Demonstration and analysis of rarefied particle motions on hillslopes

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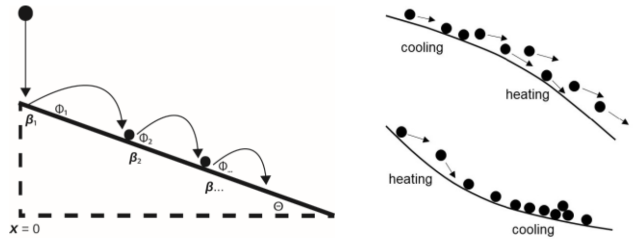
Theory

Consider the motion of a sediment particle tumbling down a rough surface inclined at an angle, having started this motion at position x = 0.



The potential energy of the particle, with a maximum value at the initial $x = 0$, is converted to kinetic energy (hereafter referred to as gravitational heating) as it moves downslope. Frictional cooling acts to counter this heating as the particle bounces down the rough surface. The particle is disentrained, or deposited, at the point on the surface where heating is overcome by cooling through collisional friction. **The distances that particles travel following entrainment directly reflect the probabilistic mechanics of motion and deposition.**

The ratio of gravitational heating to frictional cooling is defined by the Kirkby number,

$$Ki = \frac{4 \tan \varphi \, S}{\langle \beta_x \rangle},$$

where φ denotes the expected reflection angle of particles following collision with the surface, S denotes the surface slope, and β_x denotes the proportion of the translational particle energy E_p extracted by the collision, namely,

$$\beta_x = -\frac{\Delta E_p}{E_p}.$$

If we write the Kirkby number as $Ki = {}^{S}/_{\mu}$, then

$$\mu = \frac{\langle \beta_x \rangle}{4 \tan \varphi}$$

may be considered a friction coefficient. In turn, for a given initial average particle energy E_{a0}, a characteristic length scale of deposition L_c can be defined as

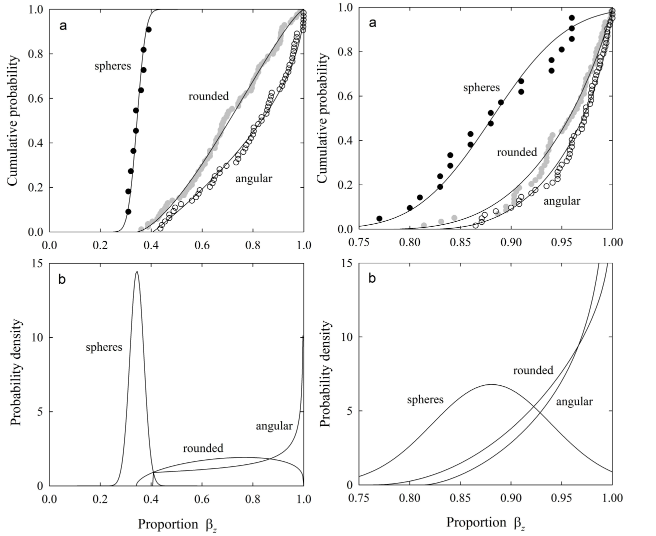
$$L_c = \alpha \frac{E_{a0}}{mg\mu \cos \theta},$$

where the factor α modulates this length scale, likely in relation to particle size, angularity and mode of motion (e.g., translational versus rotational).

Demonstration and Analysis

[1] Drop experiments aimed at demonstrating the basis for treating the proportion of energy extraction, β_x, as a random variable.

- ❑ Varied angularity, size, and drop height of particles
- ❑ Varied surface roughness on which particles were dropped



Plots of (a) cumulative distribution of β_z for glass spheres fit to a Gaussian distribution and rounded and angular gravel particles fit to a beta distribution, and (b) associated probability density functions of fitted distributions. Left set of plots is for collisions on hard slate, and right set of plots is for collisions on a rough concrete surface.

[2] Particle travel distance experiments using laboratory-scale hillslope with a concrete surface and a pendulum catapult launching device.

- ❑ Varied angularity of particles
- ❑ Varied surface slope

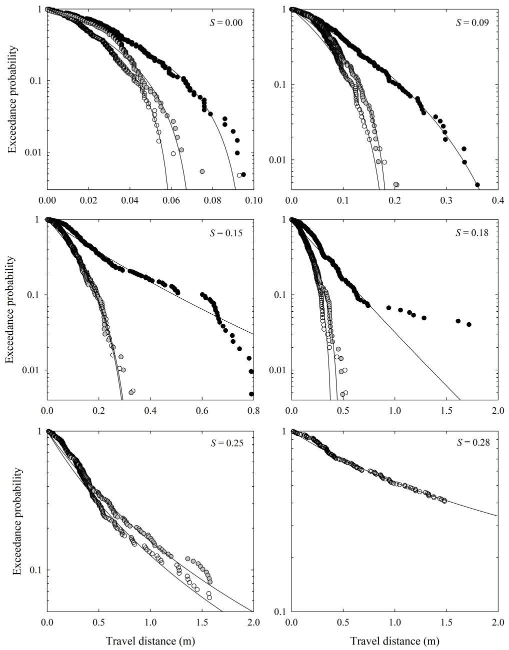
Data from travel distance experiments are consistent with the generalized Pareto distribution obtained through a formal energy balance in Furbish et al. 2019 *(in prep)*.

The fitting of distribution shape, A, and scale, B, parameters provides estimates of frictional factor μ and scaling factor α where the latter can be defined as

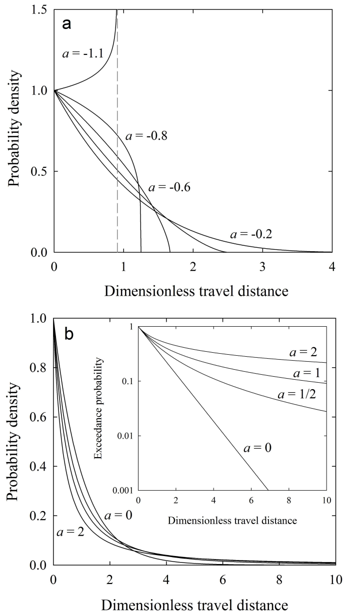
$$\alpha = \frac{B\gamma mg\mu \cos \theta}{E_{a0}}$$

with γ denoting the ratio of the average energy to the harmonic mean energy.

Particle angularity affects energy extraction and travel distances.



Plots of exceedance probability versus travel distance over six values of slope S showing **angular** (open circles), **rounded** (black circles) and small (gray circles) particles together with fitted distributions (lines).



Plots of dimensionless probability density function $f_{\hat{x}}(\hat{x})$ versus dimensionless travel distance $\hat{x}$ for scale parameter b = 1 and shape parameters (a) a < 0 and (b) a > 0.