Study landau damping of the DIA wave in a non-extensive distributed dusty plasma

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Abstract

The Landau damping of the dust ion-acoustic wave (DIAW) propagating in a dusty plasma with nonextensive distributed components is kinetically analyzed. The electron, ion, and dust particles are effectively modeled by nonextensive distributions of Tsallis statistics. For a collisionless plasma with different values of plasma components indices, the general dispersion relation is achieved, and the nonextensivity effects on the frequency, as well as the Landau damping, are studied. We show that for $q \rightarrow 1$, the preliminary results of the Maxwellian plasma are obtained. The decrease of wave damping is achieved by increasing the coefficient q index and the ion to electron density ratio. The damping rate also increases with a decreasing dust-to-electron temperature ratio.

Keywords: Landau damping, dust-ion acoustic wave, nonextensive distribution, dusty plasma, Tsallis statistics

1. Introduction

A diversity of spatial observations indicates the existence of many suprathermal particles in dusty plasma environments [1]. The laboratory and spatial observations demonstrate non-Maxwellian suprathermal tails for the velocity distribution function of most dust plasmas. Indeed an extensive formalism could not be applied to physical systems containing long-range forces or long-range memory, for example, for the dusty plasma with long-range interactions. Tsallis proposed the nonextensive form of entropy to solve the problems faced by applying the Boltzmann-Gibbs standard statistical mechanics [2-4], and later, Silva et al. introduced the q-extensive velocity distribution function [5]. The nonextensive approach has been successfully used in several spatial plasmas. Compared with more complicated models, the nonextensive distribution of an ideal classical gas presents the best overlap with the observed distribution of galaxy clusters [6].

The nonextensive distribution as an exciting topic in plasma physics has been applied in a variety of researches, such as studying the variable charge DA solitary waves [7], the transverse oscillation in relativistic plasmas [8], arbitrary amplitude kinetic Alfven solitons [9], IA solitary waves [10], electrostatic fluctuations in a two-component magnetoplasma [11], nonlinear propagation of electron-acoustic waves [12], the full Zakharov equations [13], and double layers in a warm plasma with nonextensive electrons [14].

The dust charge oscillation produces an instability state for low-frequency waves, such as DIAW, dust acoustic wave (DAW), and ion acoustic wave (IAW) [15, 16]. In the field of instability in the physics of plasma waves, Landau damping is considered as a well-known

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phenomenon. For instance, the Landau damping of DIAW for ordinary plasma and Lorentzian distribution [17, 18] were studied. In recent years, the study of instability with nonextensive distribution has received more attention, for instance studies for IAW [19-21] and DAW [22-24] in the plasma.

To the best of our knowledge, the Landau damping of DIAW in a dusty plasma with the application of nonextensive distribution has not been reported. In the present article, we have tried to present exciting aspects of the nonextensive parameter, q, in the dispersion relation and damping of DIAW for plasma particles in a dusty plasma through modeling by q-distribution.

The manuscript is organized in 5 section. In section 2, using the Vlasov equation, we derive the dispersion relation of the DIA wave in Tsallis statistics. Then, we study the effect of q, δ, and electron temperature parameters on the wave frequency and the Landau damping in section 3. Section 4 provides the conclusion of this work.

2. The dispersion relation of DIA wave

The entropy in Tsallis statistics is presented as follows [2]:

$$S_q = k_B \frac{1-\sum_i p_i^q}{q-1} \tag{1}$$

where k_B, is the Boltzmann constant, $\{p_i\}$ is the probability of the i-th microstate, and the q parameter is the degree of nonextensivity. In the limit $q \to 1$, the Tsallis entropy (S_q) is turned to the B-G entropy. The fundamental aspect of Tsallis entropy is the nonextensivity for $q \neq 1$. In other words, for a given composite system $A+B$, including two independent subsystems A and B, we have:

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B) \tag{2}$$

Generally, the one-dimensional q-equilibrium distribution function is given as follows [5]:

$$f_{q\alpha}(v) = \frac{\mathrm{n}_\alpha \mathrm{A}_{q_\alpha}}{\sqrt{\pi} v_\alpha}\left(1-(q_\alpha-1)\frac{v^2}{v_\alpha^2}\right)^{1/(q_\alpha-1)} \qquad (\alpha = e, i, d) \tag{3}$$

where n_α is the number density, $v_{\alpha(\mathrm{i,e,d})} = \sqrt{2k_B\mathrm{T}_\alpha/\mathrm{m}_\alpha}$ is the thermal speed of particles α, k_B is the Boltzmann constant, Γ is gamma function, T_α and m_α are temperature and mass of particles α, respectively. The constant $A_q = \sqrt{1-q}\,\frac{\Gamma(1/(1-q))}{\Gamma(1/(1-q)-1/2)}$ gives $A_q = \frac{1+q}{2}\sqrt{q-1}\,\frac{\Gamma(1/(q-1)+1/2)}{\Gamma(1/(q-1))}$ for $0 < q \leq 1$ and also for $q \geq 1$. It should be mentioned that the maximum value for the speed is considered $v_{max} = \sqrt{v_\alpha^2/(q-1)}$ and for $q \to 1$, the q-distribution function becomes Maxwellian.

In this research, an un-magnetized, unbounded, and collisionless plasma including electrons, ions, and dust particles with nonextensive distribution function is considered for obtaining the dispersion relation of dust ion-acoustic waves. An electrostatic state with a weak perturbation in the equilibrium state is assumed. In kinetic theory, the Vlasov equation (see Eq(4)) is considered a fundamental equation for the evolution of distribution function:

$$\frac{\partial}{\partial t} f_\alpha(\boldsymbol{r}, \boldsymbol{V}, t) + \boldsymbol{V}.\nabla f_\alpha + \frac{Q_\alpha}{m_\alpha}(\boldsymbol{E} + \boldsymbol{V} \times \boldsymbol{B}).\nabla_v f_\alpha = 0 \qquad \alpha = e, i, d \tag{4}$$

where m_α and Q_α are mass and charge of particles α, respectively. The quasi-neutrality condition at the equilibrium in a dusty plasma is evaluated as follows:

$$Q_d n_{d0} + Q_i n_{i0} - Q n_{e0} = 0 \tag{5}$$

where $Q_d = z_d e$ and $Q_i = z_i e$ are the charge of dust and ion particles, respectively. Singly charged ions and negatively charged dust, i.e., $\mathrm{Z_i} = +1$ and $\mathrm{Z_d} < 0$, are considered in this research. Furthermore, in the abovementioned equation, the charge fluctuation of dust particles is ignored. We considered the liner perturbation as $f_\alpha = f_{\alpha 0} + f_{\alpha 1}$ and $n_\alpha = n_{\alpha 0} + n_{\alpha 1}$, where indexes 0, 1 demonstrate the equilibrium and perturbation states, respectively, with the assumption of $|\mathrm{f}_{\alpha 0}| \gg |\mathrm{f}_{\alpha 1}|$ and $|\mathrm{n}_{\alpha 0}| \gg |\mathrm{n}_{\alpha 1}|$. The Poisson equation was applied for solving the Vlasov equation in this paper.

$$\boldsymbol{\nabla}^2 \varphi = -4\pi \sum_{\alpha = e,i,d} Q_\alpha \int \mathrm{f}_{\alpha 1}\, d^3 v \tag{6}$$

The Fourier transformation $\left(i.e., \mathrm{f}_{\alpha 1} \simeq \mathrm{e}^{\mathrm{i}(\mathbf{k}.\mathbf{r} - \omega t)}, \varphi \simeq \mathrm{e}^{\mathrm{i}(\mathbf{k}.\mathbf{r} - \omega t)}\right)$ is applied for Eq. (4) and Eq(6), then the combination of these equations leads to the longitudinal dielectric permittivity as Eq (7) :

$$\varepsilon_{q_\alpha}(k, \omega) = 1 + \sum_{\alpha = e,i,d} \frac{\omega_{p\alpha}^2}{k^2} \frac{1}{n_{\alpha 0}} \int \frac{\partial f_{\alpha 0}/\partial v_x}{\omega/k - v_x} d^3 v = 1 + \sum_{\alpha = i,e,d} \chi_\alpha = 0 \tag{7}$$

where $\omega_{p\alpha(\mathrm{i,d})} = (4\pi n_{\alpha 0}\, Q_\alpha^2/m_\alpha)^{1/2}$ is the plasma frequency of species α and χ_α is the plasma dielectric susceptibility.

