

1 **Characteristics of earthquake cycles: a**
2 **cross-dimensional comparison from 0D to 3D**

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7 **Key Points:**

- 8 • Models with dimension reduction simulate qualitatively similar quasi-periodic earth-
9 quake sequences with quantitative differences.
- 10 • Reduced influence of velocity-strengthening patches due to dimension reduction
11 increases recurrence interval, slip and rupture speed.
- 12 • We provide guidelines on how to interpret lower-dimensional modeling results of
13 interseismic loading and earthquake ruptures.

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Abstract

High-resolution computer simulations of earthquake sequences in three or even two dimensions pose great demands on time and energy, making lower-cost simplifications a competitive alternative. We systematically study the advantages and limitations of simplifications that eliminate spatial dimensions, from 3D down to 1D in quasi-dynamic earthquake sequence models. We demonstrate that models in any number of spatial dimensions simulate qualitatively similar quasi-periodic sequences of quasi-characteristic earthquakes. Certain coseismic characteristics like stress drop and fracture energy are largely controlled by frictional parameters and thus their overall values are observed to be comparable across models of different dimensions. However, other observations are more strongly affected by dimension reduction. We find corresponding increases in recurrence interval, coseismic slip, peak slip velocity, and rupture speed. We find that these changes are largely explained by the elimination of velocity-strengthening patches that transmit loading conditions onto the velocity-weakening fault patch, thereby reducing the interseismic loading rate and enhancing the slip deficit. This is supported by a concise theoretical framework that explains some of these findings quantitatively. Given the computational efficiency of lower-dimensional models, this contribution aims to provide qualitative and quantitative guidance on economical model design and interpretation of modeling studies.

Plain Language Summary

Computer simulations are a powerful tool to understand earthquakes and they are often simplified to save time and energy. Dimension reduction - using 1/2D models instead of 3D models - is a commonly used simplification, but its consequences are not systematically studied. Here we find that both the overall earthquake recurrence pattern and the magnitude of stress changes on the fault caused by earthquakes remain relatively unchanged by model simplification by dimension reduction. However, some key observations such as the total slip and rupture speed achieved during an earthquake, as well as the precise recurrence interval are larger in lower-dimensional models. These changes are related to the elimination of lateral creeping regions that transmit stress onto the fault, which is an unavoidable consequence of the elimination of a physical dimension. We use simple theoretical calculations to reproduce these observations and justify this causal relationship. As simplified models are still popular due to their computational ef-

46 efficiency, this contribution helps their users to understand and anticipate the potential
47 discrepancies of their results with respect to the three-dimensional situation that exists
48 in nature. Therefore users can design their models and interpret their results with this
49 work as a guideline.

50 **1 Introduction**

51 Earthquake sequences show statistical regularity in space and time (e.g., Utsu et
52 al., 1995; Uchida & Bürgmann, 2019). Despite the complex patterns of earthquake oc-
53 currence observed over our limited observational window, evolution of fault slip has its
54 internal time-dependence: stress accumulation due to persistent tectonic loading and stress
55 release due to occasional seismic events make up an earthquake cycle. Understanding
56 earthquake cycles is fundamental to recognize the recurrence of natural and induced earth-
57 quakes and ultimately helps to better assess long-term seismic hazard. Despite small to
58 intermediate-size events regularly occurring on the same fault in nature (e.g., Chlieh et
59 al., 2004; Prawirodirdjo et al., 2010) and generated quasi-periodically in scaled labora-
60 tory experiments (e.g., Rosenau et al., 2009; McLaskey & Lockner, 2014), the recurrence
61 of large destructive earthquakes is hard to monitor due to their long recurrence inter-
62 val. Moreover, natural observations are largely confined to the earth’s surface, at some
63 distance to the hypocenter. In addition, laboratory experiments are restricted by their
64 scale and thus upscaling is often necessary to interpret their findings. Therefore, numer-
65 ical models are well-suited to overcome these spatial-temporal limitations and improve
66 our understanding of the Sequences of Earthquakes and Aseismic Slip (SEAS).

67 Numerical models featuring different degrees of complexity in different dimensions
68 have been used to simulate earthquake cycles. They can be 0D (e.g., Madariaga, 1998;
69 Erickson et al., 2008), 1D (e.g., Burridge & Knopoff, 1967; Gu & Wong, 1991; Ohtani
70 et al., 2020), 2D (e.g., Lapusta et al., 2000; Van Dinther, Gerya, Dalguer, Mai, et al., 2013;
71 Herrendörfer et al., 2018), 2.5D (e.g., Lapusta, 2001; Weng & Ampuero, 2019; Preuss
72 et al., 2020) or 3D (e.g., Okubo, 1989; Lapusta & Liu, 2009; Barbot et al., 2012; Erick-
73 son & Dunham, 2014; Chemenda et al., 2016; Jiang & Lapusta, 2016). Generally, 3D mod-
74 els will produce results closest to nature among the listed methods. However, given that
75 they are still very time and energy consuming (Uphoff et al., 2017), simplified model se-
76 tups are still largely adopted by many researchers and may be the optimal choice to an-
77 swer specific research questions (e.g., Allison & Dunham, 2018; Cattania, 2019). A key

78 reason for the need of such simplifications is the extremely high resolution required in
79 both space and time, while at least exploring sensitivities in forward modeling studies
80 (Lambert & Lapusta, 2021). On top of that computational speed is particularly criti-
81 cal in situations where monotonous repetition of those forward models is required, for
82 example, for inversion, data assimilation, physics-based deep learning, uncertainty quan-
83 tification, and when dealing with probabilities, such as for probabilistic seismic hazard
84 assessment (e.g., Weiss et al., 2019; Van Dinther et al., 2019). However, also when try-
85 ing to understand coupled multi-physics or multi-scale feedback these approximations
86 can be really useful (Van Dinther, Gerya, Dalguer, Corbi, et al., 2013; Allison & Dun-
87 ham, 2018; Lotto et al., 2019; Ohtani et al., 2019; Petrini et al., 2020). To optimize com-
88 puting resources, researchers have to define suitable model complexities before and dur-
89 ing their numerical simulations. Therefore it becomes a common concern to what extent
90 lower dimensional models can reproduce nature when compared to 3D models. How are
91 the observed differences in results attributed to the corresponding dimension reduction?
92 And under what circumstances is this simplification justified?

93 These questions have not yet been systematically addressed. Nonetheless, several
94 contributions have considered various aspects of this problem, especially via the com-
95 parison between 2D and 3D models. Lapusta and Rice (2003); Kaneko et al. (2010); Chen
96 and Lapusta (2019) all suggested ways to interpret their 2D results in more realistic 3D
97 situations so that they can be directly compared to 3D results. By doing this, they could
98 compare velocity-strengthening (VS) barrier efficiency in rupture propagation, seismic
99 moment, and the scaling law for earthquake recurrence interval and seismic moment be-
100 tween 2D and 3D models in their studies. For the coseismic phase, simulations with dy-
101 namic rupture models of one single earthquake are generally conducted in 3D to obtain
102 a full view of fault plane and thus give not enough attention to 2D models, except for
103 the benchmark community. Harris et al. (2011) introduced two benchmark problems for
104 dynamic rupture modelers where 3D simulations produced smaller ground motions (peak
105 ground velocities) than the 2D simulations, in both elastic and plastic scenarios. Sev-
106 eral contributions have also been made by earthquake cycle modelers. Chen and Lapusta
107 (2009) suggested the 3D nucleation size would be larger than 2D by a factor of $\pi^2/4$. Chen
108 and Lapusta (2019) also noted that their 2D models did not produce earthquakes that
109 rupture only a part of the velocity-weakening (VW) patch, unlike what was happening
110 in 3D. However, these findings are in pieces and some of them are lacking necessary rea-

111 soning. Here we fill in this gap by comparing earthquake cycle results across all dimen-
112 sions from 0D to 3D, which includes all phases of the earthquake cycle, i.e., interseismic,
113 nucleation, coseismic and postseismic.

114 We perform a systematic investigation of limitations and advantages of each dimen-
115 sion. By doing so, we compare physical characteristics and importance of different phys-
116 ical processes across dimensions both qualitatively and quantitatively. The aim of this
117 paper is to serve as guidelines for modelers designing models and for all researchers in-
118 terpreting results developed under necessary limitations. We first introduce the numer-
119 ical method and the model setup of a strike-slip fault under rate-and-state friction. The
120 code package is validated and benchmarked by Southern California Earthquake Center
121 (SCEC) SEAS benchmark problems BP1-qd (Erickson, Jiang, Barall, Lapusta, et al., 2020)
122 and BP4-qd (Jiang et al., 2021) (see Supporting Information S1). Next, we systemat-
123 ically compare interseismic and coseismic characteristics of our models from 1D to 3D,
124 summarizing and quantifying their advantages and shortcomings. The numerical results
125 are explained and supported by a series of theoretical calculations. Finally the compu-
126 tational cost is compared. In the discussions, we first discuss under what conditions 2D
127 models can substitute 3D models. Related issues on the model choices of this research,
128 limitations and future improvements as well as possible applications are also discussed.

129 **2 Methods**

130 To readily build models in different dimensions we exploit the flexibility of *Gar-*
131 *net*, a recently developed code library for the parallel solution of coupled non-linear multi-
132 physics problems in earth sciences (C. C. Pranger, 2020). *Garnet* employs the classical
133 second-order accurate staggered grid finite difference discretization of PDEs in space, for-
134 mulated in a dimension-independent way and thus readily generalizable to higher or lower
135 spatial dimensions. It also includes adaptive time stepping schemes of various orders of
136 accuracy and other characteristics, all based on the linear multistep family of time dis-
137 cretizations. The library interfaces to PETSc (Balay et al., 1997, 2019b, 2019a) for lin-
138 ear and nonlinear solvers and preconditioners, to MPI (Forum, 2015) for coarse scale dis-
139 tributed memory parallelism and intermediate scale shared memory parallelism, and to
140 Kokkos (Edwards et al., 2014) (and in turn OpenMP, POSIX threads, or CUDA) for fine
141 scale concurrency. In this section we further introduce the equations and algorithms that
142 define our study. To enable general comparison with established implementations, we

143 take the 3D SCEC SEAS benchmark problem BP4-qd (Jiang et al., 2021) as a reference
 144 case for our models.

145 2.1 Physics

Under the assumption of static stress transfer, the momentum balance equation reads

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0, \quad (1)$$

where ρ is density, σ_{ij} denotes the stress tensor, and v_i denotes the material velocity in the direction x_i ($i = 1, 2, 3$). Both gravity and inertia are ignored in our models. We will revisit the assumption of static stress transfer throughout the remainder of this section. Hooke’s law in differential form relates stress rate $\dot{\sigma}$ to strain rate $\dot{\epsilon}$ by

$$\dot{\sigma}_{ij} = K \dot{\epsilon}_{kk} \delta_{ij} + 2G \left(\dot{\epsilon}_{ij} - \frac{1}{3} \dot{\epsilon}_{kk} \delta_{ij} \right) \quad (2)$$

with bulk modulus K , shear modulus G , and δ_{ij} Kronecker’s delta symbol. We assume infinitesimal strain rate $\dot{\epsilon}$ as defined by

$$\dot{\epsilon}_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right). \quad (3)$$

For a fault with unit normal vector \hat{n} , the (scalar) normal stress σ_n (positive in compression) is given by the projection $\sigma_n = -\hat{n} \cdot \boldsymbol{\sigma} \cdot \hat{n}$, the shear traction vector $\vec{\tau}_s$ by the projection $\vec{\tau}_s = \boldsymbol{\sigma} \cdot \hat{n} + \sigma_n \hat{n}$, the scalar shear traction τ_s by the Euclidean norm $\tau_s = \|\vec{\tau}_s\|$, and finally the unit fault tangent \hat{t} (which defines the orientation of the scalar fault slip V) by the normalization $\hat{t} = \vec{\tau}_s / \tau_s$, such that $\tau_s = \hat{t} \cdot \boldsymbol{\sigma} \cdot \hat{n}$. Further following Jiang et al. (2021), the fault is assumed to be governed by the rate-and-state friction law, which was initially proposed based on laboratory friction experiments by Dieterich (1979); Ruina (1983). We employ a regularization near zero slip velocity according to Rice and Ben-Zion (1996) and Ben-Zion and Rice (1997), so that the friction law that defines the relation between shear stress τ_s and normal stress σ_n on the fault is given by

$$\tau_s = a \sigma_n \operatorname{arcsinh} \left(\frac{V}{2V_0} \exp \left(\frac{\mu_0}{a} + \frac{b}{a} \ln \left(\frac{\theta V_0}{L} \right) \right) \right) + \eta V. \quad (4)$$

