

# Mapping Arctic Sea Ice Thickness: A New Method for Improved Ice Freeboard Retrieval from Satellite Altimetry

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## Key Points:

- Lead observations from neighboring altimeter tracks are exploited to improve the sea surface height calculated at ice-covered locations
- The interpolation is constrained with sea surface decorrelation length- and time-scales from CryoSat-2 data and ice-ocean model simulations
- Altimeter sampling of multi-track leads improves the precision of Arctic sea ice freeboards by 20% and can increase accuracy by up to 25%

## 26 **Abstract**

27       A growing number of studies are concluding that the resilience of the Arctic sea ice cover in  
28 a warming climate is essentially controlled by its thickness. Satellite radar and laser altimeters  
29 have allowed us to routinely monitor sea ice thickness across most of the Arctic Ocean for several  
30 decades. However, a key uncertainty remaining in the sea ice thickness retrieval is the error on  
31 the sea surface height (SSH) which is conventionally interpolated at ice floes from a limited  
32 number of lead observations along the altimeter's orbital track. Here, we use an objective  
33 mapping approach to determine sea surface height from all proximal lead samples located on the  
34 orbital track and from adjacent tracks within a neighborhood of 10s of kilometers. The patterns  
35 of the SSH signal's zonal, meridional, and temporal decorrelation length scales are obtained by  
36 analyzing the covariance of historic CryoSat-2 Arctic lead observations, which match the scales  
37 obtained from an equivalent analysis of high-resolution sea ice-ocean model fields. We use these  
38 length scales to determine an optimal SSH and error estimate for each sea ice floe location. By  
39 exploiting leads from adjacent tracks, we can increase the SSH precision estimated at orbital  
40 crossovers by a factor of three. In regions of high SSH uncertainty, biases in CryoSat-2 sea ice  
41 freeboard can be reduced by 25% with respect to coincident airborne validation data. The new  
42 method is not restricted to a particular sensor or mode, so it can be generalized to all present and  
43 historic polar altimetry missions.

44

## 45 **Plain Language Summary**

46 Arctic Ocean sea ice thickness has been measured with satellite altimeters for several  
47 decades by stitching together observations of the sea level at open water leads or ‘cracks’ in the  
48 ice. The height difference between the sea ice surface and sea level, known as the freeboard, can  
49 then be converted to an estimate for the ice thickness. However, open water lead observations  
50 can be hundreds of kilometers apart along the satellite’s orbit, so here we develop a new method  
51 which also uses leads on nearby orbits to improve the sea level estimate at ice-covered locations.  
52 This requires us to understand how rapidly the Arctic sea level varies over space and time, which  
53 we do using ESA’s CryoSat-2 satellite radar altimeter. With an optimal processing method that  
54 exploits 10-100s of times more observations than normal, we can treble the precision of the sea  
55 level estimated ‘under’ sea ice. Up to 25% improvement in sea ice freeboard further indicates  
56 that the new method could upgrade current and historic altimetry-derived Arctic sea ice  
57 thickness records.

58

## 59 1. Introduction

60 Sea ice extent in the Northern Hemisphere has been declining at an increasingly alarming  
61 rate for more than two decades now (Parkinson & DiGirolamo, 2016). Recent studies have  
62 recognized that trends and interannual variations in ice extent are extremely sensitive to the  
63 pan-Arctic distribution of sea ice thickness (Rae, et al., 2014; Castro–Morales, et al., 2014). The  
64 transition from a sea ice cover dominated by thicker, multi-year ice in the 1980s to an ice cover  
65 dominated by thinner, first-year ice in the present day (Tschudi, et al., 2016) has amplified  
66 interannual fluctuations in the sea ice extent (Stroeve, et al., 2018). It has been demonstrated that  
67 seasonal forecasts for the sea ice area can be strikingly improved by initializing numerical models  
68 with ice thickness observations (Msadek, et al., 2014; Massonnet, et al., 2015; Allard, et al., 2018;  
69 Blockley & Peterson, 2018; Fritzner, et al., 2019; Schröder, et al., 2019).

70 Sea ice thickness has been estimated with satellite radar altimeters, including ERS-1/-2 and  
71 Envisat RA-2, and laser altimeters, including ICESat, for more than three decades (Quartly, et al.,  
72 2019). With the launch of the European Space Agency (ESA) CryoSat-2 mission in 2010  
73 (Wingham, et al., 2006) and the National Aeronautics and Space Administration (NASA) ICESat-2  
74 mission in 2018 (Markus, et al., 2017), we are now in a position to monitor Arctic sea ice  
75 thickness up to 88 degrees latitude, covering the full basin on a monthly basis. These missions  
76 can provide sea ice thickness information for climate monitoring and sea ice trend analysis  
77 (Kwok, 2018), assimilation into Numerical Weather Prediction (NWP) systems (Blockley &  
78 Peterson, 2018), evaluating risk for polar marine vessels (Rinne & Similä, 2016), and predicting  
79 light-availability under sea ice for Arctic primary production (Stroeve, et al., 2021). The value of  
80 these sea ice thickness observations to the scientific community and commercial sector, e.g.  
81 shipping companies navigating Arctic routes, offshore marine operators and insurers (Melia, et  
82 al., 2016; Aksenov, et al., 2017), along with the success of the CryoSat-2 mission (Parrinello, et al.,  
83 2018), have motivated the European Commission to support the development of the satellite  
84 CRISTAL: Copernicus Polar Ice and Snow Topography Altimeter. If approved, the CRISTAL  
85 mission will carry a dual-frequency altimeter to measure the sea ice thickness and overlying  
86 snow depth simultaneously (Kern, et al., 2020).

87 Sea ice thickness can be estimated from measurements of the ice freeboard – the height of a  
88 sea ice floe above sea level – taken by a satellite radar or laser altimeter, such as CryoSat-2 or  
89 ICESat-2. Sea ice freeboard is converted to thickness with estimates for the sea ice density, and  
90 the depth and density of snow accumulating at the ice surface. Since the level sea ice floes are  
91 typically no more than five meters in thickness (Laxon, et al., 2013), small variations in the  
92 measured sea ice floe height, sea surface height, or estimated snow depth or density can readily  
93 introduce systematic uncertainty in the derived ice thickness order 10-30% (Landy, et al., 2020).  
94 Methodological differences in the processing chain or in the auxiliary observations used in  
95 various algorithms can therefore lead to systematic differences in derived sea ice thickness of  
96 more than a meter (Sallila, et al., 2019). This is large enough to obscure long-term climate trends  
97 in the Arctic sea ice thickness (Kwok & Cunningham, 2015).

98 One of the largest sources of uncertainty in sea ice thickness estimates from altimetry is  
99 introduced in the measurement of the sea surface height (SSH). The SSH is defined as the ocean  
100 free surface elevation with respect to a reference ellipsoid at a sea ice floe and is conventionally  
101 interpolated for all ice-covered locations from sea surface tie-points located at the closest leads,  
102 i.e. openings in the sea ice pack, along the altimeter’s orbital track (Laxon, et al., 2003; Kwok, et  
103 al., 2007). Uncertainty in the SSH can be estimated from height variations derived from altimeter  
104 returns at leads within a moving window applied along the track (Ricker, et al., 2014) (further  
105 details in Section 2). However, distances between an ice-covered sample and its closest lead can  
106 exceed 200 km along track, particularly in the compact pack ice (concentration >98%) of the  
107 Central Arctic Ocean (Wernecke & Kaleschke, 2015). In these cases, the SSH uncertainty is  
108 constrained only by the deviation of the interpolated sea surface from the local mean measured  
109 elevation and can reach 50 cm, varying considerably across the Arctic (Ricker, et al., 2014).  
110 Importantly, these interpolation uncertainties are highly correlated over distances of hundreds of  
111 kilometers (Tilling, et al., 2018), owing to the sparse distribution of leads along track (Wernecke  
112 & Kaleschke, 2015) and long-wavelength errors in the orbital or geophysical (e.g. earth and ocean  
113 tides) corrections used to process the altimeter observations (Wingham, et al., 2006). So, these  
114 errors only reduce in quadrature with the averaging of multiple tracks, rather than the total  
115 number of samples (Tilling, et al., 2018; Lawrence, et al., 2018). Averaged to a 25-km grid, the  
116 SSH error ranges from approximately 1 to 12 cm.