We aim to investigate the effect of the dispersion relation, Landau damping of DIAW, and nonextensive effects on the wave frequency as well as instability. Considering the DIA wave condition ($\mathrm{v_d}, \mathrm{v_i} \ll \omega/\mathrm{k} \ll \mathrm{v_e}$), if the nonextensive distribution function given by Eq. (3) is employed for unperturbed particles, the dispersion relation can be obtained as follows:

$$1 + \frac{1}{k^2} \sum_{\alpha = e,i,d} \frac{1}{\lambda_\alpha^2} \left[\frac{1 + q_\alpha}{2} + \xi_\alpha Z_{q_\alpha}(\xi_\alpha)\right] = 0 \tag{8}$$

where $\lambda_\alpha = \frac{v_\alpha}{\sqrt{2}\omega_{p\alpha}}$, $\xi_\alpha = \omega/\mathrm{k}v_\alpha$, and plasma dispersion function, Z_q, is [19]

$$Z_q(\xi_\alpha) = \frac{A_q}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{(1 - (q-1)t^2)^{(2-q)/(q-1)}}{t - \xi_\alpha} dt \qquad t = v/v_\alpha \tag{9}$$

$$Z_q(\xi_\alpha) \approx -\frac{1+q}{2\xi_\alpha} - \frac{1}{2\xi_\alpha^3} - \frac{3}{2(3q-1)\xi_\alpha^5} + \mathrm{i}\sqrt{\pi} A_q\, (1 - (q-1)\xi_\alpha^2)^{(2-q)/(q-1)} \qquad (|\xi_\alpha| \gg 1) \tag{10}$$

$$Z_q(\xi_\alpha) \approx \mathrm{i}\sqrt{\pi} A_q\, (1 - (q-1)\xi_\alpha^2)^{(2-q)/(q-1)} \qquad (|\xi_\alpha| \ll 1) \tag{11}$$

In limit $q \to 1, Z_q(\xi_\alpha)$ is recovered to the standard form of Z-function (the plasma dispersion function) [25]. Regarding the phase velocity much smaller (larger) than the thermal velocity of the electron (ion and dust), the large argument expansion of the Z_q-function for the ion and dust particles and the small argument expansion for the electron particles are considered. Therefore, the dispersion relation of the DIA wave can be approximately derived as follows:

$$1 + \frac{1}{k^2\lambda_e^2}\frac{1+q_e}{2} - \frac{\omega_{pi}^2}{\omega^2}\left(1 + \frac{3k^2v_i^2}{(3q_i-1)\omega^2}\right) - \frac{\omega_{pd}^2}{\omega^2}\left(1 + \frac{3k^2v_d^2}{(3q_d-1)\omega^2}\right) + i\frac{\sqrt{\pi}}{k^2}\sum_{\alpha=e,i,d} A_{q_\alpha}\frac{\xi_\alpha}{\lambda_\alpha^2}(1 - (q_\alpha - 1)\xi_\alpha^2)^{(2-q_\alpha)/(q_\alpha-1)} = 0 \tag{12}$$

3. Results and Discussion

To investigate the dispersion relation of the DIAW, the dielectric permittivity is considered as $\varepsilon_q(k,\omega) = \varepsilon_q^{re}(k,\omega) + i\varepsilon_q^{im}(k,\omega)$, where $\varepsilon_q^{im}(k,\omega)$ and $\varepsilon_q^{re}(k,\omega)$ are the imaginary and real parts of the dielectric permittivity, respectively, and $\omega = \omega_r + i\omega_i$, where ω_i and ω_r are the imaginary and real parts of the frequency, respectively. We suppose the linear perturbation that $|\varepsilon_q^{re}(k,\omega)| \gg |\varepsilon_q^{im}(k,\omega)|$ and $|\omega_r| \gg |\omega_i|$.

3.1. Frequency

For the real part of the dielectric permittivity, we have derived:

$$\varepsilon_q^{re}(k,\omega) = 1 + \frac{1}{k^2\lambda_e^2}\frac{1+q_e}{2} - \frac{\omega_{pi}^2}{\omega^2}\left(1 + \frac{3k^2v_i^2}{(3q_i-1)\omega^2}\right) - \frac{\omega_{pd}^2}{\omega^2}\left(1 + \frac{3k^2v_d^2}{(3q_d-1)\omega^2}\right) \tag{13}$$

Expanding $\varepsilon_q^{re}(k,\omega)$ at $\omega = \omega_r$, we have derived:

$$\omega_r^2 = \frac{k^2\lambda_e^2}{k^2\lambda_e^2 + \frac{1+q_e}{2}}(\omega_{pi}^2 + \omega_{pd}^2) + 3k^2\left(\frac{v_i^2}{3q_i - 1} + \frac{v_d^2}{3q_d - 1}\right) \tag{14}$$

where the condition $\varepsilon_K^{re}(k,\omega_r) = 0$, is used to drive the Eq.(14), which is the generalized dispersion relation of the DIAW in a nonextensive distributed dusty plasma. Regarding $\omega_r \gg kv_i, kv_d$, in limit $q_{\alpha(e,i,d)} \to 1$, one can easily show that Eq. (14) can be reduced to the ordinary plasma dispersion relation in B-G statistics [15, 16]. Therefore, we can write Eq. (14) as Eq. (15) as follows:

$$\frac{\omega_r^2}{\omega_{pi}^2} = \frac{k^2\lambda_e^2}{k^2\lambda_e^2 + \frac{1+q_e}{2}}\left(1 + \frac{m_i n_i}{m_d n_d}\left(1 - \frac{1}{\delta}\right)^2\right) + \frac{6}{\delta}\left(\frac{1}{3q_i - 1}\frac{T_i}{T_e} + \frac{1}{3q_d - 1}\frac{m_i}{m_d}\frac{T_d}{T_e}\right)k^2\lambda_e^2 \tag{15}$$

where, $\delta = \frac{n_{io}}{n_{eo}}$ is the ion-electron density ratio.

Within the permissible range of values for q, the frequency rate, ω_r/ω_{pi}, of the DIAWs as a function of $k\lambda_e$ is plotted as shown in Figs. 1-5. Typical parameters of the dusty plasma are considered; T_i/T_e =0.2, T_d/T_e =0.01, $m_i/m_d = 10^{-6}$, and $10^{-2} < m_i n_i/m_d n_d < 10^2$, which is considered $m_i n_i/m_d n_d$ =0.01 in this study [16, 26]. Accordingly, a decrease of the wave frequency can be obtained by increasing the q. The wave phase velocity for long wavelength ($k\lambda_e \ll 1$ or $k\lambda_e \to 0$) is found as follows:

$$v_{ph} = \frac{\omega_r}{k} \approx \sqrt{\frac{2}{1+q_e}}\lambda_e\left(\omega_{pi}^2 + \omega_{pd}^2\right)^{1/2} \approx \sqrt{\frac{2}{1+q_e}}\lambda_e\omega_{pi}(1 + \frac{m_i n_i}{m_d n_d}\left(1-\frac{1}{\delta}\right)^2)^{1/2} \quad (16)$$

In the limit of long wavelength, we can show that the phase velocity increases by decreasing the electron density. In the limit of $q_e \to 1$, the wave phase velocity is proportional to $v_{ph} \approx \lambda_e\omega_{pi} = \sqrt{\delta}(k_B T_e/m_i)^{1/2}$ and is entirely conformed to the Maxwellian plasma [15].

Interestingly, Liang, X. et al. reported the same finding in the laboratory. Their experimental observations show that decreases of the electron density as well as increases of the phase velocity of the DIAW with increasing of z (distance) [27].

For short wavelength ($k\lambda_e \gg 1$ or $k\lambda_e \to \infty$), we can write the wave phase velocity as $v_{ph} \approx \left(\omega_{pi}^2 + \omega_{pd}^2\right)^{1/2}/k$. Practically, the dust ion-acoustic wave oscillate with the frequency of ω_r, which is the summation of ions and dusts plasma frequencies, $\omega_r \approx \left(\omega_{pi}^2 + \omega_{pd}^2\right)^{1/2} \approx \omega_{pi}(1 + \frac{m_i n_i}{m_d n_d}\left(1-\frac{1}{\delta}\right)^2)^{1/2}$. Actually, for the short wavelength the effect of

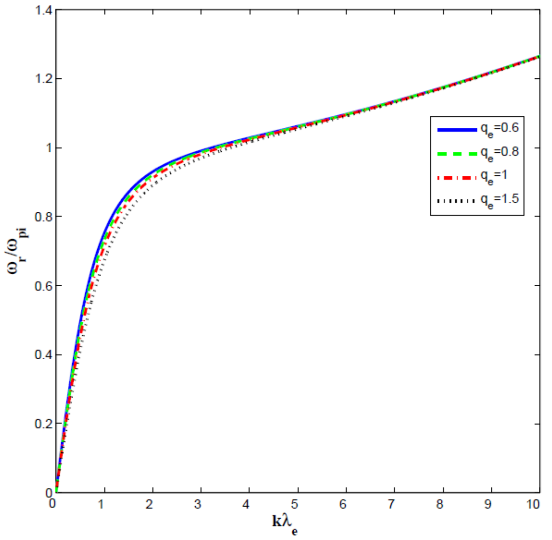

Fig. 1. The normalized frequency rate of the DIAWs as a function of the normalized wave number for various values of q_e ($q_e = 0.6$ (solid), $q_e = 0.8$ (dashed), $q_e = 1$ (dash-dot; Maxwellian) and $q_e = 1.5$ (dotted). $\delta = 100$, $q_i = q_d = 1$ and $m_i n_i/m_d n_d = 0.01$ are assumed).

