

1 **Contribution of gravity waves to universal vertical wavenumber**
2 **($\sim m^{-3}$) spectra revealed by a gravity-wave-permitting general**
3 **circulation model**

4 **Haruka Okui¹, Kaoru Sato¹, and Shingo Watanabe²**

5 ¹Department of Earth and Planetary Science, Graduate School of Science, The University
6 of Tokyo, Tokyo, Japan

7 ²Japan Agency for Marine–Earth Science and Technology, Yokohama, Japan

8 Corresponding author: Haruka Okui (okui@eps.s.u-tokyo.ac.jp)

9
10 **Key Points:**

- 11 • Using a gravity-wave-permitting general circulation model, vertical wavenumber
12 (m) spectra in the whole middle atmosphere were examined.
- 13 • Characteristics of the model-simulated m spectra of gravity waves are broadly
14 consistent with observations.
- 15 • Disturbances other than gravity waves contribute substantially to the lowest m
16 part of the $\sim m^{-3}$ range.

17 **Index Terms**

18 3255 Spectral analysis, 3332 Mesospheric dynamics, 3334 Middle atmosphere dynamics,
19 3363 Stratospheric dynamics, 3384 Acoustic-gravity waves

20 **Key Words**

21 Gravity waves, middle atmosphere, spectral analysis

22

23 **Abstract**

24 Observations with high vertical resolution have revealed that power spectra of horizontal
25 wind and temperature fluctuations versus vertical wavenumber m have a universal
26 shape with a steep slope in a high m range, approximately proportional to m^{-3} . Several
27 theoretical models explaining this spectral slope were proposed under an assumption of
28 gravity wave (GW) saturation. However, little evidence has been obtained to show that
29 these universal spectra are fully composed of GWs. To confirm the validity of this
30 assumption, two kinds of m spectra are calculated using outputs from a GW-permitting
31 high-top general circulation model. One is the spectra for GWs designated by fluctuations
32 having total horizontal wavenumbers of 21–639. The other is the spectra of fluctuations
33 unfiltered except extracting a linear trend in the vertical that were often analyzed in the
34 observational studies. Comparison between the two shows that GWs dominate the
35 observed spectra only in a higher m part of the steep slope, whereas disturbances other
36 than GWs significantly contribute to its lower m part. Moreover, geographical
37 distributions of the characteristic wavenumbers, slopes, and spectral densities of GW
38 spectra are examined for several divided height regions of the whole middle atmosphere.
39 It is shown that strong vertical shear below the zonal wind jets as well as the wave
40 saturation is responsible for the formation of the steep slopes of GW spectra.

41 **Plain Language Summary**

42 Radar, lidar, and radiosonde observations have revealed that vertical wavenumber power
43 spectra of horizontal wind and temperature fluctuations have common shape with a steep
44 slope. Several theoretical studies have explained this universality by assuming that these
45 spectra are composed of saturated gravity waves. To confirm the validity of this
46 assumption, spectral analysis of gravity waves, having short horizontal wavelengths, was
47 conducted using a gravity-wave-permitting high-top general circulation model.
48 Comparison of the spectra of gravity waves with those of all model-simulated
49 disturbances showed that disturbances other than gravity waves significantly contribute
50 to the spectra in a low vertical-wavenumber part of the steep slope. Moreover, vertical
51 and geographical variations of characteristics of gravity wave spectra were described. It
52 is inferred that strong vertical shear below the eastward and westward jets as well as the
53 wave saturation is responsible for the formation of the steep slopes of gravity wave
54 spectra. As well as deepening our understanding of gravity waves in the middle
55 atmosphere, these findings may provide useful guidelines for improving
56 parameterizations of gravity waves in climate models.

58 **1 Introduction**

59 Gravity waves (GWs) are small-scale atmospheric waves that play fundamental
60 roles in determining the large-scale dynamic and thermal structure of the middle
61 atmosphere by transporting momentum and energy (e.g., Fritts & Alexander, 2003). For
62 example, GW forcing contributes substantially to both maintaining the weak wind layer
63 near the mesopause and driving the meridional circulation in the mesosphere (e.g., Holton,
64 1983). Equatorial stratospheric and mesospheric quasi biennial oscillations (QBOs) are
65 mainly driven by GWs originating from the troposphere (e.g., Sato & Dunkerton, 1997;
66 Kawatani et al., 2010a, 2010b; Mayr et al., 1997; Ern et al., 2014). It has also been shown
67 that GWs cause extension of the deep branches of the Brewer–Dobson circulation to
68 higher latitudes, and determination of the location of the turnaround latitude of the
69 circulation (e.g., Okamoto et al., 2011; Sato & Hirano, 2019).

70 On the basis of radar (e.g., VanZandt, 1985; Fritts & Chou, 1987; Tsuda et al.,
71 1989, 1990), radiosonde (e.g., VanZandt, 1982; Allen & Vincent, 1995; Sato et al., 2003)
72 and rocket (e.g., Dewan et al., 1984; Dewan & Good, 1986) observations, it has been
73 shown that power spectra versus the vertical wavenumber (m) of horizontal wind and
74 temperature fluctuations have common shape with a steep slope. These ‘universal’ spectra
75 are roughly proportional to m^{-3} , with slight dependence on the latitude, in a m range
76 higher than the characteristic wavenumber m_* . The universal spectra have also been
77 reported by satellite observations. Some satellite observations have considerably high
78 vertical resolutions, such as GPS radio occultation data (e.g., Tsuda et al., 2011;
79 Noersomadi & Tsuda, 2016) and Constellation Observing System for Meteorology,
80 Ionosphere and Climate (COSMIC) (e.g., Yan et al. 2018). However, their horizontal
81 resolutions are in general not so high. In most observations, the higher end of the m^{-3}
82 range cannot be detected because of limitation in vertical resolution and/or in observable
83 height range. According to a temperature m spectrum observed by radiosondes shown
84 in Fig. 9a in Sato and Yamada (1994), the higher end of the steep-slope range is observed
85 at $\sim 6 \times 10^{-2} \text{ rad m}^{-1}$ (vertical wavelength of $\sim 100 \text{ m}$). By intercomparing GW spectra
86 simulated by several convection-permitting models, Stephan et al. (2019) showed that
87 model-simulated spectra also have steep slopes in a high m range similar to observations.

88 Several theories have been developed regarding these characteristic m spectra.
89 Adopting a concept of superposition of saturated GWs, Smith et al. (1987) developed a
90 theoretical model that explains that the horizontal wind spectrum is described as

91 $N^2/6m^3$ for $m \gtrsim m_*$, where N is the buoyancy (Brunt–Väisälä) frequency. They
 92 assumed that the spectral range occupied by a single saturated GW (Δm) is proportional
 93 to m (Dewan & Good, 1986). Sato and Yamada (1994) considered the change in m of
 94 a single saturated GW in the linear vertical shear dU/dz of the background wind parallel
 95 to the horizontal wavenumber vector and derived a theoretical spectrum without the
 96 assumption of $\Delta m \propto m$. The spectral form of the horizontal wind fluctuations shown by
 97 Sato and Yamada (1994) is expressed as follows:

$$P_u(m) \approx N^2 \cdot (2m^3 \cdot \Delta z |dU/dz|)^{-1} \sqrt{N^2/m^2 + f^2/k^2}, \quad (1)$$

98 where Δz is the height expanse for the spectrum calculation, k is the horizontal
 99 wavenumber, and f is the Coriolis parameter. This spectral theory succeeded in
 100 explaining the characteristic shape and level of the m spectra. However, it has not been
 101 fully confirmed that the observed m spectra are totally attributable to GWs. A useful
 102 approach to examine this issue is to calculate GW m spectra by extracting GWs as
 103 fluctuations having high total horizontal wavenumbers using a recently available GW-
 104 permitting general circulation model (GCM).

105 The theories describing the m spectra give the basis of nonorographic GW
 106 parameterizations (e.g., Hines, 1997a, 1997b; Warner & McIntyre, 1996, 1999) used
 107 widely in climate models. Descriptions of vertical and horizontal variations of GW
 108 spectra in a GW-permitting model may provide useful guidelines for the GW
 109 parameterizations (e.g., McLandress & Scinocca, 2005; Watanabe, 2008).

110 In this study, we use outputs from a hindcast of December 2018 using a GW-
 111 permitting GCM extending from the surface to the lower thermosphere (Okui et al., 2021).
 112 Fluctuations having total horizontal wavenumbers of 21–639 are designated as GWs. First,
 113 it is verified that the model reproduces the main observed spectral properties. Next, to
 114 examine the contribution of GWs to the spectra in the m^{-3} range, m spectra of GWs
 115 and all-fluctuation components obtained from each single profile, just as they would be
 116 extracted from radar or radiosonde observations, are compared. Global distributions of
 117 parameters describing the characteristics of GW spectra are also examined for each height
 118 region in the middle atmosphere. The remainder of this paper is structured as follows.
 119 Detailed descriptions of the model and the analysis method are given in Section 2. Results
 120 are discussed in Section 3. A summary and concluding remarks are presented in Section
 121 4.

122

123 **2 Method and Model Description**

124 The model used in this study is a high-resolution version of the Japanese
125 Atmospheric GCM for Upper Atmosphere Research (JAGUAR) (Watanabe & Miyahara,
126 2009). This model comprises 340 vertical layers from the surface to the geopotential
127 height of ~150 km, with a log-pressure height interval of 300 m throughout the middle
128 atmosphere, and it has a horizontal-triangularly truncated spectral resolution of T639,
129 whose minimum resolvable horizontal wavelength is ~60 km. No parameterizations for
130 subgrid-scale GWs were used in the present study. It is considered that the JAGUAR,
131 whose vertical grid interval is 300 m, realistically resolves GWs having wavelengths at
132 least longer than ~2.0 km (6–7 grids). By performing GCM simulations with different
133 vertical resolutions (Δz), Watanabe et al. (2015) examined the dependence of GW
134 momentum flux on the vertical resolution, suggesting that the vertical grid interval shorter
135 than or equal to 300 m give almost the same amount of the fluxes. This fact suggests that
136 the GWs having the most part of momentum fluxes can be resolved with $\Delta z = 300$ m,
137 which supports the validity of using a GCM having a vertical resolution of 300 m to
138 examine characteristics of GW m spectra.

