Supporting Information for Quantification of Volcano Deformation caused by Volatile Accumulation and Release"

A. Spang [1], M. Burton [2], B. J. P. Kaus [1,4], F. Sigmundsson [3]

[1]Johannes Gutenberg University, Institute of Geosciences, Johann-Joachim-Becher-Weg 21, 55128 Mainz, Germany

[2]School of Earth and Environmental Sciences, University of Manchester, Manchester M13 9PL, UK

[3]Nordic Volcanological Center, Institute of Earth Sciences, University of Iceland, Askja, Sturlugata 7, Reykjavik IS-101, Iceland

[4]TeMaS, Terrestrial Magmatic Systems Research Center, temas.uni-mainz.de

Corresponding author: A. Spang, Department of Geosciences, Johannes Gutenberg University, Johann-Joachim-Becher-Weg 21, 55128 Mainz, Germany. (arspang@uni-mainz.de)

Introduction

This file contains texts S1: Resolution and Time Stepping, S2: Volatile Volume and Density, S3: 2D Scaling Law and S4: 2D vs 3D as well as Figures S1–S6 and supplementary table S1.

Contents of this file

1. Texts S1 to S4

2. Figures S1 to S6

3. Table S1

Text S1: Resolution and Time Stepping

To maximize the resolution, we use only one quarter (half for 2D) of the perfectly symmetric domain. Figures S2a, S2b and S3a show that we do not introduce any effects through this simplification. Using 384 cells in each direction yields roughly 56.6 million cells and a vertical resolution of about 40 meters (Figure 1a). As the model extends 15 km in the vertical and 50 km in horizontal directions, the horizontal cell size is only 40 meters in the central 2.5 km and then increases towards the horizontal edges of the domain. Resolution tests confirm that this is sufficient (Figures S2a, S2b and S3b). We run the model for 40 years with a constant time step of 1 year. This domain width and time stepping allows the evaluation of reliable models (see section 3.1 and Figures S2c, S2d, S3c, S3d). Figure 1d shows that emplacing the full volatile reservoir at once or incrementally increasing its size results in the same surface response.

Text S2: Volatile Volume and Density

We use the ideal gas law to estimate the volume of the volatile reservoir:

$$V = \frac{nRT}{p} \tag{S1}$$

V is the volume, n the amount of substance, R the universal gas constant, T the temperature and p the pressure. n is calculated by

$$n = \frac{m}{M}, \tag{S2}$$

where m is the total volatile mass and M is the average molar mass of the volatile mixture. Finally, we calculate density ρ by:

$$\rho = \frac{m}{V}. \tag{S3}$$

Text S3: 2D Scaling Law

Figure S4b shows four material parameters that have a considerable effect on the surface response. The radius of the volatile reservoir (r_{vol}), the depth of the volatile reservoir (d_{vol}), the density contrast between volatiles and crust ($\Delta\rho$) and the shear modulus of the crust (G_{crust}). We performed a systematic parameter variation, testing 5 different values for each parameter (9 for d_{vol}) while keeping the other parameters constant. Figure S5 shows the results for individual parameters. From this, we are able to derive the following scaling relationship:

$$\Delta h_0 = A \frac{r^2\, \Delta\rho\, g}{d\, G}, \tag{S4}$$

where Δh_0 is the predicted subsidence upon volatile removal, A is a pre-factor with units of meters, r is the radius of the volatile reservoir, $\Delta\rho$ is the density contrast between volatiles and crust, g is the gravitational acceleration, d is the depth of the volatile reservoir and G the shear modulus of the crust. Figure S4c shows that the scaling law is able to predict the modeling results as well as 24 additional cases where multiple parameters were varied and that were not used in deriving the law. The error is on the order of 10% or lower for the majority of cases. Exceptions are models with a shallow volatile reservoir ($r\,/\,d > 0.1$, blue in Figure S4d). Analytical solutions for the gravity anomaly of buried cylinders or spheres have the same issue of only being applicable when the depth of the body is much larger than its radius (Turcotte & Schubert, 2002). The same is true for simple models relating the inflation of magma bodies to surface deformation (e.g. Mogi, 1958; Yang et al., 1988).