The ‘state’ θ in turn is governed by the evolution equation

$$\dot{\theta} = 1 - \frac{V\theta}{L}, \quad (5)$$

146 corresponding to the so-called ‘aging law’ (Ruina, 1983). Symbols used in (4) and (5)
 147 include the reference friction coefficient μ_0 , the reference slip rate V_0 , the characteris-

148 tic slip distance L , and the parameters $a > 0$ and b that control the relative influence
 149 of rate-strengthening and slip-weakening effects, respectively. The fault is velocity-weakening
 150 (VW) and potentially frictionally unstable when $a - b < 0$, and velocity-strengthening
 151 (VS) and generally frictionally stable when $a - b > 0$. Finally, the parameter η used
 152 in (4) refers to the ‘radiation damping term’ used in the quasi-dynamic (QD) approx-
 153 imation of inertia (e.g., Rice, 1993; Cochard & Madariaga, 1994; Ben-Zion & Rice, 1995;
 154 Liu & Rice, 2007; Crupi & Bizzarri, 2013) that is employed in earthquake cycle simu-
 155 lations to reduce the computational cost – even though it is known to introduce qual-
 156 itative and quantitative differences compared to fully dynamic (FD) modeling results (Thomas
 157 et al., 2014). The damping viscosity $\eta = G/(2c_s)$ is equal to half the shear impedance
 158 of the elastic material surrounding the fault.

159 The nonlinear friction law (4) and evolution law (5) are solved in a point-wise fash-
 160 ion using a Newton-Raphson iteration for the slip rate V at a given stress σ (algorithm
 161 flowchart in Fig. S1). The problem is closed with an essential velocity boundary con-
 162 ditions $\vec{v} = \frac{1}{2}V\hat{t}$ on the fault, and remaining initial and boundary conditions are given
 163 in upcoming sections.

164 2.2 Model setup

165 Over the last decade, the SCEC has supported various code comparison projects
 166 to verify numerical simulations on dynamic earthquake ruptures (e.g. Harris et al., 2009,
 167 2018). The SEAS benchmark project (Erickson, Jiang, Barall, Lapusta, et al., 2020; Jiang
 168 et al., 2021), launched in 2018, is an extension to evaluate the accuracy of numerical mod-
 169 els simulating earthquake cycles. This benchmark initiative provides us with a platform
 170 to verify the earthquake cycle implementation in *Garnet*. Therefore, we build our mod-
 171 els based on the setup of SEAS benchmark problem BP4-qd. This benchmark setup in
 172 turn facilitates the validation of our code package against other participating codes from
 173 the scientific community (see Supporting Information S1 and Jiang et al., 2021).

174 The BP4-qd describes a planar vertical fault embedded in a homogeneous, isotropic
 175 linear elastic medium, observing the physics described in section 2.1 (Fig. 1). The x, y, z
 176 axes are directions perpendicular to the fault plane, along the strike and along the dip,
 177 respectively. Following Jiang et al. (2021), the fault condition is prescribed at $x = 0$.
 178 The central part of the fault is assumed to follow the rate-and-state friction formulation

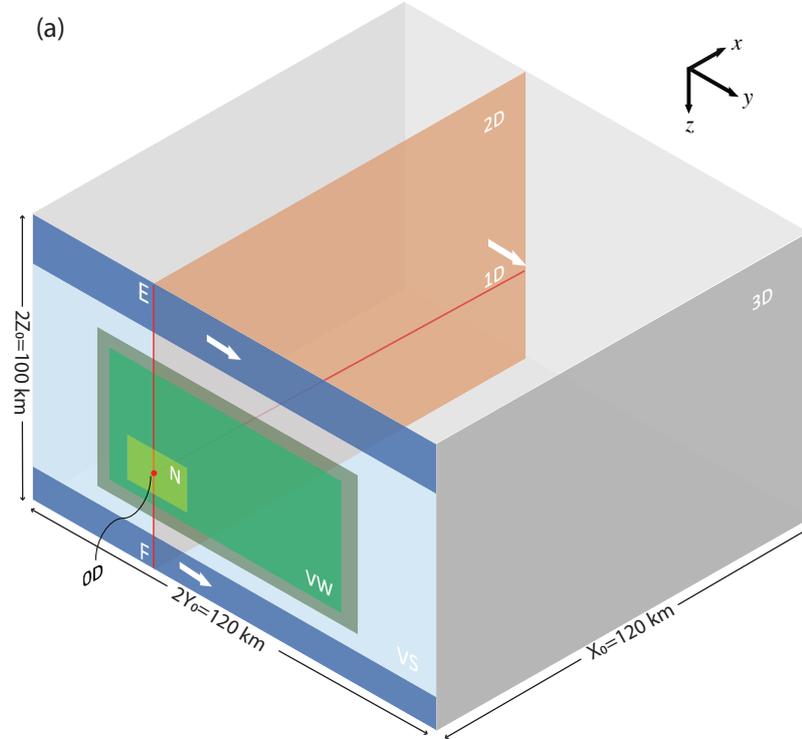


Figure 1. Numerical model setup of a vertical strike-slip fault embedded in an elastic medium: 3D setup of SEAS benchmark BP4-qd and its simplification to 2D, 1D and 0D. “VW” and “VS” denotes the VW (light green) and VS (light blue) patches, respectively. The transition between VW and VS patches is shown in dark green. Tectonic loading regions at the top and bottom of the fault (dark blue) are subjected to constant velocities (white arrows). “N” denotes the predefined nucleation zone (yellow) with higher initial slip rate and shear stress. Computational domain in 2D is reduced to xz -plane (orange) with 1D fault line “EF” along x -axis (red). Computational domain in 1D is reduced to x -axis (red) with 0D fault point “N” (red). In this case tectonic loading is applied at the far-away end with constant velocity (white arrow). Computational domain in 0D is fault point “N” without medium extent.

179 where a VW region is surrounded by a VS region. The top and bottom parts of the fault
 180 are not governed by rate and state friction and are instead subjected to a constant fault-
 181 parallel loading velocity V_p . Due to the symmetry respective to the fault plane and the
 182 resulting anti-symmetry of fault-parallel motion, the motion at the fault is taken to be
 183 relative to a fictitious oppositely moving domain that is not modeled. The computational
 184 domain is thus limited to the half space $x \geq 0$.

185 Since the benchmark proposes an infinitely large half space, the computational do-
 186 main needs to be truncated to a finite domain when using a volumetric discretization.
 187 We use the computational domain $\Omega(x, y, z) = [0, X_0] \times [-Y_0, Y_0] \times [-Z_0, Z_0]$ (Fig. 1),
 188 where X_0, Y_0, Z_0 are chosen sufficiently large to have negligible impact on the fault be-
 189 havior (Jiang et al., 2021, see also Fig. S2). The top and bottom boundaries $z = \pm Z_0$
 190 are prescribed to move at the same constant loading velocity V_p . The remaining three
 191 boundaries $x = X_0, y = -Y_0, y = Y_0$ mimic the conditions at infinity and are set to
 192 be traction-free.

The initial conditions are chosen to allow the fault to creep at the imposed slip ve-
 locity V_p in a steady state at $t = 0$ (Jiang et al., 2021), namely

$$\theta(t = 0) = \frac{L}{V_p}, \quad (6)$$

and

$$\tau_s(t = 0) = a\sigma_n \operatorname{arcsinh} \left(\frac{V_p}{2V_0} \exp \left(\frac{\mu_0}{a} + \frac{b}{a} \ln \left(\frac{V_0}{V_p} \right) \right) \right) + \eta V_p. \quad (7)$$

We define a highly stressed zone ‘‘N’’ in the VW patch with higher initial slip velocity
 V_i (Fig. 1) to ensure the first earthquake nucleates at that location when the computa-
 tion starts. For this zone, the state variable θ keeps unchanged to achieve the high pre-
 stress, namely

$$\tau_s((y, z) \in N, t = 0) = a\sigma_n \operatorname{arcsinh} \left(\frac{V_i}{2V_0} \exp \left(\frac{\mu_0}{a} + \frac{b}{a} \ln \left(\frac{V_0}{V_p} \right) \right) \right) + \eta V_i. \quad (8)$$

193 This helps us to better compare the coseismic behavior across dimensions. All physical
 194 and numerical parameters are summarized in Table 1.

195 2.3 Spatial and temporal discretization

We choose a spatial discretization that ensures that the smallest physical length
 scale in the rate and state friction model – the cohesive zone size Λ – is always well re-
 solved. This cohesive zone size Λ (Rubin & Ampuero, 2005; Day et al., 2005) is given

Table 1. Physical and numerical parameters

Parameter	Symbol	Value
Density	ρ	2.670 g/cm ³
Shear wave speed	c_s	3.464 km/s
Poisson ratio	ν	0.25
Shear modulus	G	32.0 GPa
Bulk modulus	K	53.4 GPa
Normal stress	σ_n	50 MPa
Loading rate	V_p	10 ⁻⁹ m/s
Width of rate-and-state fault	W_f	80 km
Length of uniform VW region	l	60 km
Width of uniform VW region	H	30 km
Width of VW-VS transition zone	h	3 km
Reference friction coefficient	μ_0	0.6
Reference slip rate	V_0	10 ⁻⁶ m/s
Characteristic slip distance	L	0.04 m
Rate-and-state direct effect	a	
- VW		0.0065
- VS		0.025
Rate-and-state evolution effect	b	0.013
Width of predefined nucleation zone “N”	w_i	12 km
Distance of nucleation zone to boundary	h_i	1.5 km
Initial slip rate		
- inside nucleation zone	V_i	10 ⁻³ m/s
- outside nucleation zone	V_p	10 ⁻⁹ m/s
Medium extent perpendicular to fault	X_0	<i>40/80/120</i> ^a km
Half fault extent along strike	Y_0	<i>60/90</i> ^a km
Half fault extent along dip	Z_0	<i>50/60</i> ^a km
Grid size	Δx	<i>500/1000</i> ^a m

^a Numbers in italic are used in parameter studies.

by

$$\Lambda = \Lambda_0 \sqrt{1 - \frac{V_r^2}{c_s^2}},$$

$$\Lambda_0 = \frac{9\pi}{32} \frac{GL}{b(1-\nu)\sigma_n},$$

196 where V_r is the rupture speed and c_s is the shear wave speed. Λ_0 is the upper limit of
 197 the cohesive zone size when $V_r \rightarrow 0$. The dynamic cohesive zone size Λ shrinks with
 198 increasing rupture speed V_r . We find that a high resolution is required for the seismo-
 199 genic domain and its neighboring off-fault area, while it is not required at medium to
 200 large distances to the fault. To save time and energy, we consider a grid that is stati-
 201 cally refined near the VW zone. Refinement is realized by deforming the regular grid and
 202 writing the governing equations in general rectilinear coordinates, thus preserving the
 203 2nd-order accuracy of the numerical method (C. C. Pranger, 2020).

We use adaptive time stepping to deal with the strong variation of the slip veloc-
 ity and state variables in between interseismic and coseismic phases. The critically res-
 olvable time scale is due to the evolution of the friction law (Eq. 5). Following Lapusta
 et al. (2000), we let the time step Δt be given by

$$\Delta t = \min \left\{ \zeta \frac{L}{V_{\max}}, (1 + \alpha)\Delta t_{\text{old}}, \Delta t_{\max} \right\}. \quad (9)$$

204 where ζ is a factor controlled by the material and frictional parameters. We also require
 205 the next time step not to be larger than $(1+\alpha)$ times the former time step Δt_{old} to avoid
 206 instability in the postseismic phase. A maximum time step size Δt_{\max} is further needed
 207 to keep resolving the interseismic period in sufficient detail.

