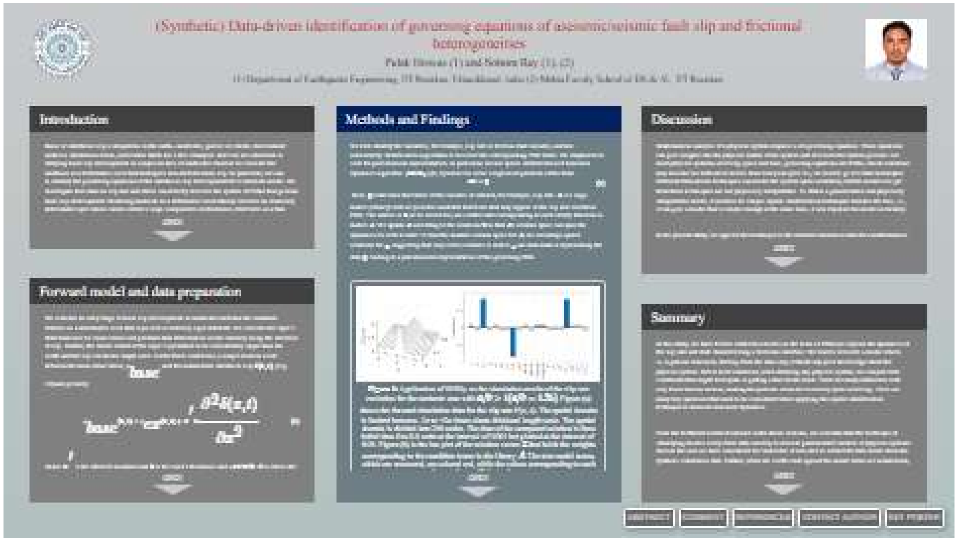
(Synthetic) Data-driven identification of governing equations of aseismic/seismic fault slip and frictional heterogeneities



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INTRODUCTION

Basal or interfacial slip is ubiquitous in the earth– landslides, glacier ice sheets, decollement surfaces, subduction zones, and crustal faults are a few examples. Here we are interested in studying basal slip development on simple models of landslides wherein we consider the landmass as a deformable solid that undergoes non-uniform basal slip. In particular, we aim to identify the governing equations of the interfacial slip development for a landslide model. We investigate how data on slip rate and stress can directly discover the system of PDEs that governs basal slip development. Modeling landslide as a deformable solid usually involves an elastically deformable layer whose lateral extent is large compared to its thickness, referred to as a thin layer. The layer slips downslope over a relatively rigid substrate; however, the magnitude of the slip is non-uniform in space leading to the deformation of the thin layer.

The interfacial strength is given by Amonton-Coulomb type friction, where the shear strength is the product of normal stress (in excess of pore fluid pressure) and friction coefficient. The friction coefficient is usually considered slip velocity- and state-dependent. Their combination, elastic deformation due to differential slip and rate-and-state friction, leads to nonlinear partial differential equations that govern the spatiotemporal evolution of slip.

