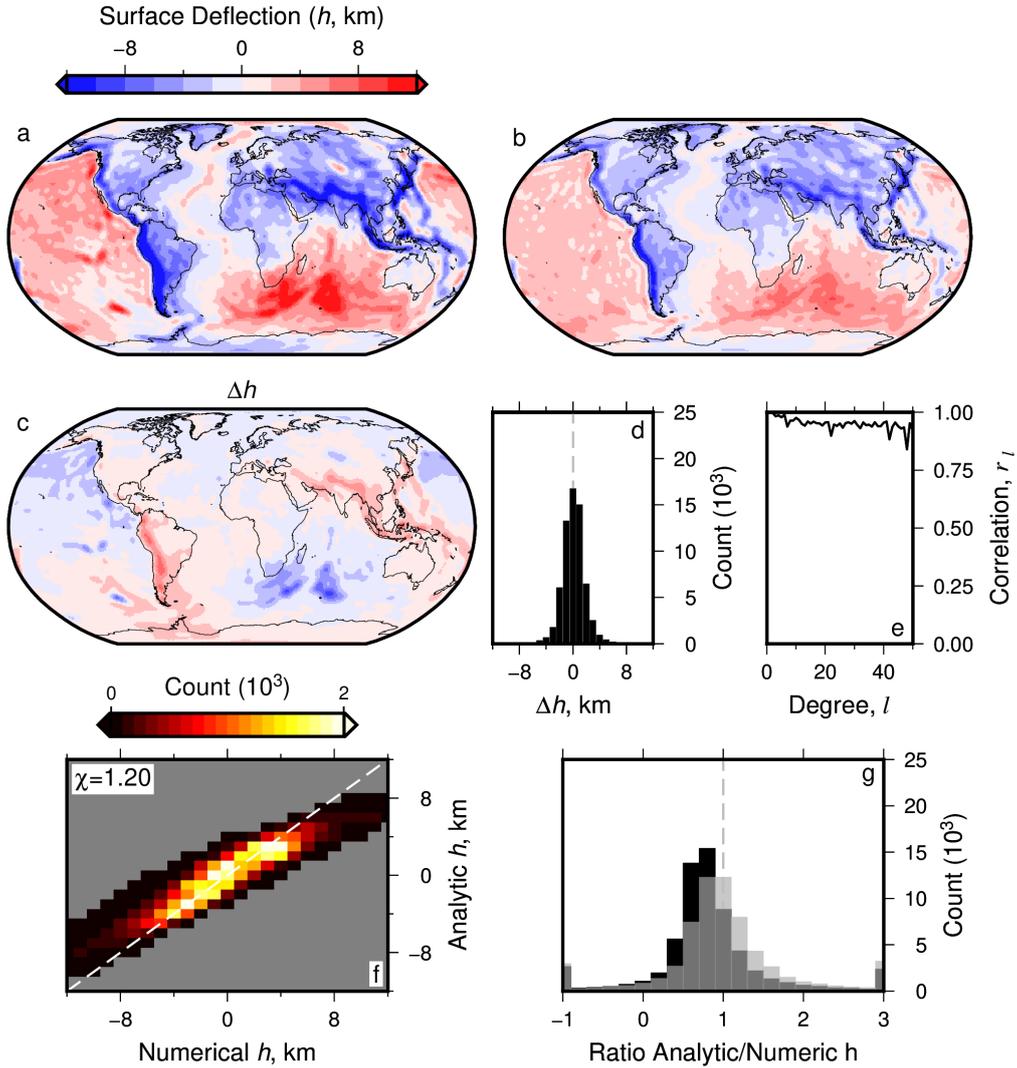
**Figure 4. Impact of self-gravitation (a–c) and gravitational potential of deflected surfaces (d–e) on surface deflections calculated analytically.** In these tests surface deflections from models with different gravity parameterizations are compared to predictions from Model 2. (a) Difference between water-loaded surface deflections calculated using the propagator matrix technique incorporating self-gravitation (Model 3; black curve in panel b) and  $g = 10 \text{ m s}^{-2}$  (dashed line in panel b; Model 2). (c) Histogram of values in panel (a). (d–e) Differences in surface deflection from models with (Model 4) and without (Model 2) stress perturbations induced by gravitational potential of the deflected surface. All histograms are weighted by latitude to correct to equal-area, they show the full extent of the results. Figures S4–S5 show extended results including maps of calculated surface deflections.