208 2.4 Model simplification by progressive elimination of dimensions

209 In this work we take a structured approach to dimension reduction, eliminating first
 210 the lateral along-strike dimension, then the vertical dimension, and finally the fault-perpendicular
 211 dimension. Each of these steps are illustrated in Fig. 1. For clarity, the assumptions and
 212 variables concerned in each dimension are summarized in Table 2.

213 In 2D, the model is simplified by excluding the along-strike fault direction (denoted
 214 in orange in Fig. 1). This means that the material and frictional properties, boundary
 215 and initial conditions are assumed to be homogeneous in this direction. That assump-
 216 tion thus omits the along-strike heterogeneity introduced by the bounding VS patches

217 as well. Furthermore, motion along the dip v_z is omitted. In this way, any half plane cut-
 218 ting the fault vertically may be taken as representative of the the entire model. The com-
 219 putational domain can thus be reduced to $\Omega(x, z) = [0, X_0] \times [-Z_0, Z_0]$. As a conse-
 220 quence, only the σ_{xy} and σ_{yz} components of the stress tensor are required to be eval-
 221 uated in this anti-plane strain model. To allow a coseismic comparison we keep there the
 222 highly stressed nucleation zone defined in 3D and choose to model the plane cutting across
 223 this zone. The fault is collapsed to the line “EF” (denoted in red in Fig. 1). Another com-
 224 mon 2D perspective includes the in-plane strain assumption that models motion in a hor-
 225 izontal plane cutting the fault. While this configuration models a more complete set of
 226 momentum balance and elastic constitutive equations than the out-of-plane configura-
 227 tion we have chosen, the differences are only expected to manifest as a slightly modified
 228 elastic loading and corresponding changes in friction and nucleation size. We therefore
 229 choose to use the vertical 2D configuration that keeps the top/bottom loading regions
 230 for better comparison.

231 In 1D, we further simplify the model by letting all fields be invariant along dip in
 232 which case only the shear stress component σ_{xy} and the velocity component v_y remain.
 233 We thus lose the possibility to model spatial variations of frictional properties as the fault
 234 reduces to a 0D point at $x = 0$ in the computational domain $\Omega(x) = [0, X_0]$. We choose
 235 the fault “point” to be velocity weakening, corresponding to a location inside the pre-
 236 defined nucleation zone at “N” to facilitate coseismic comparison (denoted in red in Fig. 1).
 237 In this model we lose the along-dip fault extent, so that the original on-fault tectonic load-
 238 ing from the top and bottom is no longer possible. Instead it is added at the far-away
 239 boundary with a constant creeping rate there. To achieve a comparable tectonic load-
 240 ing rate inside the VW patch across dimensions, we adjust the domain size X_0 so that
 241 the shortest distance between the VW patch and the creeping boundary is the same as
 242 in higher dimensional models. Namely, we let X_0 equal to $(W_f - H)/2$.

243 In 0D, both the medium and the fault become the same point. In this model with-
 244 out medium extent, physical loading at medium boundaries is also impossible. There-
 245 fore a driving force that can be chosen arbitrarily has to be added to the system instead.
 246 This model will be further discussed in section 4.2 where the equivalence of 1D and 0D
 247 models will be illustrated.

Table 2. Simplifications in different dimensional models

Dimension	Unknowns	Simplifications
3D	$V, \theta; v_x, v_y, v_z, \sigma_{xx}, \sigma_{xy}, \sigma_{xz}, \sigma_{yy}, \sigma_{yz}, \sigma_{zz}$	No fault opening
2D	$V, \theta; v_y, \sigma_{xy}, \sigma_{yz}$	+ strike-slip only, along-strike invariant
1D	$V, \theta; v_y, \sigma_{xy}$	+ along-dip invariant
0D	V, θ	+ integral perpendicular to fault

3 Results and Analysis

Following the simplifications summarized in Table 2 and Fig. 1, this section compares the 3D to 2D and 1D results, where the fault is modeled in 2D, 1D and 0D, respectively. Starting from the long-term observations of the earthquake cycles, we compare the interseismic phase across dimensions. Facilitated by the same initial conditions and predefined nucleation zone, we then compare the coseismic phase by the observations of the first earthquake.

3.1 Interseismic phase

Regardless of dimension, we observe quasi-periodic earthquake sequences (Fig. 2). In one earthquake cycle, shear stress is first accumulated from minimum 25 MPa to maximum 35-42 MPa during the interseismic phase and then released in an earthquake (Fig. 2b). Accordingly, slip velocity also increases from locked rates of 10^{-17} m/s in 2/3D and 10^{-20} m/s in 1D to seismic rate 10^0 m/s at the same time (Fig. 2a).

By dimension reduction, our simulated earthquakes become more and more characteristic. In 3D, all simulated earthquakes nucleate from one corner of the rectangular VW zone and rupture throughout it until the rupture front reaches the transition to the VS zone. However, not all earthquakes initiate from the same nucleation zone, as is suggested by the slip profile (Fig. 3a). Rather, the nucleation location alternates between the top-left and bottom-right corners, resulting in a periodic cycle of two earthquakes with slightly different slip and recurrence interval. Similar results in 3D of two or more characteristic earthquakes repeating as a group have also been reported by Barbot (2019), where several possible mechanisms are suggested for this poorly understood phenomenon.

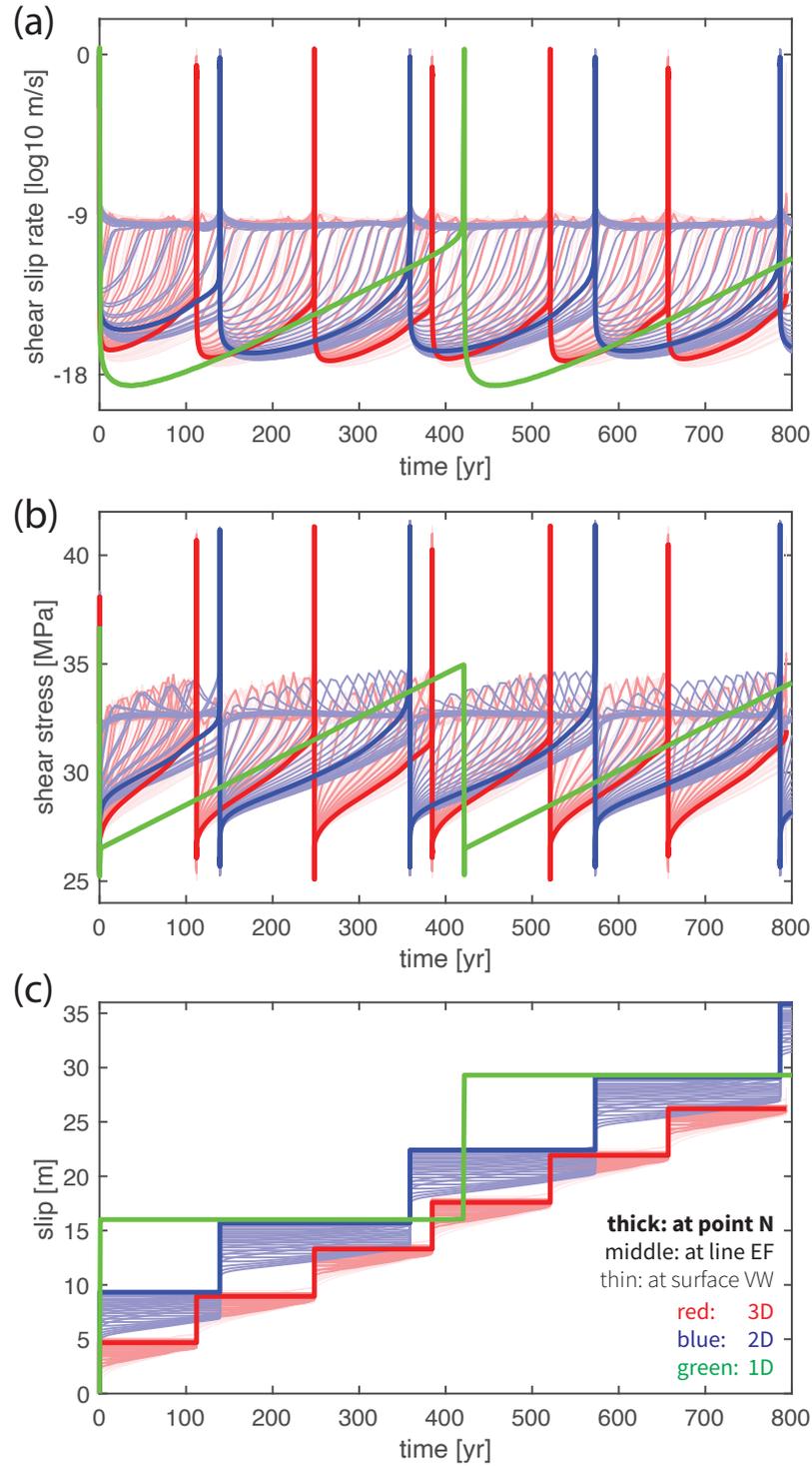


Figure 2. Comparison of the long-term time series of (a) slip rate, (b) stress and (c) accumulated slip in 1-3D models. The lines with different thicknesses and degrees of transparency are recorded at different locations on the fault, where the thick lines are recorded at the rim of the nucleation zone “N” of the sixth earthquake, the semi-thick lines along the line “EF” cutting across “N” vertically and the thin lines elsewhere in the VW patch (see Fig. 6).

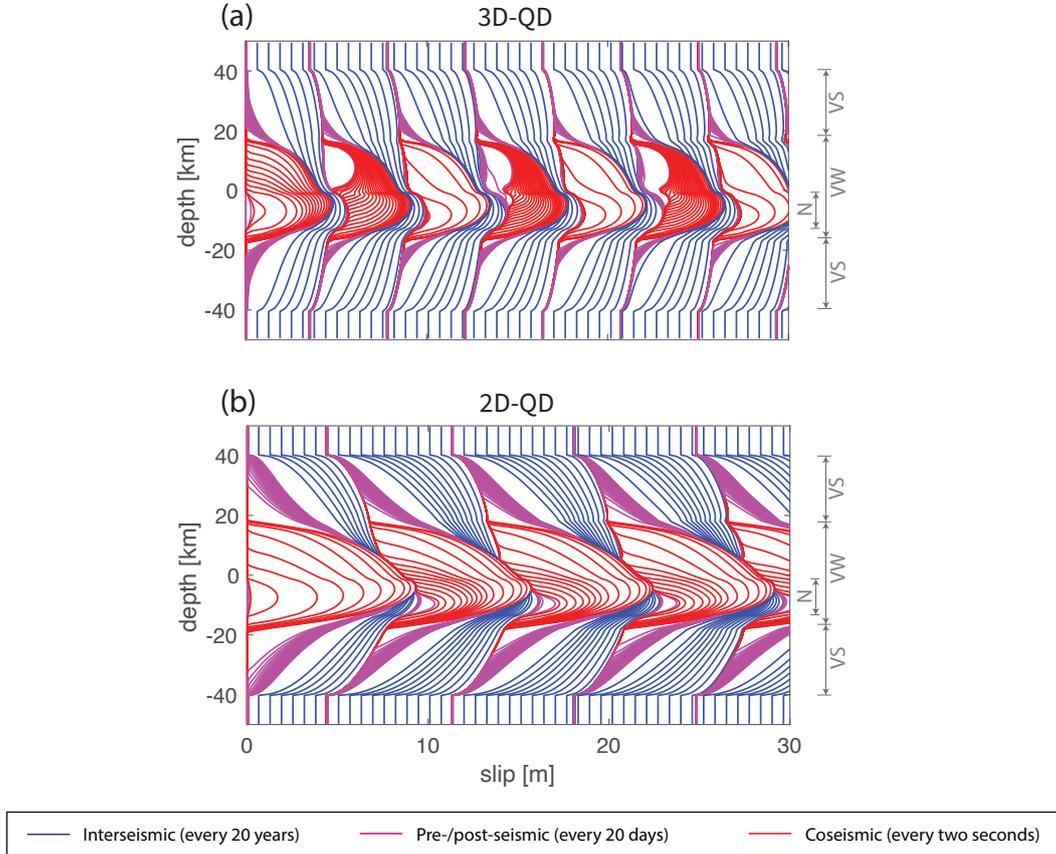


Figure 3. Cross-dimensional comparison of cumulative seismic and aseismic slip. The cumulative slip profile of (a) the 3D model and (b) the 2D model, along the dip direction “EF” cutting across the predefined nucleation zone “N” (see Fig. 1). “VW”, “VS”, “N” label the range of VW, VS and predefined nucleation zone. The interseismic phase is plotted every 20 years (blue), the pre- and post-seismic phase every 20 days (magenta) and the coseismic rupture every two seconds (red). Note that the slip contour distortions around a depth of -1.5 km and -13.5 km are introduced into these cumulative patterns by the predefined nucleation zone, whose properties increased the amount of slip in that zone for the first earthquake only.