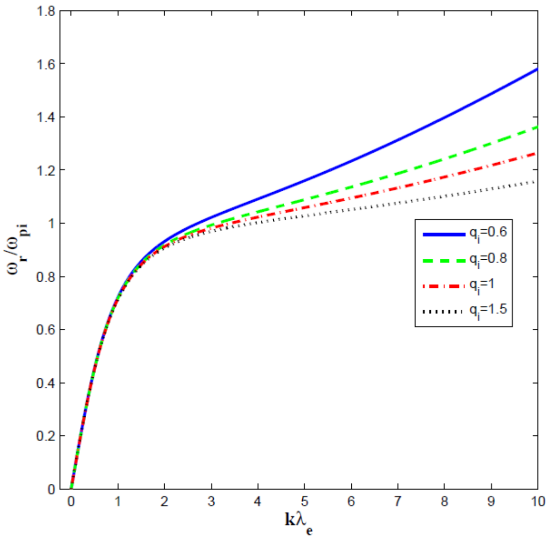

Fig. 2. The normalized frequency rate of the DIAWs as a function of the normalized wave number for various values of q_i($q_i = 0.6$ (solid), $q_i = 0.8$ (dashed), $q_i = 1$ (dash-dot; Maxwellian) and $q_i = 1.5$ (dotted). $\delta = 100$, $q_e = q_d = 1$ and $m_i n_i/m_d n_d = 0.01$ are assumed).

q is vanished.

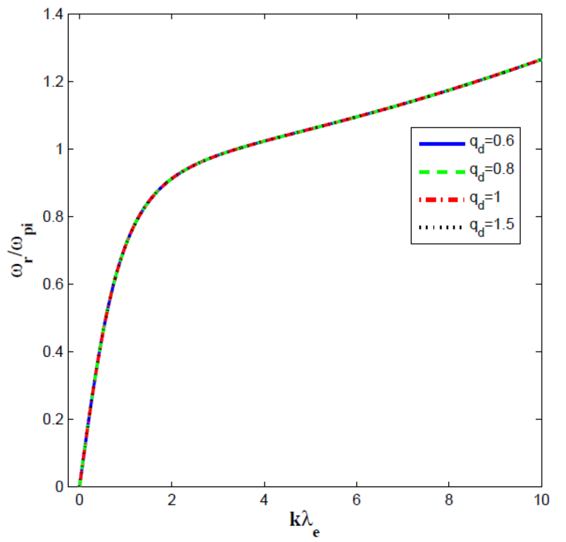

Fig. 3. The normalized frequency rate of the DIAWs as a function of the normalized wave number for various values of q_i($q_d = 0.6$ (solid), $q_d = 0.8$ (dashed), $q_d = 1$ (dash-dot; Maxwellian) and $q_d = 1.5$ (dotted). $\delta = 100$, $q_e = q_i = 1$ and $m_i n_i / m_d n_d = 0.01$ are assumed).

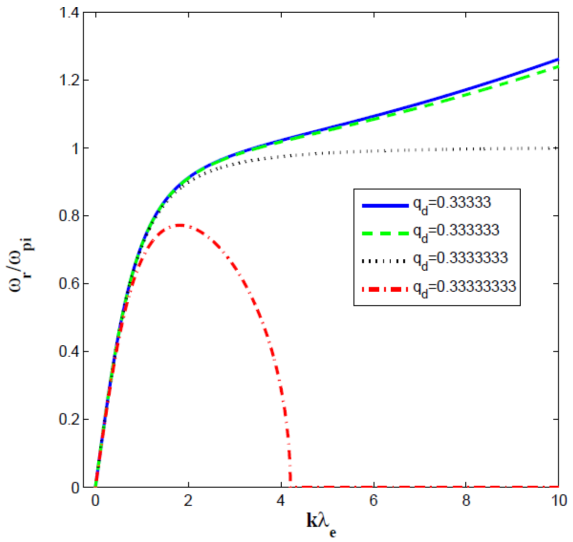

Fig. 4. The normalized frequency rate of the DIAWs as a function of the normalized wave number for various values of $q_i(q_d = 0.33333$ (solid), $q_d = 0.333333$ (dashed), $q_d = 0.3333333$ (dotted) and $q_d = 0.33333333$ (dash-dot). $\delta = 100$, $q_e = q_i = 1$ and $m_i n_i / m_d n_d = 0.01$ are assumed).

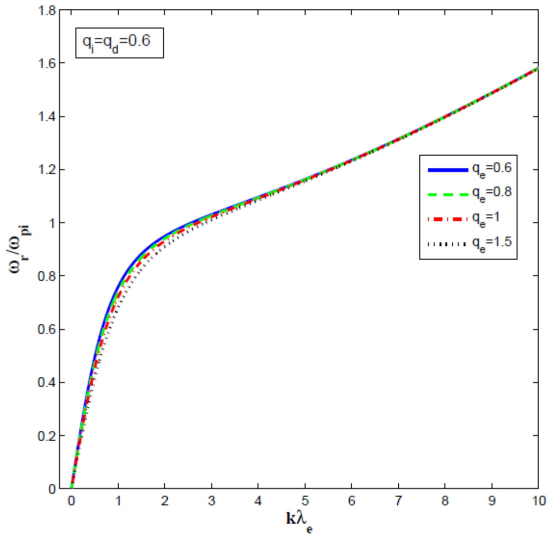

Fig. 5. The normalized frequency rate of the DIAWs as a function of the normalized wave number for various values of q_e ($q_e = 0.6$ (solid), $q_e = 0.8$ (dashed), $q_e = 1$ (dash-dot; Maxwellian) and $q_e = 1.5$ (dotted). $\delta = 100$, $q_i = q_d = 0.6$ and $m_i n_i / m_d n_d = 0.01$ are assumed).

In Fig. 1 and 2, a decrease of v_{ph} can be seen by increasing the q nonextensive index for electron and ion particles. For dust particles (see Fig. 3), an approximately constant v_{ph} is observed by varying the q nonextensive index values. However, as can be seen in Fig. 4, dust particles present a similar behavior with electron and ion particles in a tiny change of q index, though with a more intense trend. Fig. 5, shows an increase of wave amplitude as the ion and dust particles are considered as nonextensive.

3.2. Landau damping

The Landau damping can be studied through the Eq(17):

$$\gamma = -\left.\frac{\varepsilon_{\mathrm{q}}^{\mathrm{im}}(\mathrm{k},\omega)}{\frac{\partial\varepsilon_q^{\mathrm{re}}(\mathrm{k},\omega)}{\partial\omega}}\right|_{\omega=\omega_{\mathrm{r}}} \tag{17}$$

where γ is the imaginary part of frequency and $\varepsilon_{\mathrm{q}}^{\mathrm{im}}(\mathrm{k},\omega)$ for DIAW can be identified as Eq(18):

$$\varepsilon_{\mathrm{q}}^{\mathrm{im}}(\mathrm{k},\omega) = \frac{\sqrt{\pi}}{k^2}\sum_{\alpha=e,i,d} A_{q_\alpha}\frac{\xi_\alpha}{\lambda_\alpha^2}\left(1-\left(q_\alpha-1\right)\xi_\alpha^2\right)^{(2-q_\alpha)/(q_\alpha-1)} \tag{18}$$

