Comment on "Groundwater affects the geomorphic and hydrologic properties of coevolved landscapes" by Litwin et al.

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Key Points:

- We clarify that the specific contributing area is defined in the limit of an infinitesimal contour length.
- We show that not all solutions of the minimalist landscape evolution model are rescaled replicas of each other.
- Boundary conditions play an essential role in solutions of landscape evolutions models.

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Abstract

In this comment, we clarify that the specific contributing area is defined in the limit of an infinitesimal contour length. We also confirm that not all solutions obtained from the minimalist landscape evolution model of Bonetti et al. (2020) are rescaled copies of each other because of the crucial role of the boundary conditions. We use dimensional analysis and numerical simulations for a case where only one horizontal length scale enters the physical law to establish this point.

Plain Language Summary

We clarify here the definition of the specific contributing area, one of the main variables of landscape evolution models. Using dimensional analysis and numerical simulations, we also demonstrate that the solutions of these models are not rescaled copies of each other because of the effects of the boundary conditions.

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Litwin et al. (2022) wrote a very interesting paper on the important problem of the co-evolution of groundwater hydrology and land-surface dynamics. The authors are to be commended for this very nice research contribution, which quantifies the feedback between the spatial patterns in the emerging topography and sub-surface properties. Our comment is intended to rectify some statements in this paper, which regard our own work (Bonetti et al., 2018, 2020). The first one considers the definition of the specific contributing area a and the second one is related to the dimensional analysis and the scaling properties of the landscape evolution model (LEM). We feel that these corrections are necessary to avoid misinterpretations of our results as well as to clarify some aspects of the scaling of the obtained solutions. The statements in question concern the LEM

$$\frac{\partial z}{\partial t} = D\nabla^2 z - K\sqrt{a}|\nabla z| + U, \tag{1}$$

$$-\nabla \cdot \left(a \frac{\nabla z}{|\nabla z|} \right) = 1, \tag{2}$$

where $z(x, y, t)$ is elevation of the topographic surface, t is time, D is a diffusion coefficient, K is an erosion coefficient, a is the specific contributing area, and U is the uplift rate. For constant K, the model – referred to as the *NoHyd* model – is a specific version of the minimalist LEM of Bonetti et al. (2020).

Regarding the definition of the specific contributing area, Litwin et al. (2022) write on page 5: "To make the conversion between A and a, we represent A as the product of a and a characteristic contour width v_0, which is a chosen constant value". In reality, a is defined in the limit of an infinitesimal contour length (Gallant & Hutchinson, 2011; Bonetti et al., 2018), not as the product of a reference contour width. As a result, Eq. (2) is only valid for the specific contributing area defined in this limit, as discussed in Bonetti et al. (2018).

With reference to similarity and dimensional analysis, on page 4 they write: "Additionally, the nondimensionalization generalizes our results and reconciles conflicting dimensional analyses provided by Theodoratos et al. (2018) and Bonetti et al. (2020)." Further on page 27: "We show that contrary to Bonetti et al. (2020) there is a single typology of the *NoHyd* model which can be rescaled to obtain all results the model may produce". These conclusions are mistaken because the effect of the boundary conditions is not included in the dimensional analysis. While the dimensional analysis of the LEM solutions is facilitated by the fact that some of the main variables and parameters are clearly listed in the governing equations, the solutions depend on the equations as much as they do on the initial and boundary conditions (Bursten, 2021).

For concreteness, we consider the solution for a square domain with boundaries kept at a constant elevation value, so that only one horizontal length scale (the side length l) enters in the physical law (more complicated geometries will add further governing quantities) i.e.,

$$z = f(K, D, U, x, y, t, l). \tag{3}$$

Taking L, H, and T as primary dimensions for horizontal directions, vertical direction, and time, respectively (Porporato, 2022), the dimensions of the quantities involved in Eq. (3) are $[z] = \mathrm{H}$, $[K] = \mathrm{L}^{1/2}\mathrm{T}^{-1}$, $[D] = \mathrm{L}^2\mathrm{T}^{-1}$, $[U] = \mathrm{HT}^{-1}$, $[x] = \mathrm{L}$, $[y] = \mathrm{L}$, and $[l] = \mathrm{L}$. Choosing K, D, U as the repeating variables and applying the Π theorem yields