Text S4: 2D vs 3D

The surface displacement in our 2D models is larger than in the 3D models by a factor of about 30. This stems from the fact that modeling a sphere in 2D is equivalent to modeling an infinite horizontal cylinder/pipe in 3D. Our study highlights the importance of modeling magmatic systems in 3D. Yet, there are some relevant results that do not change when moving from 2D to 3D. The influence of material parameters $\Delta\rho$ and G as well as the independence of the surface deformation from the reservoir shapes and the temperature structure of the crust did not change when moving to 3D. Consequently, the scaling laws for 2D (equation S4) and 3D (equation 8) are similar. So 2D models can serve as a good tool for orientation as they are computationally much cheaper and allow to do extensive initial testing. To get quantitative results on finite bodies, it is however critical to use 3D models.

References

Mogi, K. (1958). Relations between the eruptions of various volcanoes and the deformations of the ground surfaces around them. *Earthq Res Inst*, *36*, 99–134.

Ranalli, G. (1995). *Rheology of the Earth.* Springer Science & Business Media.

Turcotte, D. L., & Schubert, G. (2002). *Geodynamics.* Cambridge university press.

Yang, X. M., Davis, P. M., & Dieterich, J. H. (1988). Deformation from inflation of a dipping finite prolate spheroid in an elastic half-space as a model for volcanic stressing. *Journal of Geophysical Research: Solid Earth*, *93*(B5), 4249–4257.