270 In 2D, we find that the earthquakes are more periodic because they all nucleate from the
 271 same down-dip limit of the VW patch and rupture towards the up-dip limit, instead of
 272 alternately nucleating from the top and bottom sides (Fig. 3b). The earthquake size is
 273 also more identical with same recurrence interval. In 1D, we observe purely periodic, char-
 274 acteristic earthquakes of the same size (Fig. 2). This trend is because with fewer dimen-
 275 sions, interseismic loading pattern to the VW patch becomes simpler, so that the poten-
 276 tial nucleation locations are also reduced. In 3D, earthquakes can potentially nucleate
 277 from four corners of the VW patch, which is reduced to two (top and bottom) in 2D and
 278 one in 1D. These observations demonstrate that as spatial dimensions are eliminated,
 279 the simulated results often exhibit a simpler spatio-temporal behavior.

280 Interseismic slip velocity and shear stress evolution depends on whether the obser-
 281 vational point is inside the nucleation zone, at the nucleation rim (denoted by “N” in
 282 Fig. 6), or outside the nucleation zone (Fig. 2). Inside the nucleation zone tectonic load-
 283 ing is faster, therefore this portion of fault starts to creep at loading rate earlier. In the
 284 meantime, shear stress reaches its peak and gradually falls back to the steady-state level.
 285 Both slip velocity and shear stress are kept at this steady-state until the nucleation zone
 286 expands large enough (middle to thin lines that are to the left and above the thickest
 287 line in Fig. 2a, b). Outside the nucleation zone, at a point closer to the central VW patch
 288 that experiences slower loading, slip velocity and shear stress increase more slowly. This
 289 fault portion remains locked before the start of the next earthquake, i.e., slip velocity
 290 is always smaller than loading rate and shear stress lower than the aforementioned steady-
 291 state stress level (middle to thin lines that are to the right and below the thickest line
 292 in Fig. 2a, b). Only at the rim of the nucleation zone slip velocity and shear stress in-
 293 crease at a unique rate that allows for earthquake to occur as soon as respectively the
 294 loading rate and interface strength (as defined by Nakatani (2001)) are reached. As a
 295 result, no aseismic slip is accumulated at this location before earthquake starts (thick-
 296 est lines in Fig. 2). These three patterns are shared in 2D and 3D models even though
 297 their nucleation zone shape and size are different. The 1D model with a 0D fault “point”
 298 mimics the nucleation rim of the higher dimensional models because slip initially becomes
 299 seismic without preceding aseismic accumulation. This is because in 1D an earthquake
 300 nucleates instantaneously as all points on the simulated fault plane reach the interface
 301 strength at the same time. We will see how the simple pattern of shear stress accumu-
 302 lation at this location helps in the later theoretical calculations.

303 By dimension reduction, simulated earthquakes reach larger slip and longer recur-
 304 rence interval (Fig. 3). Different points of the fault experiences larger or smaller seismic
 305 slip during the earthquake, but the total slip (i.e., seismic slip + aseismic slip) in one
 306 earthquake cycle is generally constant throughout the fault plane (Fig. 3). It is also equal
 307 to the maximum coseismic slip because the maximum is achieved where the fault por-
 308 tion is locked outside the coseismic phase. This makes it, together with earthquake re-
 309 currence interval, a good indicator of the long-term earthquake cycle characteristics. In
 310 3D, we observe earthquakes with average total slip of ~ 4.5 m and recurrence interval
 311 of ~ 135 yr (Fig. 3a). In 2D, the simulated values are ~ 6.8 m and ~ 215 yr, respectively,
 312 i.e., about 50% larger than in 3D (Fig. 3b). In 1D, they are 13.3 m and 420 yr, respec-
 313 tively, about three times as large as the 3D results and twice the 2D results (Fig. 2c).
 314 Note that in these numbers we excluded the slightly larger first earthquake that initi-
 315 ated at the predefined nucleation zone without tectonic loading.

316 These interseismic differences can largely be explained by the reduced presence of
 317 VS patches due to dimension reduction. During the interseismic phase, the VS patches
 318 are creeping at the loading rate so they do not accumulate stress. They only play a role
 319 in transferring the tectonic loading from the loading boundaries into the VW patch they
 320 surround. In other words, the VW patch is loaded directly by its surrounding VS patches
 321 rather than the loading boundaries, whether the bulk medium is simulated explicitly or
 322 not. This clarification is fundamental because in this way the VW patch in 3D is loaded
 323 from four sides, rather than only from the top/bottom where tectonic loading regions
 324 are located. While the VW patch in 2D is loaded from two sides, resulting in a lower in-
 325 terseismic loading rate inside the VW patch and hence a longer period before the next
 326 earthquake can nucleate. Given that the constant creeping rate in the VS patches is un-
 327 changed, the resulting larger slip deficit in the VW patch has to be made up by an earth-
 328 quake with more slip. This is why larger earthquakes are observed in 2D. Quantitative
 329 calculations based on theoretical considerations, proving the analysis above, will follow
 330 in section 3.4.

331 That clarification also implies that the tectonic loading in the VW patch depends
 332 on the size of the VW patch itself instead of the size of the VS patches or the distance
 333 of the loading boundaries. The smaller the VW patch is, on average the stronger the load-
 334 ing will be. Therefore in 2/3D models the VW patch is actually not loaded at a distance
 335 from the predefined loading boundary but from much closer. This is simply not possi-

336 ble in 1D without VS patches. Even though in this case the distance between the VW
 337 fault and the far-away loading boundary is already chosen to be the same as in higher
 338 dimensions (in section 2.4) to make the stress rate directly caused by the loading bound-
 339 aries comparable, the actual stress rate is proved to be inadequate. This is why larger
 340 slip and longer recurrence interval are still observed in 1D. Note that this explanation
 341 also suggests that different types of tectonic loading realization do not influence much
 342 the perceived stress rate inside the VW patch as long as it is surrounded by VS patches.
 343 This is supported by additional models with different types of loading at various distances
 344 from the fault, where similar earthquake recurrence intervals are obtained as long as the
 345 domain size is large enough (see Discussions and Table S1).

346 **3.2 Observations of the first earthquake**

347 We first analyze the coseismic behavior of the first earthquake where we have pre-
 348 defined the same nucleation zone as initial condition (Fig. 4a, c, e). Here we focus on
 349 the dynamic rupture behavior and use dimensional comparison to provide guidelines for
 350 dynamic earthquake rupture models. We then look at the sixth earthquake, a charac-
 351 teristic earthquake in the sequence, that experiences physical interseismic and nucleation
 352 processes (Fig. 4b, d, f). Via comparison to the first earthquake, we focus on the influ-
 353 ence of the interseismic tectonic loading to the coseismic phase.

354 For the first event (Fig. 4a, c, e), the source time function at all locations within
 355 the VW patch takes the shape of Kostrov’s solution (Kostrov & Das, 1988) with a short
 356 rise time and relatively long deceleration tail. As dimensions are reduced, the duration
 357 of the rise time decreases while the duration of the deceleration increases. The deceler-
 358 ation in 1D is the slowest, since the rupture does not interact with patches of different
 359 stress or strength properties that could decelerate it. For the same reason, it is impos-
 360 sible to observe rupture reflections in 1D. While the rupture reflection from the VW-VS
 361 boundary in 3D is clearly observable as a second slip velocity peak (Fig. 4a).

362 The peak slip velocity and the rupture speed are important earthquake character-
 363 istics that closely relate to rupture area size and seismic moment. We observe that peak
 364 slip velocities are all around the same order of magnitude around 10^0 m/s, but in 3D $<$
 365 $2D < 1D$ (Fig. 4a). The local peak slip velocity also increases while getting away from
 366 nucleation center. In 3D, the peak slip velocity is initially ~ 0.8 m/s in the predefined

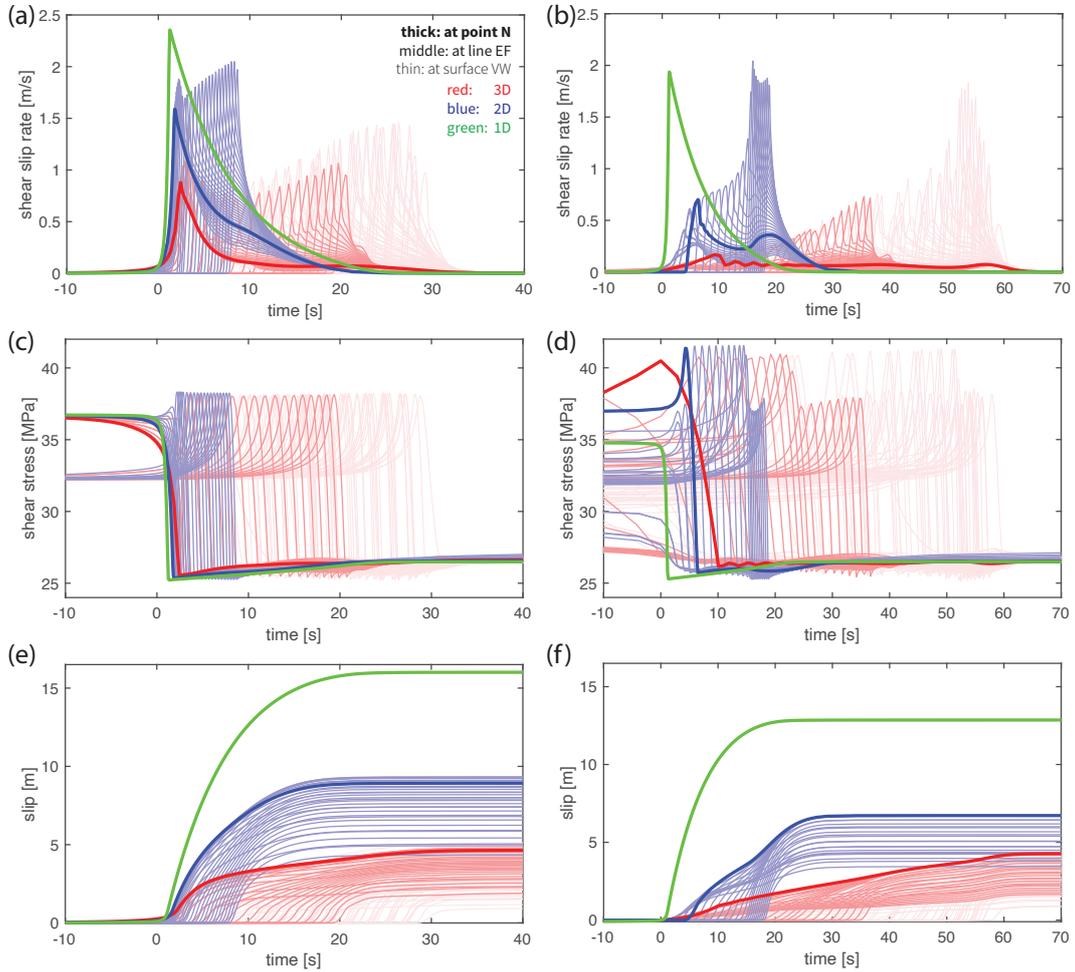


Figure 4. Comparison of the coseismic time series of (a, b) slip rate, (c, d) stress and (e, f) accumulated slip in 1-3D models. The first earthquake is shown in (a, c, e), and the sixth earthquake is shown in (b, d, f), where origin time is set at the onset of the respective earthquake. The lines with different thicknesses and degrees of transparency are recorded at different locations on the fault, where the thick lines are recorded at the nucleation location “N”, the semi-thick lines along the line “EF” cutting across “N” vertically and the thin lines elsewhere in the VW patch (see Fig. 6a, c).