117 For several applications, including the reconciliation of sea ice mass balance, polar sea level,  
118 climate, and oceanography for scientific purposes and for commercial activities such as  
119 operational navigation, accurate determination of the SSH and its uncertainty in ice-covered  
120 waters are crucial. In regions with low lead density and high SSH uncertainty, derived freeboards  
121 can include long-distance spatially correlated biases (Xia & Xie, 2018). Such biases may either  
122 amplify or cancel each other out in different locations, for instance when estimating snow depth  
123 from centimeter-scale differences between radar and laser sea ice freeboards (Kwok, et al.,  
124 2020). Here, we present a new approach for the accurate determination of the SSH which exploits  
125 all available lead observations from both the orbital track in focus and additional neighboring  
126 tracks. Although we use CryoSat-2 observations to demonstrate the method, the approach can be  
127 applied to any contemporary or historical satellite laser or radar altimetry mission (both pulse-  
128 limited and SAR). In Section 2 we give an overview of the conventional approaches for estimating  
129 the SSH and its uncertainty in sea ice-covered regions. Here we also discuss sources of random  
130 and systematic error in SSH observations. In Section 3 we introduce the satellite and airborne  
131 data sets used within our study. In Section 4 we analyze multi-year mean patterns of the Arctic  
132 Ocean SSH's spatial and temporal decorrelation length scales from CryoSat-2 lead observations.  
133 We then compare these length scales to patterns derived from a high-resolution, nominally 1/12  
134 deg. ( $\sim 4\text{km}$  in the Arctic), simulation of a coupled sea ice-ocean model within the NEMO  
135 (Nucleus for European Modelling of the Ocean) framework, where the sea ice component is LIM2  
136 (Louvain-le-Neuve sea ice model version 2) and the ocean component is OPA (Ocean Parallélisé).  
137 We combine these length scales with the error estimates for SSH observations in Section 5 to  
138 determine the optimal instantaneous SSH at sea ice samples, through objective analysis of all  
139 proximal lead observations on both the track in focus and neighboring tracks. Section 6 compares  
140 the new SSH mapping scheme with a conventional scheme for March 2013. Section 7 validates  
141 our results both at orbital crossovers of the CryoSat-2 satellite and with coincident airborne  
142 observations of the sea ice freeboard. In Section 8 we discuss the theoretical limitations of the  
143 objective mapping technique and prospects for utilizing the method for multiple altimetry  
144 missions. Section 9 presents conclusions of the study.

## 145 2. Estimating Sea Surface Height in Sea Ice-Covered Locations

146 Satellite altimeter returns from leads within the sea ice pack are identified by their  
147 reflectivity and roughness, in the case of laser altimetry (Kwok, et al., 2007; Kwok, et al., 2019), or  
148 by their microwave scattering properties in the case of radar altimetry (Laxon, et al., 2003;  
149 Laxon, et al., 2013). The classification algorithms for identifying leads are generally based on  
150 thresholds of parameters for the returning laser or radar echo (Quartly, et al., 2019). These  
151 algorithms vary in complexity, depending on the number of parameters used, e.g. between 1 and  
152 5+ (Ricker, et al., 2014; Wernecke & Kaleschke, 2015; Lee, et al., 2016; Meloni, et al., 2020), and  
153 can use machine learning for the training of thresholds (Lee, et al., 2016; Paul, et al., 2018).  
154 Alternative algorithms use, for instance, neural networks to classify echoes based on their shape  
155 (Poisson, et al., 2018). For CryoSat-2, returns from leads typically make up 1 to 15% of all the  
156 valid samples (Wernecke & Kaleschke, 2015; Passaro, et al., 2018), with higher densities of  
157 returns in zones of first-year ice at the pack ice margins where the sea ice concentration is lower.  
158 The mean distance from a sea ice sample to the nearest lead along track is approximately  $30 \pm 60$   
159 km (based on our processing chain, see Section 3). However, interpolation distances for the SSH  
160 between lead samples actually depend on the ‘strictness’ of the waveform classifier, with a trade-  
161 off between the number of lead samples available and the precision/accuracy of those height  
162 observations. For instance, in the performance analysis of (Wernecke & Kaleschke, 2015) the  
163 most liberal classifier produced a lead sample density of 26% but included 13% false positive  
164 leads and high variance between proximal SSH observations. With the same dataset, a  
165 conservative classifier produced a sample density of only 1% but included zero false positives  
166 and low variance between observations.

167 Linear interpolation (Ricker, et al., 2014; Lee, et al., 2016; Landy, et al., 2017; Guerreiro, et  
168 al., 2017; Xia & Xie, 2018) or regression (Kwok, et al., 2007; Tilling, et al., 2018; Lawrence, et al.,  
169 2018) is used to estimate the SSH between lead tie-points (Fig. 1). A low-pass filter can be used to  
170 smooth the final surface at clusters of leads, where noise may introduce artificially rough sea  
171 surface topography. Data may be discarded where insufficient lead returns are available to  
172 reliably interpolate the SSH at a sea ice location (Tilling, et al., 2018). Uncertainty on the derived  
173 SSH is estimated from the root-mean square (RMS) height of lead returns within a moving  
174 window (25 km for instance) along track (Ricker, et al., 2014), or by analyzing the RMS of SSH  
175 pairs at orbital crossovers (Tilling, et al., 2018). For CryoSat-2, the uncertainty on a single SSH  
176 measurement has been estimated in the range of 2-50 cm. However, this uncertainty is likely to

177 be correlated over wavelengths >100 km owing to the length-scale of the SSH interpolation, to  
178 the typical distances between lead observations and to errors in the satellite orbit determination  
179 or geophysical corrections (Wingham, et al., 2006). Consequently, random errors in the SSH  
180 observations in sea ice zones cannot simply be reduced by accumulating observations of the  
181 leads along the track.

182 In the current approach for estimating SSH from pulse-limited radar altimeters, significant  
183 positive biases can also be added to the radar range when leads located outside the nadir point of  
184 the satellite ‘snag’ the radar (Armitage & Davidson, 2014). This sea surface elevation bias ranges  
185 from -1 to -4 cm, depending closely on the strictness of the lead classification algorithm, and  
186 results in a 10-40 cm overestimate in sea ice thickness if uncorrected (Armitage & Davidson,  
187 2014). The bias can be reliably removed by using information on the interferometric phase  
188 difference of the radar wave travel-time to an off-nadir lead scatterer (Di Bella, et al., 2018; Di  
189 Bella, et al., 2020); however, of all the radar altimeters only the CryoSat-2 mission has had this  
190 capability, and only operating over a small part of the Arctic Ocean. Taking advantage of the  
191 interferometric SARIn-mode, around 35% of the lead returns discarded in SAR-mode can be  
192 retained, leading to a ~40% reduction in SSH uncertainty (Di Bella, et al., 2018).

### 193 **3. Data & Preprocessing**

#### 194 *3.1. CryoSat-2 Level 2 Processing*

195 We use Baseline-C Level 1B CryoSat-2 waveform observations, details in (Bouffard, et al.,  
196 2018), for the period between October 2010 and April 2019 obtained from the official ESA  
197 science server (accessed in June 2019 at <https://science-pds.cryosat.esa.int>). SAR- and SARIn-  
198 mode observations are retracked by fitting waveforms to echoes simulated from a numerical  
199 model for the delay-Doppler SAR altimeter waveform (Landy, et al., 2019), using the Lognormal  
200 Altimeter Retracking Model (LARM) algorithm described in (Landy, et al., 2020). A local  
201 interpolation of the mean sea surface (MSS) is then removed from the profile of surface heights.  
202 The MSS model is a 10-km field obtained from the linear interpolation of all CryoSat-2 lead  
203 observations between 2010 and 2019. We apply a three-parameter classification routine to  
204 separate CryoSat-2 returns from sea ice and leads, based on the calibrated backscattering  
205 coefficient ( $\sigma^0$ ), the pulse peakiness (PP) and the waveform stack standard deviation (Laxon, et



206 al., 2013; Ricker, et al., 2014; Paul, et al., 2018), as described in (Landy, et al., 2020). Surface  
207 heights at leads referenced to the MSS, i.e. sea level anomaly (SLA) observations, are retained at  
208 this point for further analysis.

209 To obtain estimates for the radar freeboard, the long-wavelength ( $>200$  km) median profile  
210 (which we assume contains residual error from the satellite orbital determination and/or  
211 geophysical corrections (Kwok & Cunningham, 2015) or largest-scale features of the dynamic  
212 ocean topography) is removed from each CryoSat-2 elevation track. SSH is estimated at sea ice  
213 locations by linear interpolation between lead tie-points (Landy, et al., 2017). We apply a 25-km  
214 low-pass filter to smooth sea surface topography at dense lead clusters and estimate the SSH  
215 uncertainty from the RMS height of leads within a 50 km window (Ricker, et al., 2014). Radar  
216 freeboard is then estimated from the sea ice floe elevation minus the SSH, and the ‘single-shot’  
217 uncertainty on a freeboard measurement is the root-sum-square of the SSH uncertainty and  
218 speckle noise (which is 11.6 cm for SAR-mode and 15.3 cm for SARIn-mode (Wingham, et al.,  
219 2006)).

220 Furthermore, we use the SSH observations at leads within the sea ice pack to estimate  
221 monthly fields of the Arctic Ocean mean geostrophic current, following the method of (Armitage,  
222 et al., 2017). Dynamic ocean topography (DOT) within the sea ice-covered zone is estimated from  
223 the difference between CryoSat-2 SSH observations and the GOCO5S geoid (Kvas, et al., 2019),  
224 referenced to the same WGS-84 ellipsoid. The DOT therefore contains the long-term offset of the  
225 SLA with respect to the geoid. Estimates of the DOT greater than  $\pm 2$  m are removed before the  
226 remaining estimates are sampled onto a 25-km Northern Hemisphere EASE2 grid and smoothed  
227 with a 300-km width Gaussian convolution filter. We calculate gradients of the smoothed DOT  
228 grid along zonal and meridional axes and convert these to  $u$  and  $v$  vectors of the surface  
229 geostrophic current following (Armitage, et al., 2017). For the purposes of this study, we  
230 calculate the average ‘climatological’ October-April Arctic Ocean surface current over the entire  
231 CryoSat-2 2010-2019 period and mask the region north of  $87^\circ$  latitude due to measurement  
232 noise (Fig. 2). The climatological field illustrates the major components of the long-term Arctic  
233 Ocean circulation in the winter, including the Beaufort Gyre, Transpolar Drift, East Greenland and  
234 Baffin Island currents, along with the Atlantic and Pacific inflows to the Arctic.