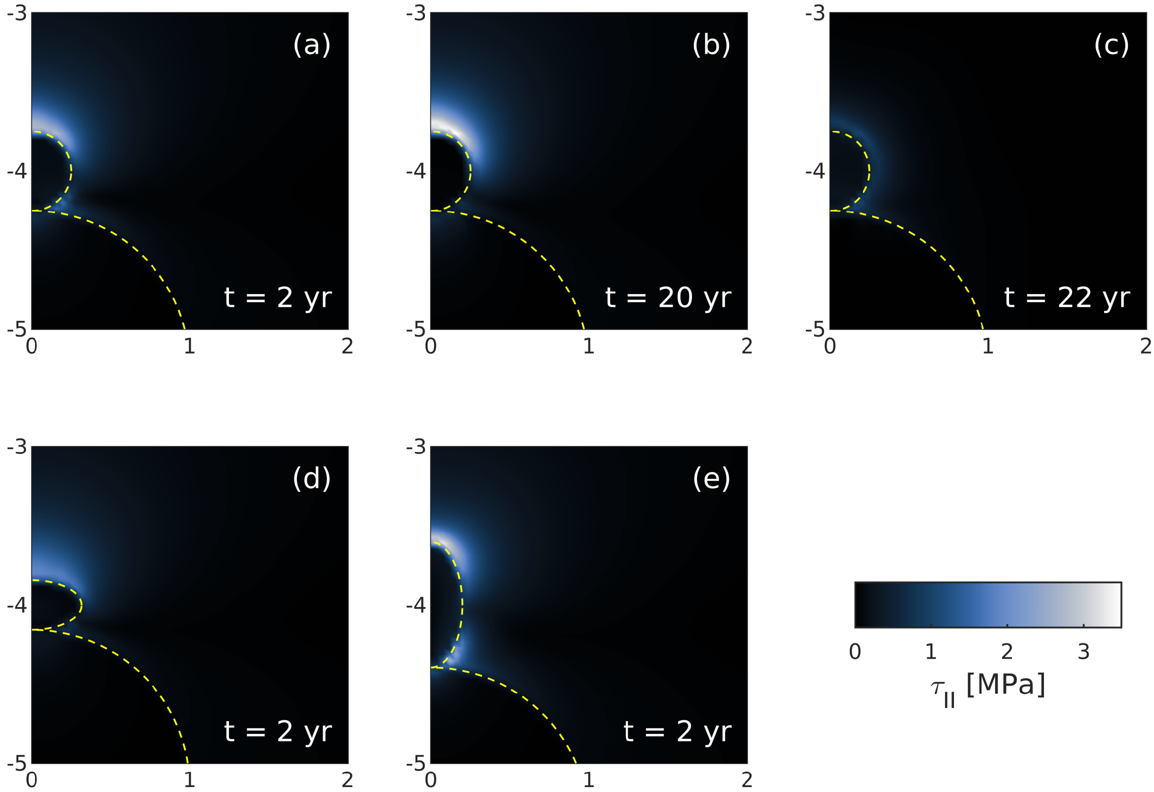

Figure S1. Magnitude of deviatoric stress (τ_{II}) around the volatile reservoir for different time steps and reservoir shapes. Dashed yellow lines follow boundaries of volatile and magma reservoir. (a) Reference model with volatiles emplaced. (b) Reference model right before the eruption. (c) Reference model after the volatiles have been released. (d) Oblate reservoir with volatiles emplaced. (e) Prolate reservoir with volatiles emplaced.

Table S1. Material parameters

Material	$\rho\,[\frac{\mathrm{kg}}{\mathrm{m}^3}]$	G [GPa]	$B_n\,[\frac{1}{\mathrm{MPa}^{-\mathrm{n}}}]$	E_n [kJ]	n
Crust	2500	2.0	3.2 x 10^{-4}	154	2.3
Magma	2500	0.1	$\eta = 10^{18}$ Pas		
Volatiles	500	0.1	$\eta = 10^{16}$ Pas		

Creep parameters ($\mathrm{B_n}$, $\mathrm{E_n}$, n) are chosen after Ranalli (1995)