367 nucleation zone and gradually increases to its global maximum of ~ 1.5 m/s. In 2D, the
 368 peak slip velocity is ~ 1.6 m/s at the beginning and gradually increases to ~ 2.0 m/s.
 369 In 1D, the maximum slip velocity is ~ 2.4 m/s. We explain this increase with dimen-
 370 sion reduction by considering the 2D models in a 3D perspective, where the 1D fault “line”
 371 is extended along strike to form a 2D fault surface in which the VW patch is infinitely
 372 long (e.g., Andrews et al., 2007). More importantly, every portion of the fault along this
 373 direction has to start to rupture at the same time and has the same rupture pattern. There-
 374 fore, contrary to 3D, no fracture energy is needed to rupture the unbroken part along
 375 strike, not to mention the inexistence of VS patches to absorb energy. Consequently en-
 376 ergy is saved to achieve higher slip velocities. The same consideration also applies when
 377 considering the formulation of the 1D model in a 3D perspective, where the 0D fault “point”
 378 is extended to form an infinitely large, 2D fully-VW fault plane. Again, every portion
 379 of the fault has to rupture simultaneously with the same slip velocity and reach the same
 380 yield stress. No energy transfer is required at all to rupture the fault in both dip and
 381 strike directions and thus propagate the earthquake across it. Similarly, higher slip ve-
 382 locities are achieved (Kanamori & Rivera, 2006).

383 This explanation also suggests that the commonly observed periodic slow slip events
 384 cannot be reproduced in 1D models. When considered in the 3D perspective, the infinitely
 385 large VW patch namely leads to an infinite ratio of VW patch size (H) over nucleation
 386 size (h^*). While it is known that large enough H/h^* ratios always lead to seismic slip
 387 rates (Liu & Rice, 2007; Herrendörfer et al., 2018). This is because in this case the nu-
 388 cleation zone suddenly becomes infinitely large as soon as the 1D fault “point” starts to
 389 nucleate. This instability unavoidably leads to an earthquake (i.e., slip at seismic rate)
 390 instead of any slow slip events without additional damping. This extension is supported
 391 by a parameter study of hundreds of models in which no suitable frictional parameters
 392 are found allowing non-decaying slow-slip event simulation in 1D (Diab-Montero et al.,
 393 in prep).

394 Rupture speed across dimensions shows larger variation than peak slip velocity. In
 395 3D, the coseismic rupture lasts for ~ 30 s. The rupture propagates faster in the hori-
 396 zontal direction than in the vertical direction and it experiences an acceleration in the
 397 last ~ 10 s to reach near-shear speed (Fig. 5a). The rupture front takes ~ 20 s to prop-
 398 agate along the vertical line “EF”, at a near-constant speed of ~ 0.83 km/s except for
 399 the first several seconds and the arrest. In 2D, the rupture takes only ~ 10 s to reach

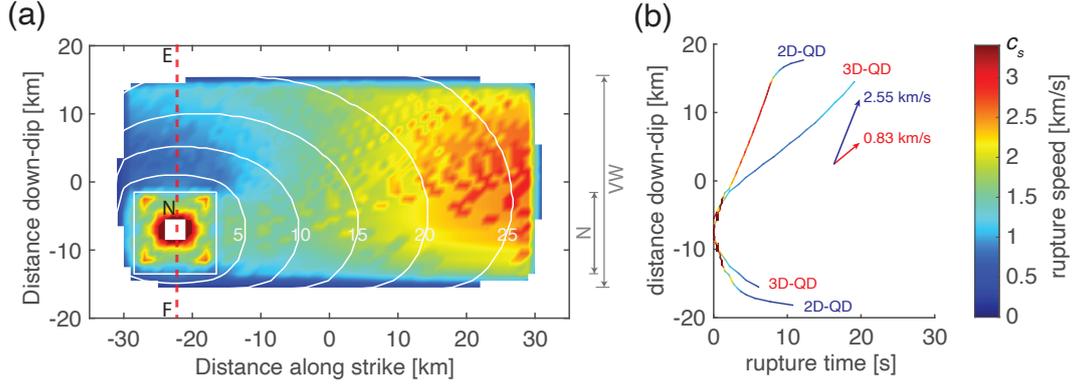


Figure 5. Comparison of coseismic rupture propagation. (a) The coseismic rupture speed of the first earthquake in 3D. The arrival time of the coseismic rupture front, which is measured when slip velocity reaching the seismic limit, is plotted every five seconds as contours. The central part of the fault plane is shown where white color means no seismic slip is observed. The red dashed line labels the observation line “EF” introduced in Fig. 1. Note that no reliable rupture speed is measured at rupture onset (left white near “N”). (b) The coseismic rupture front arrival time along the vertical line “EF” in 2D and 3D. The line color indicates the rupture speed under the same color scale as (a). Lines end at where slip rates drop below seismic threshold. The average rupture speed in the middle of propagation (i.e., except during nucleation and arrest) is measured as stated.

400 the up-dip limit starting from the same nucleation region (Fig. 5b). Thus coseismic du-
 401 ration is about 50% shorter than in 3D. Accordingly, the rupture speed of the stable part
 402 is ~ 2.55 km/s, almost three times higher than in 3D. To explain these differences in rup-
 403 ture speed the same consideration used to explain peak slip velocities differences can be
 404 applied. In 2D models, no fracture energy needs to be overcome to rupture into the strike
 405 direction and hence more energy can be directed along dip, which allows the rupture to
 406 achieve higher speeds and thus shortens the rupture duration. For this reason 2D mod-
 407 els are also seen rupturing deeper into the surrounding VS patches than 3D models. Given
 408 the difference between 2D and 3D models locate in the horizontal direction while the ver-
 409 tical direction keeps identical, our results suggest that the (in)existence of the horizon-
 410 tal VS patches has influence on the coseismic rupture behavior inside the VW patch, even
 411 in the vertical direction. We find this even evident in additional models where a second
 412 rupture deceleration can be observed if the length of the VW patch is shortened to one
 413 fourth (see Discussions, Fig. 9c, h, i).

414 The stress drop $\Delta\tau$, the stress difference between the start and the end of an earth-
 415 quake, and the fracture energy G_c , the surface area below the stress w.r.t slip profile (Fig. 6b,
 416 d) are important earthquake parameters. Given the same initial condition, the stress drop
 417 and fracture energy of the first earthquake are comparable in all dimensional models, both
 418 inside and outside the prestressed zone (Fig. 6b). Regardless of dimension and at all VW
 419 locations we first observe the shear stress increasing up to the yield stress and then it
 420 drops to a constant level corresponding to dynamic friction (Fig. 4c). Both the yield stress
 421 and the dynamic stress are comparable across dimensions. Therefore the difference be-
 422 tween the two (i.e., strength excess + stress drop, also called breakdown stress drop $\Delta\tau_b$)
 423 is also similar. Note that the initial stress increase is not as large when getting close to
 424 the nucleation zone and it is nearly zero inside it. This shows that the nucleation zone
 425 has to reach the yield stress before the coseismic phase, which is usually lower compar-
 426 ing to the maximum achievable yield stress elsewhere. After the stress drop we then im-
 427 mediately observe a small stress increase that is similar in size across dimensions (Fig. 4c),
 428 as a result of momentum conservation following the stress drop at neighbouring locations.
 429 This is the transition from dynamic stress drop to static stress drop (e.g., Madariaga,
 430 1976). The dynamic stress drop at different locations is accompanied by a similar size
 431 of slip (Fig. 6b), regarded as the characteristic weakening distance by the slip-weakening
 432 theory. After this distance, coseismic slip continues to accumulate until earthquake ar-

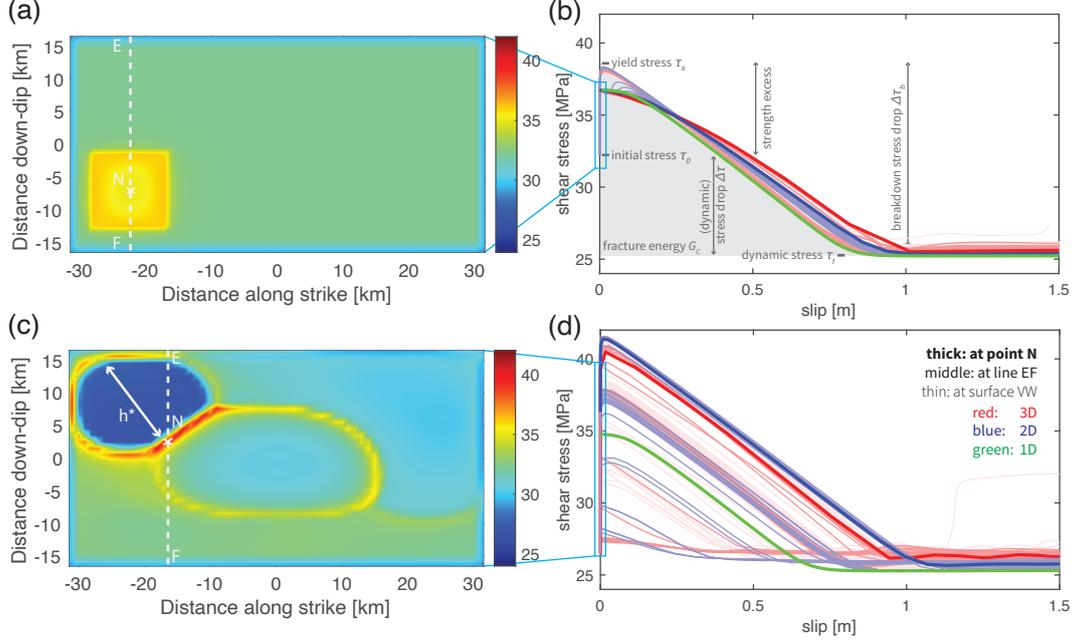


Figure 6. Cross-dimensional comparison of (a, c) the initial stress and (b, d) the coseismic stress evolution w.r.t. slip in 1-3D models for (a, b) the first earthquake and (c, d) the sixth earthquake. (a, c) The initial stress is measured when the maximum slip velocity reaches the seismic threshold. The nucleation size is denoted as h^* . Due to the high prestress, the coseismic slip of the first earthquake begins from the center of the nucleation zone. Whereas the coseismic slip of the sixth earthquake begins at the rim of the nucleation zone. Both are denoted by label “N”. (b, d) The lines with different thicknesses and degrees of transparency are recorded at different locations on the fault, where the thick lines are recorded at point “N”, the semi-thick lines along the line “EF” and the thin lines elsewhere in the VW patch (see panels a, c, respectively).

433 rests. The slip-weakening distance we measured here is between 0.8 m and 1.1 m, with
 434 3D the longest and 1D the shortest. As a result, the fracture energy G_c (Fig. 6b) is also
 435 measured to be similar at all VW locations, and with $3D > 2D > 1D$.

436 The differences of stress drop and fracture energy across dimensions are not strong,
 437 which is in line with expectations, since these earthquake parameters are largely controlled
 438 by the frictional properties and the normal stress (e.g., Rubin & Ampuero, 2005) that
 439 are homogeneous in this model. However, the modest systematic differences in, for ex-
 440 ample, the effective slip weakening distance shortening with dimension reduction, still
 441 indicates that the dynamics on the fault play a role in redistributing the earthquake en-
 442 ergy budget, so that the stress drop and the slip weakening distance can change accord-
 443 ingly. This is more evident when the fault is shorted to one fourth its width where yield
 444 stress is observed decreasing while rupture propagates (see Discussions, Fig. 9j-m). In
 445 this case we also find that the observed yield stress decrease is accompanied with rup-
 446 ture deceleration. Lapusta and Liu (2009) observed the other two scenario's: they showed
 447 that the yield stress and slip-weakening distance increase with rupture acceleration while
 448 they are near constant when rupture propagates steadily.