### 235 3.2. Airborne OIB Ku-Band Data Level 2 Processing

236 To validate the CryoSat-2 sea ice freeboard observations derived from our new method, we  
237 use geolocated Level 1B echograms from the Center for Remote Sensing of Ice Sheets (CReSIS)  
238 ultrawideband (UWB) Ku-band airborne radar altimeter, operated on Arctic campaigns by NASA  
239 Operation IceBridge (OIB), to generate airborne estimates of radar freeboard coinciding with the  
240 satellite. The data were accessed from <https://data.cresis.ku.edu/#KBRA> in January 2020. We  
241 selected five airborne campaigns in 2011, 2012 and 2014 (all in March) that were flown to  
242 coincide in space and time with CryoSat-2 overpasses and covered both first-year and multi-year  
243 sea ice in the Chukchi and Lincoln Seas, respectively. The CReSIS Ku-band radar has a central  
244 frequency of 15 GHz (Rodriguez-Morales, et al., 2013) and therefore should, in theory, produce a  
245 comparable estimate for the radar scattering horizon over snow-covered sea ice to the 13.6 GHz  
246 CryoSat-2 radar (Willatt, et al., 2011). The flat-surface range resolution of the UWB radar is  
247 approximately 4.9 cm in snow and the sensor has an along track sample spacing of approximately  
248 5 m (Paden, et al., 2017).

249 Our processing methodology for the CReSIS radar is built on the algorithm detailed in  
250 (Landy, et al., 2020) to derive snow-ice interface elevation, with several additional steps required  
251 to determine the sea ice radar freeboard which we introduce here. We exclude all aircraft  
252 segments where the variability of the detrended aircraft altitude is  $>0.6$  m, or where the mean  
253 aircraft pitch or roll is  $>6^\circ$ . The local CryoSat-2 MSS is removed from the retracked elevation  
254 profile. Radar returns from leads are classified by thresholding waveforms with  $\sigma^0$  and PP above  
255 dynamic thresholds (Fig. 3a). Each threshold is determined from the 99<sup>th</sup> percentile of  $\sigma^0$  or PP  
256 samples, but are no higher than 34 dB or 0.25, respectively, calculated recursively over groups of  
257 twelve radar segments (60 km total length). The SSH is estimated at ice floes using the method  
258 described in Section 3.1 and radar freeboard is obtained from ice floe elevations minus the SSH  
259 (Fig. 3b). We exclude all samples located more than 5 km from their nearest lead to prevent the  
260 introduction of correlated freeboard biases away from leads. A single airborne freeboard  
261 estimate is calculated per  $\sim 300$  m coinciding CryoSat-2 footprint (Fig. 3b) following (Di Bella, et  
262 al., 2018).

### 263 3.3. Auxiliary data

264 The EUMETSAT Ocean and Sea Ice Satellite Application Facility (OSI SAF) global sea ice  
265 concentration climate data record (OSI-450) daily EASE2 gridded observations (accessed from  
266 <ftp://osisaf.met.no/reprocessed/ice/conc/v2p0> in January 2020) (Lavergne, et al., 2019), are  
267 used to filter valid CryoSat-2 observations from the sea ice zone. The OSI SAF global sea ice type  
268 record (OSI-403-c) daily polar stereographic gridded observations (accessed from  
269 [ftp://osisaf.met.no/archive/ice/type\\_in\\_June\\_2019](ftp://osisaf.met.no/archive/ice/type_in_June_2019)) (Breivik, et al., 2012), are used to identify  
270 whether CryoSat-2 or OIB airborne observations are located over first-year or multi-year sea ice.

## 271 4. Correlation length scales for the Arctic sea level anomaly from satellite altimetry

272 To determine which leads can be used for interpolating the local SLA at sea ice floe  
273 locations, we must first define the typical spatial and temporal length scales of the Arctic SLA.  
274 The CryoSat-2 lead observations present several challenges for accurately resolving  
275 characteristic wavelengths of the SLA signal. Generally, the observations are strongly clustered  
276 into groups of 1-10 consecutive valid specular lead returns along the track. The distances  
277 between clusters of valid lead returns can also be in excess of 100 km along track, using our lead  
278 classification routine. Adjacent tracks are sampled every 1-2 hours and generally spaced  
279 hundreds of kilometers apart. Consequently, at small time and distance lags, we are limited by  
280 these sampling considerations and cannot accurately resolve the higher-frequency scales of the  
281 SLA. One might expect this to include SLA signatures of ocean circulation features (such as  
282 mesoscale eddies and meanders) caused by instability of ocean currents at the scale of the local,  
283 first-mode Rossby deformation radius, estimated to be around 5-15 km in the Arctic (Nurser &  
284 Bacon, 2014). These mesoscale features can alias the SLA signal, increasing the uncertainty in  
285 SLA predicted for nearby leads. However, through the present analysis we will demonstrate that  
286 the Arctic Ocean SLA spatial decorrelation length scales are generally much larger than the local  
287 Rossby deformation radius.

288 Using CryoSat-2 observations for the Arctic SLA at leads within the sea ice pack, we map the  
289 winter decadal average spatial (in zonal and meridional directions) and temporal decorrelation  
290 length scales of the Arctic Ocean SLA signal. We map the length scales onto a 50 km EASE2 grid  
291 (Brodzik, et al., 2012) covering the Northern Hemisphere above 50 degrees latitude. We select a

292 minimum lag distance of 2.5 km and lag time of 0.5 days based on the sampling limits of the  
 293 CryoSat-2 data, although following our analysis the smallest scales identified were several times  
 294 larger than these values. The covariance  $\rho$  of the SLA at lag distance or time  $r$  is defined as:

$$\rho(r) = \frac{1}{n(r)-1} \sum_{i=1}^{n(r)} [z(x_i) - \bar{z}] [z(x_i + r) - \bar{z}] \quad (1)$$

295 Where  $n$  is the number of paired observations at lag distance or time  $r$ ,  $z$  is the  
 296 instantaneous SLA,  $x_i$  is the location or time of observation  $i$ . The SLA signal is modelled with a  
 297 Gaussian function, following previous studies in the equatorial oceans, e.g. (Jacobs, et al., 2001).  
 298 This model can account for non-zero covariance between observation pairs within the few  
 299 shortest lag bins, which we expect due to the uncorrelated speckle noise properties of 20 Hz  
 300 CryoSat-2 observations, and asymptotic limit at the covariance amplitude, i.e. the random  
 301 variance of the field. We fit the following Gaussian model to the empirical zonal, meridional, and  
 302 time-dependent covariance functions obtained from Eq. (1):

$$\rho(r) = (a-s)e^{\frac{-3r^2}{L^2}} + s \quad (2)$$

303 Where  $a$  is the covariance amplitude,  $s$  is the covariance at  $r = 0$ , and  $L/\sqrt{3}$  is the e-folding  
 304 scale of the SLA.  $L$  is the ‘effective range’, which defines the lag where  $\rho$  drops to 5% of the  
 305 covariance amplitude and is applied as the first zero-crossing of the imposed SLA signal  
 306 decorrelation in Eq. 6 (Section 5). The model in Eq. (2) is fit to the empirical covariance functions  
 307 with a bounded nonlinear least-squares optimization algorithm. The lower bound for  $s$  is zero  
 308 and  $L$  is bounded at the maximum lag distance or time. The quality of fit is determined from the  
 309 optimized coefficient of determination between empirical and model covariance functions.

#### 310 *4.1. Spatial and temporal length scales*

311 To obtain spatial patterns for the characteristic SLA spatial length scale over the entire  
 312 Arctic Ocean, we perform the following analysis at monthly intervals for the entire Oct-Apr 2010-  
 313 2019 CryoSat-2 lead observation dataset. For every cell of the 50-km EASE2 grid we identify all  
 314 SLA observations in the month within 500 km. Lag distances are employed at 5 km intervals from  
 315 2.5 to 502.5 km, including pairs from the same and different tracks. A time limit of 3 days  
 316 between observation pairs is imposed to maximize the likelihood that observations are

317 correlated in time (Pujol, et al., 2016) with sufficient observations remaining available for  
318 analysis. By doing this we are limiting the chances of decorrelation in time but searching for  
319 spatial correlations over a very wide range. The derived length scales are therefore  
320 representative of averaged conditions over a 3-day time window. We construct a matrix of the  
321 zonal and meridional distances of all valid observation pairs and sample the covariance for each  
322 lag bin along both directional axes using Eq. (1). We then fit the Gaussian model in Eq. (2) to each  
323 empirical function and determine the e-folding length, covariance amplitude and minimum  
324 covariance for the grid cell. Only grid cells with a model  $r^2$  fit  $>0.3$  are retained.