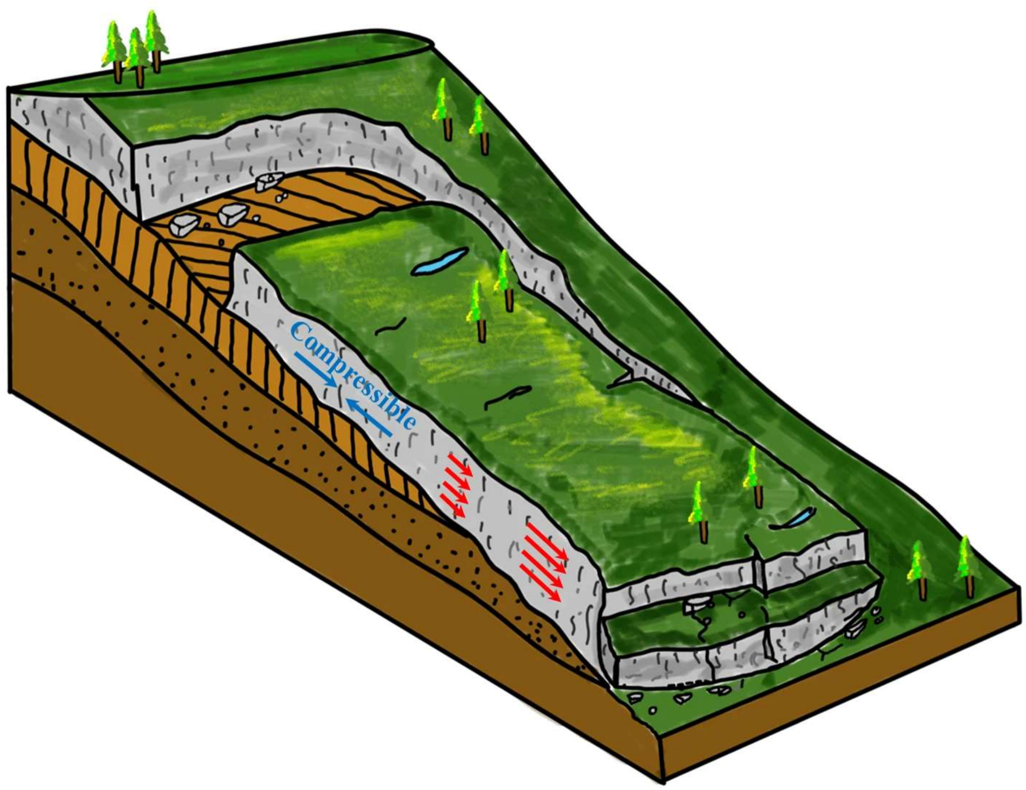
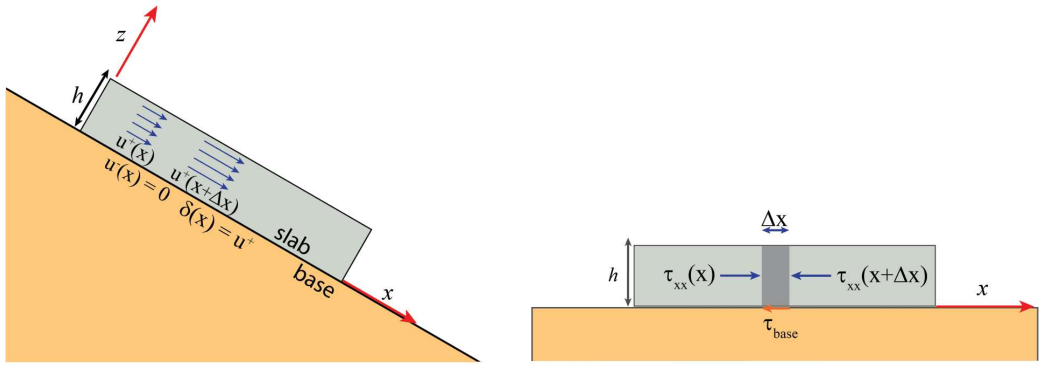


Figure (1): Initial stage of a landslide where we consider the deformation within the sliding block elastic.



(a) (b)

Figure 2: (a) A laterally deformable thin-slab model for early-stage basal slip development in landslides, (b) various stress components acting on the element Δx.

Considering an elastic deformation within the sliding block with frictional strength at the base, we try to capture the slip evolution. To capture a detailed picture of the sliding and deformation process, we need to consider a complete 3-dimensional model. In this study, we started with 1D modeling of the slip development on landslide and applied the nonlinear dynamics identification technique to rediscover the model.