449 **3.3 Later earthquakes**

450 For later earthquakes experiencing physical interseismic and nucleation processes,
 451 the comparison regarding the rupture speed and slip velocity remains qualitatively valid
 452 (Fig. 4b, d, f). However, we should point out that the rupture speed is overall about 50%
 453 slower than the first earthquake, resulting in twice as long rupture duration in both 2D
 454 and 3D models (Fig. 4b vs. a). This is because the central VW patch has been locked
 455 during the preceding interseismic phase during which it is healed to a much higher in-
 456 terface strength than its surrounding (Fig. 6c). The high interface strength limits rup-
 457 ture propagation into it. Not only the rupture speed is slowed down, but also the peak
 458 slip velocity is suppressed during this period of passing a high strength patch. Only un-
 459 til the rupture front has passed by it and is closer to the VW-VS transition can we ob-
 460 serve the rupture speed and peak slip velocity increasing again. Combining lower slip
 461 velocity and longer coseismic duration, the accumulated seismic slip is observed to be
 462 smaller than the first earthquake (Fig. 3, 4e vs. f). Given that the initial pre-stressed
 463 zone increases the average initial stress of the first earthquake and thus increases stress

464 drop, it is expected the first event can have higher seismic moment and thus larger slip
 465 than later ones.

466 Furthermore, the stress-slip profile and fracture energy are no longer near-identical
 467 throughout the VW patch due to heterogeneous initial stress and yield stress distribu-
 468 tion (Fig. 4d, 6d). Instead of being predefined uniform, the initial stress in later earth-
 469 quakes are the result of the uneven interseismic tectonic loading and the nucleation pro-
 470 cess. The nucleation zone thereby has the lowest initial stress, whereas its rim has the
 471 highest values close to yield stress (Fig. 6c). Given the same level of dynamic stress af-
 472 ter the earthquake (Fig. 6d), this nonuniform initial stress field also results in a nonuni-
 473 form stress drop $\Delta\tau$. In addition, we observe the yield stress changes during rupture prop-
 474 agation, making the breakdown stress drop $\Delta\tau_b$ nonuniform as well (Fig. 4d, 6d). Com-
 475 pared to the first earthquake, the yield stress becomes higher near the central VW patch
 476 and lower closer to the VW-VS transition and it becomes lower on average. The frac-
 477 ture energy varies accordingly. This observation is consistent with the explanation above
 478 where we mentioned the central VW patch is hard to rupture into due to its high inter-
 479 face strength. By comparing to the first earthquake we have illustrated the importance
 480 of earthquake/loading history to the coseismic process in modifying the stress and en-
 481 ergy profile. We see that even the yield stress cannot be simply defined by the frictional
 482 properties. The 1D model, lacking the space for nucleation and dynamic rupture, never
 483 reaches the predefined high stress again in later earthquakes. Although we stated ear-
 484 lier that 1D model mimics the nucleation rim of 2/3D models in the long term, lacking
 485 high enough yield stress makes it dissimilar to 2/3D simulations in the coseismic phase.

486 **3.4 Theoretical considerations**

487 To better analyze the similarities and understand the differences from 1D to 3D,
 488 we utilize theoretical calculations that can estimate the aforementioned characteristic
 489 observables to the first order.

490 **3.4.1 Earthquake cycle parameters**

491 We calculate earthquake recurrence interval and total slip by extending the 3D the-
 492 oretical formulation in Chen and Lapusta (2019) to all other dimensions using the an-
 493 alytical crack models of Knopoff (1958) and Keilis-borok (1959).

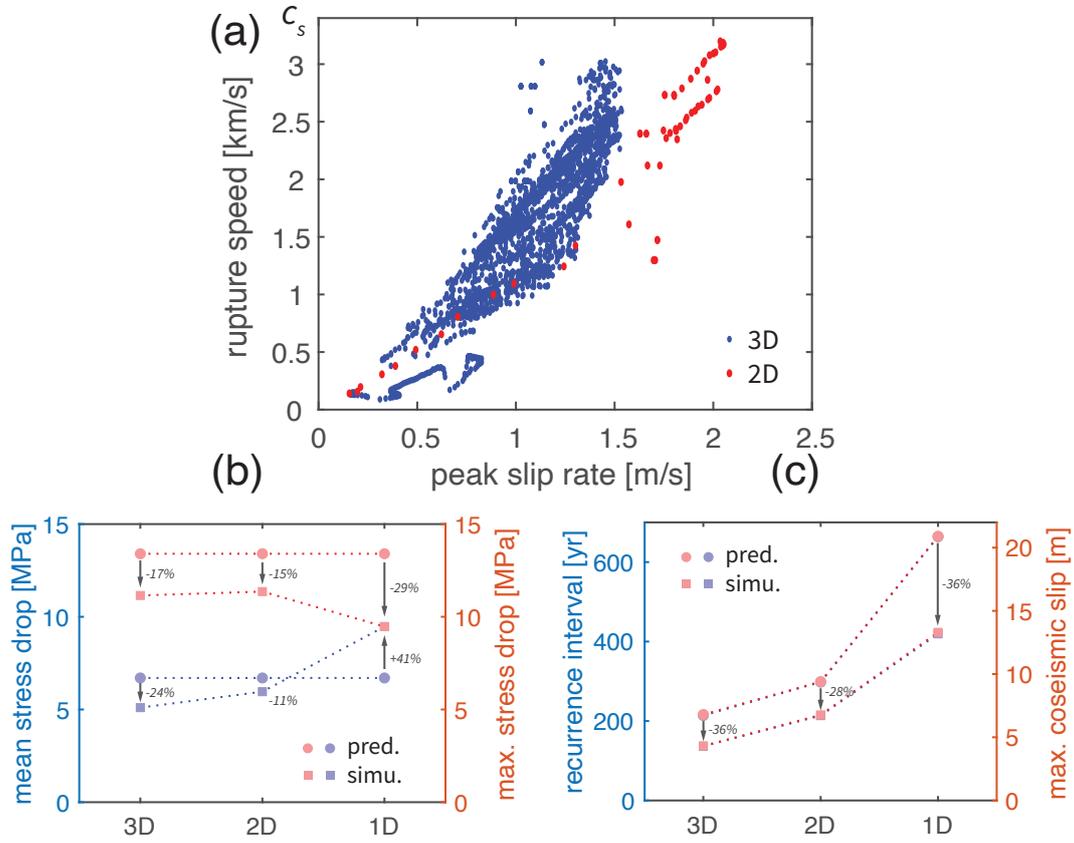


Figure 7. Comparison between theoretical predicted and numerically simulated results. (a) Interrelation between rupture speed and peak slip velocity in 3D (blue) and 2D (red) models. The local values are measured at different locations inside the VW patch. (b) Comparison between theoretically predicted (circle) and numerically simulated (square) average stress drop (blue) and maximum stress drop (red). The difference (in percentage) between calculated and simulated values is labeled aside. (c) Comparison between theoretically predicted (circle) and numerically simulated (square) recurrence interval (blue) and maximum coseismic slip (red). Same labels as in (b). Note that the markers in blue and red are largely overlapped in this panel.

494 The earthquake recurrence interval can be estimated when it is known how much
 495 stress will accumulate and what the stress rate is. However, as observed in the long-term
 496 time series (Fig. 2c), the stress accumulation pattern can be complicated and the stress
 497 level at the start of the next earthquake (i.e., the initial stress) is not uniform across the
 498 fault plane. In addition, the interseismic stress loading rate is not homogeneous within
 499 the VW zone. It is always loaded faster near the VW-VS transition and slower towards
 500 the central area, because the local stress rate is determined by the local strain rate that
 501 is largest at the strongest velocity contrasts. This means that both the accumulated stress
 502 and the interseismic stress rate vary from one point to another. Fortunately we notice
 503 that at some specific locations both can be calculated easily. At the end of the nucle-
 504 ation phase, the nucleation zone is expanded to its largest area at whose rim the high-
 505 est initial stress is achieved that is close to the yield stress (e.g., location “N” in Fig. 6c).
 506 Since the accumulated stress is always released by the (dynamic) stress drop in the co-
 507 seismic phase, it is also at these locations that the maximum stress drop is acquired. As
 508 the strength excess is small, the stress drop $\Delta\tau$ is close to the breakdown stress drop $\Delta\tau_b$
 509 (Fig. 4c, d). Moreover, the interseismic stress loading there is close to linear as well (Fig. 2c).
 510 Therefore, by analyzing the stress accumulation pattern at location “N”, we can esti-
 511 mate the maximum stress drop $\Delta\tau_{\max}$ and the recurrence interval T at the same time.
 512 This location is at the distance of h^* inside the VW patch since an earthquake can only
 513 nucleate when the creep penetrates this distance into the VW patch, where h^* is the nu-
 514 cleation size. In the end, the total slip D (i.e., aseismic + seismic slip) in one earthquake
 515 cycle, which equals to the maximum seismic slip, is estimated from the amount of the
 516 interseismic slip accumulated on the surrounding creeping VS patches.

First, the maximum stress drop $\Delta\tau_{\max}$ is approximated by the breakdown stress
 drop $\Delta\tau_b$, which is estimated from the stress difference between the two steady-state fric-
 tion level during the interseismic and coseismic phase (Cocco & Bizzarri, 2002)

$$\Delta\tau_{\max} \approx \Delta\tau_b = b\sigma \ln \frac{V_{\text{dyn}}}{V_p}, \quad (10)$$

where dynamic slip velocity V_{dyn} is approximated as 1 m/s for simplicity. Second, the
 stress rate is calculated at the desired location that is at the distance of h^* inside the
 VW patch (in 2D and 3D models, respectively, Rubin & Ampuero, 2005)

$$\begin{aligned} h_{2\text{D}}^* &= \frac{2GLb}{\pi\sigma(b-a)^2} \\ h_{3\text{D}}^* &= \frac{\pi^2}{4} h_{2\text{D}}^* = \frac{\pi GLb}{2\sigma(b-a)^2} \end{aligned} \quad (11)$$

for mode III deformation in our models. The stress rate $\dot{\tau}_{h^*}$ there can be expressed as (Chen & Lapusta, 2019; Keilis-borok, 1959; Knopoff, 1958)

$$\dot{\tau}_{h^*} = C \frac{GV_p}{\sqrt{r^2 - (r - h^*)^2}} . \quad (12)$$

For a fault segment of half-width r in 2D models or a circular fault of radius r in 3D models it has the same form with C a dimension-dependent constant being either $C_{3D} = 7\pi/24$ (Keilis-borok, 1959) or $C_{2D} = 1/2$ (Knopoff, 1958). This expression is directly applicable to our 2D models with $r = H/2$. While in 3D models, taken into consideration that the width of VW patch H is shorter than its length l , we can still apply this expression to our rectangular fault by assuming $r \approx H/2$. In 1D, the tectonic loading is applied from the far-away boundary. In this case we replace the whole denominator $\sqrt{r^2 - (r - h^*)^2}$ by X_0 , the distance between fault and the far-away loading boundary, with $C_{1D} = 1$. Third, by combining the interseismic stress rate and coseismic stress drop together we approximate the recurrence interval T by

$$T = \Delta\tau_{\max}/\dot{\tau}_{h^*} . \quad (13)$$

Finally, the total slip D , or the maximum coseismic slip, is estimated by

$$D = V_p T . \quad (14)$$

517 The theoretically predicted and numerically simulated maximum stress drop, re-
 518 currence interval and maximum coseismic slip are in agreement for all dimensions (Fig. 7b,
 519 c). This confirms the observed trend that longer recurrence interval and larger coseismic
 520 slip are acquired due to dimension reduction. It also justifies our explanation that the
 521 larger coseismic slip is caused by the larger slip deficit during longer recurrence inter-
 522 val and the longer recurrence interval is caused by the lower interseismic stress rate.