139 A hindcast was performed for 5 December 2018 to 17 January 2019 using global
140 analysis data produced by the JAGUAR–Data Assimilation System (JAGUAR-DAS)
141 (Koshin et al., 2020, 2022) in a medium-resolution (T42L124) version of the JAGUAR
142 as initial data. A four-dimensional local ensemble transform Kalman filter and a filter
143 called incremental analysis updates (Bloom et al., 1996) are used in the JAGUAR-DAS.
144 The PrepBUFR observational dataset provided by the National Centers for Environmental
145 Prediction (NCEP), satellite temperature data from the Aura Microwave Limb Sounder
146 (MLS) and the Sounding of the Atmosphere using Broadband Emission Radiometry
147 (SABER) on the Thermosphere, Ionosphere, Mesosphere Energetics Dynamics (TIMED)
148 satellite, and brightness temperature data from the Special Sensor Microwave
149 Imager/Sounder (SSMIS) were assimilated. The hindcast period was divided into
150 consecutive 4-day intervals, for each of which an independent run was performed using
151 the high-resolution JAGUAR. Each model run consisted of a spectral nudging run over 3
152 days and a free run over the subsequent 4 days. We analyzed the outputs at 1-hour
153 intervals from the 4-day free runs only for the 5–20 December 2018. We did not use data
154 from later periods because substantial modulation of GW fields was expected in
155 association with the onset of major sudden stratospheric warming on 1 January 2019.
156 Detailed analysis of this sudden stratospheric warming is presented in Okui et al. (2021).

157 In the present study, GWs were extracted as fluctuations having total horizontal

158 wavenumbers of 21–639 (horizontal wavelengths of $\lambda_h < 2000$ km). We did not use any
159 vertical filter for extraction of GWs. Note that some GWs may have longer horizontal
160 wavelengths than this cutoff wavelength (e.g., Chen et al., 2013, 2016; Chen and Chu,
161 2017). However, several radiosonde and radar observations showed that dominant
162 horizontal wavelengths of GWs in the lower stratosphere are hundreds of kilometers
163 except at low latitudes, where they can be ~ 1000 km or longer (e.g., Sato, 1994; Wang et
164 al., 2005). Analyzing satellite observation data, Ern et al. (2018) showed that dominant
165 horizontal wavelengths of GWs in the stratosphere and mesosphere along the satellite
166 orbit, which would always overestimate their true values, are 500–2000 km. Based on
167 these results from previous studies, we chose ~ 2000 km as the cutoff wavelength.

168 To imitate the extraction methods of fluctuations in radar and radiosonde
169 observations, in addition to GW spectra, we calculated m spectra for fluctuations with
170 all horizontal wavenumbers using vertical profiles in which only linear trends in the
171 vertical were removed. Hereafter, these profiles are referred to as “all fluctuations”. It is
172 possible that all fluctuations include not only GWs but also vertical variations of larger-
173 scale waves, such as Rossby waves and equatorial waves, and mean fields. Comparison
174 between the m spectra of GWs and all fluctuations allows us to examine the GW
175 contribution to observed m spectra by radars, lidars, and radiosondes. Temperature
176 fluctuations were multiplied by g/T_0N , where T_0 is the background temperature
177 extracted as a linear trend from each unfiltered temperature profile, and g is
178 gravitational acceleration. Because the obtained power spectra are expected to have steep
179 slopes, i.e., proportional to $\sim m^{-3}$, prewhitening and recoloring processes were performed
180 before and after the calculation of spectra, respectively (e.g., Sato et al. 2003). The degree
181 of prewhitening β was taken as 0.95 in this study. Profiles having finite data length were
182 tapered using a 10% cosine-tapered window for the first and last tenth of the data series.
183 Using a Fast Fourier Transform, power spectra were calculated from these processed
184 profiles of GWs (i.e., components with $n = 21\text{--}639$) and all fluctuations. The spectra
185 calculated in this way were multiplied by an energy correction factor of $1/0.875$ to
186 compensate the spectral reduction due to the 10 % cosine-tapered window.

187

188 **3 Results and Discussion**

189 **3.1 Contribution of GWs to m^{-3} spectra**

190 The zonal wind, meridional wind, and temperature spectra from 5–20 December
191 2018 at Shigaraki (35°N, 136°E), Japan, where the MU radar is located, are shown in Fig.

192 1. Figs. 1a–c are spectra in the lower stratosphere and Figs. 1d–f are those in the middle
 193 and upper mesosphere, for which the m spectra from the MU radar observations were
 194 reported by Tsuda et al. (1989). The height regions for the spectra were determined such
 195 that N^2 is approximately constant, as assumed in Smith et al. (1987). Note that the MU
 196 radar observation shown by Tsuda et al. (1989) were consistent with the theory of Smith
 197 et al. (1987).

198 It is important that the model-simulated spectra of all fluctuations have a shape
 199 with a steep slope of $\sim m^{-3}$ in the high m range in both the lower stratosphere and the
 200 mesosphere. This feature is consistent with the spectra calculated from MU radar
 201 observations (e.g., Tsuda et al., 1989). The $\sim m^{-3}$ range of the spectra is in good
 202 agreement with the theoretical spectral model derived by Smith et al. (1987). The GW
 203 spectra are bent at a specific value of m of $3\text{--}5 \times 10^{-4} \text{ m}^{-1}$ (a vertical wavelength λ_z of
 204 $\sim 2\text{--}3$ km) in the lower stratosphere and $1\text{--}2 \times 10^{-4} \text{ m}^{-1}$ ($\lambda_z = \sim 5\text{--}10$ km) in the mesosphere.
 205 In the m range above the bending point, the GW spectra are nearly proportional to m^{-3}
 206 and they agree well with the spectra of all fluctuations. However, at lower m s, the
 207 spectral density of the GW spectra is smaller than that of all fluctuations, even within the
 208 $\sim m^{-3}$ range of the all-fluctuation spectra. These facts suggest that the observed spectral
 209 slopes of $\sim m^{-3}$ mainly consist of GWs in the high m range, but that disturbances other
 210 than GWs also contribute substantially to the spectra at lower m part of the $\sim m^{-3}$ range.
 211 The highest m in the range where there is notable disagreement between the spectra of
 212 GWs and those of all fluctuations is lower in the mesosphere than in the lower
 213 stratosphere. This difference shows that GWs having longer vertical wavelengths are
 214 dominant in the mesosphere than in the stratosphere.

215 A series of meridional wind spectra in the middle stratosphere for $z = 18\text{--}25$ km,
 216 averaged over the latitudinal range of $\pm 5^\circ$ around each latitude, are shown in Fig. 1g. To
 217 compare them with observed spectra, we chose the same height region as Sato et al.
 218 (2003), in which the m spectra were obtained as a function of the latitude using
 219 radiosonde observations performed over a research vessel for the middle Pacific. To
 220 estimate the parameters describing the characteristics of these spectra, the obtained
 221 spectral curves were fitted to the following equation (Allen & Vincent, 1995) using a
 222 trust-region algorithm (Conn et al., 2000):

$$P_{\text{ALL}}(m) = F_0 \frac{m/m_*}{1 + (m/m_*)^{t+1}}, \quad P_{\text{GW}}(m) = F_0 \frac{m/m_{g*}}{1 + (m/m_{g*})^{t+1}}, \quad (2a, 2b)$$

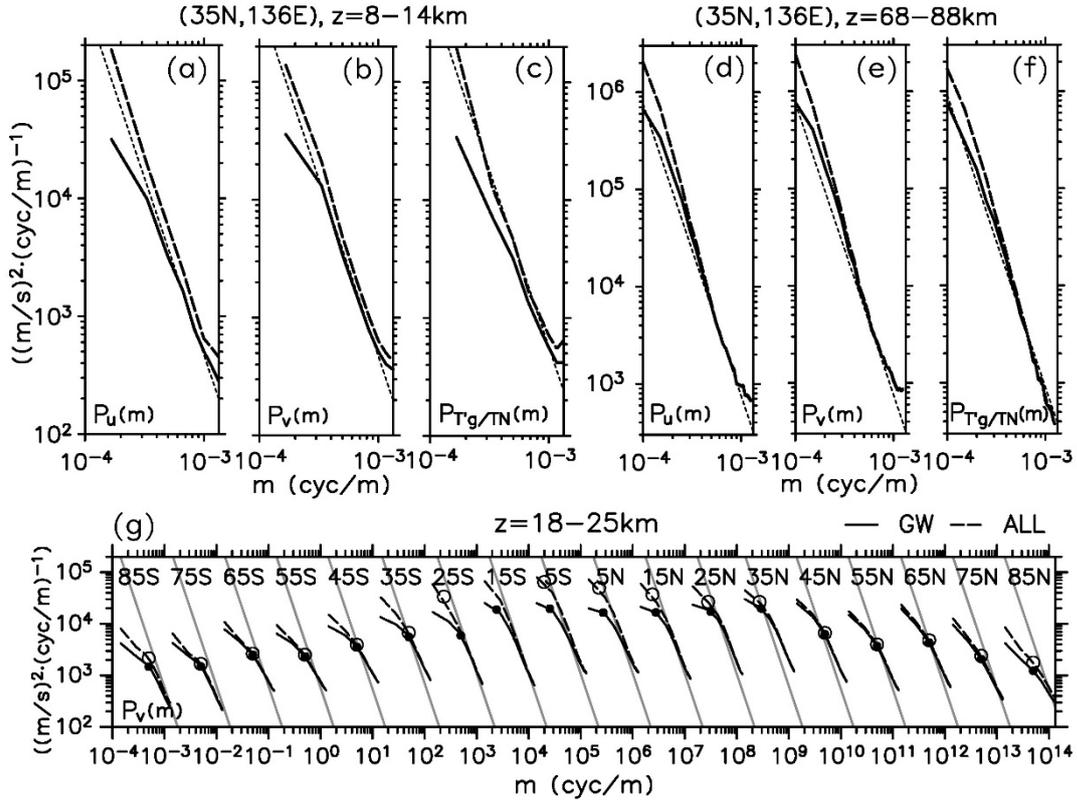
223 where m_* in Eq. (2a) and m_{g*} in Eq. (2b) are the characteristic wavenumbers
 224 (approximate bending wavenumbers) of an all-fluctuation spectrum $P_{\text{ALL}}(m)$ and the

225 GW spectrum $P_{\text{GW}}(m)$, respectively; F_0 is a parameter representing the amplitude at
226 $m = m_*$ or m_{g*} ; and t represents the spectral slope of opposite sign. The closed (open)
227 circles in Fig. 1g represent m_{g*} (m_*) estimated for each spectrum.

228 The spectra at latitudes of 25° S–25° N have shape with an $\sim m^{-3}$ slope at a higher
229 m range. However, the spectral slopes at higher latitudes are relatively gentle and almost
230 proportional to $\sim m^{-2}$, even in the high m range. This feature of steeper spectral slope at
231 lower latitudes is consistent with the observations by Sato et al. (2003). Similar to the
232 comparison with the MU radar observations at Shigaraki (Figs. 1a–f), the difference in
233 spectral density between the GWs and all fluctuations is substantial at $m < m_{g*}$ except
234 at 45°–75° S and 45°–75° N, where GWs are dominant, even in a low m range. The
235 difference is particularly large at low latitudes of 25° S–25° N and in the polar regions at
236 85° S and 85° N. The folding wavenumber for the all-fluctuation spectra is slightly (~ 1.1 –
237 2 times) smaller than m_{g*} at 25° S–25° N.