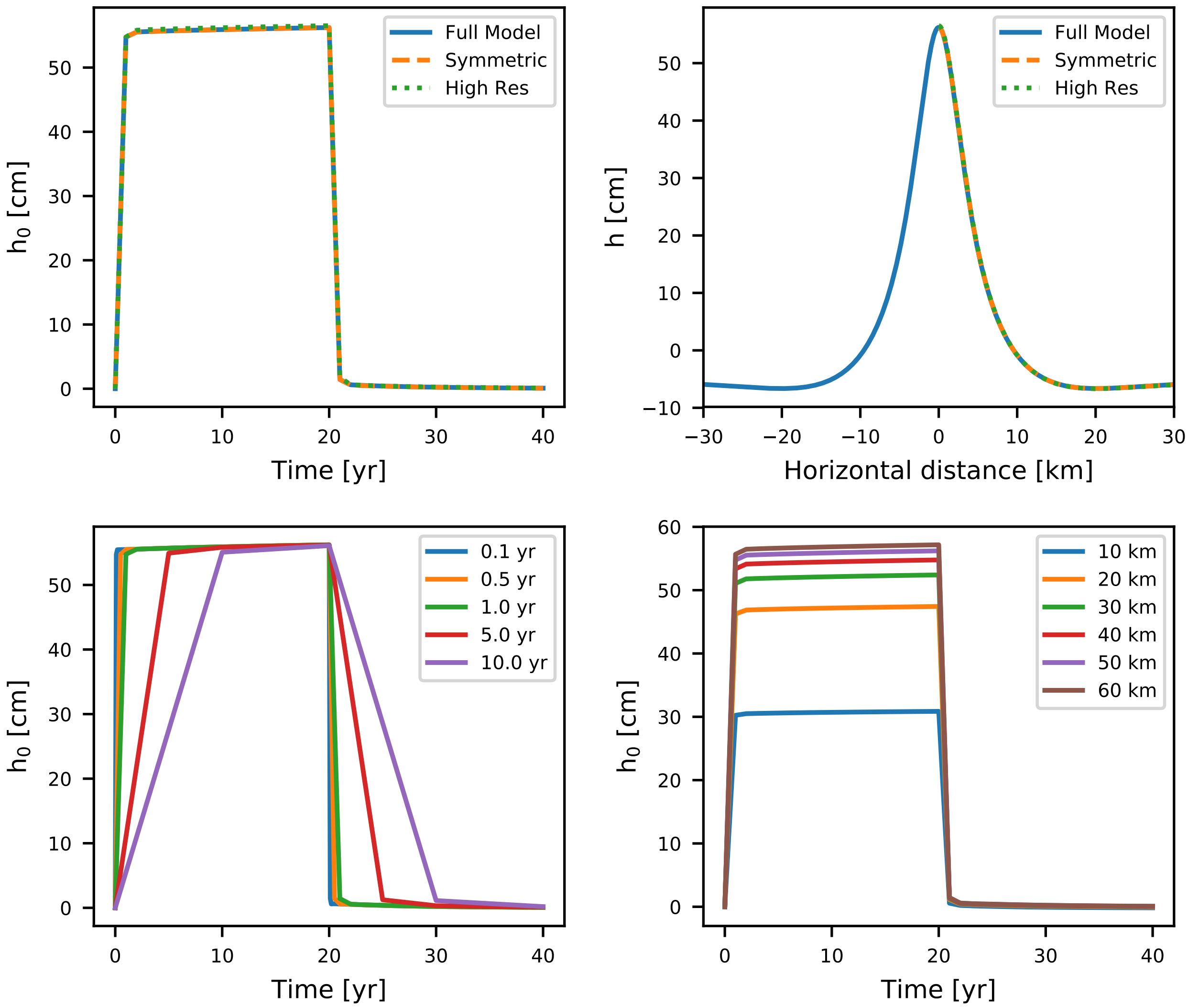

Figure S2. Results of 2D models. (a) Comparison of the maximum surface uplift through time between the full size model, the symmetric version and the symmetric version at doubled resolution. (b) Comparison of a surface profile at 20 years between the full size model, the symmetric version and the symmetric version at doubled resolution. (c) Maximum surface uplift through time in the reference model using different time steps. (d) Maximum surface uplift through time in the reference model using different model widths.