FORWARD MODEL AND DATA PREPARATION

We consider an early stage of basal slip development in landslides such that the landmass behaves as a deformable solid that slips over a relatively rigid substrate. We consider the layer's deformation to be linear elastic and presume that deformation occurs laterally along the direction of slip. Further, the lateral extent of the layer is presumed to be considerably larger than the width and the slip variations length scale. Under these conditions, a simple relation exists between the basal shear stress, τ_{base}, and the nonuniform interfacial slip $\delta(x,t)$, (Fig. 2.b)and given by

$$\tau_{base}(x,t) = \tau_{ex}(x,t) + E^{'}h\frac{\partial^2\delta(x,t)}{\partial x^2} \tag{1}$$

where $E^{'}$ is the effective modulus and h is the layer's thickness, and $\tau_{ex}(x,t)$ is stress due to any external source.

The shear strength of the base is frictional given by Coulomb friction, i.e.,

$$\tau_{base} = (\sigma - p)f(V,\theta) \tag{2}$$

where σ is the fault stress and p is the pore fluid pressure distribution along the base, f is the friction coefficient.

For the friction coefficient, we consider the laboratory derived rate- and state-dependent constitutive formulation along with common state evolution laws, namely aging and slip laws.

$$f(V,\theta) = f_0 + a\ln\left(\frac{V}{V_0}\right) + b\ln\left(\frac{\theta}{\theta_0}\right) \tag{3}$$

$$\begin{aligned} \text{Aging law:} \quad & \frac{\partial\theta}{\partial t} = 1 - \frac{V\theta}{D_c} \\ \text{Slip law:} \quad & \frac{\partial\theta}{\partial t} = -\frac{V\theta}{D_c}\ln\left(\frac{V\theta}{D_c}\right) \\ \text{Combination:} \quad & \frac{\partial\theta}{\partial t} = \varepsilon\left(1 - \frac{V\theta}{D_c}\right) + (\varepsilon - 1)\frac{V\theta}{D_c}\ln\left(\frac{V\theta}{D_c}\right) \end{aligned} \tag{4}$$

Incorporating the Coulomb friction and the state evolution law into the relation (1), we get a system of coupled nonlinear PDEs that governs the spatio-temporal evolution of slip rate [6, 7, 10, 11], given by,

$$\begin{aligned} \frac{\partial V}{\partial t} &= \nu(V,\theta;a/b) \\ \frac{\partial\theta}{\partial t} &= S(V,\theta) \end{aligned} \tag{5}$$

Here, we have taken the variables as,

$$\begin{aligned} V &\to V/V_o \\ \theta &\to \theta/\theta_o \\ x &\to x/L_{bh} \end{aligned} \tag{6}$$

where, V_o and θ_o are normalizing constants, θ_o being equal to D_c/V_o.

We numerically solve this PDE system with various initial conditions. The numerical solutions in the form of slip rate and stress time series in the spatial domain will serve as the data input for further investigation.

METHODS AND FINDINGS

We first identify the variables, for example, slip rate or friction state variable, and use nonlinearity identification algorithms to discover the corresponding PDE terms. We emphasize to look for parsimonious representation. In particular, we use sparse identification of nonlinear dynamics algorithm (**SINDy**) [8, 9] where we solve a regression problem of the form

$$Ax = y \tag{7}$$

Here, y is the time derivative of the variable of interest, for example, slip rate. A is a large matrix (library) with all possible candidate functions that may appear in the slip rate evolution PDE. The entries in x, to be solved for, are coefficients corresponding to each library function in matrix A. We update A according to the solutions x so that A's column space can span the dynamics we seek to find. To find the suitable column space for A, we encourage sparse solutions for x, suggesting that only a few columns in matrix A are dominant in representing the data y, leading to a parsimonious representation of the governing PDE.