523 The theoretically predicted values are a systematic overestimation by tens of per-
 524 cent. We notice that the relative difference is close for the recurrence interval and the
 525 total slip, indicating that the error in slip calculation (14) may be inherited from the re-
 526 currence interval calculation (13). The overestimation of the maximum stress drop $\Delta\tau_{\max}$
 527 is a main contributor to this error, which is caused by using the breakdown stress drop
 528 as its approximation. Our simulations show that even for the locations at the nucleation
 529 rim (point “N” in Fig. 6c), a small stress increase still precedes the coseismic stress drop,
 530 resulting in $\Delta\tau_{\max} < \Delta\tau_b$ (Fig. 6d). Another source of the error is the underestima-
 531 tion of the interseismic loading rate $\dot{\tau}_{h^*}$. This is because when it is near the end of the

532 interseismic phase, with the expanding nucleation zone that creeps, the stress rate at point
 533 “N” is much higher than in the original assumption (Fig. 2b). Despite the errors, these
 534 theoretical considerations well explained the simulated earthquake cycle parameters and
 535 their trend with dimension reduction as a first order approximation.

536 *3.4.2 Dynamic rupture parameters*

537 Unlike the earthquake cycle parameters, dynamic rupture parameters are variables
 538 on the fault. Therefore our theoretical calculations only serve as an approximation of
 539 their average values.

Our theoretical calculations cannot provide an estimate of the rupture speed. However, both laboratory experiments (Ohnaka et al., 1987) and theoretical considerations (Ida, 1973; Ampuero & Rubin, 2008) suggest that the peak slip velocity V_{peak} and the rupture speed V_r are interrelated by

$$V_r = \alpha_r V_{\text{peak}} \frac{G}{\Delta\tau_b}, \quad (15)$$

540 where α_r is a factor on the order of 1. This positive correlation is confirmed by our sim-
 541 ulations (Fig. 7a). We measured on average α_r of 0.82 in 3D and 0.65 in 2D for the first
 542 earthquake respectively, which is similar to what Hawthorne and Rubin (2013) measured
 543 (0.50-0.65) in their 2.5D simulations. The lower value of α_r in 2D shows that with di-
 544 mension reduction higher slip velocity can be achieved under the same rupture speed.

The stress drop $\Delta\tau$ is not uniform across the simulated VW patch. Whereas the calculated stress difference from rate-and-state friction between the two steady states in the interseismic and coseismic phase is independent of dimension and location. Therefore it only provides an estimation of the average stress drop (Chen & Lapusta, 2019)

$$\begin{aligned} \overline{\Delta\tau} &\approx \tau(V_p) - \tau(V_{\text{dyn}}) \\ &\approx \sigma[\mu_0 + (a - b)\ln(V_p/V_0)] - \sigma[\mu_0 + (a - b)\ln(V_{\text{dyn}}/V_0)] \\ &= \sigma(b - a)\ln(V_{\text{dyn}}/V_p). \end{aligned} \quad (16)$$

545 The calculated average stress drop is slightly higher than the simulated results in 2D and
 546 3D (Fig. 7b). However, it is still satisfying as a first order approximation for both mod-
 547 els given that the contribution of the changing state has been ignored. The 1D model
 548 has a higher simulated average stress drop because the “average” loses its meaning in
 549 this case and the simulated value only represents where the earthquake nucleates. It is

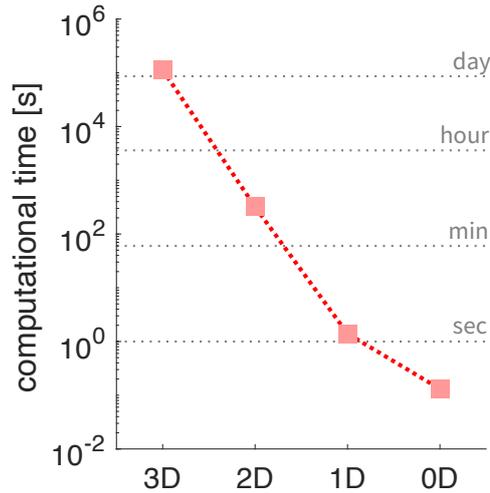


Figure 8. The average computational time of one earthquake cycle in 0D to 3D models, under the same resolution and domain size, with 12 CPUs Kokkos level parallelization.

550 well expected that higher stress drop is achieved here following the explanation in the
 551 previous section.

552 3.5 Computational efficiency

553 Lower dimensional models are computationally more efficient without losing the
 554 qualitative characteristics and the accuracy of certain earthquake parameters such as max-
 555 imum slip velocity, maximum or average stress drop and fracture energy. To evaluate the
 556 computational efficiency of each model we measure the average computational time per
 557 earthquake cycle (Fig. 8). The 3D model takes 10^3 times longer time than 2D and 10^5
 558 times longer than 1D. In the following discussions we will see that the 1D model can be
 559 further simplified to its 0D equivalent by removing the medium content. The 0D model
 560 will again save more than 90% running time compared to 1D. Note that these compu-
 561 tations do not use distributed memory and therefore ignore related parallel scaling is-
 562 sues.

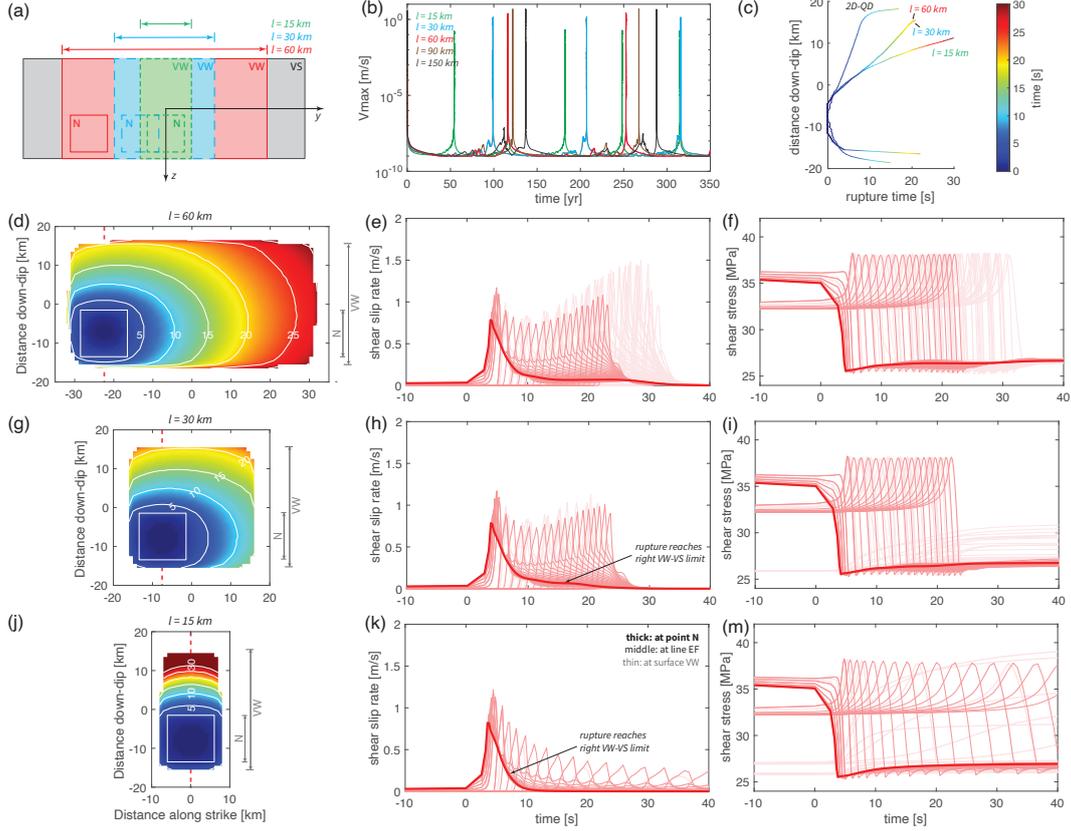


Figure 9. Comparison of the effects of fault length l (15 - 150 km) in 3D models: (d-f) 60 km, (g-i) 30 km, and (j-m) 15 km. (a) The varied VW patch sizes and varied locations of the predefined nucleation zone in three testing models with l from 15 km to 60 km. (b) The maximum slip velocity in multiple earthquake cycles for models with l from 15 km to 150 km. (d, g, j) The arrival time of the coseismic rupture front of the first earthquake, which is measured when slip velocity reaching the seismic limit. Only the central part of the fault plane is shown, where white color means no seismic slip is observed. Contours are plotted every five seconds. The red dashed line labels the observation line “EF” introduced in Fig. 1. (c) The coseismic rupture front arrival time along the vertical line “EF” under the same color scale. Lines end at where no seismic slip is observed. The rupture time of the corresponding 2D model is plotted as reference. (e, h, k) The time series of slip velocity in the coseismic phase of the first seismic event, in which origin time is set at the onset of this event. The lines with different thicknesses and degrees of transparency are recorded at different locations on the fault, where the thick lines are recorded at point “N”, the semi-thick lines along the line “EF” and the thin lines elsewhere (see Fig. 1). (f, i, m) The time series of shear stress in the coseismic phase of the first seismic event, with the same line property.

4 Discussions

4.1 Under what conditions can 2D models substitute 3D models?

We have summarized model similarities over dimensions as well as analyzed how model discrepancies give rise to result differences. It is apparent that both quantitative and qualitative changes to these findings can take place more or less when a different model setup is used. Therefore it is meaningful to discuss in which situations dimension reduction can be used without considerable side effects or should be avoided even if computational efficiency is a factor. To simplify the question, we restrict our research to the most common discussion point: under what conditions can a 3D model be substituted by a 2D model? Obviously there is no such 2D model that can represent all 3D model setups, even if they can be simplified to the same model following our dimension reduction procedure (section 2.4). The reduced dimension always plays a role. To analyze the role of the along-strike dimension that is ignored in dimension reduction, we vary the VW patch length l and keep the VW patch width H fixed. By varying the VW patch length from 150 km to 15 km, we change the aspect ratio from 5:1 to 0.5:1 (Fig. 9). The fault (VW+VS patches) size and the computational domain (X_0, Y_0, Z_0) are kept unchanged as well as the predefined nucleation zone as an initial condition, which is always set at the left bottom corner with fixed distance h_i to the VW-VS boundary (Fig. 9a). This configuration benefits the coseismic comparison along the vertical line “EF” crossing this zone (Fig. 9c-m) to our 2D simulations (Fig. 4, 5).

In the long term, longer VW patches result in longer recurrence intervals (Fig. 9b). This is because the stress rate is lower comparing to a fault with a shorter VW patch. Given that the nucleation always starts from a corner of the rectangular VW patch, longer VW patches are mainly loaded from three directions as the tectonic loading from the other horizontal direction is farther away. From the perspective of theoretical considerations, the elongated fault geometry deviates from the circular fault assumption we used in 3D, while it is closer to the infinitely long fault assumption in 2D. Therefore theoretical calculations also suggest longer recurrence intervals to be expected. Consequently, by prolonging the VW patch length, we achieve longer recurrence intervals to fit better what is observed in 2D. In other words, higher aspect ratio faults in 3D are better represented by 2D models in the long term. However, even extending the 3D patch to 210 km still

594 leads to shorter recurrence intervals in what is observed in 2D (Fig. 2), as interseismic
 595 loading remains more effective from three lateral sides than two.

596 On the other hand, a longer VW patch requires longer rupture propagation time
 597 along strike and thus longer coseismic duration, if the rupture speed remains unvaried
 598 (Fig. 9d, g). As explain before, 2D models can be seen as 3D models where theoretically
 599 no time is required to rupture along strike. In this sense, a longer VW patch length is
 600 not preferred to fit the short coseismic duration observed in 2D. However, even the short-
 601 est coseismic duration, observed with aspect ratio 1:1, is still much longer than 2D due
 602 to its low rupture speed. The rupture propagation time is not further shortened when
 603 the fault becomes even shorter. On the contrary, rupture speed is even largely decreased
 604 in the case with aspect ratio 0.5:1, resulting in a fairly long coseismic duration (Fig. 9c,
 605 j). This speed change happens after the rupture front reaches the horizontal VW-VS tran-
 606 sition, confirming again that horizontal VW-VS interaction can change vertical rupture
 607 speed. Accompanying the rupture speed reduction, the slip velocity and the stress drop
 608 are reduced at the same time (Fig. 9k-m vs. e-f & h-i). This is dissimilar to how they
 609 are observed in 2D (Fig. 4a, c). In this sense, a shorter VW patch length is not favored
 610 either. In other words, medium aspect ratio (close to 1:1) fault is better represented by
 611 2D models in the coseismic phase. Additionally, if only what happens along the verti-
 612 cal line “EF” in 3D is taken into consideration when compared to 2D, then all models
 613 with aspect ratio higher than 1:1 can be accepted. This is because we notice that the
 614 rupture propagation along the vertical line “EF” does not change much with respect to
 615 the fault length when the aspect ratio is larger than 1:1 (Fig. 9c). Nor do the slip ve-
 616 locity and coseismic slip change along this line (Fig. 9d-i).