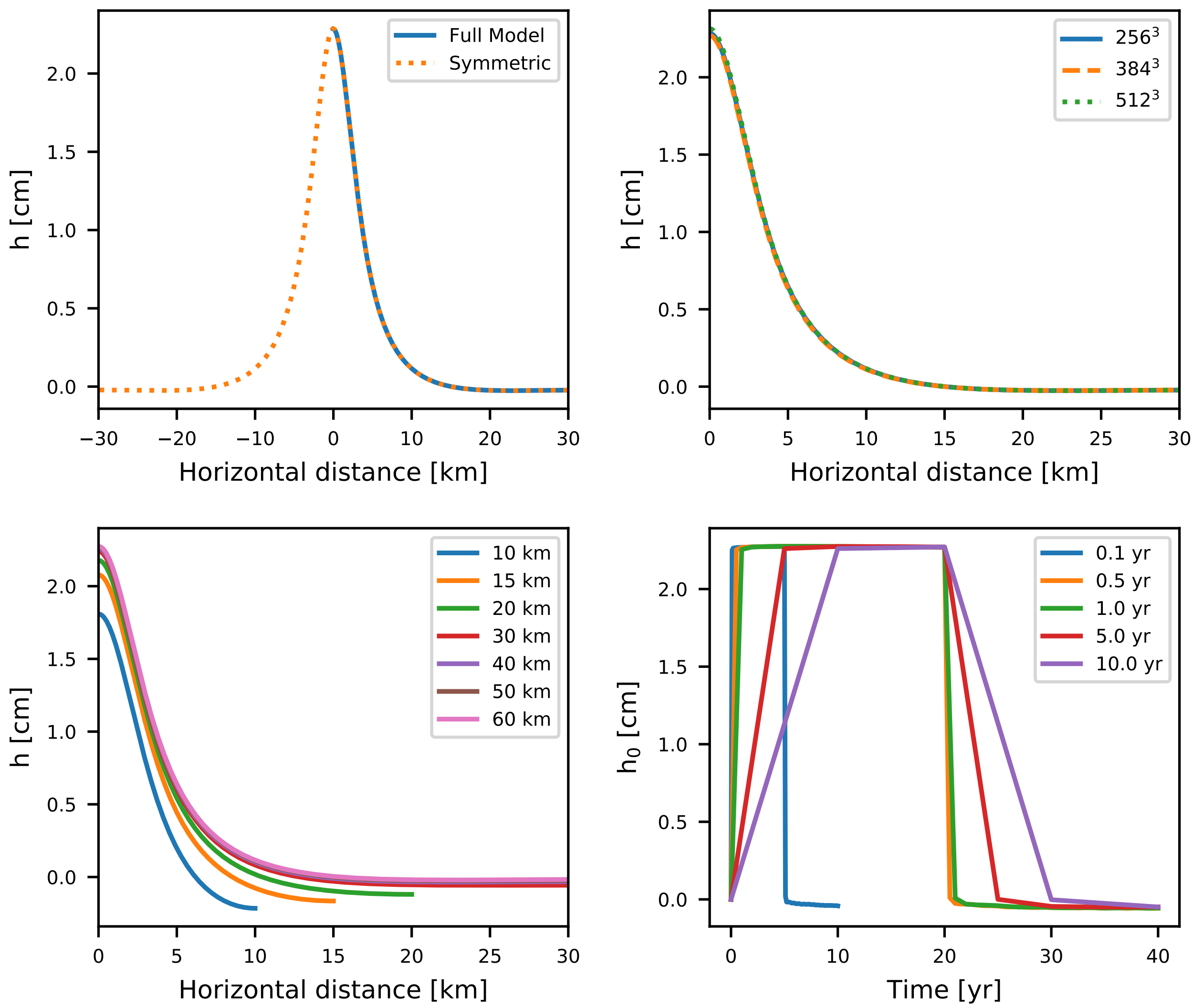

Figure S3. Results of 3D models. (a) Surface profile after 20 years (right before removal of the volatiles) for the full size model and the symmetric version (used in the study). (b) Surface profile after 20 years for different model resolutions. (c) Surface profile after 20 years for different model widths. (d) Maximum surface uplift through time in the reference model using different time steps.