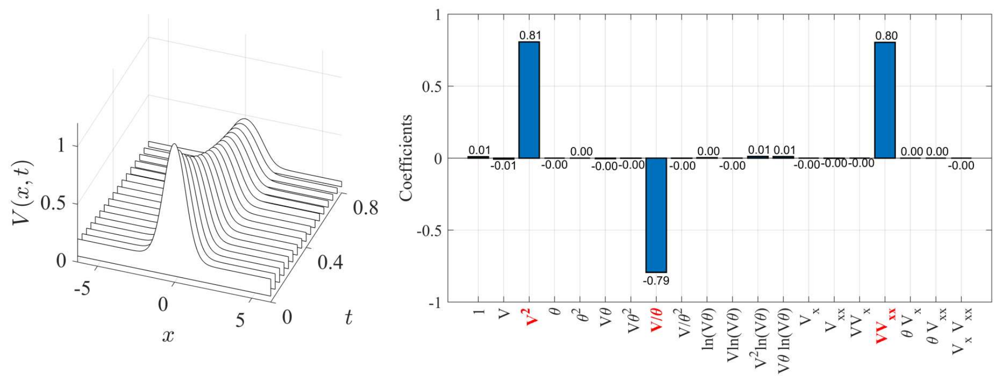
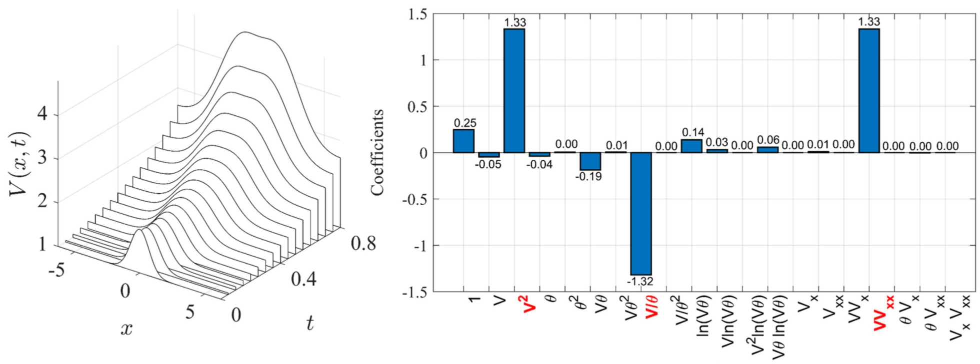


Figure 3: Application of SINDy on the simulation results of the slip rate evolution for the aseismic case with $a/b > 1(a/b = 1.25)$. Figure (a) shows the forward simulation data for the slip rate $V(x,t)$. The spatial domain is limited between -2π to +2π times elasto-frictional length scale. The spatial domain is divided into 256 nodes. The time of the computed solution is from initial time 0 to 0.8 units at the interval of 0.001 but plotted at the interval of 0.05. Figure (b) is the bar plot of the solution vector x that holds the weights corresponding to the candidate terms in the library A. The true model terms, which are recovered, are colored red, while the values corresponding to each bar are the coefficient values.



(a) (b)

Figure 4: Application of SINDy on the simulation results of the slip rate evolution for the seismic case with $a/b < 1(a/b = 0.75)$. Figure (a) shows the forward simulation data for the slip rate $V(x, t)$. The spatial domain is limited between -2π to +2π times elasto-frictional length scale. The spatial domain is divided into 256 nodes. The time of the computed solution is from initial time 0 to 0.8 units at the interval of 0.001 but plotted at the interval of 0.05. Figure (b) is the bar plot of the solution vector x that holds the weights corresponding to the candidate terms in the library A. The true model terms, which are recovered, are colored red, while the values corresponding to each bar are the coefficient values.