The Landau damping of the DIAW, in the range of $\mathrm{v_d}, \mathrm{v_i} \ll \omega/\mathrm{k} \ll \mathrm{v_e}$, can be obtained in the form Eq(19) as follow:

$$\begin{aligned}\gamma \cong -\sqrt{\frac{\pi}{8}}\delta^{3/2}\frac{\omega_{pi}k\lambda_e}{1+\frac{m_i n_i}{m_d n_d}\left(1-\frac{1}{\delta}\right)^2}&\left(\frac{1+\frac{m_i n_i}{m_d n_d}\left(1-\frac{1}{\delta}\right)^2}{k^2\lambda_e^2+\frac{1+q_e}{2}}+\frac{1}{\delta}\frac{T_i}{T_e}\frac{6}{3q_i-1}+\frac{1}{\delta}\frac{m_i}{m_d}\frac{T_d}{T_e}\frac{6}{3q_d-1}\right)^2\\
\times\Bigg[A_{q_e}\frac{1}{\delta}\left(\frac{m_e}{m_i}\right)^{1/2}+A_{q_i}\left(\frac{T_e}{T_i}\right)^{3/2}&\left(1-(q_i-1)\left(\delta\frac{T_e}{2T_i}\frac{1+\frac{m_i n_i}{m_d n_d}\left(1-\frac{1}{\delta}\right)^2}{k^2\lambda_e^2+\frac{1+q_e}{2}}+\frac{3}{3q_i-1}+\frac{3}{3q_d-1}\frac{m_i}{m_d}\frac{T_d}{T_i}\right)\right)^{\frac{2-q_i}{q_i-1}}\\
+A_{q_d}\left(1-\frac{1}{\delta}\right)\left(\frac{T_e}{T_d}\right)^{3/2}\left(\frac{m_d}{m_i}\right)^{1/2}&\left(1-(q_d-1)\left((\delta-1)\frac{T_e}{2T_d}\frac{1+\frac{m_d n_d}{m_i n_i}\frac{1}{\left(1-\frac{1}{\delta}\right)^2}}{k^2\lambda_e^2+\frac{1+q_e}{2}}+\frac{3}{3q_d-1}+\frac{3}{3q_i-1}\frac{m_d}{m_i}\frac{T_i}{T_d}\right)\right)^{\frac{2-q_d}{q_d-1}}\Bigg]\end{aligned} \tag{19}$$

Fig. 6-17 show the change of the damping rate, γ/ω_{pi}, as a function of wavenumber, $k\lambda_e$, for the various parameters such as q, δ, and electron temperature changes. An increase of the Landau damping can be observed by decreasing q parameter. Typical parameters of the dusty plasma are considered $T_i/T_e = 0.2$, $T_d/T_e = 0.01$, $T_i/T_d = 20$, $m_i/m_d = 10^{-6}$, $m_e/m_i = 0.00054$, and $10^{-2} < m_i n_i/m_d n_d < 10^2$, which is considered $m_i n_i/m_d n_d = 0.01$ in this study [16, 26].

If we consider the limit $q_e = q_i = q_d \to 1$ (Maxwellian), $\delta \to 1$, and $T_e \gg T_i, T_d$, the Landau damping rate reduces to the Maxwellian description in B-G statistics [16,17].

$$\gamma \cong -\sqrt{\frac{\pi}{8}}\frac{\omega_{pi}k\lambda_e}{(1+k^2\lambda_e^2)^2}\left(\left(\frac{T_i}{T_e}\right)^{3/2}+\left(\frac{m_e}{m_i}\right)^{1/2}\right) \tag{20}$$

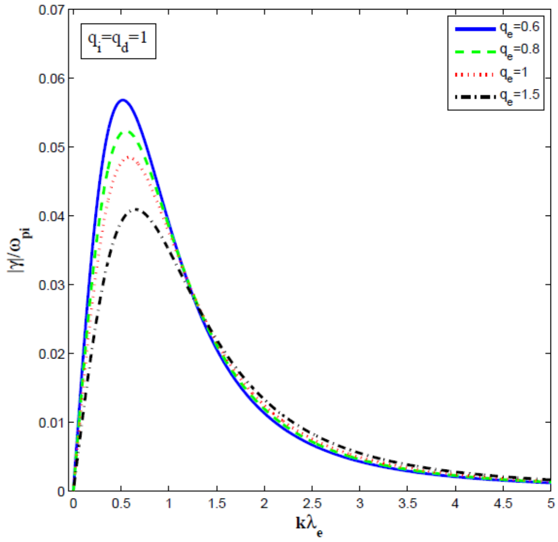

Fig. 6. The plot of Landau damping of the DIAWs in terms of the wavenumber for various values of q_e. $\delta = 100$ and $m_i n_i / m_d n_d = 0.01$ are assumed.

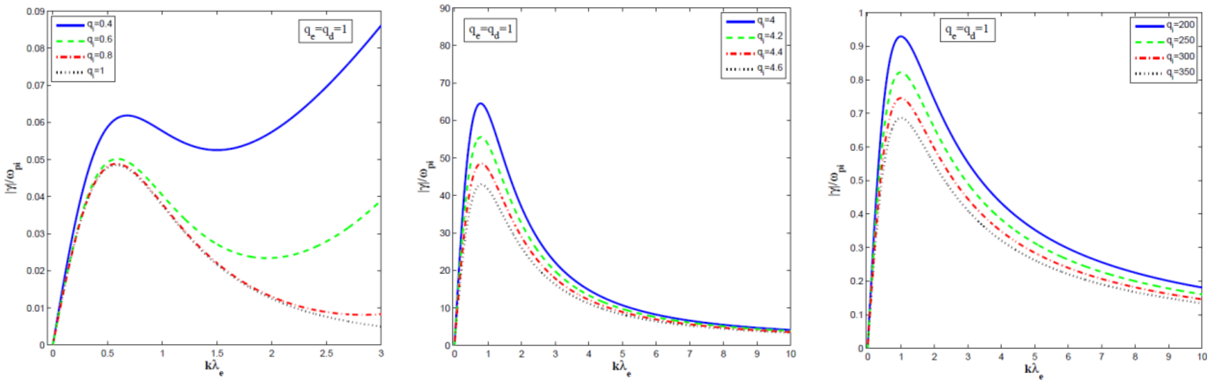

Fig. 7. Plots of Landau damping of the DIAWs in terms of the wavenumber for various values of q_i. $q_e = q_d = 1$, $\delta = 100$ and $m_i n_i / m_d n_d = 0.01$ are assumed.

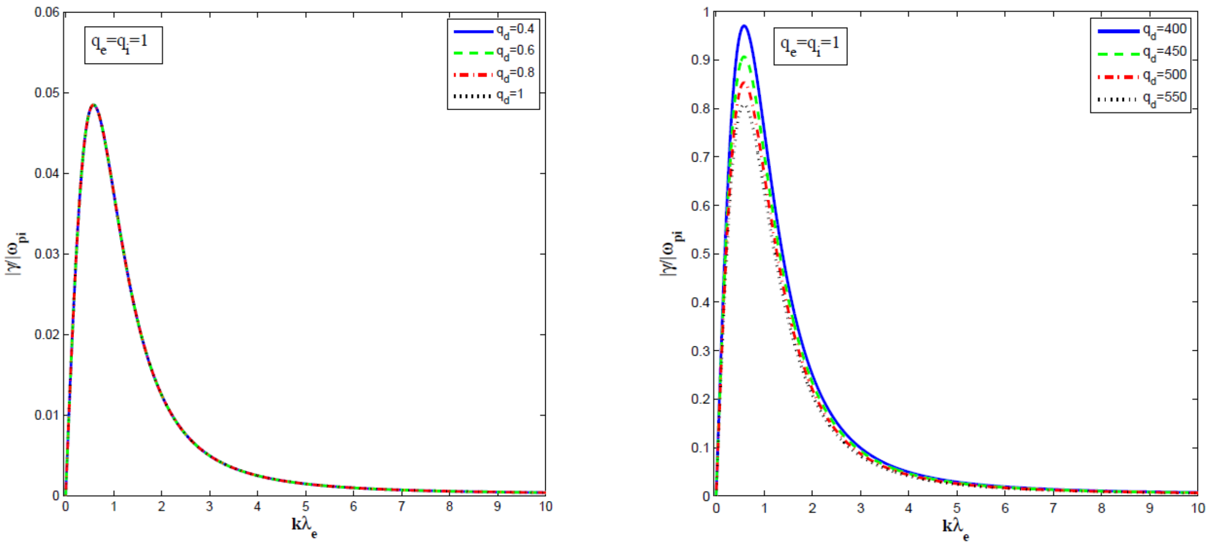

Fig. 8 Plots of Landau damping of the DIAWs in terms of the wavenumber for various values of q_d. $q_e = q_i = 1$, $\delta = 100$ and $m_i n_i / m_d n_d = 0.01$ are assumed.