325 To determine the SLA temporal length scales, we again perform the following analysis at  
326 monthly intervals of the CryoSat-2 data. For every cell of the grid, we identify all SLA  
327 observations within 100 km and  $\pm 30$  days of the 15<sup>th</sup> of the month. The spatial limit of only 100  
328 km is chosen to maximize the likelihood that observations are correlated in space (Pujol, et al.,  
329 2016), with sufficient observations remaining available for analysis. By doing this we are limiting  
330 the chances of decorrelation in space but searching for temporal correlations over a very wide  
331 range. The derived time scales are therefore representative of averaged conditions over a 100 km  
332 radius. Since the orbit time for a single CryoSat-2 pass over the Arctic Ocean is a matter of  
333 minutes, we do not analyze the time-dependent correlation between SLA observations along the  
334 track. In contrast, we apply a low-pass median filter to observations within 100-km window  
335 clusters along the orbital track, to reduce the impact of small-scale signal noise along track on the  
336 time-dependent decorrelation of the SSH between tracks. Lag times are employed at 1-day  
337 intervals from 0.5 to 30.5 days. We construct a matrix of the time difference between all valid  
338 observation pairs and sample the covariance for each lag bin using Eq. (1). Applying Eq. (2) we  
339 then fit the Gaussian model to each empirical function and again determine the e-folding length,  
340 covariance amplitude and minimum covariance for the grid cell. Only grid cells with a model  $r^2$  fit  
341  $>0.3$  are retained.

#### 342 *4.2. Mean decorrelation scales for the sea level anomaly*

343 From the analysis of monthly-mean SLA covariance fields, we find no clear seasonal or  
344 interannual patterns in the variability of both the spatial and temporal correlation scales.  
345 Therefore, as a first estimate we calculate 2010-2019 ‘climatological’ zonal, meridional, and  
346 temporal decorrelation length scales from the weighted mean of the e-folding lengths of all 62

347 fields (i.e., the total number of analyzed months), with the optimized model fit statistics  
348 providing the weights. We use these climatological scales for all remaining analysis. Another  
349 reason why climatological scales must be used is because there are typically too few lead  
350 observations proximal to sea ice samples from which to calculate local contemporary  
351 spatiotemporal correlation length scales, which can lead to high uncertainty in the derived SLA.  
352 Finally, we smooth the climatological fields with a 3 x 3 grid cell median filter to remove a few  
353 remaining anomalies.

354 The derived zonal, meridional, and temporal e-folding scales compare closely to the  
355 estimates of (Pujol, et al., 2016) obtained from multiple altimeter missions for sub-polar oceans.  
356 For example, (Pujol, et al., 2016) estimated zonal length scales of 45-100 km for the latitude band  
357 between 50 and 70 degrees north, which are comparable to our estimates of 40-120 km for the  
358 Arctic peripheral seas, the Barents, Kara, and Laptev Seas (Fig. 4a and b). Temporal scales (for  
359 the same latitude band) of 3-7 days (Pujol, et al., 2016) are marginally higher than our estimates  
360 from CryoSat-2 of 1-5 days for the peripheral seas (Fig. 4c). Our estimates for the zonal and  
361 meridional decorrelation scales (Fig. 4a and b) match patterns for the first-mode baroclinic  
362 Rossby radius obtained from hydrographic observations (Nurser & Bacon, 2014), with higher  
363 scales in the Western Arctic (Beaufort Sea region) than on the eastern side north of Svalbard (Fig.  
364 4a and b). However, the CryoSat-2 e-folding scales of 50-200 km are an order-of-magnitude  
365 higher than the baroclinic deformation radius (see Section 4.4) supporting the sub-polar  
366 observations of (Chelton, et al., 2011). For instance, (Chelton, et al., 2011) show that eddies can  
367 be three times larger than the Rossby radius, suggesting that deformation radii cannot be directly  
368 associated with the size of eddies. Our CryoSat-2 data appear to characterize mesoscale  
369 anomalies at a scale between baroclinic instabilities and larger features of the geostrophic  
370 circulation field. However, the CryoSat-2 data do appear to resolve smaller 10s km features of the  
371 SLA signal over the shelf seas, for instance. The SLA decorrelation timescales (Fig. 4c) match the  
372 typical 1-7 day synoptic period of passing weather systems (Hutchings, et al., 2011).

373 Variations in the characteristic spatial and temporal length scales of the SLA are controlled  
374 by the Arctic Ocean's bathymetry, with shallower bathymetry on the shelves introducing  
375 additional tidal signals to the SSH that may be uncorrected and cause the signal to decorrelate  
376 more rapidly (Armitage, et al., 2017). Generally, the patterns of the zonal and meridional length  
377 scales are quite similar, although it is particularly evident in the Siberian Seas and also in Hudson

378 Bay and the Greenland Sea that the meridional scale is significantly shorter than the zonal scale  
379 (Fig. 4a and b). This makes sense as the SSH will be less well correlated across the shelf-break  
380 than along it and emphasizes the need to apply these two scales independently in the analyses  
381 below. It is also interesting that the space and time decorrelation scales appear to be  
382 considerably longer in regions covered by ice for most of the year (i.e. the perennial ice zone)  
383 than areas of the marginal ice zone (MIZ) with lower sea ice concentrations.

384 The covariance amplitude (Fig. 5a) characterizes the standard deviation of the SLA outside  
385 the correlation timescale shown in Figure 4c, i.e. the variability present in the SSH signal over  
386 long time periods. It ranges from approximately 6 cm over the Central Arctic Ocean to 15+ cm on  
387 the shelf seas. If the SSH is estimated at a location from leads exclusively outside the correlated  
388 zone, the uncertainty on the SSH estimate can be no better than this value, which is a salient point  
389 because the conventional methods for interpolating SLA (Section 2) have often used length scales  
390 well above those shown in Figure 4. The covariance at zero lag ranges from around 2 cm over the  
391 central ocean to 6 cm on the shelf seas (Fig. 5b). These values represent the characteristic  
392 uncertainty on an estimate for the SSH using only lead observations in the immediate vicinity of a  
393 location and close in time, from all available tracks. Generally, this includes only a small number  
394 of lead observations but with a low sample variance, and the Arctic Ocean mean of 3.8 cm is  
395 similar to the estimate of ~4 cm SSH uncertainty derived from orbit crossover analysis (Tilling, et  
396 al., 2018).

### 397 *4.3. Interpreting the decorrelation scales*

398 We can expect the ocean surface to be ‘flat’ over a length scale defined by the first mode  
399 baroclinic Rossby radius of vertical deformation, which characterizes the approximate scale of  
400 boundary currents, eddies, and fronts. In the weakly-stratified Arctic Ocean and shallow shelf  
401 seas, the baroclinic Rossby radius has been determined as only 2-16 km from a climatology of  
402 hydrographic observations (Nurser & Bacon, 2014). This is around an order-of-magnitude  
403 smaller than the length scales over which SLA is conventionally interpolated along the altimeter’s  
404 orbital track when deriving sea ice freeboard (see Section 2). Therefore, small-scale dynamic  
405 features of the ice-covered Arctic Ocean surface topography cannot reliably be resolved from  
406 dispersed lead observations in along-track altimeter data (let alone in adjacent time-lagged  
407 tracks). However, sea ice floes can interact with and suppress dynamic features such as eddies

408 (Meneghello, et al., 2017), so the SLA in ice-covered waters may – in reality – covary over much  
409 longer distances than the baroclinic Rossby radius predicts (Chelton, et al., 2011; Nurser &  
410 Bacon, 2014).

411 To examine whether this is likely to be the case, we have further analyzed the covariance of  
412 SSH fields from a  $1/12^\circ$  global simulation (the ORCA0083-N06 run) of the coupled ocean-sea ice  
413 model OPA-LIM2 (Madec, et al., 1998; Fichet & Morales Maqueda, 1997; Goosse & Fichet,  
414 1999), applying an identical method to the one we applied here for CryoSat-2 (Fig. S3) every 5  
415 days between 2011 and 2015. The model uses the quasi-uniform, tri-polar ORCA grid (Madec &  
416 Imbard, 1996) to avoid the singularity associated with convergence of meridians at the north  
417 pole. The grid has 75 vertical levels and a lateral resolution of 2-5 km in the Arctic region (Fig.  
418 S1), which should be sufficient to capture decorrelation length scales of the SLA of order 10s km,  
419 indicative of dynamic features (such as eddies, e.g. see Fig. S2) that we may be missing with  
420 CryoSat-2. The uniform model SSH fields also do not suffer from the same nonuniform clustered  
421 sampling limitations of the altimeter data. This configuration of NEMO has been widely used for  
422 Arctic Ocean studies, e.g. (Bacon, et al., 2015; Tsubouchi, et al., 2018; Kelly, et al., 2019). The SLA  
423 is calculated from the SSH fields with reference to a mean sea surface height model derived from  
424 all time slices between 2011 and 2015.

425 We find the smallest e-folding length scales from NEMO are 10-20 km in the North Atlantic  
426 (Fig. S4, which suggests the model can resolve small-scale dynamical features if they are present.  
427 Patterns of the zonal and meridional length scales are remarkably similar to CryoSat-2, with the  
428 largest scales in the Central Arctic and much smaller scales in the sub-polar seas. The range of  
429 length scales between NEMO and CryoSat-2, of around 20-200 km, are almost identical. There are  
430 relatively higher length scales in the East Siberian Sea, Central Arctic and Hudson Bay, and  
431 relatively lower scales in the Southern Beaufort Sea and Baffin Bay, between NEMO and the  
432 CryoSat-2 data. These model findings support previous idealized simulations of the Beaufort Gyre  
433 that resulted in eddies emerging with about 100 km scale (Manucharyan & Spall, 2016). Large-  
434 scale variability can still dominate the SLA due to basin and gyre scale mechanisms that  
435 exaggerate the correlation lengths (Jacobs, et al., 2001). To examine whether our CryoSat-2  
436 observations may be picking up only the largest gyre-scale features of the SSH, we try low-pass  
437 filtering the NEMO SLA to remove features greater than 250 and 125 km and recalculating the  
438 length scales (Fig. S5). Even after removing features  $>125$  km, the derived scales remain 20-100



439 km within the Arctic and do not reduce to Rossby-like radii (despite these decorrelation scales  
440 appearing at other locations, such as the North Atlantic where the model grid is actually  
441 coarsest). This implies that length scales obtained from our analysis of the NEMO and CryoSat-2  
442 data without filtering are the dominant length scales of the SLA.