238 It is considered that the contribution of equatorial waves to the spectral densities
239 is large at low latitudes near the equator. Because meridional wind fluctuations are
240 examined here, waves except Kelvin waves are possible candidates. However, typical
241 meridional scale of equatorial waves such as Rossby-gravity waves and equatorial inertia-
242 gravity waves is ~ 1500 km ($\sim 7^\circ$ S– 7° N). Therefore, these waves hardly account for the
243 additional spectral density at 25° S and 25° N. Inertial instability is another possible
244 candidate for the meridional wind structure having a vertical wavenumber of ~ 1 – 2×10^{-4}
245 m^{-1} corresponding a vertical wavelength of ~ 10 km (e.g., Dunkerton, 1981; Rapp et al.,
246 2018; Strube et al., 2020). However, anomalous potential vorticity, which is the inertially
247 unstable condition, is rarely observed in the region of $z = 18$ – 25 km in the analyzed
248 model data (not shown). Secondary circulation associated with the QBO has a larger
249 vertical scale than the range of m^{-1} discussed here. The cause of the differences
250 between GW and all-fluctuation spectra at low latitudes is left for future studies. As for
251 the spectra at 85° S and 85° N, the difficulty in handling zonal and meridional wind
252 fluctuations near the poles may have affected the results. Since it may be useful for
253 comparison with lidar and radar observations in the future, zonal mean and $\pm 5^\circ$ latitude
254 mean v spectra for $z = 30$ – 60 km and 80 – 100 km are shown in Fig. S1 in Supporting
255 Information.

256



257 **Figure 1** Vertical wavenumber spectra from 5–20 December 2018 of (a, d) zonal wind,
 258 (b, e) meridional wind, and (c, f) temperature fluctuations at Shigaraki (35° N, 136° E),
 259 Japan, in the height regions of (a–c) $z = 8\text{--}14$ and (d–f) $68\text{--}88$ km. Solid and dashed
 260 curves show the spectra of GWs and all-fluctuation components, respectively. Theoretical
 261 spectra from Smith et al. (1987) are indicated by thin dotted lines. (g) Meridional wind
 262 spectra from 5–20 December 2018 for $z = 18\text{--}25$ km averaged zonally and over the
 263 respective latitude regions of $\pm 5^\circ$ of the center of the latitudes shown in the figure. Closed
 264 (open) circles indicate the folding point of GW (all-fluctuation) spectra. Gray lines are
 265 the theoretical spectra from Smith et al. (1987). The scale of the horizontal axis is for the
 266 spectra at 85° S and curves for the other latitudes are shifted by an order of magnitude
 267 one by one.

268

269 3.2 Characteristics of GW spectra in the middle atmosphere

270 To examine the behavior of GWs in the middle atmosphere, the vertical and
 271 geographical distributions of parameters m_{g*} , F_0 , and t in Eq. (2b) were estimated for
 272 $P_{GW}(m)$. Figure 2 shows the zonally averaged parameters at each height region as
 273 functions of latitude. The height regions used for calculation were determined such that

274 N^2 was almost constant in each region. The main features of the parameter distributions
275 for the zonal wind spectra (Figs. 2a–c), meridional wind spectra (Figs. 2d–f), and
276 temperature spectra (Figs. 2g–i) are generally consistent. The following discussion is
277 based on the parameters of meridional wind spectra, but similar results were obtained for
278 both the zonal wind and the temperature spectra.

279 The characteristic wavenumber m_{g^*} is lower at higher altitudes (Fig. 2a). This is
280 consistent with the theoretical expectation of Smith et al. (1987). For $z=60\text{--}90$ km in the
281 middle and upper mesosphere, m_{g^*} are $1/3\text{--}2/3$ of those in the lower stratosphere ($z=$
282 $18\text{--}33$ km). Parameter F_0 is larger in higher altitude regions (Fig. 2c). The values of F_0
283 for $z=60\text{--}90$ km are $50\text{--}200$ times larger than those for $z=18\text{--}33$ km. Adopting the
284 concepts of wave amplitude growth with height and GW saturation, it is theoretically
285 estimated that the ratio of m_{g^*} for $z=60\text{--}90$ km to that for $z=18\text{--}33$ km is $\sim 1/6\text{--}1/3$,
286 and that the ratio of F_0 for $z=60\text{--}90$ km to that for $z=18\text{--}33$ km is $\sim 40\text{--}240$ (Smith et
287 al., 1987). The model results roughly agree with these theoretical estimates. Thus, the
288 vertical variations of the parameters are mostly explained by growth in the amplitude of
289 saturated GWs due to the exponential decrease in atmospheric density. However, vertical
290 variation in m_{g^*} is slightly more moderate than that of the theoretical estimates. One
291 possible explanation for the departure from the theory is that the assumption of GW
292 saturation is not necessarily fulfilled. In the low-latitude region of $15^\circ\text{S}\text{--}25^\circ\text{N}$, t is ~ 2.5
293 and approximately constant with height. At mid- and high latitudes, t is $1.5\text{--}1.8$ for $z=$
294 $18\text{--}33$ km and approaches ~ 3 with height. This wide distribution of t is consistent with
295 Lidar observations at McMurdo, Antarctica (78°S , 167°E) (Lu et al., 2015; Zhao et al.,
296 2017; Chu et al., 2018) and at Urbana (40°N , 88°W) (Senft & Gardner, 1991).

297 In terms of latitudinal variation, m_{g^*} has a maximum value of $2.2\text{--}2.5 \times 10^{-4} \text{m}^{-1}$
298 at $20^\circ\text{S}\text{--}40^\circ\text{N}$ in the lower and middle stratosphere (i.e., $z=18\text{--}33$ km and $z=33\text{--}45$
299 km, respectively). It is almost homogeneous in the uppermost stratosphere and
300 mesosphere ($z=45\text{--}60$ km and $z=60\text{--}90$ km, respectively). The value of t shows large
301 latitudinal variations. For $z=45\text{--}60$ km in the uppermost stratosphere and lowermost
302 mesosphere, t has two significant peaks of ~ 2.7 at $\sim 15^\circ\text{S}$ and ~ 3.0 at $\sim 50^\circ\text{N}$. It is
303 expected that GWs tend to be saturated in a high m range in regions of weak background
304 wind. However, the spectra may not be due to saturated GWs below and near the strong
305 eastward or westward jet in the middle atmosphere, since intrinsic phase velocity becomes
306 large due to the Doppler shift and thus m becomes small in a strong background wind.
307 Below the jets, the spectral slopes are highly affected by the strong vertical shear. Due to
308 its m dependency, this shear effect steepens GW spectra (see Section 3.3). At 50°N , t

309 increases with height from ~ 1.8 in the lower stratosphere (i.e., $z=18\text{--}33$ km), which is
310 much smaller than 3, to ~ 3.0 in the height region of $45\text{--}60$ km, where the eastward jet
311 core is located. The shear effect on GW spectra below the jet core also prevents GW
312 saturation, which is discussed in detail in Section 3.3. This difference in the factors
313 controlling GW spectral slopes t among different latitudes is a possible reason for the
314 large latitudinal variation in t .

315 As for the F_0 distribution, there are two peaks of $\sim 3 \times 10^4 \text{ m}^3 \text{ s}^{-2}$ at $\sim 15^\circ \text{ S}$ and
316 $\sim 4 \times 10^4 \text{ m}^3 \text{ s}^{-2}$ at $50^\circ\text{--}75^\circ \text{ N}$ in the lower stratosphere ($z=18\text{--}33$ km). The former peak
317 at $\sim 15^\circ \text{ S}$ shifts to higher latitudes at higher altitudes, as is consistent with the poleward
318 propagation of eastward GWs due to refraction toward the summer westward jet (e.g.,
319 Sato et al., 2009). The latter peak in the Northern Hemisphere (NH) corresponds to the
320 region near the eastward jet in the middle atmosphere. This peak is sharpest at $\sim 55^\circ \text{ N}$ in
321 the region of $z=45\text{--}60$ km, where the jet core exists, and spreads over a broader latitude
322 region in the region of $z=60\text{--}90$ km above the jet. These sharpening and broadening may
323 be a result of lateral propagation of GWs from their source.

324 Note that the F_0 peak at northern mid- and high latitudes observed for $z=18\text{--}33$
325 km in Fig. 2f is not clear in the spectra for $z=18\text{--}25$ km shown in Fig. 1g. This apparent
326 inconsistency in lower-stratospheric GW spectra between the two height regions is likely
327 due to the difference in the background wind condition. The height region of $18\text{--}25$ km
328 corresponds to the region far below the middle atmosphere eastward jet, while the region
329 of $z=18\text{--}33$ km includes the lower part of the jet. In the eastward jet region, it is
330 considered that GWs tend to have longer vertical wavelengths, which makes a steep
331 spectral slope extend toward lower m and thus F_0 larger. Spectra averaged zonally and
332 over a latitudinal region of $\pm 5^\circ$ for $z=18\text{--}33$ km showed better consistency with F_0 in
333 Fig. 2f (not shown).