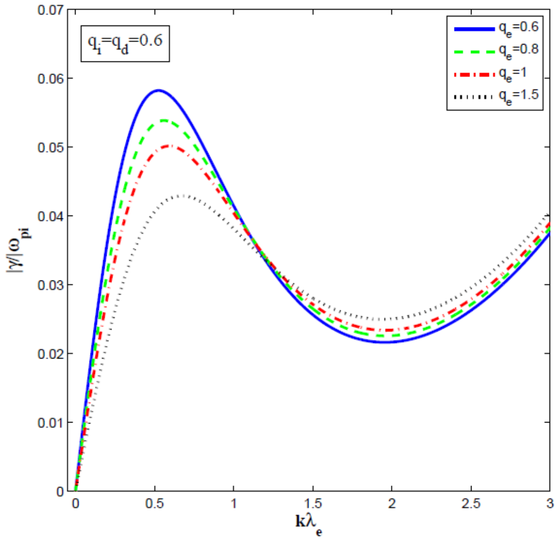

Fig. 9 Plot of Landau damping of the DIAWs in terms of the wavenumber for various values of q_e. $q_i = q_d = 0.6$, $\delta = 100$ and $m_i n_i / m_d n_d = 0.01$ are assumed.

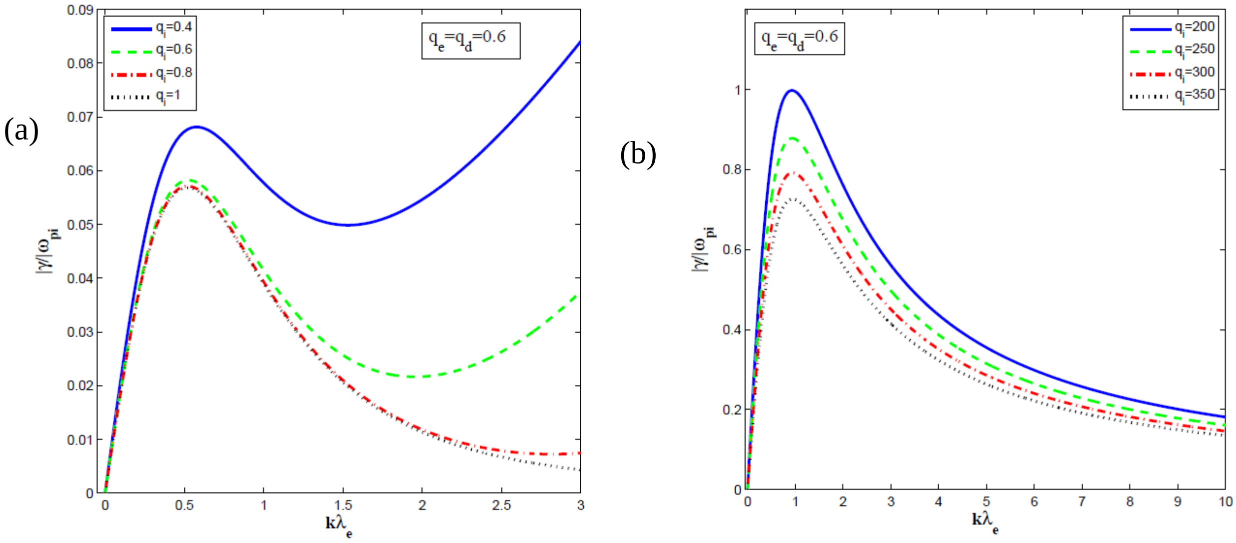

Fig. 10. Plots of Landau damping of the DIAWs in terms of the wavenumber for various values of q_i. $q_e = q_d = 0.6$, $\delta = 100$ and $m_i n_i / m_d n_d = 0.01$ are assumed.

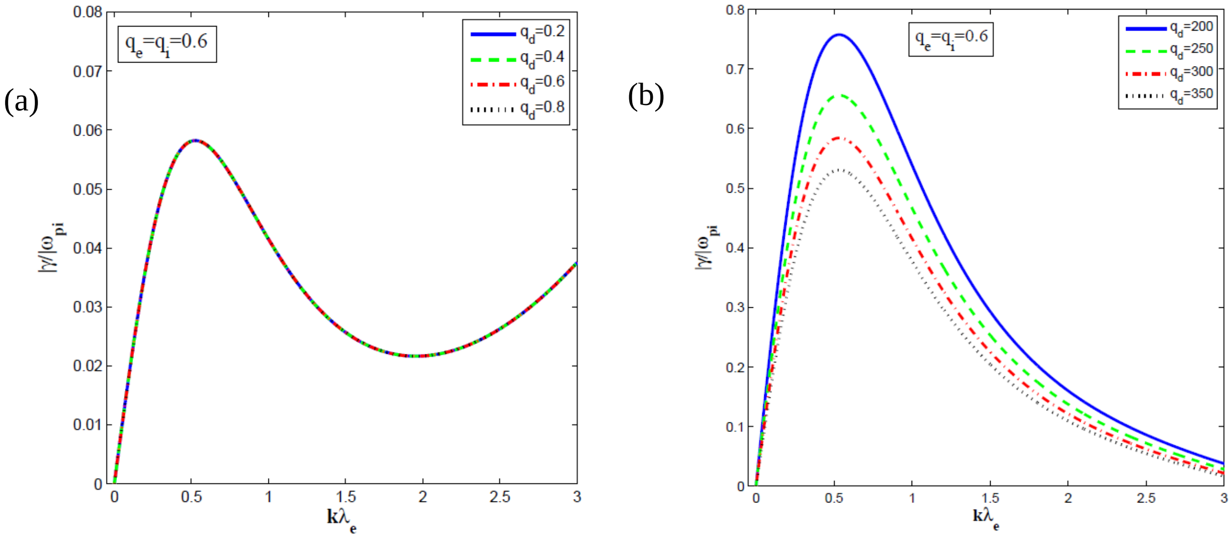

Fig. 11. Plots of Landau damping of the DIAWs in terms of the wavenumber for various values of q_d. $q_e = q_i = 0.6$, $\delta = 100$ and $m_i n_i / m_d n_d = 0.01$ are assumed.

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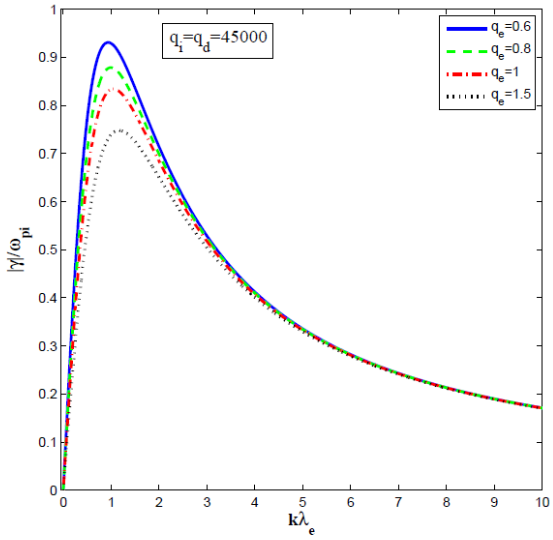

Fig. 12. The plot of Landau damping of the DIAWs in terms of the wavenumber for various values of q_e. $q_i, q_d > 1$, $\delta = 100$ and $m_i n_i / m_d n_d = 0.01$ are assumed.

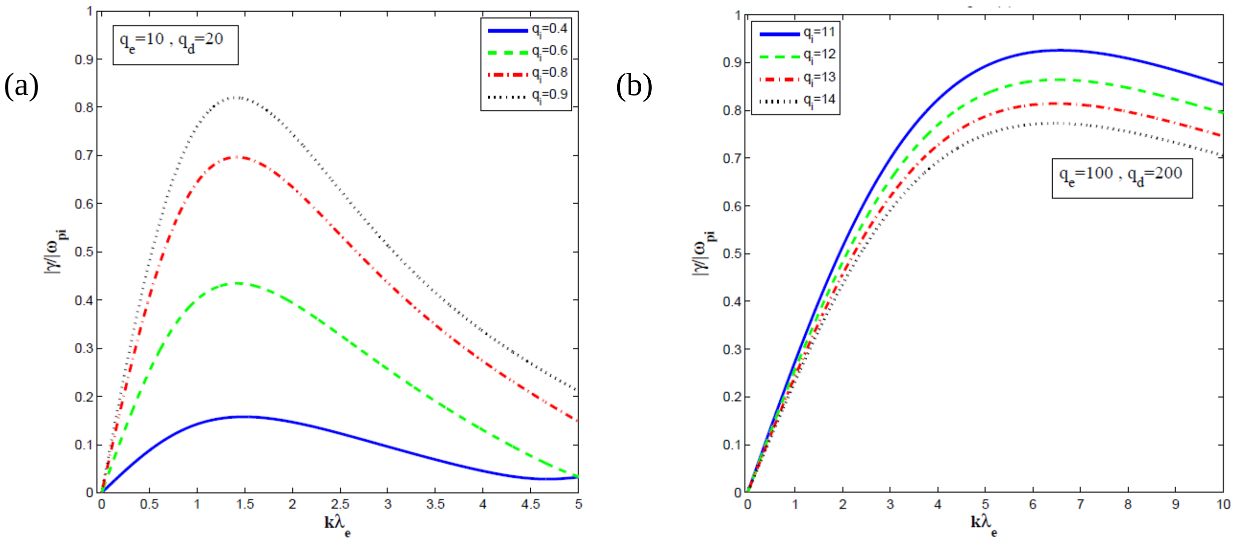

Fig. 13. Plots of Landau damping of the DIAWs in terms of the wavenumber for various values of q_i. $q_e, q_d > 1$, $\delta = 100$ and $m_i n_i / m_d n_d = 0.01$ are assumed.