617 To summarize, 2D models can better represent high aspect ratio faults in 3D for
 618 long-term observations and medium-to-high aspect ratio faults for coseismic observations.
 619 Whereas for coseismic observations there are definitely inevitable qualitative differences
 620 in between. Our conclusion suggests that when using empirical scaling relations to in-
 621 terpret 2D results from a 3D perspective, it is crucial to assume a suitable aspect ratio
 622 according to the corresponding research objective. Wesnousky (2008) summarized 36 his-
 623 torical natural earthquakes and found that they have similar rupture width but varied
 624 rupture length, resulting in varied aspect ratio from 0.7 to 12. The analysis in this study,
 625 covering the range 0.5 - 5, can therefore be useful to refer to when interpreting 2D sim-
 626 ulations to 3D natural observations.

627 4.2 Model setup choices

628 We are going to discuss several model setup adjustments in this section to ensure
 629 that the conclusions drawn from our simulations are further supported and they can be
 630 generalized when located in a broader context. However, we should also acknowledge that
 631 there are research questions whose answers inherently require higher-dimensional spa-
 632 tial or geometrical complexity. We are not aiming at finding substitutes for such cases
 633 but rather to present the essential differences that are apparent in the simplest setup.

634 We started to build our models following the SEAS benchmark BP4-qd (Erickson,
 635 Jiang, Barall, Abdelmeguid, et al., 2020) and inherited their frictional parameter a, b, L
 636 choice that was aimed to facilitate the benchmarking under rather low resolution (500
 637 - 1000 km). To make up for this somewhat unnatural choice we have implemented (part
 638 of) this study using the frictional parameters in benchmark BP1 (Erickson, Jiang, Bar-
 639 all, Lapusta, et al., 2020). Those simulations confirmed the results presented in this pa-
 640 per under high resolution (25 - 50 m), indicating the possibility to generalize our con-
 641 clusions to a broader frictional parameter range. These benchmarks also helped us to
 642 validate the code library *Garnet* and our models for usage in earthquake cycle model-
 643 ing by comparing to other participated modelers (see Supporting Information S1).

644 Our choice of computational domain size is aimed to set all boundaries far away
 645 from the fault so that the influence of those artificial boundary conditions is kept min-
 646 imal. This is justified by implementing our models with different domain sizes (X_0, Y_0, Z_0
 647 as in Table 1). We show that the simulated earthquake cycles in both long term and co-
 648 seismic phase are converging upon enlarging the medium thickness X_0 and the differ-
 649 ence is negligible when $X_0 > 40$ km (Fig. S2).

650 Tectonic loading is usually applied in two different ways: directly on the fault plane
 651 (e.g., Kaneko et al., 2011) or indirectly at the far-away boundaries (e.g., Herrendörfer
 652 et al., 2018). Both types have been adopted by studies for different research purposes.
 653 We adopted tectonic loading at the top/bottom of the fault plane for 2D and 3D mod-
 654 els following BP4-qd, but at the far-away boundary for 1D models due to dimensional
 655 restriction. During the coseismic period, the influence of tectonic loading is not notice-
 656 able because of the short duration. To test the influence in the interseismic phase we ap-
 657 plied tectonic loading conditions (a) only on fault surface at top/bottom region with fixed
 658 fault width, (b) only on far-away boundary surface, (c) both (a) and (b). We modeled

659 in 2D with gradually enlarged computational domain (Table S1). We find that the re-
 660 currence interval converges to a set value as the computational domain is enlarged and
 661 is hardly affected by the type of loading when the computational domain is large enough.
 662 This invariance with respect to loading condition is supported by our theoretical calcu-
 663 lations (section 3.4). Because there we explained that the main loading force to the locked
 664 VW patch is from its surrounding creeping VS patches. No matter how the loading is
 665 applied, the stress rate inside the VW patch is only related to its dimension and inde-
 666 pendent of the size of the VS patches or the fault as a whole (Eq. 12). The velocity gra-
 667 dient perpendicular to the fault contributes to the loading process as well, but it is min-
 668 imized for large enough computational domain where on-fault loading becomes domi-
 669 nant. Therefore both the interseismic and coseismic characteristics are not sensitive to
 670 what kind of loading boundary condition is applied.

671 As for the initial condition, we have also adopted a predefined highly-stressed zone
 672 within the VW patch following BP4-qd. Since the later earthquakes do not necessarily
 673 occur from the same location, this predefined zone facilitated the quantitative coseismic
 674 comparison across dimensions by forcing the first earthquake to nucleate from this same
 675 region. It is suggested by former studies that initial conditions have little effect on sub-
 676 sequent earthquakes (e.g., Takeuchi & Fialko, 2012; Allison & Dunham, 2018), therefore
 677 this special initial condition should not harm our findings in terms of earthquake cycle
 678 characteristics. In this study we observe that the accumulative slip contour distortions
 679 around a depth of -1.5 km and -13.5 km are introduced by the predefined nucleation zone,
 680 whose properties increased the amount of slip in that zone for the first earthquake (Fig. 3).
 681 However, for non-accumulative variables no influence from the initial condition is observed
 682 in later earthquakes. Nevertheless, the first earthquake is not relatively characteristic
 683 in an earthquake cycle even though some qualitative characteristics are still shared by
 684 later earthquakes. We have added analysis of the sixth earthquake across dimensions to
 685 make up for this.

We mentioned that since physical tectonic loading becomes unavailable in 0D mod-
 els, an arbitrary “driving force” has to be added to the system instead. To facilitate com-
 parison, we can integrate the strain rate along the x direction in 1D models and use it
 to drive the 0D system. This is how the well-known “spring-slider” model is built (Burridge
 & Knopoff, 1967). Such a 0D model is mathematically equivalent to the 1D model. This
 is because the static momentum balance equation (Eq. 1) in 1D reduces to $\partial\sigma_{xy}/\partial x =$

0, i.e. $\sigma_{xy}(x, t) = \sigma_{xy}(t)$ – shear stress is time-varying but spatially constant. Combined with the 1D elastic constitutive equation, the time derivative of stress is given by

$$\dot{\sigma}_{xy} = G \frac{V_p - V}{X_0}. \quad (17)$$

686 Since this is an analytical simplification, the resulting model behavior is expected to re-
 687 main the same. In this case 0D models are to replace 1D models due to their compu-
 688 tational efficiency (Fig. 8). Nevertheless, when heterogeneity, inelasticity and/or inertia are
 689 considered, the explanation above no longer holds, then 0D and 1D models have to be
 690 treated separately (e.g., C. Pranger et al., 2021).

691 4.3 Implications

692 We are the first to systematically study and quantify similarities and differences
 693 of how models in different dimensions simulate earthquake sequences. While large-scale
 694 parallel computing can be exploited to reduce the time to solution of 3D applications,
 695 this does not significantly lower the power consumption and consequently the monetary
 696 and environmental burden. Moreover, we find that the orders of magnitude difference
 697 of speed-up by dimensional reduction are so large, and can only be even larger when higher
 698 resolution is necessary, that they readily make the difference between being feasible for
 699 scientific and exploratory research or not. Hence lower dimensional models will likely re-
 700 main essential for scientific exploration in the coming decades (Lapusta et al., 2019). Es-
 701 pecially when the researcher’s interest falls into the scope of what the lower dimensional
 702 models can handle, they are encouraged to use them as they could be hundreds to mil-
 703 lions times faster than a 3D model with the same resolution. Even if 3D models are nec-
 704 essary for certain studies (e.g., Ulrich et al., 2019), simpler models can always be a use-
 705 ful starting point of an exploration. These results should serve as guidelines as to how
 706 to interpret the lower-dimensional modeling results with the effect of dimensional reduc-
 707 tion always taken into account, rather than being regarded restricting model simplifi-
 708 cations being adopted.

709 5 Conclusions

710 In this paper, we addressed a common concern of numerical modelers: how com-
 711 plex should my model be to answer my research question? Will dimension reduction qual-
 712 itatively and quantitatively affect my results? And how? For this purpose we have sys-

713 tematically investigated different dimensional models from 0D to 3D in terms of their
714 interseismic and coseismic characteristics and computational time for earthquake sequences
715 and individual quasi-dynamic ruptures.

716 Our results demonstrate that all dimensional models simulate qualitatively sim-
717 ilar quasi-periodic earthquake sequences. The stress accumulation pattern is much the
718 same when observed at the rim of the nucleation zone. As for the earthquake cycle pa-
719 rameters, lower dimensional models produce longer recurrence intervals and hence larger
720 coseismic slip. This trend is supported by our theoretical calculations where the effect
721 of dimension reduction is well quantified. We observe that the VS patches play a cru-
722 cial role in causing differences in the interseismic phase, because tectonic loading is ef-
723 fectively realized at the VW-VS transition by the velocity contrast between the creep-
724 ing VS patches and the locked VW patch. As VS patches are removed when fault di-
725 mension is reduced, their absence reduces the interseismic stress rate inside the VW patch
726 and thus increases the recurrence interval. The larger slip deficit built in this period will
727 be transferred to a larger coseismic slip.

728 In the coseismic phase, we find that certain earthquake parameters such as the break-
729 down stress drop, (dynamic) stress drop and fracture energy can be accurately repro-
730 duced in each of these simpler models, because they are mainly governed by material fric-
731 tional parameters. This finding is especially valid for the first earthquake without phys-
732 ical tectonic loading. For later earthquakes, the statement is only true on average of the
733 VW patch. This is because the yield stress and effective slip weakening distance can change
734 due to tectonic loading history. For the dynamic rupture parameters, lower dimensional
735 models generally produce higher maximum slip velocities and higher rupture speeds. This
736 is because less energy consumption will be required when fewer directions need to be rup-
737 tured into thus higher kinetic energy is reserved. Furthermore, we demonstrate that this
738 interaction at the VW-VS transition can modify rupture speed, which is another cru-
739 cial role the VS patches play in the coseismic phase. We find that the vertical rupture
740 speed along the line “EF” in 3D is slowed down compared to 2D. It can be further slowed
741 down when the fault length is shortened to one fourth its original length, proving the
742 vertical rupture behavior is influenced by horizontal properties.

743 Finally, we highlight the power of lower dimensional models in terms of their com-
744 putational efficiency. We find that under the same resolution 3D models require 10^3 times

745 longer computational time than 2D, 10^5 times longer than 1D and 10^6 times longer than
746 0D models to simulate one earthquake cycle. Therefore dimension reduction can not only
747 relieve the heavy energy-consuming simulations, but also improve the efficiency of projects
748 that require monotonous repetitions of forward models. All the aforementioned findings
749 are confirmed by our theoretical calculations, which suggest that differences during load-
750 ing in the interseismic phase affect the subsequent coseismic phase. This paper may serve
751 as guidelines to check in simplified models what results can be expected to be accurately
752 modeled as well as what physical aspects are missing and how they are related to the
753 discrepancies observed in the results. Not restrictive to this study, those theoretical con-
754 siderations can be generally applied to other earthquake cycle models as well.

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770 [.org/cpranger/garnet](https://bitbucket.org/cpranger/garnet). The models for this study can be found in the respective fold-
771 ers under [https://bitbucket.org/cpranger/garnet/src/meng-eejit/experiments/](https://bitbucket.org/cpranger/garnet/src/meng-eejit/experiments/comparison/)
772 [comparison/](https://bitbucket.org/cpranger/garnet/src/meng-eejit/experiments/comparison/). The data produced and analyzed in this study is available via [http://](http://doi.org/10.5281/zenodo.4890931)
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