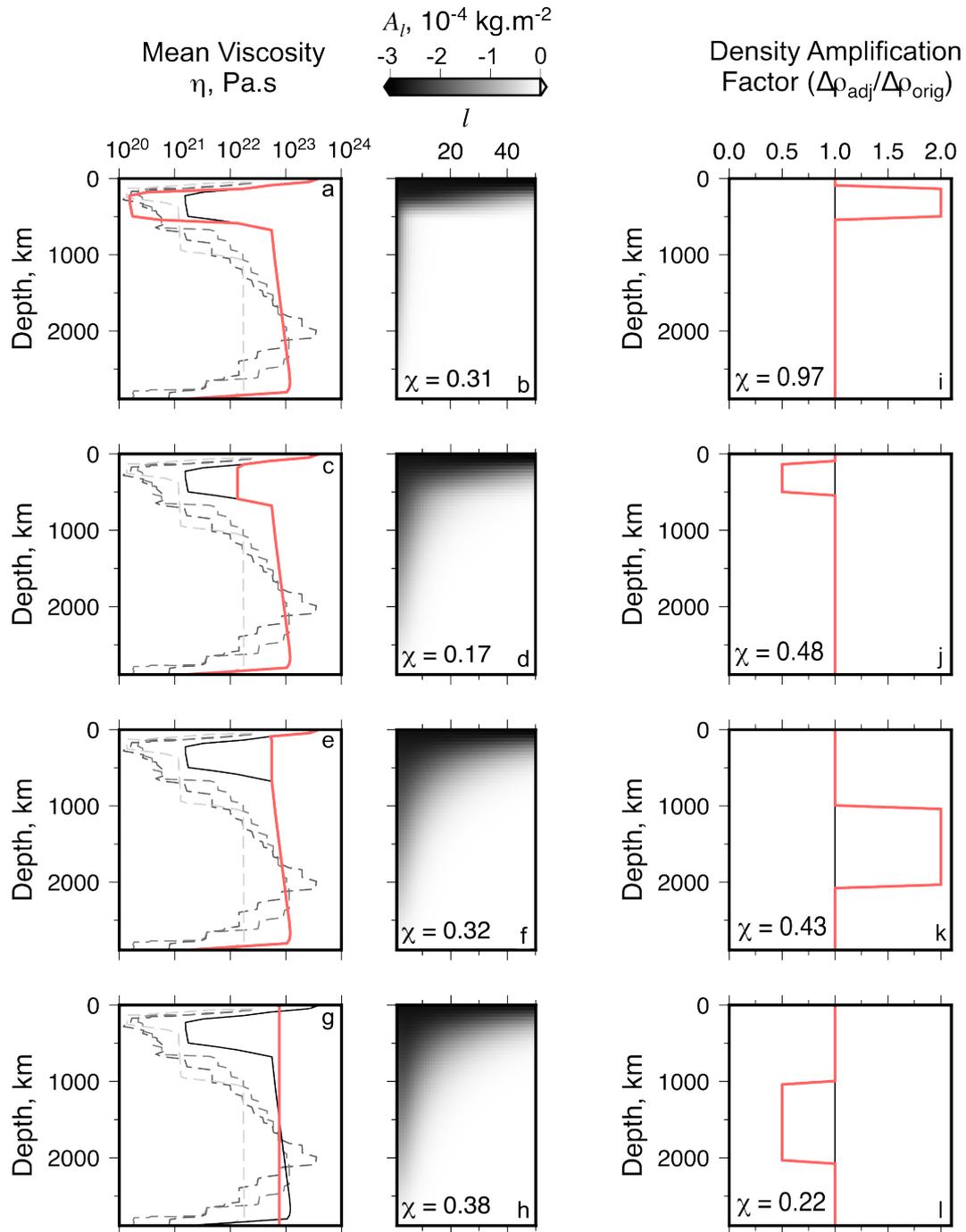
**Figure 5. Effect of removing shallow structure on surface deflections calculated analytically.** Surface deflections in models with shallow structure removed are compared to those predicted by Model 2. (a) Black line = Power spectra of predicted water-loaded surface deflection from propagator matrix solution for Model 2 (Figure 3b), but with effect of upper 50 km of density anomaly structure ignored in calculation (Model 5). Gray line and band = expected dynamic topography from Kaula’s rule using admittance  $Z = 12 \pm 3 \text{ mGal km}^{-1}$  (Kaula, 1963). Orange dashed line = expected power spectrum for water-loaded residual topography from Holdt et al. (2022), via analytic solution of special case of Equation 15.  $\chi_p$  = root-mean-squared difference between calculated (black) and independent (orange & grey) surface deflection power (see Equation 20). (b) Spectral correlation coefficient,  $r_l$ , of surface deflections in Models 5 and 2 (see Equation 19). Inset  $\chi$  = root-mean-squared difference in surface deflections of Models 5 and 2 (see Equation 18). (c–d) and (e–f) as (a–b) but for depth cut-offs of 100 (Model 6) and 200 km (Model 7), respectively. Figure S7 show extended results including maps of calculated surface deflections and differences with Model 2.