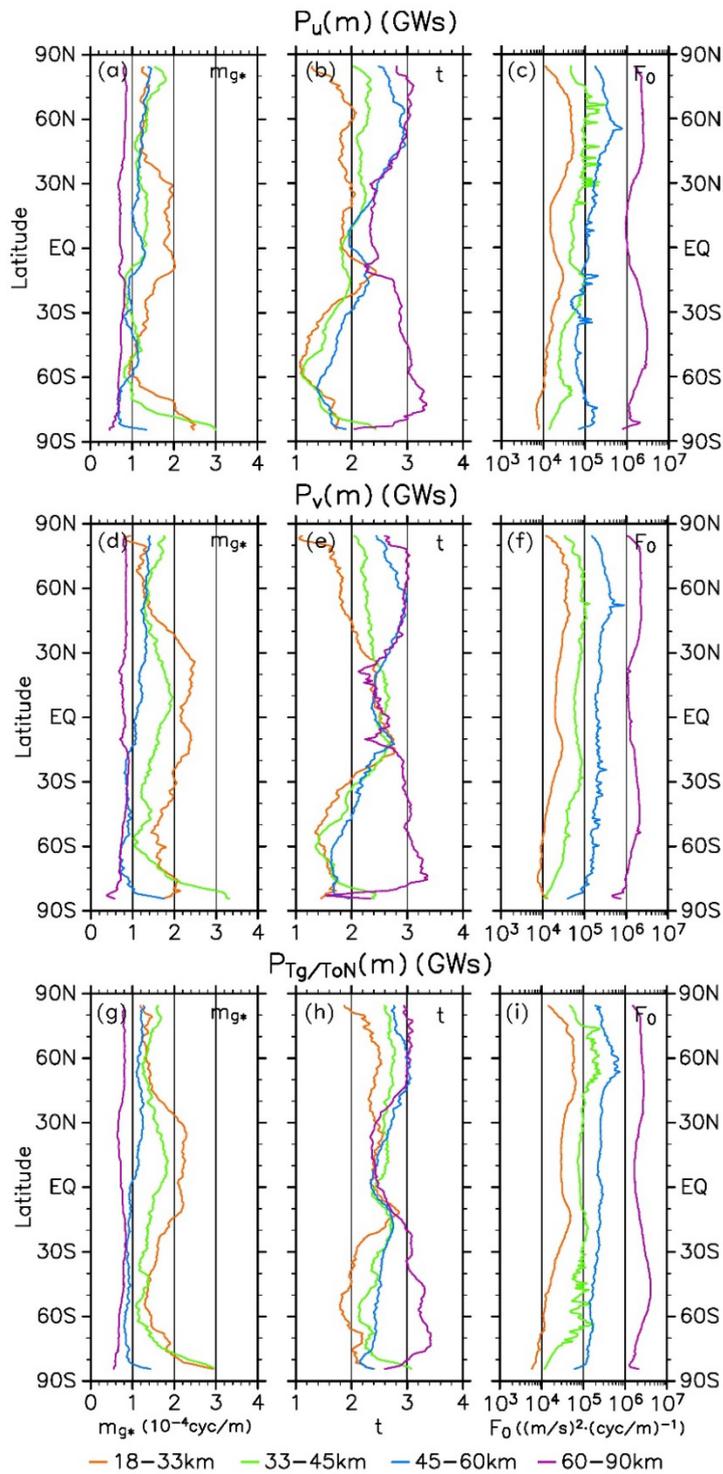
334 Figure 3 illustrates horizontal maps of m_{g^*} , t , and F_0 for $z=18\text{--}33$ km in the
335 lower stratosphere. The contours represent zonal wind. In the high-latitude region of the
336 NH, m_{g^*} is small along the eastward jet. This small m_{g^*} can be explained by the
337 Doppler shift in the jet as follows. The ground-based phase velocity of a GW c is
338 conserved in the background field that is steady and homogeneous in the horizontal
339 wavenumber vector direction. However, the intrinsic phase velocity \hat{c} ($\equiv c - U$, where
340 U denotes background horizontal wind) varies when U changes in the vertical. Above a
341 weak-wind layer near $z=20$ km, the background zonal wind U has an eastward vertical
342 shear below the NH middle atmosphere jet core. On the other hand, a major part of GWs
343 reaching the weak-wind layer near $z=20$ km should have small or westward c because

344 they need to pass through the tropospheric eastward jet below. Thus, the intrinsic phase
345 velocity \hat{c} of the GWs near the weak wind layer is westward and becomes larger while
346 they propagate upward in the eastward shear below the middle atmosphere jet core due
347 to the Doppler shift. The linear gravity wave theory indicates that stronger U makes m
348 smaller, because $m^2 \approx N^2/(c - U)^2$. Thus, m_{g^*} near the eastward jet is expected to get
349 small.

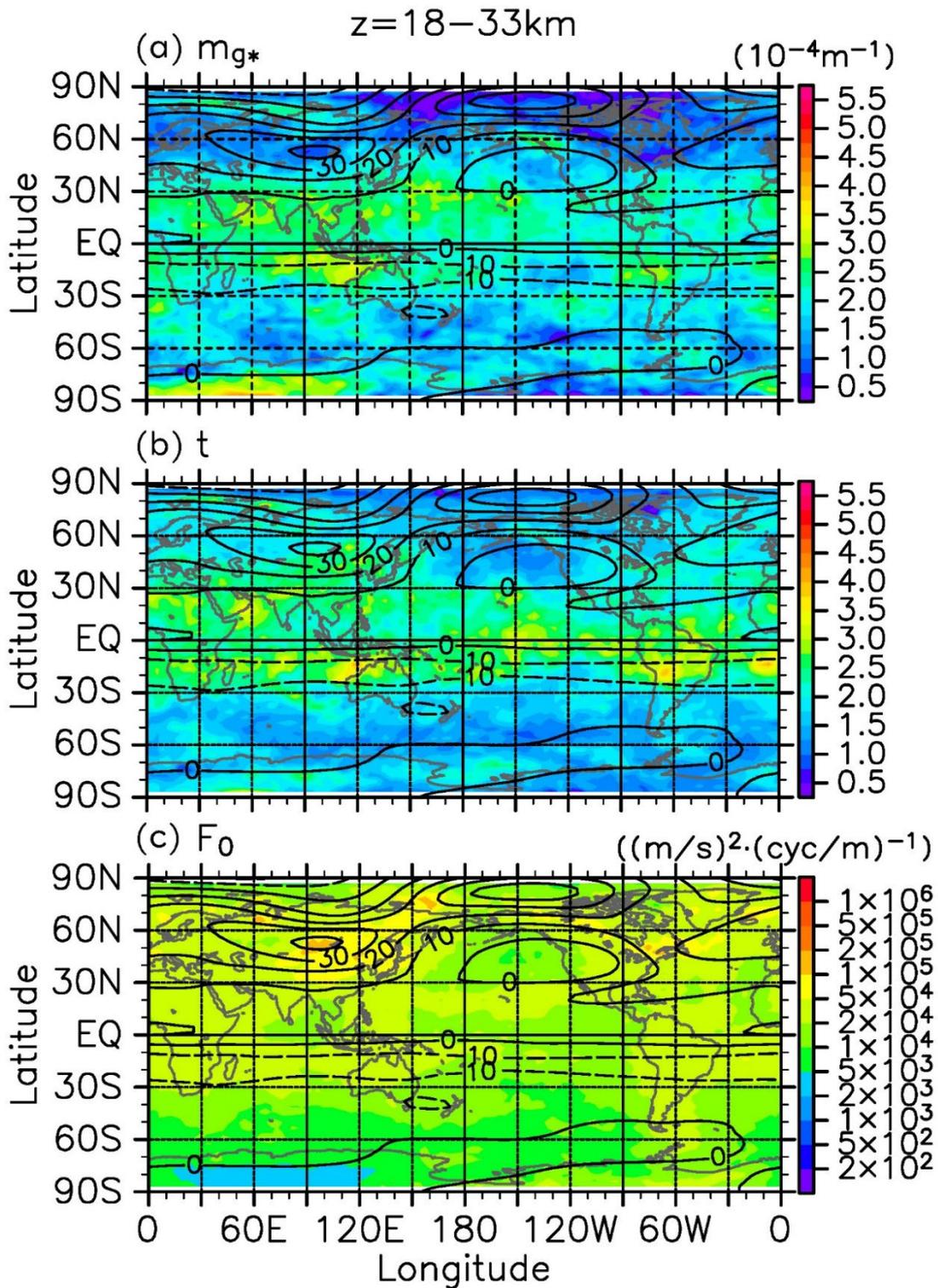
350 In contrast, in the low-latitude region, especially 20° S– 10° N, both m_{g^*} and t
351 are large. This feature can be attributable to a zero-wind phase of the QBO over a latitude
352 region of 15° S– 15° N (not shown). Assuming that c is small, in the background
353 condition of small U , \hat{c} ($= c - U$) is small, m is large, and the saturation condition
354 $|\hat{c}| = |u'|$, where $|u'|$ is the horizontal wind amplitude of the GW, easily holds. It is
355 worth noting that the geographical distribution of m_{g^*} is consistent (i.e., has negative
356 correlation) with the satellite observations for dominant GW vertical wavelengths shown
357 by Ern et al. (2018). There are F_0 peaks along the middle atmosphere jet in the Northern
358 Hemisphere, at low latitudes in the Southern Hemisphere, and around South America.
359 Such geographical distribution of the F_0 peaks is also roughly consistent with that of the
360 GW amplitude peaks observed by satellites shown by Ern et al. (2018). Sato et al. (2009)
361 suggested that steep mountains, jet-front systems in winter, and subtropical monsoon
362 convection in summer are dominant GW sources. The distribution of such GW sources
363 are likely responsible to the observed F_0 peaks.

364 Maps of the spectral parameters in the uppermost stratosphere and lowermost
365 mesosphere (i.e., $z=45$ – 60 km) are shown in Fig. 4. Interestingly, t in the eastward jet
366 region in the Northern Hemisphere is notably large, ranging from 3–3.75. At low latitudes
367 in the Southern Hemisphere, t is also relatively large along the westward jet. As
368 mentioned above, the m of a GW becomes small where U is strong. Because
369 $N^2/m^2 \gg f^2/k^2$ in Eq. (1) derived by Sato and Yamada (1994), which is a theoretical
370 saturated spectrum of a GW propagating in linear wind shear, the spectral slope
371 approaches -4 . The distributions of m_{g^*} and F_0 in the lower mesosphere (Figs. 4a and
372 4c, respectively) have less spatial variability than those in the lower stratosphere (Figs.
373 3a and 3c, respectively). This spatial uniformity was also observed in the height region of
374 60–90 km in the middle and upper mesosphere (not shown).