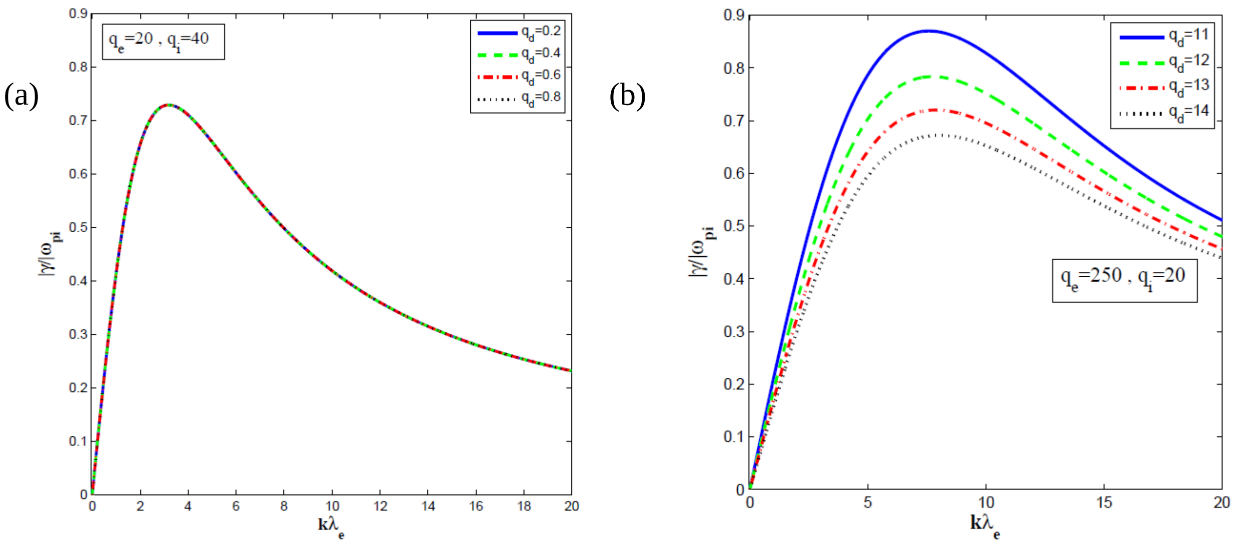

Fig. 14. Plots of Landau damping of the DIAWs in terms of the wavenumber for various values of q_d. $q_e, q_i > 1$, $\delta = 100$ and $m_i n_i / m_d n_d = 0.01$ are assumed.

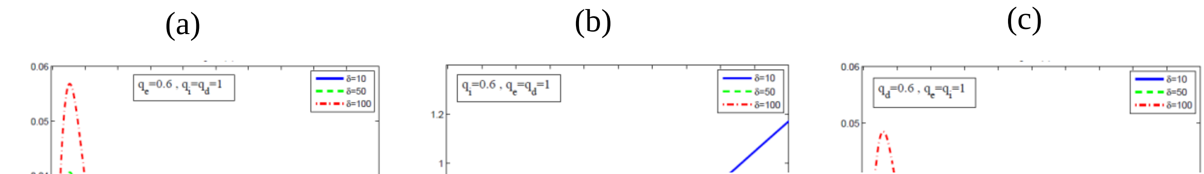

Fig. 15. Plot of Landau damping of the DIAWs in terms of the wave
$\delta = 10$ (solid), $\delta = 50$ (dashed), $\delta = 100$(dash-dot). $m_i n_i/m_d n_d$

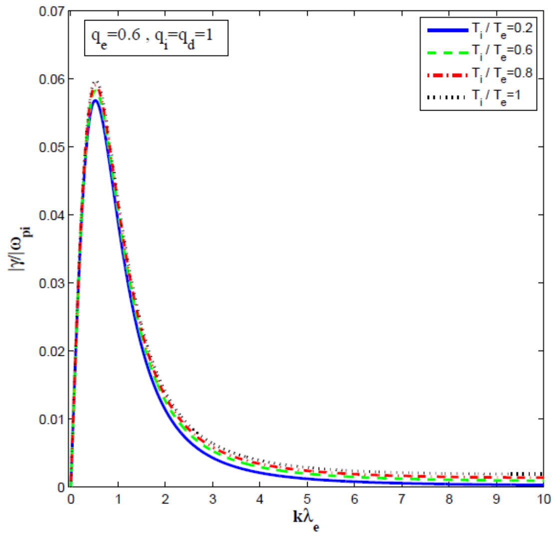

Fig. 16. Plot of Landau damping of the DIAWs in terms of the wavenumber for various values of T_i/T_e ($T_i/T_e = 0.2$(solid), $T_i/T_e = 0.6$ (dashed), $T_i/T_e = 0.8$ (dash-dot) and $T_i/T_e = 1$(dotted). $T_d/T_e = 0.01$,$q_e = 0.6$,$q_i = q_d = 1$, $\delta = 100$ and $m_i n_i/m_d n_d = 0.01$ are assumed).

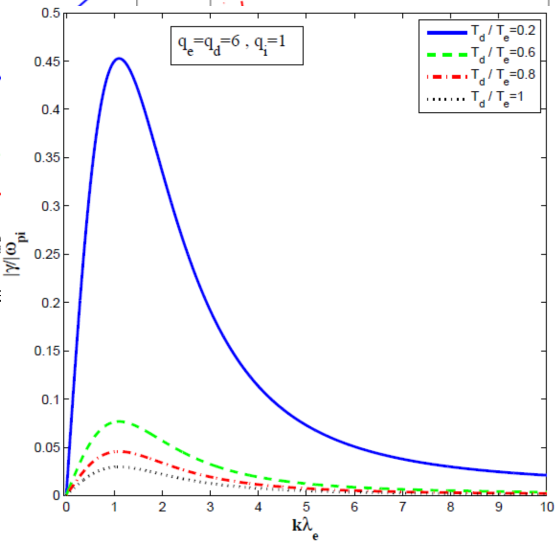

Fig. 17. Plot of Landau damping of the DIAWs in terms of the wavenumber for various values of T_d/T_e ($T_d/T_e = 0.2$ (solid),$T_d/T_e = 0.6$(dashed), $T_d/T_e = 0.8$(dash-dot) and $T_d/T_e = 1$ (dotted). $T_i/T_e = 0.2$, $q_i = 1$, $q_e = q_d = 6$, $\delta = 100$ and $m_i n_i/m_d n_d = 0.01$ are assumed).

If it is considered that $q_i, q_d \leq 1$ for various values of q_e, in the limit of long-wavelength (short wavelength), the Landau damping rate decreases (increases) by increasing the q_e index, as can be seen in Fig. 6 and Fig. 9. Furthermore, for $q_i, q_d > 1$, the Landau damping of the DIA wave generally decreases by increasing the q_e nonextensive index (see Fig.12). Regarding that the damping rate is normalized by ion plasma frequency, it should be less than one. Accordingly, the data obtained from Fig. 7 (b) is not acceptable regarding physics concepts, therefore, the q_i values larger than 200 are acceptable.

In Fig. 7 and Fig. 10, for $q_e, q_d \leq 1$, an increase of the Landau damping is observed by decreasing of the q_i nonextensive index. However, for $q_e, q_d > 1$, Fig. 13 shows that the damping rate decreases (increases) by increasing the nonextensive index q_i for $q_i > 1$ ($q_i < 1$). Furthermore, the damping rate is approximately constant for $q_d < 1$, though for $q_d > 1$ it increases by decreasing the q_d index (see Fig. 8, Fig. 11, and Fig. 14).

Generally, as can be seen in Fig. 15, the increase of δ enhances the damping rate, expect for ion particles that the Landau damping rate increases (decreases) by increasing the q_i index in the limit of long-wavelength (short wavelength) (see Fig. 15 (b)). Furthermore, the electron temperature changes on the Landau damping were investigated (see Fig. 16), which represents an increase of the damping rate by increasing the ion-electron temperature ratio. Interestingly, a decrease of the damping rate was observed by increasing the dust-electron temperature ratio even with the slightest changes in the electron temperature. This attractive findings is pointed to the essential concept in the dusty plasma.