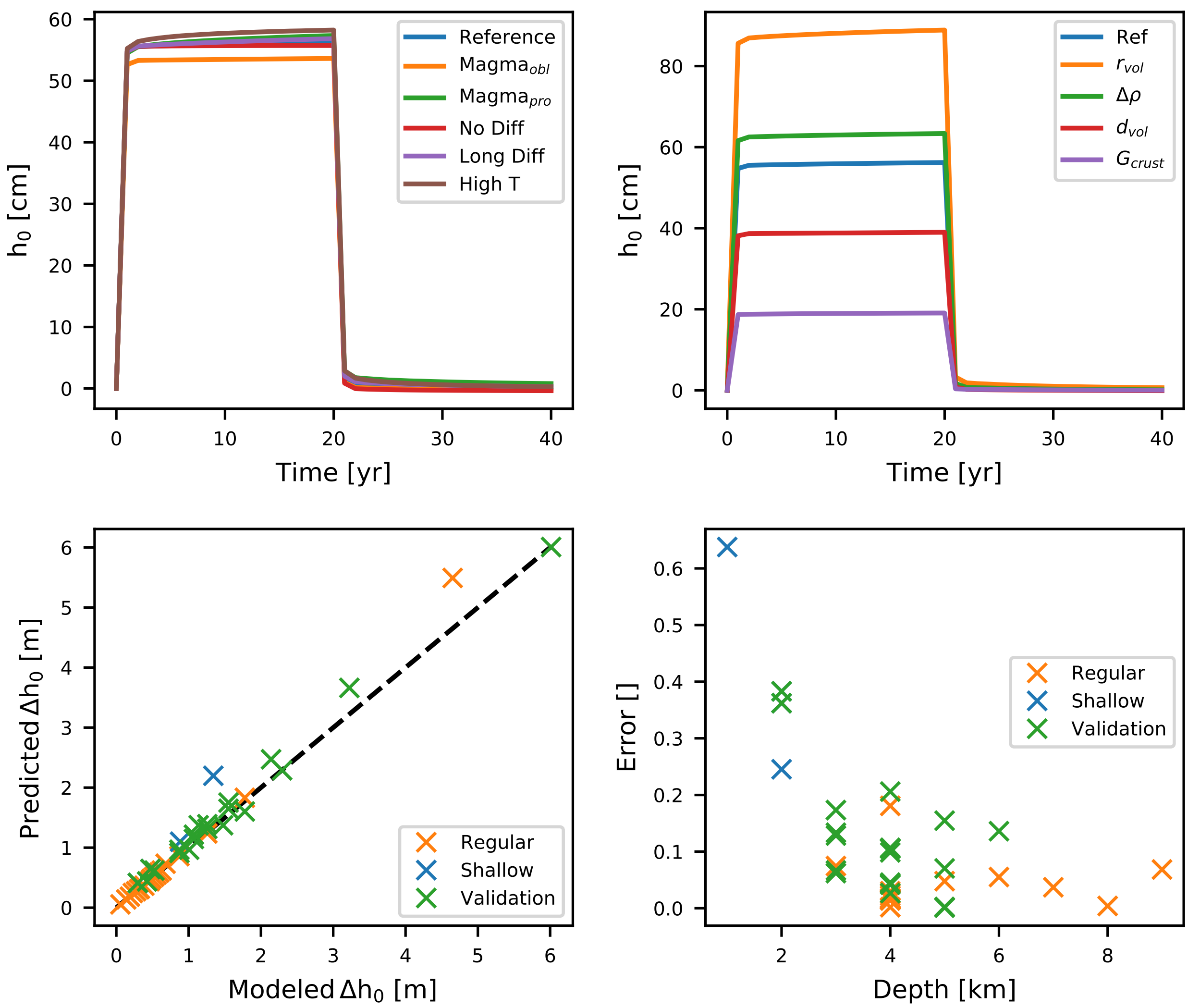

Figure S4. Results of 2D models. (a) Maximum surface uplift through time for different magma body shapes and temperature structures. All models show similar results. (b) Effect of perturbing one crucial material parameter compared to the reference model. (c) Comparison between modeled subsidence and subsidence predicted by equation S4. Black dashed line shows 1:1 correlation. Orange crosses show models that were used to derive equation S4, green crosses show models that were used to test it and blue crosses show models with a shallow (<= 2 km) volatile reservoir. (d) Error between the modeled subsidence and the predicted subsidence in dependence of reservoir depth. Error is only > 20% for the models with a shallow volatile reservoir.