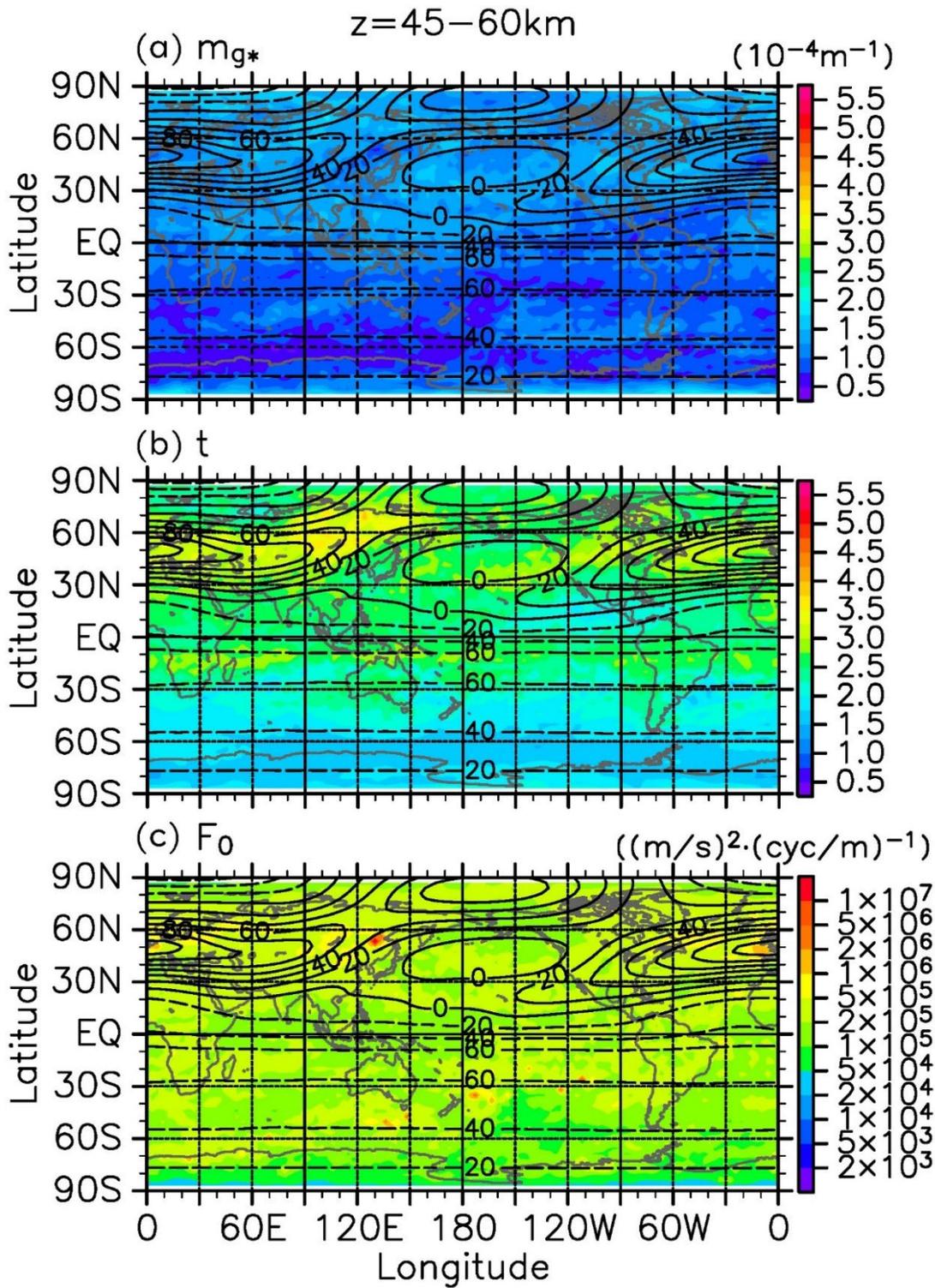
375



376 **Figure 2** Zonal mean m_{g^*} , t , and F_0 of (a–c) zonal wind, (d–f) meridional wind, and
 377 (g–i) temperature spectra of GWs from 5–20 December 2018 as functions of latitude. The
 378 color of the curves represents the height region: $z = 18\text{--}33$ km (orange), $33\text{--}45$ km (green),
 379 $45\text{--}60$ km (blue), and $60\text{--}90$ km (purple).
 380



381 **Figure 3** Maps of (a) m_{g^*} , (b) t , and (c) F_0 for meridional wind spectra of GWs in
 382 the lower stratosphere for $z=18-30$ km from 5–20 December 2018. Contours show zonal
 383 wind.
 384



385 **Figure 4** Similar to Fig. 3 but for the uppermost stratosphere and lowermost
 386 mesosphere for $z=45-60$ km from 5–20 December 2018. Note that the colormap used
 387 in Fig. 4c and the contour interval are different from those used in Fig. 3d.
 388

389 3.3 Shear effect on vertical variation of GW spectra

390 At mid- and high latitudes below the height region of 45–60 km, the spectral slope
 391 becomes steeper with height (Figs. 2b and 2e). Additionally, the decrease in m_{g^*} with
 392 height is much more modest than that at low latitudes (Figs. 2a and 2d). These
 393 characteristic vertical changes of the spectral parameters (i.e., increase in t and modest
 394 decrease in m_{g^*} with height) at mid- and high latitudes at $z < 60$ km is particularly
 395 remarkable near the middle atmosphere eastward and westward jets (Fig. 4). Here, the
 396 effect of strong vertical shear below the jets on the shape of a GW spectrum is examined.

397 Consider a GW propagating in a background wind $U(z)$ that is parallel to the
 398 horizontal wavevector and varies only in the vertical. The dispersion relation for a
 399 hydrostatic and nonrotational internal GW is as follows (e.g., Andrews et al., 1987; Fritts
 400 & Alexander, 2003):

$$\hat{\omega}^2 = \frac{k^2 N^2}{m^2}, \quad (3)$$

401 where $\hat{\omega}$ is the intrinsic frequency and k is the horizontal wavenumber. We can take
 402 the sign of $\hat{\omega}$ as positive without loss of generality. Under this setting, m is negative
 403 for a GW having an upward group velocity. Thus, based on the Wentzel–Kramers–
 404 Brillouin approximation, the variation of $m = m(U; z)$ is described as follows:

$$m(U; z) = -\frac{N}{|c - U(z)|}. \quad (4)$$

405 This equation yields the z derivative of $m(U; z)$:

$$\frac{dm}{dz} \left(= -\frac{d|m|}{dz} \right) = \frac{dm}{dU} \frac{dU}{dz} = \frac{m^2}{N} \frac{U - c}{|U - c|} \frac{dU}{dz}. \quad (5)$$

406 Here, we assume that dominant GWs have westward intrinsic phase velocities \hat{c} in the
 407 middle atmosphere eastward jet, i.e., $|U - c| = U - c$, as before. In the summer
 408 hemisphere, there is a wind-reversal layer between the eastward jet in the troposphere and
 409 westward jet in the middle atmosphere. Convection at low latitudes (e.g., Sato et al., 2009)
 410 and shear instability just above the tropospheric jet (e.g., Bühler et al., 1999; Okui & Sato,
 411 2020) are possible sources of GWs, which generally have eastward \hat{c} ($= c - U$) in the
 412 middle atmosphere westward jet. Thus, both $U - c$ and dU/dz are positive (negative)
 413 below the middle atmosphere eastward (westward) jet. The rightmost side of the Eq. (5)
 414 is positive and hence dm/dz is larger for higher $|m|$. This fact shows that the absolute
 415 wavenumber $|m|$ decreases more rapidly with height in a higher $|m|$ range. In addition,

416 the horizontal wind amplitude $|u'|$ is modulated by the modulation of m assuming the
 417 momentum flux conservation:

$$|u'|^2 = \left| \frac{m}{k} \right| |u'w'| \propto |m|. \quad (6)$$

418 Both changes in m and in $|u'|$ shown in Eqs. (5) and (6) by the vertical shear below the
 419 peak of a jet acts to increase in t for the GW spectra.

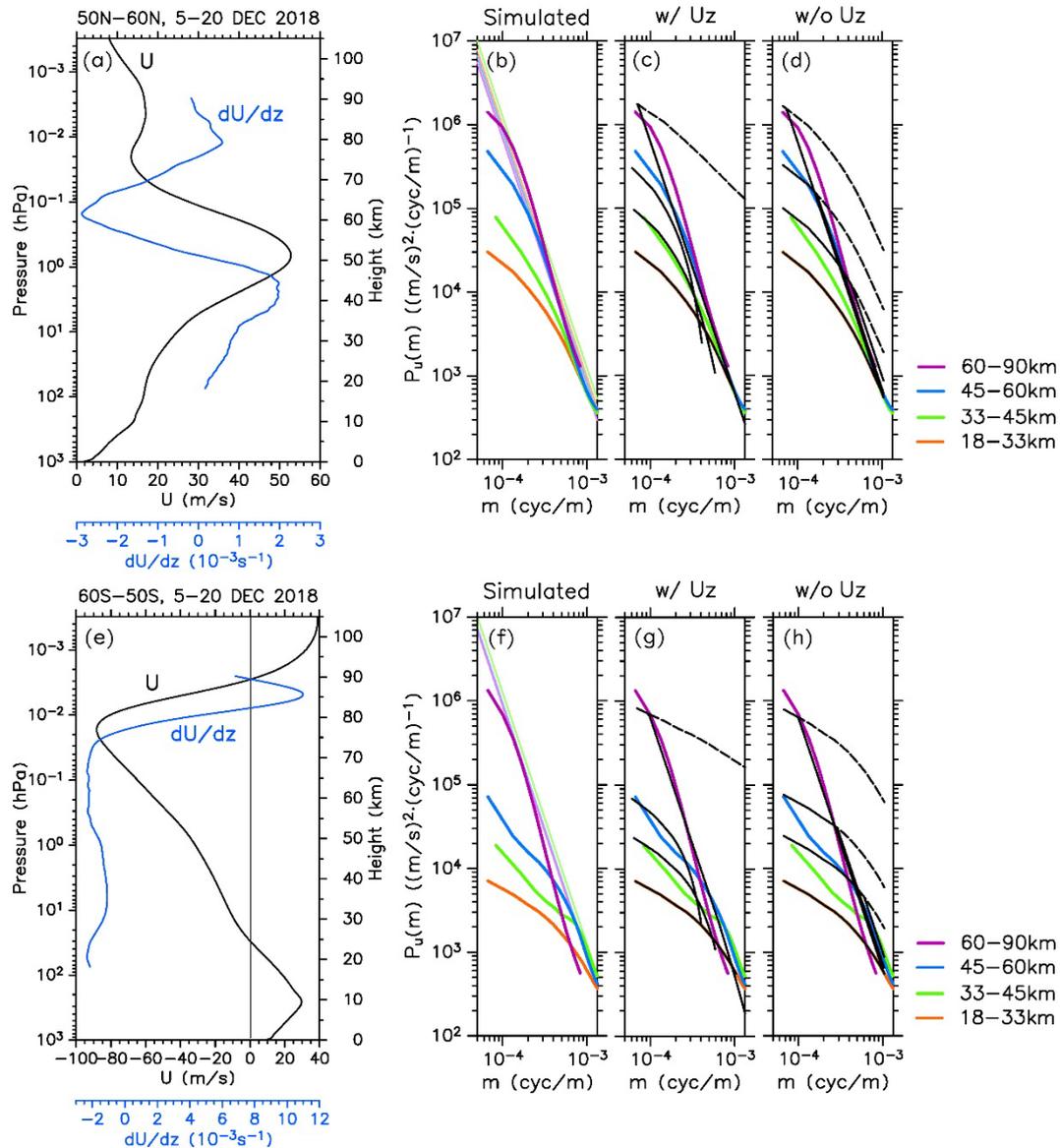
420 To evaluate the shear effect on a spectral shape quantitatively, we obtain
 421 theoretical spectra by integrating Eq. (5) numerically in the vertical for the zonal wind u'
 422 spectra of GWs at 50° – 60° N around the middle atmosphere eastward jet and at 50° – 60°
 423 S around the westward jet. We take the representative heights z_{ri} ($i=1, \dots, 4$), whose
 424 definition is described below, as the upper and lower limits for the integration. Here, i
 425 denotes each of four height regions in which the model-simulated spectra were calculated
 426 (i.e., $z=18$ – 33 km, 33 – 45 km, 45 – 60 km, and 60 – 90 km). The height z_{ri} is defined as
 427 the average height weighted by the product of $u'(z)^2$ and 10% cosine-tapered window,
 428 taking a possible large vertical dependence of the GW amplitudes into account. We took
 429 $z_{r1}=25.47$ km for $z=18$ – 33 km, $z_{r2}=40.01$ km for $z=33$ – 45 km, $z_{r3}=54.30$ km for
 430 $z=45$ – 60 km, and $z_{r4}=73.86$ km for $z=60$ – 90 km for 50° – 60° N, and $z_{r1}=24.98$ km
 431 for $z=18$ – 33 km, $z_{r2}=39.80$ km for $z=33$ – 45 km, $z_{r3}=53.46$ km for $z=45$ – 60 km,
 432 and $z_{r4}=81.41$ km for $z=60$ – 90 km for 50° – 60° S. We also consider the wave saturation
 433 using the theory by Smith et al. (1987).

