Supporting Information for "Excess ground ice profiles in continuous permafrost mapped from InSAR subsidence"

S. Zwieback[1,2], G. Iwahana[3], S. Sakhalkar[1,2], R. Biessel[1,2], S. Taylor[1], F.J. Meyer[1,2]

[1]Geophysical Institute, University of Alaska Fairbanks, AK, USA

[2]Department of Geosciences, University of Alaska Fairbanks, AK, USA

[3]International Arctic Research Center, University of Alaska Fairbanks, AK, USA

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Introduction The supporting information provides extended model descriptions. Specifically, section 1 describes how the forward model evaluates the heat flux into the frozen ground; section 2 specifies stratigraphy parameterization.

Corresponding author: S. Zwieback, Geophysical Institute and Department of Geosciences, University of Alaska Fairbanks, USA. (szwieback@alaska.edu)

Text S1. Heat-balance integral solution for heat flux into frozen soil

The heat-balance integral approach was used to model the heat flux Q_f into the frozen soil below the thaw front and the temperature profile below the thaw front. We postulated a quadratic temperature profile between $y_\mathrm{f}(t)$ and $y_\mathrm{p}(t)$. We model

$$y_\mathrm{p}(t) - y_\mathrm{f}(t) = w\, y_\mathrm{f}(t) \tag{1}$$

using an unknown time-invariant parameter w.

The heat-balance integral approach posits that the heat equation holds for the assumed temperature profile in an integral sense. By integrating over time (entire thaw period t_t) and depth (y_p and y_t), we find

$$w = -\frac{3}{2} + \left(\frac{9}{4} + \frac{12 k_\mathrm{f} t_\mathrm{t}}{c_\mathrm{f}\, y_\mathrm{f}(t_\mathrm{t})^2}\right)^{\frac{1}{2}}, \tag{2}$$

with a uniform frozen conductivity k_f and volumetric heat capacity $_\mathrm{f}$. The end-of-season thaw depth $y_\mathrm{f}(t_\mathrm{t})$ can be refined iteratively, but here we took the first guess from the simplified energy balance.

The heat flux into the frozen substrate Q_f at any time only depends on y_f through

$$Q_\mathrm{f}(t) = -k_\mathrm{f} \frac{2(T_0 - T_\mathrm{m})}{w\, y_\mathrm{f}(t)}, \tag{3}$$

where T_0 is the temperature at depth.

Text S2. Stratigraphies

The prior distribution of the stratigraphies was modeled generatively from prior distributions of soil characteristics. These described the soil properties of the frozen soil column prior to the onset of thaw.

Each stratigraphy was decomposed into an organic layer at the surface and a mineral layer underneath. The organic layer thickness y_o followed a uniform distribution $\mathcal{U}(y_{\mathrm{o,min}}, y_{\mathrm{o,max}})$ in the interval $[y_{\mathrm{o,min}}, y_{\mathrm{o,max}}]$.

The volume not occupied by excess ice was partitioned into mineral particles, organic matter, non-excess ice and air. For each y cell, the fraction of mineral particles was set to m_o and m_m in the organic and mineral layer, respectively; that of organic matter to o_o and o_m. The degree of saturation S – the fraction of the pore space occupied by non-excess ice – in the organic layer was drawn from a uniform distribution $\mathcal{U}(S_{\mathrm{o,min}}, S_{\mathrm{o,max}})$ in the organic layer. That of the mineral layer was modeled analogously.

The thaw period n-factor n was drawn from a beta distribution on the interval $[n_{\mathrm{min}}, n_{\mathrm{max}}]$, viz.

$$n \sim \mathrm{Beta}\left(\alpha_n, \beta_n; n_{\mathrm{min}}, n_{\mathrm{max}}\right) . \tag{4}$$

The parameters governing the prior distributions that we adopted for this study are summarized in Tab. S1.

The parameters used to evaluate the soil thermal properties as a function of soil composition are summarized in Tab. S2.

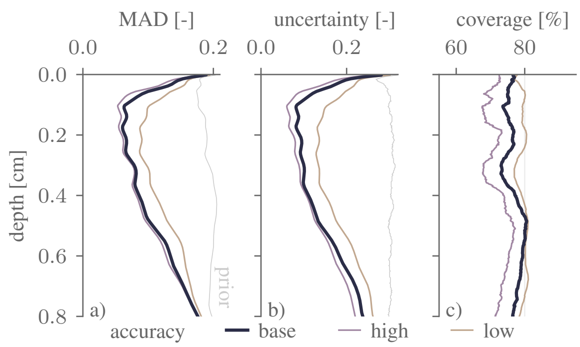

Figure S1. Synthetic performance assessment for $N = 10^4$ ensemble members and three settings of the observational accuracy. Same as Fig. 6 except smaller N.

	Description	Value	Rationale
$\mu_{\mathbf{l_e}}$	mean of logit(e) at knots	−3.0	non-informative prior
$\sigma_{\mathbf{l_e}}$	standard deviation of logit(e) at knots	3.0	non-informative prior
$\rho_{\mathbf{l_e}}$	lag-1 autocorrelation of logit(e) at knots	0.7	non-informative prior
$y_{\mathrm{o,min}}$	minimum organic layer thickness	0.1 m	HV and IC study sites
$y_{\mathrm{o,max}}$	maximum organic layer thickness	0.2 m	HV and IC study sites
m_{o}	mineral fraction of non-excess-ice volume in organic layer	0.00	HV and IC study sites
m_{m}	mineral fraction of non-excess-ice volume in mineral layer	0.35	HV and IC study sites
o_{o}	organic fraction of non-excess-ice volume in organic layer	0.10	HV and IC study sites
o_{m}	organic fraction of non-excess-ice volume in mineral layer	0.05	HV and IC study sites
$S_{\mathrm{o,min}}$	minimum degree of saturation in organic layer	0.4	HV and IC study sites
$S_{\mathrm{o,max}}$	maximum degree of saturation in organic layer	0.8	HV and IC study sites
$S_{\mathrm{m,min}}$	minimum degree of saturation in mineral layer	0.8	HV and IC study sites
$S_{\mathrm{m,max}}$	maximum degree of saturation in mineral layer	1.0	HV and IC study sites
n_{min}	minimum n-factor	0.85	(Klene et al., 2001; Cable et al., 2016)
n_{max}	maximum n-factor	1.00	see n_{min}
α_n	α shape parameter of n-factor distribution	2.0	gradual decay from mean
β_n	β shape parameter of n-factor distribution	2.0	$\alpha_n = \beta_n$ gives symmetric distribution

Table S1. Parameters governing the prior distribution of soil characteristics along with the values used in this study.

	Description	Value
L_i	volumetric enthalpy of phase change of ice	$3.34 \times 10^8\,\mathrm{J m^{-3}}$
k_m	thermal conductivity of mineral particles	$3.80\,\mathrm{W m^{-1} K^{-1}}$
k_o	thermal conductivity of organics	$0.25\,\mathrm{W m^{-1} K^{-1}}$
k_i	thermal conductivity of ice	$2.20\,\mathrm{W m^{-1} K^{-1}}$
k_w	thermal conductivity of water	$0.57\,\mathrm{W m^{-1} K^{-1}}$
k_a	thermal conductivity of air	$0.02\,\mathrm{W m^{-1} K^{-1}}$

Table S2. Parameters for evaluating the soil thermal properties along with the values used in this study, taken from (Zwieback et al., 2019).

References

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