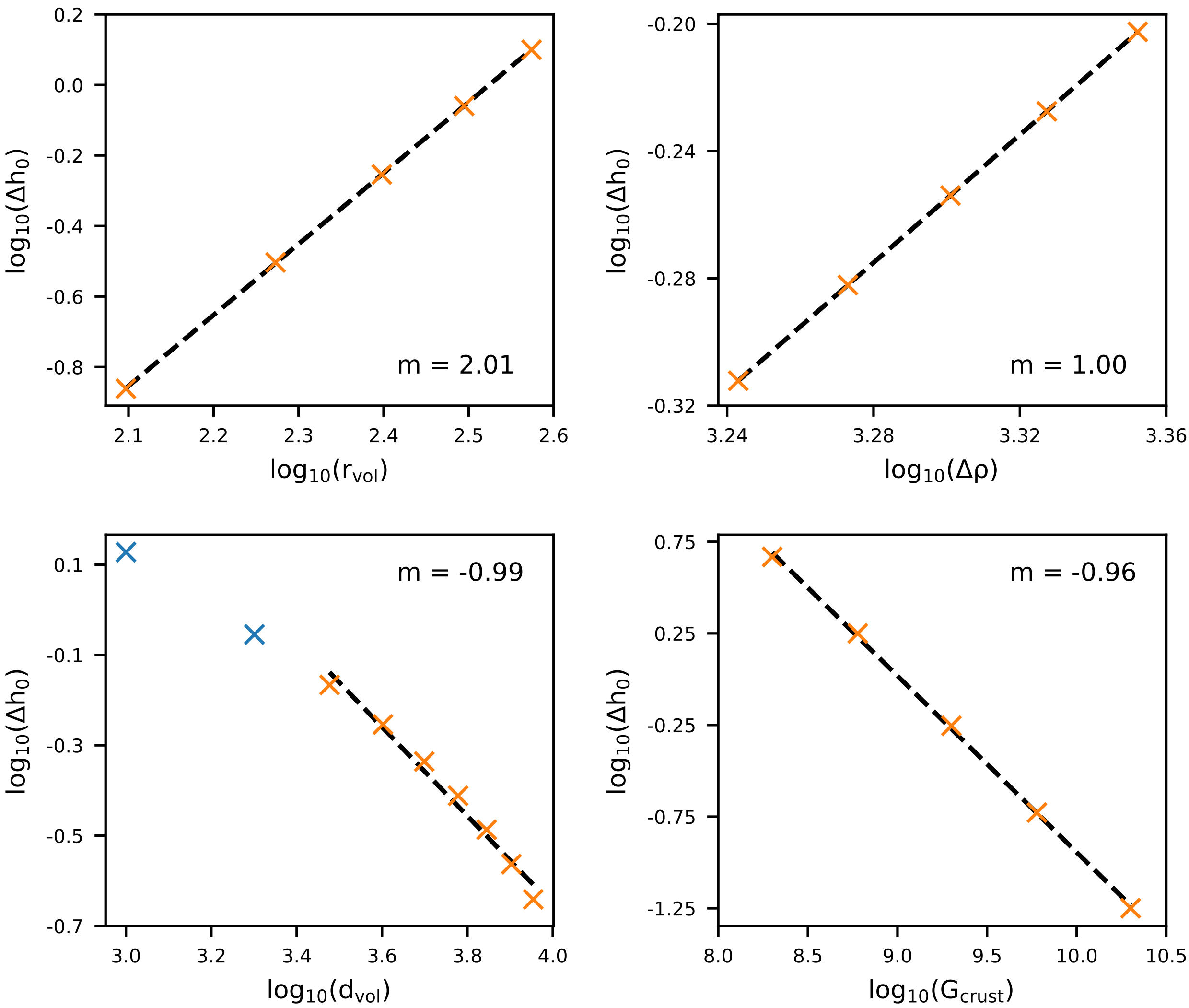

Figure S5. Results of scaling law analysis in 2D. Surface subsidence upon volatile removal (Δh_0) in dependence of the 4 material parameters, we found to be significant. m denotes the slope of the linear fits and equates to the exponent of the parameter in the scaling law (equation S4). Blue crosses in c denote two shallow models ($d <= 2\,km$) that were excluded to allow a linear fit.