Also, we can see in Fig. 17 that the damping rate decreases when the dust-electron temperature ratio increases, even with the slightest changes in the electron temperature, and this is essential point in a dusty plasma.

4. Conclusion

In this study, we investigate the phase velocity and the Landau damping of DIAW propagating in an un-magnetized, unbounded, and collisionless dusty plasma modeled q-distribution in Tsallis statistics. The effect of various q parameters index for dust, ion, and electron particles were considered. Using the kinetic theory and Vlasov-Poisson equations, the dispersion relation and the Landau damping rate (γ) were obtained. In the limit $q \to 1$, the results were conformed to Maxwellian duty plasma in the framework of B-G statistics. Generally, we showed that the phase velocity of the dust-ion acoustic decreases with an increase in the q-nonextensive index, applying different amplitudes for electron, ion, and dust particles. It was shown that the wave normalized Landau damping rate $\left(|\gamma|/\omega_{pi}\right)$ can be analyzed for various values of the nonextensive index q, the ion-electron number density ratio δ, and the change of the electron temperature ($\mathrm{T}_i/\mathrm{T}_e$ and $\mathrm{T}_d/\mathrm{T}_e$). It was found that for electron particles, the Landau damping rate increases with a decrease in the q nonextensive index. Our investigation showed a considerable variation of the DIAW damping rate by the dust nonextensivity (q_d) as well as the ion nonextensivity (q_i). The result is very exciting for $q_i, q_d < 1$, with regarding various values for q_e, the damping rate increases with an increases in the ion-electron number density ratio δ. It was shown that the wave normalized Landau damping rate decreases by decreasing the ion-electron temperature ratio, though the reduction of the dust-electron temperature ratio enhances the damping.

Considering the ubiquitous form of the dust particles in space, our results may be helpful in the perception of the effects of nonextensivity on the frequency as well as the Landau damping of the DIAW in the nonextensive distributed dusty plasma, such as planetary areas, magnetospheres, and space plasma environments.

Acknowledgments

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References

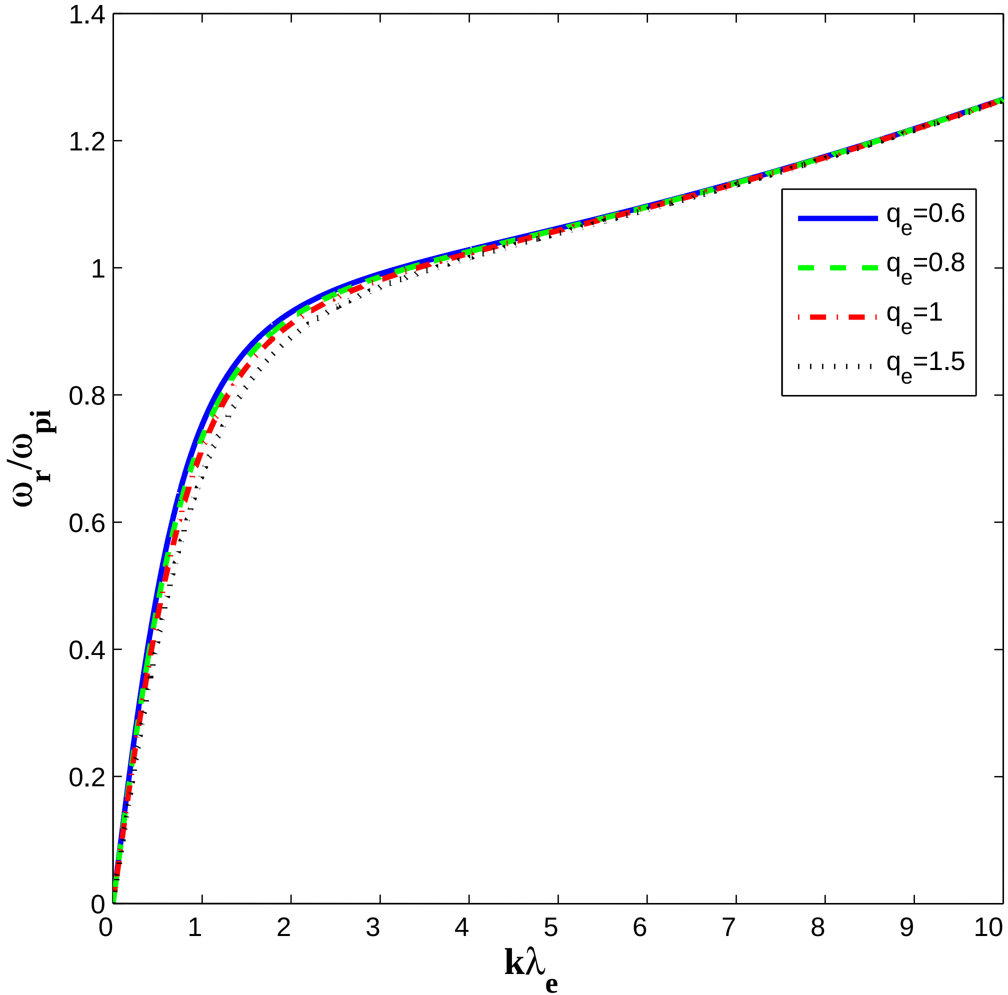
1. Mendis, D.A. and M. Rosenberg, *Cosmic Dusty Plasma*. Annual Review of Astronomy and Astrophysics, 1994. **32**(1): p. 419-463.
2. Tsallis, C., *Possible generalization of Boltzmann-Gibbs statistics*. Journal of Statistical Physics, 1988. **52**(1): p. 479-487.

3. Curado, E.M. and C. Tsallis, *Generalized statistical mechanics: connection with thermodynamics.* Journal of Physics a: mathematical and general, 1991. **24**(2): p. L69.
4. Tsallis, C., *Non-extensive thermostatistics: brief review and comments.* Physica A: Statistical Mechanics and its Applications, 1995. **221**(1-3): p. 277-290.
5. Silva Jr, R., A. Plastino, and J. Lima, *A Maxwellian path to the q-nonextensive velocity distribution function.* Physics Letters A, 1998. **249**(5-6): p. 401-408.
6. Lavagno, A., et al., *Non-extensive thermostatistical approach of the peculiar velocity function of galaxy clusters.* Astrophysical Letters and Communications, 1998. **35**: p. 449.
7. Amour, R. and M. Tribeche, *Semi-analytical study of variable charge dust acoustic solitary waves in a dusty plasma with a q-nonextensive ion velocity distribution.* Communications in Nonlinear Science and Numerical Simulation, 2011. **16**(9): p. 3533-3539.
8. Liu, S.-Q. and X.-C. Chen, *Dispersion relation of transverse oscillation in relativistic plasmas with non-extensive distribution.* Journal of Plasma Physics, 2011. **77**(5): p. 653-662.
9. Liu, Y., S. Liu, and B. Dai, *Arbitrary amplitude kinetic Alfvén solitons in a plasma with a q-nonextensive electron velocity distribution.* Physics of Plasmas, 2011. **18**(9): p. 092309.
10. Roy, K., T. Saha, and P. Chatterjee, *Effect of ion temperature on ion-acoustic solitary waves in a plasma with a q-nonextensive electron velocity distribution.* Physics of Plasmas, 2012. **19**(10): p. 104502.
11. Rufai, O.R. and R. Bharuthram, *Electrostatic fluctuations in a magnetized two-component plasma with q-nonextensive electrons and thermal ions.* Physics of Plasmas, 2016. **23**(9): p. 092306.
12. Rahman, M.M., et al., *Nonlinear propagation of Electron-acoustic waves in a nonextensive electron-positron-ion plasma.* Journal of the Korean Physical Society, 2015. **66**(6): p. 941-946.
13. Liu, X.-L. and X.-Q. Li, *The full Zakharov equations in nonextensive q-plasma.* Physics of Plasmas, 2014. **21**(2): p. 022306.
14. Akhtar, N., W.F. El-Taibany, and S. Mahmood, *Electrostatic double layers in a warm negative ion plasma with nonextensive electrons.* Physics Letters A, 2013. **377**(18): p. 1282-1289.
15. Shukla, P. and V. Silin, *Dust ion-acoustic wave.* PhyS, 1992. **45**(5): p. 508.
16. Shukla, P. and A. Mamun, *Introduction to Dusty Plasma Physics IOP*. 2002, Bristol.
17. Lee, M.-J., *Effects of suprathermal particles on the dust ion-acoustic waves in a complex plasma.* Current Applied Physics, 2010. **10**(5): p. 1340-1344.
18. Abbaszadeh, T., et al., *Study of dust ion-acoustic wave propagation in the Lorentzian magnetized dusty plasma.* Current Applied Physics, 2013. **13**(9): p. 1933-1942.
19. Liyan, L. and D. Jiulin, *Ion acoustic waves in the plasma with the power-law q-distribution in nonextensive statistics.* Physica A: Statistical Mechanics and its Applications, 2008. **387**(19-20): p. 4821-4827.
20. Liu, Z., L. Liu, and J. Du, *A nonextensive approach for the instability of current-driven ion-acoustic waves in space plasmas.* Physics of Plasmas, 2009. **16**(7): p. 072111.
21. Bouzit, O., M. Tribeche, and A.S. Bains, *Modulational instability of ion-acoustic waves in plasma with a q-nonextensive nonthermal electron velocity distribution.* Physics of Plasmas, 2015. **22**(8): p. 084506.