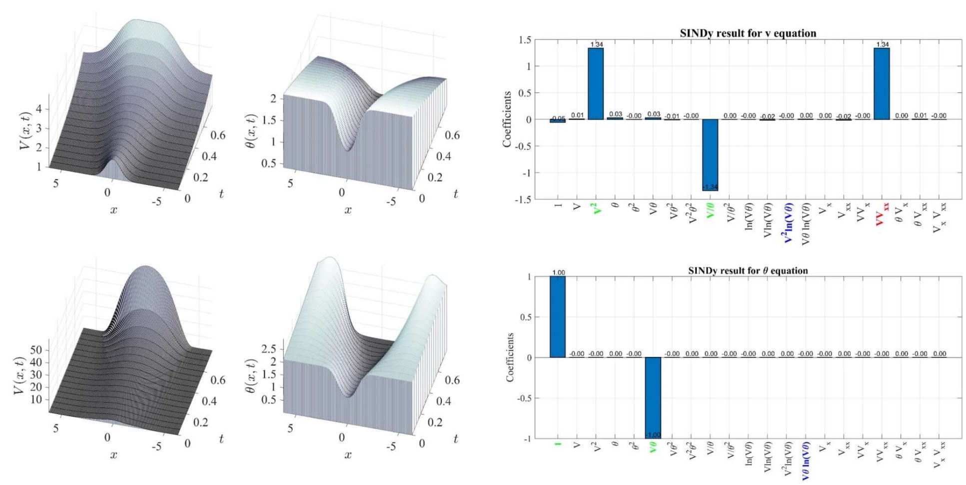


Figure 5: SINDy applied to the dataset of slip rate and state with two initial conditions. SINDy can successfully recover the governing state evolution law, in this case, Aging law $a/b < 1(a/b = 0.75)$.

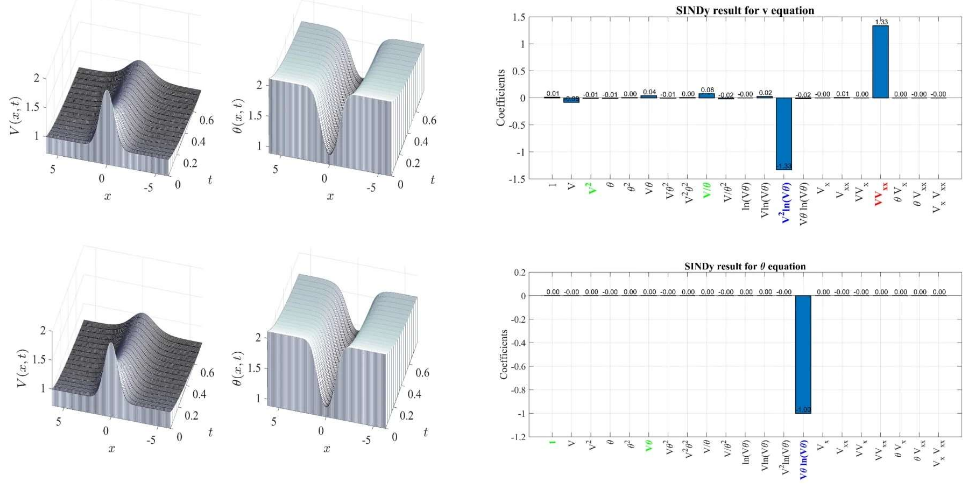


Figure 6: SINDy applied to the dataset of slip rate and state with two initial conditions. SINDy can successfully recover the governing state evolution law, in this case, Slip law $a/b < 1 (a/b = 0.75)$.