443 The covariance amplitudes of the NEMO SLA also has a similar pattern to those derived  
444 from CryoSat-2 (Fig. S4) but are consistently  $\sim 5$  cm lower reflecting the absence of measurement  
445 noise in model SSH fields. The NEMO amplitudes also underestimate the high CryoSat-2  
446 amplitudes measured in Hudson Bay and the Canadian Arctic, for example. One final notable  
447 result from the NEMO analysis is that length scales are almost always higher when a grid cell is  
448 ice-covered than ice-free. Presence of sea ice reduces covariance amplitudes by 65% and  
449 increases decorrelation scales by 20% on average when we test the same locations with and  
450 without sea ice (Fig. S6). This may partly explain the enhanced decorrelation scales measured by  
451 CryoSat-2 in the perennially ice-covered Central and Western Arctic.

452 The apparent decorrelation length and time scales observed by CryoSat-2 are also  
453 supported by previous observations of sea ice motion from ice-mounted buoy arrays. Multi-scale  
454 drifter arrays deployed in the Beaufort Sea as part of the 2007 SEDNA experiment showed little  
455 coherence in ice deformation patterns across spatial scales of 10-100 km, with coherence only  
456 appearing at scales exceeding 100 km (Hutchings, et al., 2011). The observed coherence between  
457 buoys is also typically only lost over synoptic time periods longer than 3-8 days (Hutchings, et al.,  
458 2011). These evident spatial scales of coherent sea ice motion are  $>10$  times larger than the first  
459 mode baroclinic Rossby radius of deformation, reflecting more closely the apparent decorrelation  
460 scales of the SSH signal observed by CryoSat-2 (Figure 4). For example, we find characteristic  
461 scales for the SSH signal of 100-150 km and 2-5 days in the Beaufort Sea.

## 462 **5. Objective mapping for estimating the SLA at sea ice floes**

463 We use an objective mapping methodology to estimate the instantaneous sea level anomaly  
464 at all sea ice floe locations along the CryoSat-2 altimeter track. The method is a suboptimal space-  
465 time objective analysis based on the Gauss-Markov theorem (Le Traon, et al., 1998) that takes  
466 into account both random uncorrelated errors of the altimeter range measurement (e.g. speckle  
467 noise) and long-wavelength along-track correlated errors such as those related to the satellite  
468 orbit or L1B tidal corrections (Wingham, et al., 2006). The SLA is obtained at any location from

469 the best linear estimate of a given irregularly distributed sample of CryoSat-2 SLA observations  
 470 at proximal leads (on the orbital track in focus and adjacent tracks), their errors, and an assumed  
 471 covariance function of the SLA space-time signal.

472 The best least-squares linear estimator  $\theta_{est}$  and associated error field  $\epsilon^2$  for the *a priori*  
 473 unknown sea level anomaly at a sea ice floe location are (Le Traon, et al., 1998; Ducet, et al., 2000;  
 474 Pujol, et al., 2016):

$$\theta_{est} = \sum_{i=1}^n \sum_{j=1}^n A_{ij}^{-1} C_{xj} \Phi_{obs} \quad (3)$$

$$\epsilon^2 = C_{xx} - \sum_{i=1}^n \sum_{j=1}^n C_{xi} C_{xj} A_{ij}^{-1} \quad (4)$$

475 Where  $\Phi_{obs}$  is an observation, i.e. the true SLA  $\Phi_i$  and its observation error  $\epsilon_i$ .  $A$  is the  
 476 covariance matrix of all  $n$  selected observations, and  $C$  is the covariance vector between the  
 477 observations and field to be estimated:

$$\begin{aligned} A_{ij} &= \langle \Phi_{obs} \Phi_{obs} \rangle = \langle \Phi_i \Phi_j \rangle + \langle \epsilon_i \epsilon_j \rangle \\ C_{xi} &= \langle \theta(x) \Phi_{obs} \rangle = \langle \theta(x) \epsilon_i \rangle \end{aligned} \quad (5)$$

478 Where  $\theta(x)$  is the SLA at the ice floe location  $x$ . The zonal, meridional, and temporal  
 479 decorrelation scales and propagation velocities characteristic of the SSH signal to be retrieved  
 480 are defined by the covariance function (Arhan & De Verdière, 1985):

$$\begin{aligned} C(r, t) &= \left[ 1 + ar + \frac{1}{6} (ar)^2 - \frac{1}{6} (ar)^3 \right] e^{-ar} e^{-\frac{t^2}{T^2}} \\ a &= 3.337 \\ r &= \sqrt{\left( \frac{dx - P_x dt}{L_x} \right)^2 + \left( \frac{dy - P_y dt}{L_y} \right)^2} \end{aligned} \quad (6)$$

481  $dx$ ,  $dy$  and  $dt$  are the distance in space (zonal and meridional directions) and time to the  
 482 observation or estimator location under consideration,  $L_x$ ,  $L_y$  and  $T$  are the zonal, meridional,  
 483 and temporal decorrelation length scales defined by the effective range in Eq. (2) (Section 4.3),  
 484 and  $P_x$  and  $P_y$  are propagation velocities of the SSH signal in zonal and meridional directions  
 485 (Section 3.1). We use the long-term average propagation velocities, obtained from the  
 486 climatological geostrophic currents (Figure 2), for  $P_x$  and  $P_y$ . This covariance function has been  
 487 regularly applied to model the SSH signal in sub-polar seas (Le Traon, et al., 1998; Le Traon, et al.,

2003; Pujol, et al., 2016) and its properties are illustrated in Figure 6. The observation errors have two components: an uncorrelated random component with variance  $b^2$  which contributes to the diagonal of the  $\langle \varepsilon_i \varepsilon_j \rangle$  matrix and a long-wavelength correlated component  $E_{LW}$ . The latter is added to non-diagonal terms of the  $\langle \varepsilon_i \varepsilon_j \rangle$  matrix as  $\delta_{ij}b^2 + E_{LW}$  if observations  $i$  and  $j$  are on the same track, where  $\delta_{ij}$  is the Kronecker delta. The field  $\epsilon^2$  is expressed as a fraction of the error variance, so a final estimate for the total SSH uncertainty  $\theta_{unc}$  is obtained from:

$$\theta_{unc} = \sqrt{\epsilon^2 b^2} \quad (7)$$

Which can be related directly to estimates for the sea level uncertainty at sea ice floes obtained through conventional methods, such as from the root-mean square of lead elevations within a defined window along the altimeter track.

For each CryoSat-2 return classified as a sea ice floe along track, we first sample the zonal, meridional, and temporal decorrelation length scales, and geostrophic currents, from the mean fields shown in Figures 1 and 3 at this ‘estimator location’. We identify all available SSH observations (leads) within three times the spatial and temporal correlation scales from this location, including observations both on and off the estimator track. However, only one of four observations is retained outside one correlation length to reduce the size of the matrix inversion, i.e. (Pujol, et al., 2016). The number of valid observations meeting these criteria can still exceed 10,000 for locations close to the pole. Therefore, we determine the covariance vector between all observations and the estimator location using Eq. 6 and retain only  $N$  points with highest absolute correlation  $|C|$ . Increasing  $N$  theoretically improves the accuracy of the retrieved SSH and reduces the uncertainty, but we use  $N = 2001$  hereafter for this study in order to limit the size of the matrix to be inverted in Eq. 3. One month of CryoSat-2 Arctic Ocean observations takes approximately four days to process on a 56 core 256 GB RAM cluster with this criterion.

The covariance matrix in Eq. 5 is constructed between all SLA observations. The ‘single-shot’ random error  $b$  associated with a 20 Hz CryoSat-2 observation is 11.6 cm for SAR mode and 15.3 cm for SARIn mode (Wingham, et al., 2006). This is combined with the long-wavelength error  $E_{LW}$  estimated as 25% of the signal variance  $E_{LW} = 0.25 \text{Var}(\Phi_{obs})$ , based on results from previous studies (Le Traon, et al., 1998; Ducet, et al., 2000; Le Traon, et al., 2003), to construct the error matrix in Eq. 5. An optimal estimate for the SLA at the sea ice floe is then obtained from the

integrated inverse sum of the observation covariance and error matrices, through Eq. (3), and the SSH uncertainty is obtained from Eqs. (4) and (7). Finally, after deriving individual SLA estimates for every CryoSat-2 footprint classified as sea ice along a track, we smooth the resulting profile with a low-pass filter whose window is limited to the mean of the local SSH e-folding scales  $\frac{(L_x+L_y)}{2\sqrt{3}}$ . This removes noise introduced by anomalous leads for a few samples.