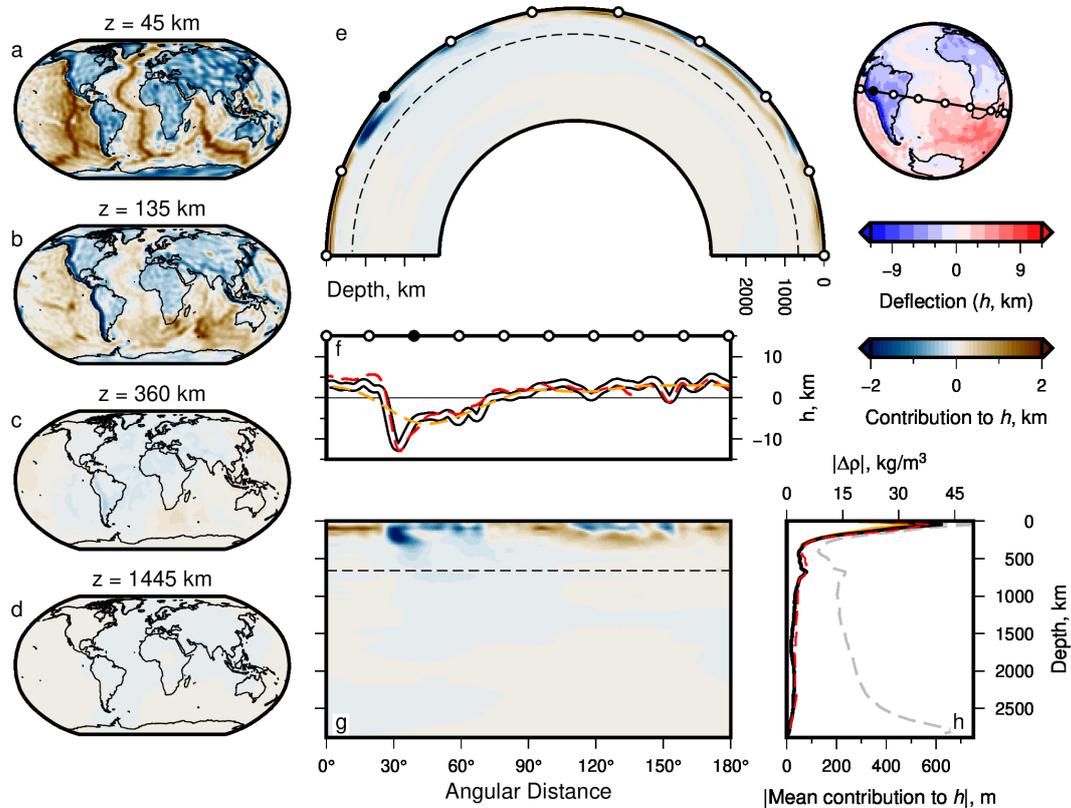
**Figure 6. Impact of free- and no-slip surface and core-mantle boundary boundary conditions on surface deflections.** This figure shows comparisons of surface deflections from models with different assumed boundary conditions and Model 2. (a) Water-loaded surface deflection sensitivity kernel  $A_l$ , for Model 8, which has a no-slip surface boundary condition, but otherwise is parameterised the same as Model 2. (b) Sensitivity kernel of Model 8 minus sensitivity kernel of Model 2. Note, positive difference implies reduced sensitivity compared to Model 2, and vice versa, since  $A_l$  is negative. (c) Predicted water-loaded surface deflection for Model 8. (d) Difference between surface deflection predictions for Model 8 and Model 2. (e–h) as (a–d) but for Model 9: free-slip surface boundary, no-slip CMB. (i–l) as (a–d) but for Model 10: no-slip surface and CMB boundaries.