434 Detailed calculation steps are as follows: (i) The model-simulated spectra for $z=$
 435 18 – 33 km are used for initial values. The initial heights for the integration were chosen
 436 as $z_{r1}=25.47$ km for 50° – 60° N and 24.98 km for 50° – 60° S. Background zonal wind
 437 U was derived by averaging a zonal wind profile zonally and over the respective latitude
 438 region. Following steps (ii)–(iv) are repeated for the three height regions of $i=2$ – 4 from
 439 below. (ii) By numerically integrating Eq. (5) from z_{ri-1} to z_{ri} ($i=2$ – 4) for each height
 440 region, m was updated. (iii) At each step of the m integral, amplitude growth due to
 441 decrease in atmospheric density at higher altitudes was included for the spectral density
 442 using Eq. (16) of Smith et al. (1987), i.e., $P(m) \propto e^{z/(2H_E/3)}$, where H_E ranges from
 443 14 – 21 km. Here, we took 18 km for the value of H_E . In addition, the spectral density at
 444 each m is multiplied by the ratio of two m s before and after the m -integral step
 445 considering Eq. (6). (iv) When an integrated spectrum exceeds the spectral density of the
 446 theoretical spectrum of Smith et al. (1987) (i.e., $N^2/12m^3$) in a specific m range, we
 447 regarded GWs in this m range as saturated and replaced the spectral density with that of
 448 the Smith et al. (1987)'s theoretical spectrum.

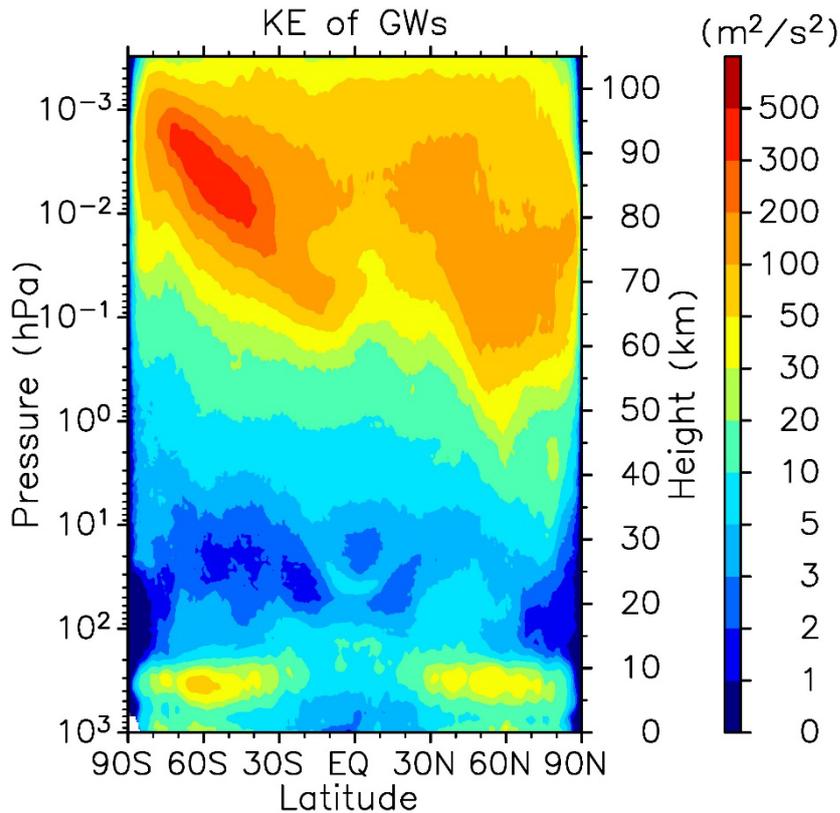
449 The results are shown in Fig. 5. The profile of the background zonal wind U in
450 each latitude region is plotted in Figs. 5a and 5e. The spectra obtained by the above
451 calculation are denoted by black curves in Figs. 5c and 5g, which are overlaid on the
452 model-simulated spectra denoted by colored curves (same as in Figs. 5b and 5f). Note
453 that m for the horizontal axes means $|m|$. Each black curve indicates the result at z_{ri} .
454 The results of the calculation without the shear effect on m are shown in Figs. 5d and
455 5h for comparison. The light-colored curves in Figs. 5b and 5f indicate theoretical spectra
456 of Smith et al. (1987). The dashed curves show the spectra without the treatment of wave
457 saturation (i.e., without the step (iv) in the previous paragraph).

458 For both latitude regions, the estimated spectra with shear effect (Figs. 5c and 5g)
459 accord better with the model-simulated spectra than the spectra estimated without shear
460 effect (Figs. 5d and 5h). This accordance of the estimated spectra with shear effect is
461 especially significant for $z=33\text{--}45$ km at $50^\circ\text{--}60^\circ$ N, and $z=33\text{--}45$ km and $45\text{--}60$ km
462 at $60^\circ\text{--}50^\circ$ S. However, there are a few exceptions. In the uppermost stratosphere and
463 lowermost mesosphere ($z=45\text{--}60$ km) at $50^\circ\text{--}60^\circ$ N, the estimated spectral density with
464 shear effect is ~ 1.5 times smaller than the model-simulated spectral density (denoted by
465 the blue curve). The estimated spectral density was calculated based on the assumption
466 that GWs propagate only vertically. In-situ generation from the eastward jet in the middle
467 atmosphere and/or lateral propagation toward the jet (e.g., Sato et al., 2009, 2012) give
468 possible explanation for the difference between the model-simulated spectrum and the
469 estimated spectrum in the region of $z=45\text{--}60$ km at $50^\circ\text{--}60^\circ$ N.

470 Even in the m range where the wave-saturation threshold is not fulfilled (i.e.,
471 without dashed curves), these estimates of shear-affected spectra show the increase in
472 spectral slope that is also seen in the model-simulated spectra. The most interesting
473 suggestion obtained from this simple investigation is that the steep slope of the GW m
474 spectrum is likely formed by strong vertical shear, even in the absence of wave saturation.
475 In contrast, the resultant spectral densities for $z=60\text{--}90$ km exceed the Smith et al.
476 (1987)'s one at almost all m s as shown in Fig. 5 for both the $50^\circ\text{--}60^\circ$ N and $60^\circ\text{--}50^\circ$ S
477 cases, regardless of whether the shear effect was taken into account or not. In these
478 regions, it is considered that wave saturation is the main cause of the steep spectral slope.
479



481 **Figure 5** Estimates of the shear effect on GW spectra at (a–d) 50° – 60° N and (e–h) 60° –
 482 50° S from 5–20 December 2018. (a, e) Vertical profiles of background zonal winds U
 483 averaged zonally and over the shown latitude regions. Blue curves represent dU/dz ,
 484 which was used for the calculation. (b, f) Model-simulated spectra (colored curves;
 485 legends are shown on the right). Curves with the same but lighter colors show theoretical
 486 spectra (Smith et al., 1987), which almost overlap with each other. (c, g) Estimated spectra
 487 with shear effect (black curves). (d, h) Estimated spectra without shear effect (black
 488 curves). These spectra are overlaid on the model-simulated spectra in the respective
 489 height regions. Dashed curves represent the results with wave saturation effect ignored.
 490



491 **Figure 6** Latitude-height section of zonal mean kinetic energy of GWs from 5–20
 492 December 2018.

493

494 **4 Summary and Concluding Remarks**

495 Using the output of a hindcast of the middle atmosphere in December 2018
 496 performed by a GW-permitting high-top GCM, we examined the contribution of GWs to
 497 the universal vertical wavenumber ($\sim m^{-3}$) spectra. The results of this study are as follows.

- 498 1. Model-simulated spectra in the stratosphere and mesosphere have shape with a
 499 steep slope of $\sim m^{-3}$ in a high m range, consistent with observations shown by
 500 previous studies.
- 501 2. In most regions of the middle atmosphere, GWs do not contribute to the entire
 502 steep-slope part of the m spectra obtained by the previous observational studies.
 503 GWs make a dominant contribution to the m spectra only in a high m part of
 504 the steep-slope range. Disturbances other than GWs also contribute to the spectra
 505 in its low m part.
- 506 3. The lowest end of the m spectral range in which GWs are dominant is lower in

507 the mesosphere than in the stratosphere.

508 4. Contribution of the disturbances other than GWs is especially large in equatorial
509 and polar regions.

510 Parameters describing the characteristics of GW spectra were also examined. The
511 m_{g^*} value of GW spectra is lower at higher altitudes. The spectral density at m_{g^*} (i.e.,
512 $F_0/2$) is larger at higher altitudes. These vertical variations are consistent with wave
513 saturation and the exponential decrease in density. Parameter t increases with height and
514 approaches ~ 3 in mid- and high-latitude regions. In general, t is not necessarily ~ 3 (i.e.,
515 $\sim m^{-3}$), which is consistent with several previous studies (e.g., Sato et al. 2003; Lu et al.
516 2015). The spectral slope in the high m range is steeper than that in the low m range.
517 In the lower stratosphere, the geographical distribution of F_0 is roughly consistent with
518 the observations reported in previous studies. We also examined the shear effect on GW
519 spectra below the eastward and westward jets in the middle atmosphere. The results
520 showed that strong vertical shear, in addition to wave saturation, is significantly
521 responsible for making the slope of the GW spectra steeper.