## 6. Results from March 2013

We compare our results obtained for the SLA at sea ice floes with a conventional method and the new objective mapping approach for March 2013. The conventional method applied uses the external DTU18 MSS model for deriving SLA, linear interpolation between leads along track, smoothed with a low-pass filter, with the SSH uncertainty obtained from the RMS height of leads within a 25-km moving window along track (Landy, et al., 2017).

### 6.1. Case study track on March 3<sup>rd</sup>

We first select a single ascending-orbit CryoSat-2 SAR-mode track at 03:46:51 on 3<sup>rd</sup> March 2013 to illustrate the advantages of the new method. This track crosses the Arctic Ocean from the Lincoln Sea to the East Siberian Sea. Although the Eastern Arctic sector of the track contains dense lead clusters, our waveform classification algorithm produces only five valid lead returns for the remaining 1800 km (Fig. 7a). This track represents a case with particularly low lead density and requires interpolation of the SLA over distances of up to 500 km to ice floes from their nearest lead (if all floes are to be included in the analysis). Owing to the low lead density, uncertainty on the derived SLA is >6 cm for the majority of the track (Fig. 7a), representing 20-50% of the final derived radar freeboard (Fig. 7d). In areas with sparse leads, the estimated SLA can be tied to single lead observations (Fig. 7a) despite each observation having a random uncertainty up to ~15 cm (Wingham, et al., 2006) and possible bias >4 cm (Armitage & Davidson, 2014).

By applying the objective mapping approach, we sample up to 2001 local observations at leads for every sea ice floe along track and estimate the SLA from the optimal interpolation of them all. Figure 7b illustrates the covariance between the location of every 80<sup>th</sup> sea ice floe along track and its local sample of SSH observations. Generally, the SLA of the distribution of lead

544 observations around a single ice floe ranges from approximately -0.2 to +0.2 m but is higher in  
545 the shallower East Siberian Sea sector (1800-2500 km along profile). For this track, 56% of all  
546 SLA observations used in the analysis are within half the distance of both  $L_x$  and  $L_y$  correlation  
547 length scales and 83% of observations are within the whole distance of  $L_x$  and  $L_y$ .

548 The final optimal interpolation (Fig. 7c) predictably coincides with most of the lead  
549 observations on the focus track because they have a time lag close to zero. However, the  
550 covariance matrix between the up to 2001 neighboring lead observations in a local sample  
551 provides a weighting on the SLA estimate that reduces the influence of anomalous observations,  
552 i.e., leads with high estimated measurement error with respect to their neighbors. For instance,  
553 the objective SLA estimate is 5 cm lower than the lead at (i) in Figure 7c, indicating this SLA  
554 observation may contain significant error. Single isolated leads or lead clusters do not over-  
555 influence the objective SLA estimate (Fig. 7c) in the same way they can for the conventional  
556 method (Fig. 7a). In such instances where the objective analysis indicates a lead is under- or  
557 over-estimated, as it does at (i), the derived radar freeboards between the new and conventional  
558 approaches contain long wavelength correlated offsets, typically of between -20 and +20 cm (Fig.  
559 7d). The uncertainty estimate for the objective analysis (from Eq. 7) is generally <2 cm,  
560 representing <15% of the final derived radar freeboard (Fig. 7d), because the SLA is estimated  
561 from tens-to-hundreds of times more observations than in the conventional approach (Fig. 7c).  
562 The uncertainty is notably higher at (ii) in Figure 7c because  $L_x$  and  $L_y$  are <150 km at this  
563 location, so the number of available SLA observations is significantly lower than the maximum  
564 2001 permitted and their variance is larger (Fig. 7b).

565 The new scheme for determining SLA enables the radar freeboard to contain greater along-  
566 track variability than the conventional scheme (Fig. 7d) because the estimated SLA is not fixed  
567 over long (>100 km) distances along track by isolated single or clusters of leads. The new scheme  
568 appears to be particularly successful resolving discontinuities in SLA (and its uncertainty) at the  
569 shelf break and other areas of complex bathymetry. For instance, the SLA does not become  
570 aliased when there are insufficient leads to resolve the detailed ocean surface topography, e.g., at  
571 (iii) in Fig. 7a and c.

## 572 6.2. Analysis of entire month

573 We complete the same comparison between the conventional SLA estimated from a linear  
574 interpolation along-track and from the objective analysis of all proximal leads from adjacent  
575 tracks, for every CryoSat-2 SAR and SARIn mode track in March 2013. Pairwise differences in the  
576 radar freeboard obtained from the conventional and objective methods are normally distributed  
577 (Fig. 8a and b) but comprise long-wavelength (10-500 km) correlated offsets between the  
578 methods in either direction. (Note we do not discard any freeboard observations based on their  
579 distance to the nearest along-track lead for this analysis). The radar freeboards can diverge by >5  
580 cm along large segments of individual tracks (Fig. 8a), where the conventional SLA estimate is  
581 poorly constrained through biased observations or a low density of lead observations along track  
582 (Section 6.1). The conventional method is essentially as likely to underestimate the objectively  
583 mapped SLA as overestimate it (Fig. 8a). On average, the conventional method underestimates  
584 the objective mapping method by  $\sim 1$  cm (Fig. 8b), constituting an estimated sea ice thickness  
585 difference of only  $\sim 10$  cm. However, the mean absolute radar freeboard difference is 3.3 cm,  
586 which represents a 27% local uncertainty on the mean freeboard and constitutes >30 cm  
587 uncertainty in sea ice thickness estimated from these freeboards. The biases between SSH  
588 mapping techniques appear to be independent of location, although there is a pattern of positive  
589 freeboard differences (mean = +2.5 cm) in the multi-year ice zone north of Canada and the largest  
590 differences are evident in coastal regions (Fig. 8a). These areas coincide with shallower tidal  
591 zones that have high SLA variability over short temporal and spatial scales (e.g. Fig. 5) and/or  
592 have the lowest available density of SSH observations at leads (Wernecke & Kaleschke, 2015).

593 By utilizing many times (typically 1-2 orders-of-magnitude) more SSH observations to  
594 determine the optimal SSH at a sea ice floe, the objective mapping method produces a factor of  
595 three reduction in the estimated SSH uncertainty (Fig. 8c). The objective analysis accounts for  
596 uncorrelated random errors in the observations, as well as long-wavelength correlated errors  
597 along the altimeter's orbital track caused by observation biases or errors in the orbit  
598 determination or geophysical corrections. Their reduction to the error estimate at a single ice  
599 floe scales directly with the number of observations and tracks, weighted by the covariance of the  
600 observations to the floe location and the covariance matrix between neighbors (Section 5). This  
601 objective estimate for the uncertainty is therefore based entirely on the observations themselves  
602 and does not suffer from the assumptions or conditions of the conventional method, for instance

603 that the SSH uncertainty is uniform across the Arctic or depends only on the RMS of SSH  
604 observations along track (Section 2).

## 605 **7. Validation of SLA and radar freeboard estimates**

### 606 *7.1. Analysis at orbital crossovers*

607 As a first assessment of the precision of the new objective mapping method for deriving SLA  
608 at ice floes we identify all crossovers of the CryoSat-2 orbit over the Arctic Ocean sea ice pack in  
609 March 2013. Around 13,000 unique crossovers are identified where orbits intersect within 24  
610 hours and valid CryoSat-2 measurements for each track are no more than 5 km apart. The  
611 crossover locations are clustered around the north pole, because the polar-orbiting satellite  
612 disproportionally crosses itself within a small region north of ~84 degrees latitude; however,  
613 there are rings of crossovers at around 66 and 79 degrees too (Fig. 9c).

614 All pairwise differences in the SLA or radar freeboard estimated at crossover locations are  
615 normally distributed (Fig. 9). The widths of the distributions represent a combination of random  
616 noise in the measurements, orthogonal sensing footprints for crossing orbits, aliased tidal  
617 signals, and – in the case of the radar freeboards – additional errors relating to ice motion and  
618 possibly radar signal penetration e.g. (Willatt, et al., 2011). The new objective mapping scheme  
619 reduces the RMS of the SSH estimated at crossover locations by 70% compared to the  
620 conventional approach, from 4.6 down to 1.4 cm (Fig. 9a). The RMS of crossovers for the  
621 conventional scheme is close to the 4 cm reported by (Tilling, et al., 2018). It is not surprising  
622 that the RMS is reduced through objective analysis, as the SSH is estimated at ice floes from all  
623 available leads on all proximal tracks. However, with our new scheme the SSH compared at a  
624 crossover is still, in all cases, from an optimal interpolation of nearby observations rather than  
625 actual lead observations, so the improvement remains impressive. The objective mapping  
626 scheme reduces the RMS of the radar freeboard measured at crossover locations by 19%  
627 compared to the conventional approach, from 6.9 down to 5.5 cm (Fig. 9b). The improved SSH  
628 estimation reduces a portion of the radar freeboard uncertainty. However, because the new  
629 scheme reduces the RMS of radar freeboard at crossovers by significantly less than it reduces the  
630 RMS of SSH at crossovers, this suggests around three quarters of the total uncertainty in

631 freeboard measurements at crossovers is ice-related (i.e. including the effects of ice motion,  
632 signal penetration, speckle noise and retracking uncertainties).