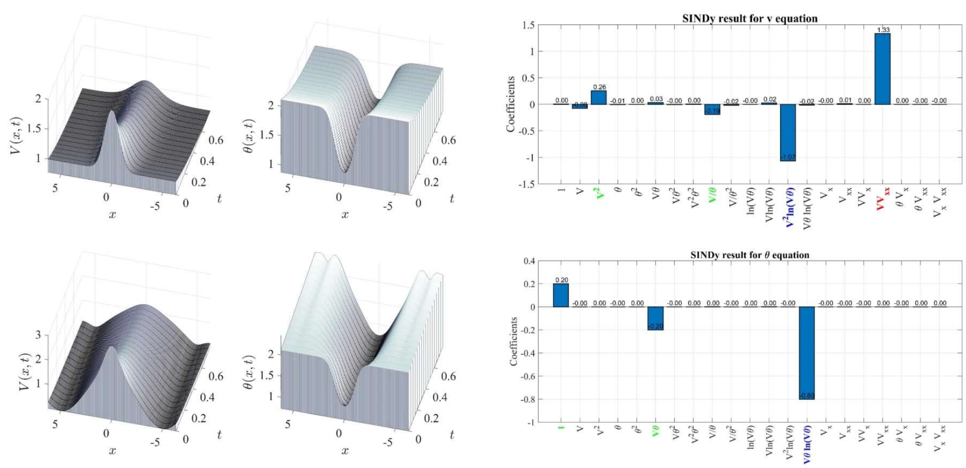


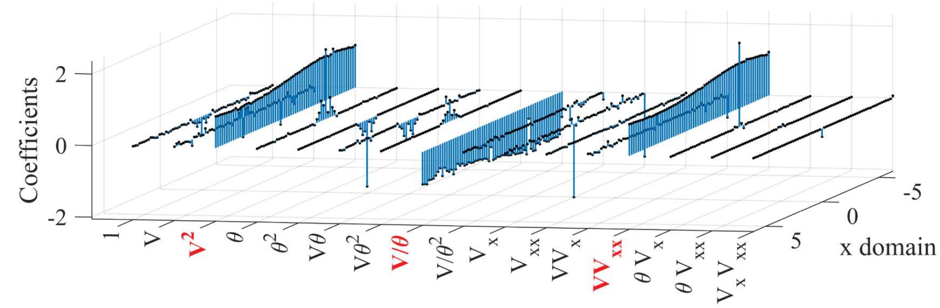
Figure 7: SINDy applied to the dataset of slip rate and state with two initial conditions. SINDy can successfully recover the governing state evolution law, in this case, a combination of aging and slip laws (eq. 4_3)$a/b < 1 (a/b = 0.75)$.

One instance of the recovered governing equation, for instability case:

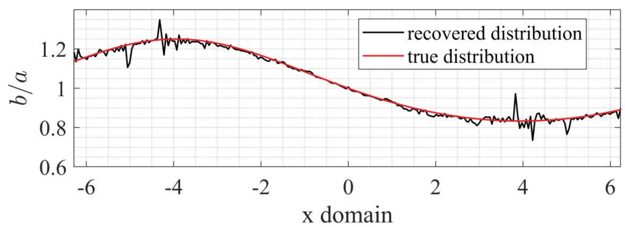
$$\left.\frac{\partial V}{\partial t} = 1.33\, V\frac{\partial^2 V}{\partial X^2} - 1.33\frac{V}{\theta} + 1.33\, V^2 \right\} \text{True model}$$

$$\left.\frac{\partial V}{\partial t} = 1.33\, V\frac{\partial^2 V}{\partial X^2} - 1.32\frac{V}{\theta} + 1.33\, V^2 \right\} \text{Discovered model}$$

We show that the algorithm successfully recovers the terms of the PDE, governing fault slip, and could also find the frictional parameter, for example, a/b, where a and b, respectively, are the magnitudes that control direct and evolution effects. Moreover, the algorithm can also determine whether the associated state variable evolves as aging- or slip-law types or their combination. Further, with the data set prepared from distinct initial conditions, we show that the algorithm can also determine the problem parameter's spatial distribution (heterogeneities) from fault slip rate and stress data.



(a)



(b)

Figure 8: (a) shows the solution vector x computed for each spatial node. For each x node, if we plot the solution vector x corresponding to each ode problem, we get the spatial distribution of the a/b parameter. Here, the true distribution of the a/b parameter is given by $a/b = 1 + 0.2 \times \sin(5x/L)$ **(b)** shows the true distribution of the b/a parameter along with the recovered distribution.