522 It is expected that the characteristics of m spectra have seasonal and interannual
523 variations following different background conditions such as the QBO in the equatorial
524 region, for example. To examine the universality of the results, it would be useful to
525 perform similar simulations for other seasons and different years. The JAGUAR hindcasts,
526 containing three-dimensional and global data of GWs in the middle atmosphere, is a
527 strong tool for quantitative elucidation of GW behavior in the middle atmosphere.

528 Acknowledgments

529 All figures in this paper were created using the Dennou Club Library (DCL). This study
530 benefitted from stimulating discussions at the International Space Science Institute (ISSI)
531 Gravity Wave activity. The study was supported by JST CREST (grant JPMJCR1663) and
532 JSPS KAKENHI (grant JP21J20798). The hindcasts were performed using the Earth
533 Simulator at the Japan Agency for Marine-Earth Science and Technology (JAMSTEC).
534 The processed model data are available at the following website: [https://pansy.eps.s.u-](https://pansy.eps.s.u-tokyo.ac.jp/archive_data/Okui_etal_GW-spectra/)
535 [tokyo.ac.jp/archive_data/Okui_etal_GW-spectra/](https://pansy.eps.s.u-tokyo.ac.jp/archive_data/Okui_etal_GW-spectra/). We thank James Buxton, MSc, from
536 Edanz (<https://jp.edanz.com/ac>) for editing a draft of this manuscript.

537

538 **References**

- 539 Andrews, D.G., Holton, J.R., & Leovy, C.B. (1987). *Middle Atmosphere Dynamics*. San
540 Diego, CA: Academic Press.
- 541 Allen, S. J., & Vincent, A. (1995). Gravity wave activity in the lower atmosphere:
542 Seasonal and latitudinal variations. *Journal of Geophysical Research*, **100**, 1327–
543 1350. <https://doi.org/10.1029/94JD02688>
- 544 Bloom, S. C., Takacs, L. L., DaSilva, A. M., & Levina, D. (1996). Data assimilation using
545 incremental analysis updates. *Monthly Weather Review*, **124**, 1256–1271.
546 [https://doi.org/10.1175/1520-0493\(1996\)124<1256:DAUIAU>2.0.CO;2](https://doi.org/10.1175/1520-0493(1996)124<1256:DAUIAU>2.0.CO;2)
- 547 Bühler, O., McIntyre, M. E., & Scinocca, J. F. (1999). On shear-generated gravity waves
548 that reach the mesosphere. Part I: Wave generation. *Journal of Atmospheric Science*,
549 **56**, 3749–3763. [https://doi.org/10.1175/1520-
550 0469\(1999\)056<3749:OSGGWT>2.0.CO;2](https://doi.org/10.1175/1520-0469(1999)056<3749:OSGGWT>2.0.CO;2)
- 551 Chen, C., Chu, X., McDonald, A. J., Vadas, S. L., Yu, Z., Fong, W., & Lu, X. (2013).
552 Inertiagravity waves in Antarctica: A case study using simultaneous lidar and radar
553 measurements at McMurdo/Scott Base (77.8°S, 166.7°E). *Journal of Geophysical
554 Research: Atmospheres*, **118**, 2794–2808. <https://doi.org/10.1002/jgrd.50318>
- 555 Chen, C., Chu, X., Zhao, J., Roberts, B. R., Yu, Z., Fong, W., et al. (2016). Lidar
556 observations of persistent gravity waves with periods of 3–10 h in the Antarctic
557 middle and upper atmosphere at McMurdo (77.83°S, 166.67°E). *Journal of
558 Geophysical Research: Space Physics*, **121**, 1483–1502.
559 <https://doi.org/10.1002/2015ja022127>
- 560 Chen, C., & Chu, X. (2017). Two-dimensional Morlet wavelet transform and its
561 application to wave recognition methodology of automatically extracting two-
562 dimensional wave packets from lidar observations in Antarctica. *Journal of
563 Atmospheric and Solar-Terrestrial Physics*, **162**, 28–47.
564 <https://doi.org/10.1016/j.jastp.2016.10.016>
- 565 Chu, X., Zhao, J., Lu, X., Harvey, V. L., Jones, R. M., Becker, E., et al. (2018). Lidar
566 observations of stratospheric gravity waves from 2011 to 2015 at McMurdo
567 (77.84°S, 166.69°E), Antarctica: 2. Potential energy densities, lognormal
568 distributions, and seasonal variations. *Journal of Geophysical Research:
569 Atmospheres*, **123**, 7910–7934. <https://doi.org/10.1029/2017JD027386>
- 570 Conn, A. R., Gould, N. I., & Toint, P. L. (2000). *Trust region methods*, Philadelphia, PA:
571 Society for Industrial and Applied Mathematics.
572 <https://doi.org/10.1137/1.9780898719857>

- 573 Dunkerton, T. J. (1981). On the inertial stability of the equatorial middle atmosphere.
574 *Journal of the Atmospheric Sciences*, **38**, 2354–2364. <https://doi.org/10.1175/1520->
575 0469(1981)038%3C2354:OTISFT%3E2.0.CO;2
- 576 Dewan, E. M., & Good, R. E. (1986). Saturation and the “universal” spectrum for vertical
577 profiles of horizontal scalar winds in the atmosphere. *Journal of Geophysical*
578 *Research*, **91**(D2), 2742–2748. <https://doi.org/10.1029/JD091iD02p02742>
- 579 Dewan, E. M., Grossbard, N., Quesada, A. F. & Good, R. E. (1984). Spectral analysis of
580 10m resolution scalar velocity profiles in the stratosphere. *Geophysical Research*
581 *Letters*, **11**, 80–83. <https://doi.org/10.1029/GL011i001p00080>
- 582 Ern, M., Ploeger, F., Preusse, P., Gille, J. C., Gray, L. J., Kalisch, S., Mlynczak, M. G.,
583 Russell III, J. M., & Riese, M. (2014). Interaction of gravity waves with the QBO:
584 A satellite perspective. *Journal of Geophysical Research: Atmospheres*, **119**, 2329–
585 2355. <https://doi.org/10.1002/2013JD020731>
- 586 Ern, M., Trinh, Q. T., Preusse, P., Gille, J. C., Mlynczak, M. G., Russell Iii, J. M., &
587 Riese, M. (2018). GRACILE: a comprehensive climatology of atmospheric gravity
588 wave parameters based on satellite limb soundings. *Earth System Science Data*, **10**,
589 857–892. <https://doi.org/10.5194/essd-10-857-2018>
- 590 Fritts, D. C., & Alexander, M. J. (2003). Gravity wave dynamics and effects in the middle
591 atmosphere. *Reviews of Geophysics*, **41**, 1003.
592 <https://doi.org/10.1029/2001RG000106>
- 593 Fritts, D. C., & Chou, H. (1987). An Investigation of the Vertical Wavenumber and
594 Frequency Spectra of Gravity Wave Motions in the Lower Stratosphere. *Journal of*
595 *the Atmospheric Sciences*, **44**(24), 3610–3624. <https://doi.org/10.1175/1520->
596 0469(1987)044%3C3610:AIOTVW%3E2.0.CO;2
- 597 Hines, C. O. (1997a). Doppler-spread parameterization of gravity-wave momentum
598 deposition in the middle atmosphere. Part 1: Basic formulation. *Journal of*
599 *Atmospheric and Solar-Terrestrial Physics*, **59**, 371–386.
600 [https://doi.org/10.1016/S1364-6826\(96\)00079-X](https://doi.org/10.1016/S1364-6826(96)00079-X)
- 601 Hines, C. O. (1997b). Doppler-spread parameterization of gravity-wave momentum
602 deposition in the middle atmosphere. Part 2: Broad and quasi monochromatic spectra,
603 and implementation. *Journal of Atmospheric and Solar-Terrestrial Physics*, **59**(4),
604 387–400. [https://doi.org/10.1016/S1364-6826\(96\)00080-6](https://doi.org/10.1016/S1364-6826(96)00080-6)
- 605 Holton, J. R. (1983). The influence of gravity wave breaking on the general circulation of
606 the middle atmosphere. *Journal of the Atmospheric Sciences*, **40**, 2497–2507.
607 [https://doi.org/10.1175/1520-0469\(1983\)040%3C2497:TIOGWB%3E2.0.CO;2](https://doi.org/10.1175/1520-0469(1983)040%3C2497:TIOGWB%3E2.0.CO;2)

- 608 Kawatani, Y., Sato, K., Dunkerton, T. J., Watanabe, S., Miyahara, S., & Takahashi, M.
609 (2010a). The roles of equatorial trapped waves and internal inertia-gravity waves in
610 driving the quasi-biennial oscillation. Part I: Zonal mean wave forcing. *Journal of*
611 *the Atmospheric Sciences*, **67**, 963–980. <https://doi.org/10.1175/2009JAS3222.1>
- 612 Kawatani, Y., Sato, K., Dunkerton, T. J., Watanabe, S., Miyahara, S., & Takahashi, M.
613 (2010b). The roles of equatorial trapped waves and internal inertia-gravity waves in
614 driving the quasi-biennial oscillation. Part II: Three-dimensional distribution of
615 wave forcing. *Journal of the Atmospheric Sciences*, **67**, 981–997.
616 <https://doi.org/10.1175/2009JAS3223.1>
- 617 Koshin, D., Sato, K., Miyazaki, K., & Watanabe, S. (2020). An ensemble Kalman filter
618 data assimilation system for the whole neutral atmosphere. *Geoscientific Model*
619 *Development*, **13**, 3145–3177. <https://doi.org/10.5194/gmd-13-3145-2020>
- 620 Koshin, D., Sato, K., Kohma, M., & Watanabe, S. (2022). An update on the 4D-LETKF
621 data assimilation system for the whole neutral atmosphere. *Geoscientific Model*
622 *Development*, **5**, 2293–2307. <https://doi.org/10.5194/gmd-15-2293-2022>
- 623 Lu, X., Chu, X., Fong, W., Chen, C., Yu, Z., Roberts, B. R., & McDonald, A. J. (2015).
624 Vertical evolution of potential energy density and vertical wave number spectrum of
625 Antarctic gravity waves from 35 to 105 km at McMurdo (77.8°S, 166.7°E). *Journal*
626 *of Geophysical Research: Atmospheres*, **120**, 2719–2737.
627 <https://doi.org/10.1002/2014JD022751>
- 628 Mayr, H. G., Mengel, J. G., Hines, C. O., Chan, K. L., Arnold, N. F., Reddy, C. A., &
629 Porter, H. S. (1997). The gravity wave Doppler spread theory applied in a numerical
630 spectral model of the middle atmosphere 2. Equatorial oscillations. *Journal of*
631 *Geophysical Research*, **102**, 26,093–26,105. <https://doi.org/10.1029/96JD03214>
- 632 McLandress, C., & Scinocca, J. F. (2005). The GCM Response to Current
633 Parameterizations of Nonorographic Gravity Wave Drag. *Journal of the*
634 *Atmospheric Sciences*, **62**(7), 2394–2413. <https://doi.org/10.1175/JAS3483.1>
- 635 Noersomadi & Tsuda, T. (2016). Global distribution of vertical wavenumber spectra in
636 the lower stratosphere observed using high-vertical-resolution temperature profiles
637 from COSMIC GPS radio occultation. *Annals of Geophysics*, **34**, 203–213.
638 <https://doi.org/10.5194/angeo-34-203-2016>.
- 639 Okamoto, K., Sato, K., & Akiyoshi, H. (2011). A study on the formation and trend of the
640 Brewer-Dobson circulation. *Journal of Geophysical Research*, **116**, D10117.
641 <https://doi.org/10.1029/2010JD014953>