**Figure 7. Comparison of surface deflections calculated numerically (Model 11b) and analytically (Model 12) using results from simulation with temperature dependent viscosity.** (a) Model 11b: Spherical harmonic expansion of predicted present-day water-loaded surface deflection converted from stress output from numeric model TERRA (Model 11a), to maximum degree  $l = 50$ . (b) Model 12: As (a) but for prediction made using propagator matrix method. (c) Difference,  $\Delta h$ , between Models 11b and 12 (panels a and b). (d) Histogram of difference values shown in (c), weighted by latitude to correct to equal-area. (e) Spectral correlation coefficient,  $r_l$ , between predictions shown in panels (a) and (b); Equation 8. (f) Numeric (Model 11b) versus analytic (Model 12) predictions of surface deflection;  $\chi$  = root-mean-squared difference between predictions, Equation 7; gray dashed line = 1:1 ratio. (g) Histogram of ratios between analytic:numeric solutions for surface deflection as in (f), weighted by latitude. Gray dashed line = 1 (i.e., identical values). Gray bars = as black bars, but for propagator matrix solution amplitudes scaled up by optimal factor to fit numeric solution (24%).



**Figure 8. Sensitivity of calculated analytic surface deflection to adjusted radial viscosity (a–h) and density anomalies (i–l).** This figure shows comparisons of surface deflections calculated in models with modified viscosity and density to the results from Model 12 (see Table 1). (a) Black curve = unadjusted prediction of present-day radial mean viscosity from Model 11; red line = adjusted radial profile with viscosity decreased by a factor of 10 between depths of  $\sim 300$ – $500$  km (Model 13); gray dashed lines = viscosity profiles used in other studies (see Figure 2). (b) Sensitivity kernel for the viscosity profile indicated by the red curve in panel a. Value of root-mean-squared difference,  $\chi$ , between calculated surface deflections for unadjusted and adjusted viscosity is stated (see Equation 7). (c–h) Results from testing alternative radial viscosity (Models 14–16). Figure S13 shows extended results including maps of surface deflections and their differences. (i–l) Density anomalies (red line) adjusted by directly scaling spherical harmonic coefficients ( $l > 0$ ) up or down by a factor of 2 (Models 17 & 19: panels e & g) or  $\frac{1}{2}$  (Models 18 & 20: f & h). Viscosity structure applied in each case is same as that used to generate Figure 7b. Sensitivity kernels for surface deflections are not shown since they are invariant with respect to density anomalies,  $\Delta\rho$ , depending only on viscosity structure. Figure S14 shows extended results including maps of surface deflections and their differences.



**Figure 9. Effective density; contributions from density anomalies to surface deflection.** (a–d) Maps of net contribution to present-day water-loaded surface deflection calculated using propagator matrix approach (Model 12; see body text for details). Depth slices at 45, 135, 360 and 1445 km depth incorporating all spherical harmonic degrees  $l$  and orders  $m$ , up to  $l = 50$ . (e) Great-circle slice ( $180^\circ$ ) showing contributions to surface deflection; globe to right shows transect location and calculated surface deflection (Model 12). White circles =  $20^\circ$  intervals; note filled black circle for orientation; dashed line = 660 km depth contour. (f) White-black curve = total surface deflection along transect shown atop globe in panel (e); abscissa aligned with panel g; orange dashed line = same but for maximum  $l = 10$  (see Supporting Information Figure S18); red dashed curve = surface deflection from Model 2. (g) Cartesian version of panel (e); ordinate aligned with panel (h). (h) Grey dashed curve = mean absolute value of density anomalies in Model 12—see top axis for values. Black curve = global mean amplitude (modulus) of contribution from density structure in Model 12 to total surface deflection  $h$ , across all  $l$  and  $m$ ; orange line = same but for maximum  $l = 10$ ; red dashed line = results for Model 2 (see Section 6.3). See Figures S15–S19 for extended results, demonstrating sensitivity of surface deflections to maximum spherical harmonic degree.