## 633 *7.2. Independent validation of radar freeboards*

634 We use independent radar freeboards derived from the CReSIS airborne Ku-band radar  
635 flown on OIB Arctic campaigns (Section 3.2) to compare the accuracies of the conventional and  
636 objective SSH mapping techniques. The Ku-band radar freeboards are used here rather than the  
637 official OIB L4 total (snow plus sea ice) freeboard and thickness product, so that we do not have  
638 to correct freeboards for snow depth and delayed radar wave propagation through the snow  
639 layer (Landy, et al., 2020). The OIB L4 snow depths contain known biases (Newman, et al., 2014)  
640 and fixed snow densities may introduce further systematic uncertainties (Mallett, et al., 2020).  
641 So, we use airborne radar freeboards to mimic the CryoSat-2 radar measurements as closely as  
642 possible and limit the chances of introducing further systematic biases into our comparisons.  
643 Satellite radar freeboards are obtained with both the objective and conventional SSH  
644 interpolation methods along CryoSat-2 tracks coinciding with five processed OIB campaigns in  
645 2011, 2012 and 2014. Of the five coinciding tracks, three produced similar radar freeboard  
646 profiles between the objective and conventional methods suggesting that the conventional along-  
647 track approach was sufficient to resolve the SSH in these cases. The more sophisticated but less  
648 computationally efficient objective analysis is not always necessary. However, for two of the  
649 campaigns, on 26<sup>th</sup> March 2012 (CryoSat-2 in SARIn mode, with a 1-hour time difference between  
650 aircraft and satellite passes) and on 26<sup>th</sup> March 2014 (SAR mode, with a 4.5-hour time  
651 difference), satellite radar freeboards from the objective analysis and conventional approaches  
652 diverged significantly. Here, we want to analyze which, if either, method accurately reproduces  
653 the airborne radar freeboards.

654 The OIB campaign on 26<sup>th</sup> March 2012 measured mostly multi-year sea ice with some first-  
655 year ice in the ‘Wingham Box’ (Fig. 10). The SSH estimated for this track with objective analysis  
656 was between 4 and 10 cm lower (Fig. 10a) than the SSH estimated with the conventional along-  
657 track approach but, owing to a low density of leads in the region, both methods included a  
658 relatively high uncertainty estimate (Fig. 10b). (Note the uncertainty in Figure 10a and b  
659 characterizes the precision of the estimated SSH, whereas the remaining analysis here  
660 characterizes its accuracy). Figure 10c illustrates the airborne and two satellite radar freeboard



661 profiles, after a moving average filter with 2 km width is applied. There is some correlation  
662 between the airborne and satellite observations in places, but it is very clear that the distribution  
663 of radar freeboards from the objective mapping method matches the airborne observations far  
664 better than the distribution obtained from the conventional along-track method (Fig. 10d and e).  
665 The conventional method appears to underestimate the airborne freeboards by 8.9 cm (mean  
666 difference, MD), because it overestimates the SSH (Fig. 10b). In comparison, the MD between the  
667 objectively mapped CryoSat-2 freeboards and OIB is -3.4 cm. The accuracy of the new method  
668 (RMSE = 11.2 cm) is improved by around 25% versus OIB relative to the conventional method  
669 (RMSE = 14.9 cm), at the 2-km length-scale of our averaged freeboard observations.

670 The OIB campaign on 26<sup>th</sup> March 2014 measured predominantly multi-year ice in the  
671 Lincoln Sea (Fig. 11). The SSH estimated for this track with objective analysis was between 0 and  
672 8 cm lower (Fig. 11a) than the SSH estimated with the conventional along-track approach and  
673 both methods produced lower uncertainty estimates at one end of the section, owing to a cluster  
674 of leads to the north (Fig. 11b). Figure 11c illustrates the airborne and two satellite radar  
675 freeboard profiles, after a moving average filter with 2 km width is applied. Like in the 2012  
676 comparison, the CryoSat-2 freeboards from each method exhibit long-wavelength (>100 km)  
677 correlated differences (Fig. 11b). Again, it is clear that the distribution of radar freeboards from  
678 the objective mapping method match the airborne observations better than the distribution  
679 obtained from along-track interpolation between leads (Fig. 11d and e). The conventional  
680 method underestimates radar freeboard by MD = 11.1 cm, in comparison to 4.8 cm for the new  
681 method. The accuracy of the objective mapping method (RMSE = 13.8 cm) is improved by around  
682 20% versus OIB relative to the conventional method (RMSE = 17.1 cm), at the 2-km length-scale  
683 of our averaged freeboard observations.

684 Our independent evaluation of the CryoSat-2 radar freeboards for both OIB campaigns  
685 demonstrates that long-wavelength errors, caused for example by a low density of valid lead  
686 returns along track, off-nadir lead errors, or errors in the L1B CryoSat-2 orbital/geophysical  
687 corrections, can introduce significant biases into derived radar freeboards using the conventional  
688 SSH mapping approach. In both cases where the conventional and objective SSH mapping  
689 techniques diverged, the objective estimate more accurately reproduced the radar freeboards  
690 observed from OIB aircraft observations.

## 691 8. Discussion

### 692 8.1. Prospects for further improvement

693 There are several avenues worth exploring to further improve the objective mapping of SSH  
694 in ice-covered seas. It may be valuable to use leads at adjacent tracks for mapping SLA at ice floes  
695 with ICESat-2, because regions of sea ice further than 10 km from their nearest lead reference  
696 along track are currently discarded (Kwok, et al., 2019). This leaves some areas such as the  
697 densely-concentrated multi-year ice of the Central Arctic occasionally missing valid observations  
698 e.g., (Petty, et al., 2020). However, the higher density of SSH observations from ICESat-2 may  
699 enable an improved characterization of the spatiotemporal characteristics of the SLA signal, and  
700 possibly also its seasonal variation, for other altimetry missions. It may also enable discarded  
701 ICESat-2 segments in lead-sparse regions like the Canadian Arctic Archipelago or Lincoln Sea to  
702 be retained (Kwok, et al., 2019). Now that Sentinel-3A and -3B are operating together with  
703 CryoSat-2 over a portion of the Arctic Ocean (Lawrence, et al., 2019), there is strong potential for  
704 characterizing the SLA signal in more detail combining all three sensors. Moreover, CryoSat-2 has  
705 been maneuvered to coincide more frequently with the ground track of ICESat-2 (as part of the  
706 CRYO2ICE Project) which could enable the direct intercomparison of SLA characteristics.

707 It is unlikely our assumption that systematic error between altimeter tracks is a maximum  
708 25% of the signal variance holds in all situations (Section 5). The systematic offset between  
709 tracks will be greater when (i) orbital errors are higher (Wingham, et al., 2006), (ii) geophysical  
710 corrections for tides and atmospheric effects are lower quality or aliased by the satellite orbit,  
711 and/or (iii) target-dependent biases such as the snagging of off-nadir leads (Di Bella, et al., 2018)  
712 or variable radar penetration depths into snow e.g. (Willatt, et al., 2011) are greater. The  
713 objective SLA mapping results would be improved if the long-wavelength correlated component  
714  $E_{LW}$  in Eq. (5) was determined from these error contributions or their spatiotemporal variability.

715 Finally, our current implementation of the objective mapping scheme takes approximately  
716 five days to process one month of CryoSat-2 SLA estimates at ice floes over the Arctic Ocean. This  
717 is around 3-4 orders-of-magnitude longer than conventional along-track SSH interpolation  
718 methods. However, it may be possible to obtain results with similar accuracy but only processing  
719 one in  $n$  sea ice floe samples along track, or using a reduced sample size of SSH observations, with

720 considerable improvements in computation speed. Equivalent results may also be possible but  
721 using faster and less data intensive optimization algorithms.

## 722 *8.2. Implications of the new method for deriving inter-mission data products and to the* 723 *reanalysis of historic altimeter missions*

724 The depth of snow on Arctic sea ice has been estimated from the offset between laser  
725 freeboards from ICESat-2 (Kwok, et al., 2020) or radar freeboards from the Ka-band AltiKa  
726 (Lawrence, et al., 2018) and radar freeboards from CryoSat-2. Whilst we do not expect errors in  
727 the determination of SLA to introduce pan-Arctic uniform biases between satellites (Fig.7b), the  
728 along-track correlated errors from interpolation between leads (Fig. 7a) could realistically  
729 introduce local biases to the derived inter-mission snow depths. These biases may either amplify  
730 or cancel each other out. Objective mapping therefore offers the prospect of combining SSH  
731 observations from multiple altimeter missions (Le Traon, et al., 2003; Pujol, et al., 2016):  
732 calculating constant inter-mission biases if present but, more importantly, preventing local biases  
733 where leads are sparse or have high uncertainty. Errors will be limited at mission crossover  
734 locations (Fig. 9) and systematic uncertainties should also be reduced on gridded freeboard  
735 differences.

736 The objective SLA mapping scheme offers most improvement over conventional techniques  
737 where sea ice concentrations are highest and/or SLA observations at leads have largest height  
738 uncertainty. For instance, the most obvious changes in gridded freeboard in Figure 8a occur in  
739 the perennially-ice covered zone north of Greenland and Canada. Historic pulse-limited radar  
740 altimeter missions, such as Envisat or ERS-1 and -2 (and the ongoing mission AltiKa), have an  
741 effective footprint of 2-8 km and are therefore more sensitive to ‘snagging’ than the SAR-focused  
742 CryoSat-2 (Section 2). The instruments on ERS-1/-2 also had a larger bandwidth than recent  
743 missions, meaning their range resolution was lower with specular lead reflections more likely to  
744 be aliased in the recorded waveforms. By estimating the optimal local SLA from a greater number  
745 of proximal lead observations, accumulated from multiple tracks, our new scheme should  
746 effectively reduce the random uncertainty from noise and waveform aliasing and the systematic  
747 uncertainty from snagging. Since a higher number of leads are used for each SLA interpolation, a  
748 more conservative lead classification can be employed for a smaller sample of higher accuracy

749 SLA observations. Improvements on the conventional SLA interpolation schemes for these  
750 missions should, in theory, be larger than we have found for CryoSat-2.