- 642 Okui, H., & Sato, K. (2020). Characteristics and Sources of Gravity Waves in the Summer
643 Stratosphere Based on Long-Term and High-Resolution Radiosonde Observations.
644 *SOLA*, **16**, 64–69. <https://doi.org/10.2151/sola.2020-011>
- 645 Okui, H., Sato, K., Koshin, D., & Watanabe, S. (2021). Formation of a mesospheric
646 inversion layer and the subsequent elevated stratopause associated with the major
647 stratospheric sudden warming in 2018/19. *Journal of Geophysical Research:
648 Atmospheres*, **126**, e2021JD034681. <https://doi.org/10.1029/2021JD034681>
- 649 Rapp, M., Dörnbrack, A., & Preusse, P. (2018). Large midlatitude stratospheric
650 temperature variability caused by inertial instability: A potential source of bias for
651 gravity wave climatologies. *Geophysical Research Letters*, **45**, 10,682–10,690.
652 <https://doi.org/10.1029/2018GL079142>
- 653 Sato, K. (1994). A statistical study of the structure, saturation and sources of inertio-
654 gravity 905 waves in the lower stratosphere observed with the MU radar. *Journal of
655 Atmospheric and Terrestrial Physics*, **56**(6), 755–774. [https://doi.org/10.1016/0021-
656 9169\(94\)90131-7](https://doi.org/10.1016/0021-9169(94)90131-7)
- 657 Sato, K., & Dunkerton, T. J. (1997). Estimates of momentum flux associated with
658 equatorial Kelvin and gravity waves. *Journal of Geophysical Research*, **102**,
659 26,247–26,261. <https://doi.org/10.1029/96JD02514>
- 660 Sato, K. & Hirano, S. (2019). The climatology of the Brewer–Dobson circulation and the
661 contribution of gravity waves. *Atmospheric Chemistry and Physics*, **19**, 4517–4539.
662 <https://doi.org/10.5194/acp-19-4517-2019>
- 663 Sato, K., Tateno, S., Watanabe, S., & Kawatani, Y. (2012). Gravity Wave Characteristics
664 in the Southern Hemisphere Revealed by a High-Resolution Middle-Atmosphere
665 General Circulation Model. *Journal of the Atmospheric Sciences*, **69**, 1378–1396.
666 <https://doi.org/10.1175/JAS-D-11-0101.1>
- 667 Sato, K., Watanabe, S., Kawatani, Y., Tomikawa, Y., Miyazaki, K., & Takahashi, M.
668 (2009). On the origins of mesospheric gravity waves. *Geophysical Research Letters*,
669 **36**, L19801. <https://doi.org/10.1029/2009GL039908>
- 670 Sato, K., & Yamada, M. (1994). Vertical structure of atmospheric gravity waves revealed
671 by the wavelet analysis. *Journal of Geophysical Research*, **99**(D10), 20,623–20,631.
672 <https://doi.org/10.1029/94JD01818>
- 673 Sato, K., Yamamori, M., Ogino, S., Takahashi, N., Tomikawa, Y., & Yamanouchi, T.
674 (2003). A meridional scan of the stratospheric gravity wave field over the ocean in
675 2001 (MeSSO2001). *Journal of Geophysical Research*, **108**(D16), 4491.
676 <https://doi.org/10.1029/2002JD003219>

- 677 Senft, D. C., & Gardner, C. S. (1991). Seasonal variability of gravity wave activity and
678 spectra in the mesopause region at Urbana. *Journal of Geophysical Research*, **96**,
679 17,229–17,264. <https://doi.org/10.1029/91JD01662>
- 680 Smith, S. A., Fritts, D. C., & Vanzandt, T. E. (1987). Evidence for a Saturated Spectrum
681 of Atmospheric Gravity Waves. *Journal of the Atmospheric Sciences*, **44**(10), 1404–
682 1410. [https://doi.org/10.1175/1520-
683 0469\(1987\)044%3C1404:EFASSO%3E2.0.CO;2](https://doi.org/10.1175/1520-0469(1987)044%3C1404:EFASSO%3E2.0.CO;2)
- 684 Stephan, C. C., Strube, C., Klocke, D., Ern, M., Hoffmann, L., Preusse, P., & Schmidt,
685 H. (2019). Intercomparison of gravity waves in global convection-permitting
686 models. *Journal of the Atmospheric Sciences*, **76**, 2739–2759.
687 <https://doi.org/10.1175/JAS-D-19-0040.1>
- 688 Strube, C., Ern, M., Preusse, P., & Riese, M. (2020). Removing spurious inertial
689 instability signals from gravity wave temperature perturbations using spectral
690 filtering methods. *Atmospheric Measurement Techniques*, **13**, 4927–4945.
691 <https://doi.org/10.5194/amt-13-4927-2020>
- 692 Tsuda, T., Inoue, T., Kato, S., Fukao, S., Fritts, D. C., & VanZandt, T. E. (1989). MST
693 Radar Observations of a Saturated Gravity Wave Spectrum. *Journal of the
694 Atmospheric Sciences*, **46**(15), 2440–2447. [https://doi.org/10.1175/1520-
695 0469\(1989\)046%3C2440:MROOAS%3E2.0.CO;2](https://doi.org/10.1175/1520-0469(1989)046%3C2440:MROOAS%3E2.0.CO;2)
- 696 Tsuda, T., Kato, S., Yokoi, T., Inoue, T., Yamamoto, M., Vanzandt, T. E., Fukao, S., &
697 Sato, T. (1990). Gravity waves in the mesosphere observed with the middle and
698 upper atmosphere radar. *Radio Science*, **25**(5), 1005–1018.
699 <https://doi.org/10.1029/RS025i005p01005>
- 700 Tsuda, T., Lin, X., Hayashi, H., & Noersomadi (2011). Analysis of vertical wave
701 number spectrum of atmospheric gravity waves in the stratosphere using COSMIC
702 GPS radio occultation data. *Atmospheric Measurement Techniques*, **4**, 1627–1636.
703 <https://doi.org/10.5194/amt-4-1627-2011>
- 704 VanZandt, T. E. (1982). A universal spectrum of buoyancy waves in the atmosphere.
705 *Geophysical Research Letters*, **9**, 575–578.
706 <https://doi.org/10.1029/GL009i005p00575>

- 707 VanZandt, T. E. (1985). A model for gravity wave spectra observed by Doppler sounding
708 systems. *Radio Science*, **20**(6), 1323–1330.
709 <https://doi.org/10.1029/RS020i006p01323>
- 710 Wang, L., Geller, M. A., & Alexander, M. J. (2005). Spatial and Temporal Variations of
711 Gravity Wave Parameters. Part I: Intrinsic Frequency, Wavelength, and Vertical
712 Propagation Direction. *Journal of the Atmospheric Sciences*, **62**(1), 125–142.
713 <https://doi.org/10.1175/JAS-3364.1>
- 714 Warner, C. D., & McIntyre, M. E. (1996). On the Propagation and Dissipation of Gravity
715 Wave Spectra through a Realistic Middle Atmosphere. *Journal of the Atmospheric
716 Sciences*, **53**(22), 3213–3235. [https://doi.org/10.1175/1520-
717 0469\(1996\)053%3C3213:OTPADO%3E2.0.CO;2](https://doi.org/10.1175/1520-0469(1996)053%3C3213:OTPADO%3E2.0.CO;2)
- 718 Warner, C. D. & McIntyre, M. E. (1999). Toward an ultra-simple spectral gravity wave
719 parameterization for general circulation models. *Earth, Planets and Space*, **51**, 475–
720 484. <https://doi.org/10.1186/BF03353209>
- 721 Watanabe, S. (2008). Constraints on a Non-orographic Gravity Wave Drag
722 Parameterization Using a Gravity Wave Resolving General Circulation Model.
723 *SOLA*, **4**, 61–64. <https://doi.org/10.2151/sola.2008-016>
- 724 Watanabe, S., & Miyahara, S. (2009). Quantification of the gravity wave forcing of the
725 migrating diurnal tide in a gravity wave-resolving general circulation model. *Journal
726 of Geophysical Research: Atmospheres*, **114**, D07110.
727 <https://doi.org/10.1029/2008JD011218>
- 728 Watanabe, S., Sato, K., Kawatani, Y., & Takahashi, M. (2015). Vertical resolution
729 dependence of gravity wave momentum flux simulated by an atmospheric general
730 circulation model. *Geoscientific Model Development*, **8**, 1637–1644.
731 <https://doi.org/10.5194/gmd-8-1637-2015>
- 732 Yan, Y. Y., Zhang, S. D., Huang, C. M., Huang, K. M., Gong, Y., & Gan, Q. (2018).
733 The vertical wave number spectra of potential energy density in the stratosphere
734 deduced from the COSMIC satellite observation. *Quarterly Journal of the Royal
735 Meteorological Society*, **145**, 318–336. <https://doi.org/10.1002/qj.3433>
- 736 Zhao, J., Chu, X., Chen, C., Lu, X., Fong, W., Yu, Z., Jones, R. M., Roberts, B. R., &
737 Dörnbrack, A. (2017). Lidar observations of stratospheric gravity waves from 2011
738 to 2015 at McMurdo (77.84°S, 166.69°E), Antarctica: 1. Vertical wavelengths,
739 periods, and frequency and vertical wave number spectra. *Journal of Geophysical
740 Research: Atmospheres*, **122**(10), 5041–5062.
741 <https://doi.org/10.1002/2016jd026368>