DISCUSSION

Mathematical analysis of a physical system requires a set governing equation. These equations can give insights into the physical nature of the system and tells how the system governs. For most physical systems involving space and time, governing equations are PDEs, which sometime may become too difficult to derive from first principles. So, we need to go for other techniques. Different data-driven techniques can model the system quite accurately, but the models we get from these techniques are not physically interpretable. To obtain a generalizable and physically interpretable model, it needs to be simple. Sparse identification techniques balance the two, i.e., it can give a model that is simple enough at the same time, it also explains the model accurately.

In the present study, we applied this technique to the forward simulation data for a mathematical analog of the landslide problem. The down-slip movement of the landslide was mathematically replicated by considering a simple model of a thin slab sliding down
a slope under the combined influence of gravity and basal friction. Sparse identification quite nicely discovered the actual model for the forward simulation data.

Sparse identification may not always need regularization techniques such as LASSO. A simple least square technique can sometimes produce a good result when the data are clean.

The results discussed in this poster highlight that the spatial distribution of frictional parameters can be determined from time series data of slip rate and stress data. To determine the spatial distribution of the problem parameters, we pose the model discovery problem as a separate ODE system (in contrast to a PDE system) identification at each spatial location.

SUMMARY

In this study, we have tried to establish a model (in the form of PDEs) to explain the dynamics of the slip rate and state characterizing a frictional interface. We tried to discover a model which is, in general nonlinear, starting from the data only, without any prior knowledge about the physical system. But in most situations, when studying any physical system, we can put some constraints that might be helpful in getting a true model back. There are many unknowns with only fewer known factors, making the problem of model discovery quite involving. There are many key questions that need to be considered while applying this sparse identification technique to discover nonlinear dynamics.

From the Different results discussed in the above sections, we conclude that the technique of identifying models solely from data can help to recover generalizable models of physical systems such as the case we have considered for landslides. It was able to extract the true model from the synthetic simulation data. Further, when the coefficients against the model terms are nonuniform, we can still find both the model and the distribution of the variable coefficients.

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ABSTRACT

Models of fault slip development generally consider interfacial strength to be frictional and deformation of the bounding medium to be elastic. The frictional strength is usually considered as sliding rate- and state-dependent. Their combination, elastic deformation due to differential slip and rate-state frictional strength, leads to nonlinear partial differential equations (PDEs) that govern the spatio-temporal evolution of slip. Here, we investigate how data on fault slip rate and stress **can directly discover the complex system (of PDEs) that governs aseismic slip development.** We first prepare (synthetic) data sets by numerically solving the forward problem of slip rate and fault stress evolution with models, such as a thin laterally deformable layer over a thick substrate. We now identify the variables, for example, slip rate or friction state variable, and use **nonlinearity identification algorithms to discover the governing PDE of the chosen variable.**

In particular, we use sparse identification of nonlinear dynamics algorithm (SINDy, Brunton et al., 2016) where we solve a regression problem, **Ax=y**. Here, **y** is the time derivative of the variable of interest, for example, slip rate. **A** is a large matrix (library) with all possible candidate functions that may appear in the slip rate evolution PDE. The entries in **x**, to be solved for, are coefficients corresponding to each library function in matrix **A**. We update **A** according to the solutions **x** so that **A**'s column space can span the dynamics we seek to find. To find the suitable column space for **A**, we encourage sparse solutions for **x**, suggesting that only a few columns in matrix **A** are dominant, leading to a parsimonious representation of the governing PDE.

We show that the algorithm successfully recovers the terms of the PDE governing fault slip and could also find the frictional parameter, for example, a/b, where a and b, respectively, are the magnitudes that control direct and evolution effects. Moreover, the algorithm can also determine whether the associated state variable evolves as aging- or slip-law types or their combination. Further, with the data set prepared from distinct initial conditions, we show that the SINDy can also determine the **problem parameter’s spatial distribution (heterogeneities)** from fault slip rate and stress data.

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