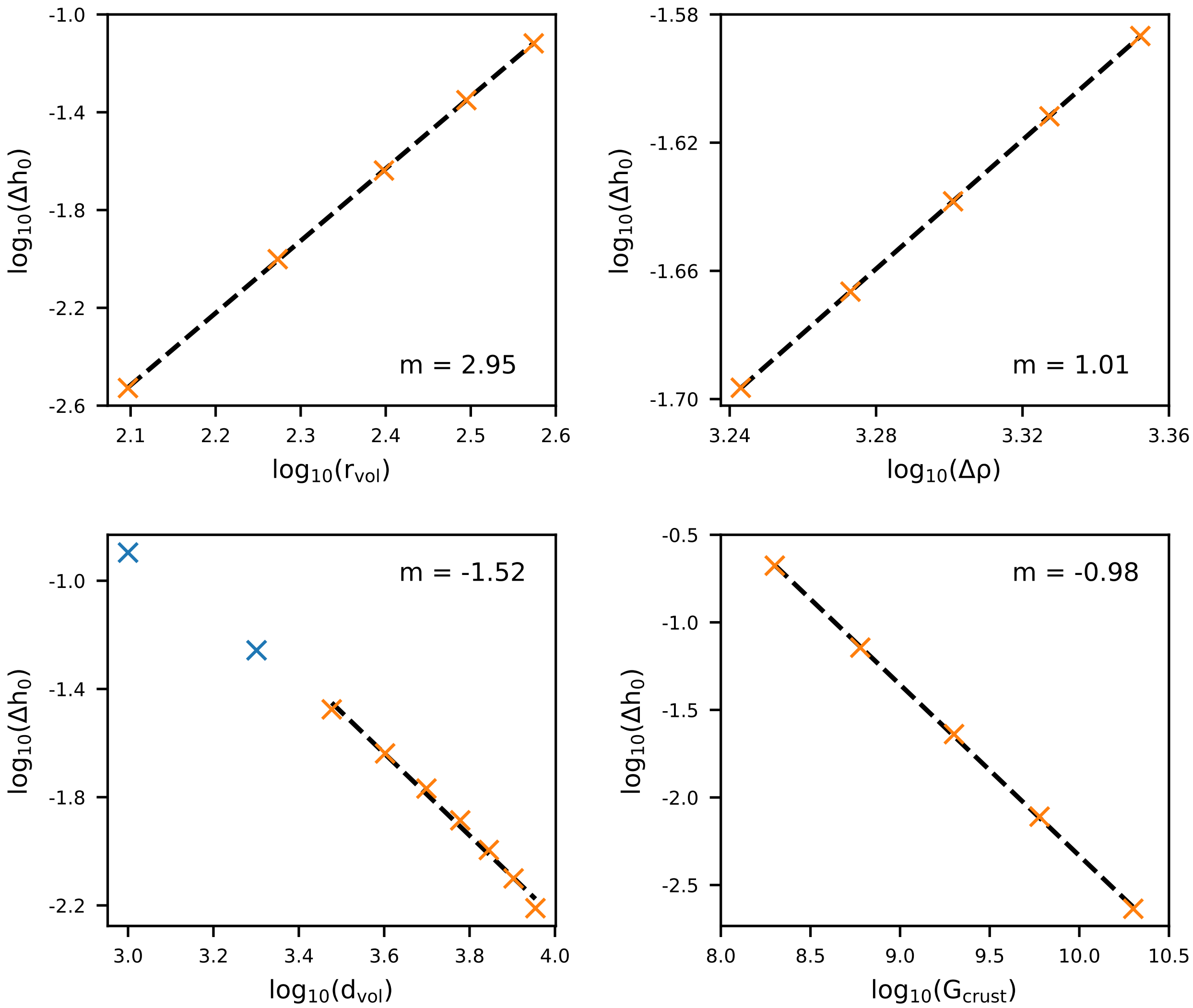

Figure S6. Results of scaling law analysis in 3D. Surface subsidence upon volatile removal (Δh_0) in dependence of the 4 material parameters, we found to be significant. m denotes the slope of the linear fits and equates to the exponent of the parameter in the scaling law (equation 8). Blue crosses in c denote two shallow models ($d <= 2\,km$) that were excluded to allow a linear fit.