$$\Pi_z = \Phi(\Pi_x, \Pi_y, \Pi_t, \Pi_l), \tag{4}$$

with

$$\Pi_z = \frac{z}{\left(\frac{DU^3}{K^4}\right)^{1/3}} = \frac{z}{\mathcal{H}} = \hat{z}, \tag{5}$$

$$\Pi_x = \frac{x}{\left(\frac{D^2}{K^2}\right)^{1/3}} = \frac{x}{\mathcal{L}} = \hat{x}, \tag{6}$$

$$\Pi_y = \frac{y}{\left(\frac{D^2}{K^2}\right)^{1/3}} = \frac{y}{\mathcal{L}} = \hat{y}, \tag{7}$$

$$\Pi_t = \frac{t}{\left(\frac{D}{K^4}\right)^{1/3}} = \frac{t}{\mathcal{T}} = \hat{t}, \tag{8}$$

$$\Pi_l = \frac{l}{\left(\frac{D^2}{K^2}\right)^{1/3}} = \frac{l}{\mathcal{L}} = {\mathcal{C}_\mathcal{I}}^{2/3}, \tag{9}$$

where $\mathcal{C}_\mathcal{I}$ is the channelization index derived in Bonetti et al. (2020).

With these dimensionless variables, the governing equations can be written in the form (Litwin et al., 2022)

$$\frac{\partial \hat{z}}{\partial \hat{t}} = \hat{\nabla}^2 \hat{z} - \sqrt{\hat{a}}|\hat{\nabla}\hat{z}| + 1, \tag{10}$$

$$-\hat{\nabla} \cdot \left(\hat{a} \frac{\hat{\nabla}\hat{z}}{|\hat{\nabla}\hat{z}|} \right) = 1. \tag{11}$$

However, the absence of the Π_l in these equations does not mean that the solutions do not contain it, as shown by Eq. (4). Moreover, with this formulation the effects of the parameters become hidden in the scaling ; for example, considering the case with negligible uplift means dealing with $\Pi_z \to \infty$, as shown by Eq. (5), which is certainly not very practical. Thus, while there is freedom in selecting the repeating variables (Porporato, 2022), choosing l, D, and U instead of K, D, and U and keeping the Π group of the boundary conditions explicitly in the dimensionless form of the equations, as in Bonetti et al. (2020), allows one to gauge the relative importance of the different terms (here creep, erosion, and uplift) for the specific problem at hand.

The dependency of the non-dimensional solutions on the boundary conditions through Π_l signifies that not all the solutions that the model can produce are the rescaled copies of each other. This is true only for those in which the non-dimensional group Π_l remains invariant. A set of steady-state simulations for a square domain demonstrates this point (Fig. 1). In panels a, b, and c, the value of Π_l is changed by increasing the erosion coefficient K for a domain of fixed side length $l = 100$ m. This is the case discussed in Bonetti et al. (2020), which produces different solutions ranging from the unchannelized case Fig. 1(a) to a case with multiple channels Fig. 1(c). On the contrary, panels b, d, and e have different values of K but, because of the different length scale l, they preserve the same Π_l and are therefore scaled models of each other. Theodoratos et al. (2018) consider this case. Clearly, there is no conflict to resolve between these two analyses.

In closing, it is important to note that the reason why many solutions of LEMs look alike is that they have an interesting property of complete self-similarity that emerges at very large $\mathcal{C}_\mathcal{I}$ values (Bonetti et al., 2020; Hooshyar et al., 2020, 2021; Porporato, 2022). For these conditions, several spatially averaged quantities become invariant with respect to $\mathcal{C}_\mathcal{I}$, having reached a self-similar regime. This is similar to what happens to turbulent flows in the fully rough regime as they become independent of the Reynolds number or to regular polygons, which tends to circles as the number of sides tends to infinity (Barenblatt, 1996).