## 751 **9. Conclusions**

752 The conventional method for estimating sea ice freeboard with altimetry uses only lead  
753 observations along the satellite orbital track to interpolate the local sea surface height at ice floes.  
754 The SSH uncertainty is typically estimated from the root-mean-square of lead height  
755 observations within a moving window along track. Here we have introduced a new method to  
756 determine the optimal interpolation of local SLA at sea ice floes using all valid proximal lead  
757 observations both on the orbital track in focus and other adjacent tracks. The objective mapping  
758 method assumes that spatial and temporal properties of the SLA in ice-covered waters can be  
759 predicted with a characteristic Gaussian covariance function. The decorrelation length scales and  
760 signal propagation velocities that constrain this function in the Arctic Ocean are obtained by  
761 analyzing historic SLA measurements acquired by the CryoSat-2 radar altimeter from lead  
762 locations between 2010 and 2019. The best linear least-squares solution for the SLA at each ice  
763 floe is determined from all valid SLA observations, weighted by their covariance with the floe  
764 location, their covariance with neighbors (i.e. to identify anomalies), and their random and  
765 systematic observation errors.

766 By exploiting a greater number of leads for interpolating the SLA, it is possible to use a  
767 stricter pulse peakiness classification threshold – discarding more ambiguous lead waveforms  
768 without compromising the height estimate and its uncertainty. For instance, the objective  
769 mapping method can effectively reduce off-nadir lead biases on the derived CryoSat-2 SLA when  
770 corrections from the interferometric phase are not available. Applying the new method to the  
771 Arctic Ocean in March 2013, our results demonstrate that the SSH uncertainty can be reduced by  
772 around three times in comparison to conventional uncertainty estimates. The root-mean square  
773 of interpolated SSH pairs at orbital crossovers is reduced by a factor of three and radar freeboard  
774 crossover RMS is reduced by 20%. Where independent airborne observations are available and  
775 the coinciding new and conventional SSH estimates from CryoSat-2 give different results, we find  
776 the objective method improves satellite-derived freeboard accuracy by 20-25%. The new method  
777 is also capable of resolving much finer-scale detail of the SSH signal in areas of complex ocean  
778 topography such as the circumpolar shelf break. However, inversion of the SSH observation

779 matrix is computationally expensive, so our current software takes around five days on a cluster  
780 to process SSH at ice floes for one month of pan-Arctic CryoSat-2 data.

781       Objective SSH mapping produces the largest improvements at local scales and may  
782 therefore enable accurate sea ice freeboards to be estimated at kilometer-scale resolutions along  
783 the satellite track. With CryoSat-2 maneuvered to align with ICESat-2 from August 2020, it will be  
784 valuable to inter-compare the SSH between these two satellites and test whether objective  
785 mapping can reduce local biases in the freeboard offsets. Furthermore, the scheme offers  
786 considerable potential for new missions such as CRISTAL and for reprocessing ice freeboards  
787 from historic pulse-limited radar altimetry missions, including AltiKa, Envisat, ERS-1 and -2,  
788 where SSH observations are more likely to have off-nadir lead biases, higher noise and are  
789 regularly spaced >100 km along track.

790

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978

979 **Figure 1.** Schematic diagram illustrating the conventional and proposed new methods for  
 980 interpolating the sea surface height at sea ice locations. In the conventional approach only the  
 981 four leads along the central track (yellow footprints) are used to interpolate SSH at the sea ice  
 982 floe location (green footprint). In the proposed approach, all 14 leads (yellow plus blue  
 983 footprints) acquired within  $\pm 2$  days at neighboring tracks inside a prescribed sea surface height  
 984 covariance limit (green circle) around the ice floe are used to compute the SSH. Background is a  
 985 SAR image from Sentinel-1.

986 **Figure 2.** Mean October-April surface geostrophic currents [km/day] for the sea ice-  
 987 covered region of the Arctic Ocean

988 **Figure 3.** (a) Echogram from the OIB Ku-band radar over MYI in the Lincoln Sea on March  
 989 26<sup>th</sup> 2014, compensated for aircraft altitude changes and relative to the WGS-84 ellipsoid,  
 990 including the retracked elevation of the snow-sea ice interface, samples classified as leads, and an  
 991 estimate for the sea surface height. (b) Radar freeboards derived from the difference between  
 992 snow-ice interface elevation and sea level, averaged onto the footprint locations of a coincident  
 993 CryoSat-2 overpass.

994 **Figure 4.** Mean e-folding decorrelation length scales of the Arctic Ocean sea level anomaly  
 995 (SLA) for (a) the zonal direction, (b) meridional direction, and (c) time, calculated from the full  
 996 2010-2019 archive of CryoSat-2 sea surface height estimates at leads. (d) First mode of the  
 997 annual-mean baroclinic Rossby radius derived from the Polar Science Center Hydrographic  
 998 Climatology and reproduced from (Nurser & Bacon, 2014).

999 **Figure 5.** Mean (a) covariance amplitude and (b) covariance at zero lag for the time-  
 1000 dependent sea surface height signal obtained from CryoSat-2 2010-2019. These maps illustrate  
 1001 the standard deviation of the SLA outside the correlation timescale (Fig. 4c) and, in contrast, the  
 1002 measurement noise when there is no time lag, respectively.

1003 **Figure 6.** Theoretical covariance function of the sea surface height (SSH) signal imposed  
 1004 within the objective mapping method (a) as a function of time and distance, and (b) distance only  
 1005 for  $t = 0$ . Here  $L_x = L_y = 200$  km,  $T = 10$  days,  $P_x = P_y = 0$ .

1006       **Figure 7.** a) Profile of retracked surface elevation estimates from a SAR-mode CryoSat-2  
1007 track on 3<sup>rd</sup> March 2013 (03:46:51 UTC), with respect to the locally computed mean sea surface  
1008 (i.e. SLA), with an estimate for the local instantaneous sea level and uncertainty derived from a  
1009 conventional approach. (b) Covariances between the CryoSat-2 observation location and nearby  
1010 leads on- and off-track, with the final objective estimate for the sea level and uncertainty. (c)  
1011 Final objective sea level and uncertainty over the CryoSat-2 elevation estimates, as in (a), and  
1012 inset map of the track location (annotations referred to in text). (d) Sea ice radar freeboards  
1013 derived with the conventional and new objective methods for deriving the SSH, and the long  
1014 wavelength correlated differences between them.

1015       **Figure 8.** Pan-Arctic analysis of the conventional and new methods for deriving ice  
1016 freeboards in March 2013. (a) 25-km gridded distribution of CryoSat-2 radar freeboards from the  
1017 objective mapping SSH method minus the conventional approach, including the limit of the multi-  
1018 year sea ice area in black. (b) Radar freeboard differences between the two methods, from the  
1019 raw along-track CryoSat-2 sea ice observations. (c) Ratio of the SSH uncertainty estimate from  
1020 the objective mapping method to the conventional along-track uncertainty estimate, also from  
1021 the along-track observations.

1022       **Figure 9.** Analysis of paired (a) sea surface height and (b) radar freeboard differences at  
1023 orbital crossover locations of CryoSat-2 in March 2013. All crossover locations within one day  
1024 and a maximum distance of 5 km are illustrated in (c).

1025       **Figure 10.** a) Profile of retracked surface elevation estimates from a SARIn-mode CryoSat-  
1026 2 track on 26<sup>th</sup> March 2012 (15:45:42 UTC), with respect to the mean sea surface (i.e. SLA);  
1027 covariances to local lead observations on- and off-track; and the objective sea level and  
1028 uncertainty estimates. (b) Conventional and objective SSH estimates with their uncertainty  
1029 (precision) envelopes. (c) Coincident radar freeboards from the CReSIS airborne Ku-band radar  
1030 and CryoSat-2 processed with the two methods, including a map of the coinciding section (in red)  
1031 inset. (d) and (e) Probability density functions (PDFs) of the airborne and satellite radar  
1032 freeboard observations, processed with conventional and objective methods for deriving the SSH.

1033       **Figure 11.** a) Profile of retracked surface elevation estimates from a SAR-mode CryoSat-2  
1034 track on 26<sup>th</sup> March 2014 (09:06:49 UTC), with respect to the mean sea surface (i.e. SLA);



1035 covariances to local lead observations on- and off-track; and the objective sea level and  
1036 uncertainty estimates. (b) Conventional and objective SSH estimates with their uncertainty  
1037 (precision) envelopes. (c) Coincident radar freeboards from the CReSIS airborne Ku-band radar  
1038 and CryoSat-2 processed with the two methods, including a map of the coinciding section (in red)  
1039 inset. (d) and (e) Probability density functions (PDFs) of the airborne and satellite radar  
1040 freeboard observations, processed with conventional and objective methods for deriving the SSH.