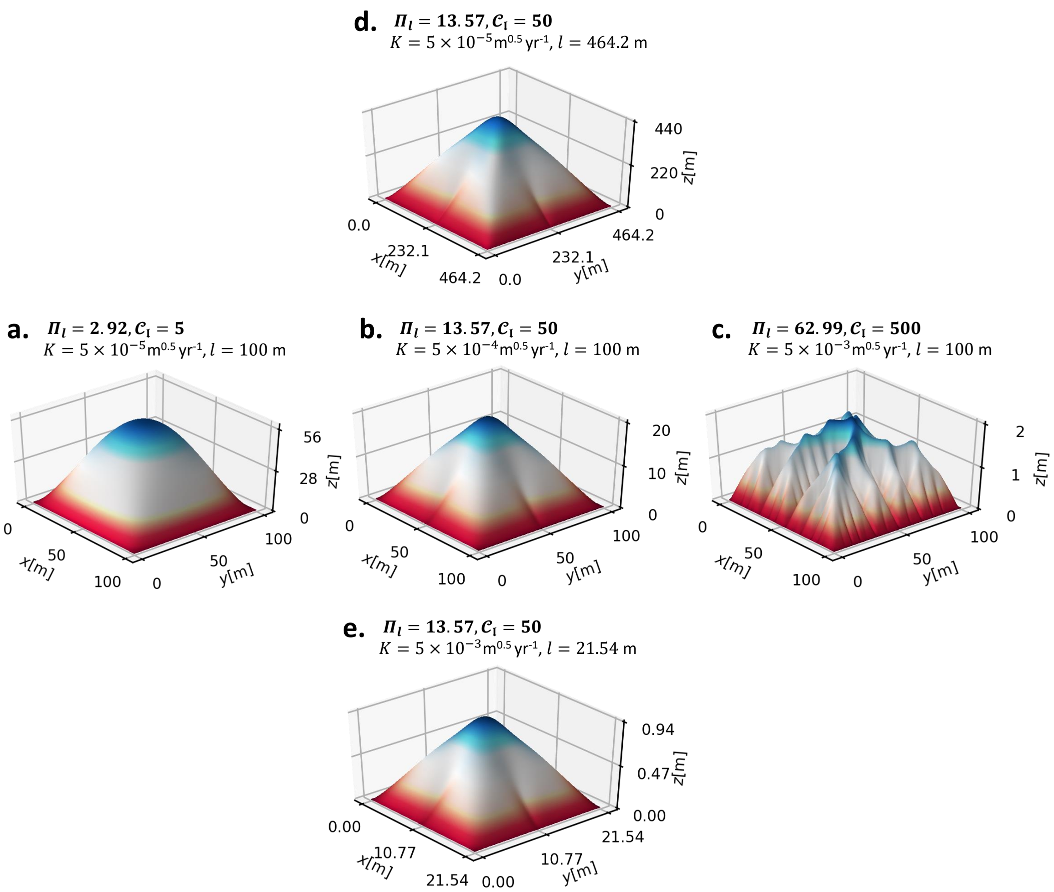

Figure 1. Steady-state solutions for a square domain with fixed elevation boundary conditions, $D = 10^{-2}$ m^2 yr^{-1} and $U = 10^{-3}$ m yr^{-1}. In panels a, b, and c (from left to right), the domain length is fixed as $l = 100$ m with increasing values of K that lead to distinct topographies; also explained by the varied values of $\Pi_l = 2.92$, 13.57, and 2.92, in these cases. In panels d, e, and f (from top to bottom), increasing values of the erosion coefficient K are offset by the decreasing values of domain length l so that the Π_l (= 13.57) or $\mathcal{C}_\mathcal{I}$ (= 50) remains invariant and the topographies appear as rescaled copies of each other. The simulations were performed by using the algorithm presented in Anand et al. (2020).

Open Research

Well-commented Python code used for the simulation results is also accessible at `https://github.com/ShashankAnand1996/LEM`.

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