22. Liu, S.-Q., H.-B. Qiu, and X.-Q. Li, *Landau damping of dust acoustic waves in the plasma with nonextensive distribution.* Physica A: Statistical Mechanics and its Applications, 2012. **391**(23): p. 5795-5801.
23. SAN QIU, L. and H.B. Qiu, *Dust acoustic instability with non-extensive distribution.* Journal of Plasma Physics, 2013. **79**(1): p. 105.
24. Amour, R. and M. Tribeche, *Collisionless damping of dust-acoustic waves in a charge varying dusty plasma with nonextensive ions.* Physics of Plasmas, 2014. **21**(12): p. 123709.
25. Fried, B. and S. Conte, *The Plasma Dispersion Function Academic Press.* Inc. New York, 1961.
26. Huba, J.D., *NRL plasma formulary*. Vol. 6790. 1998: Naval Research Laboratory.
27. Liang, X., et al., *Experimental observation of ion-acoustic waves in an inhomogeneous dusty plasma.* Physics of Plasmas, 2001. **8**(5): p. 1459-1462.

Fig1.

Fig. 1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1.2
1
0.8
0.6
0.4
0.2
0
ω_r/ω_{pi}
0
1
2
3
4
5
6
7
8
9
10
$k\lambda_e$
q_e=0.6
q_e=0.8
q_e=1
q_e=1.5

Fig2.

Fig. 2

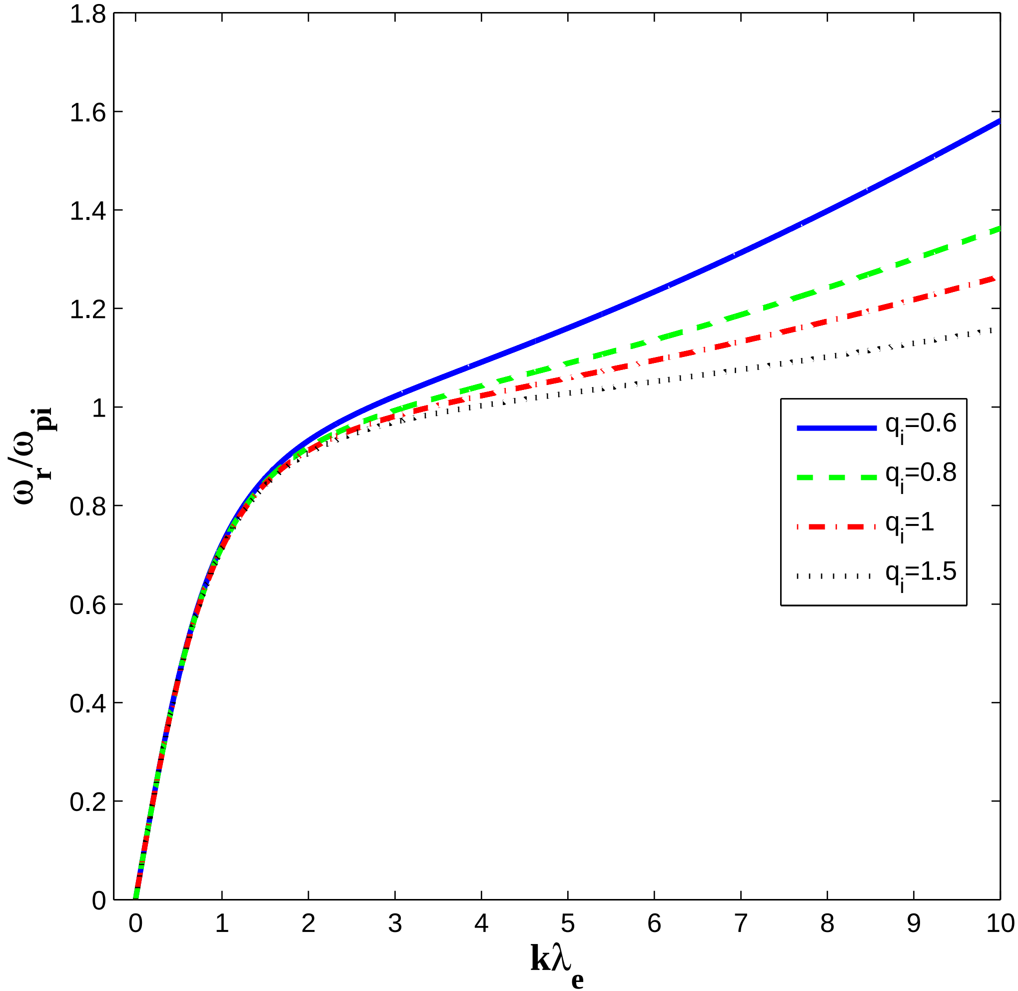
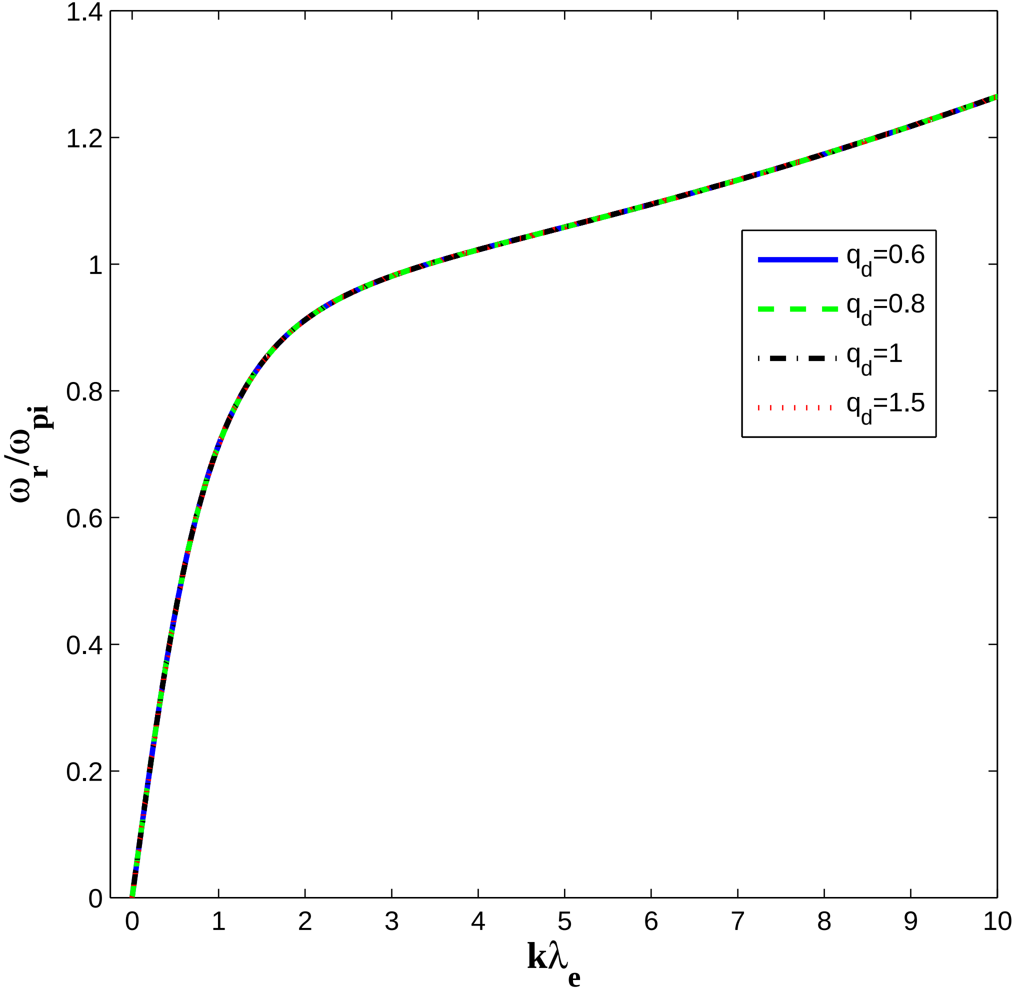


Fig3.

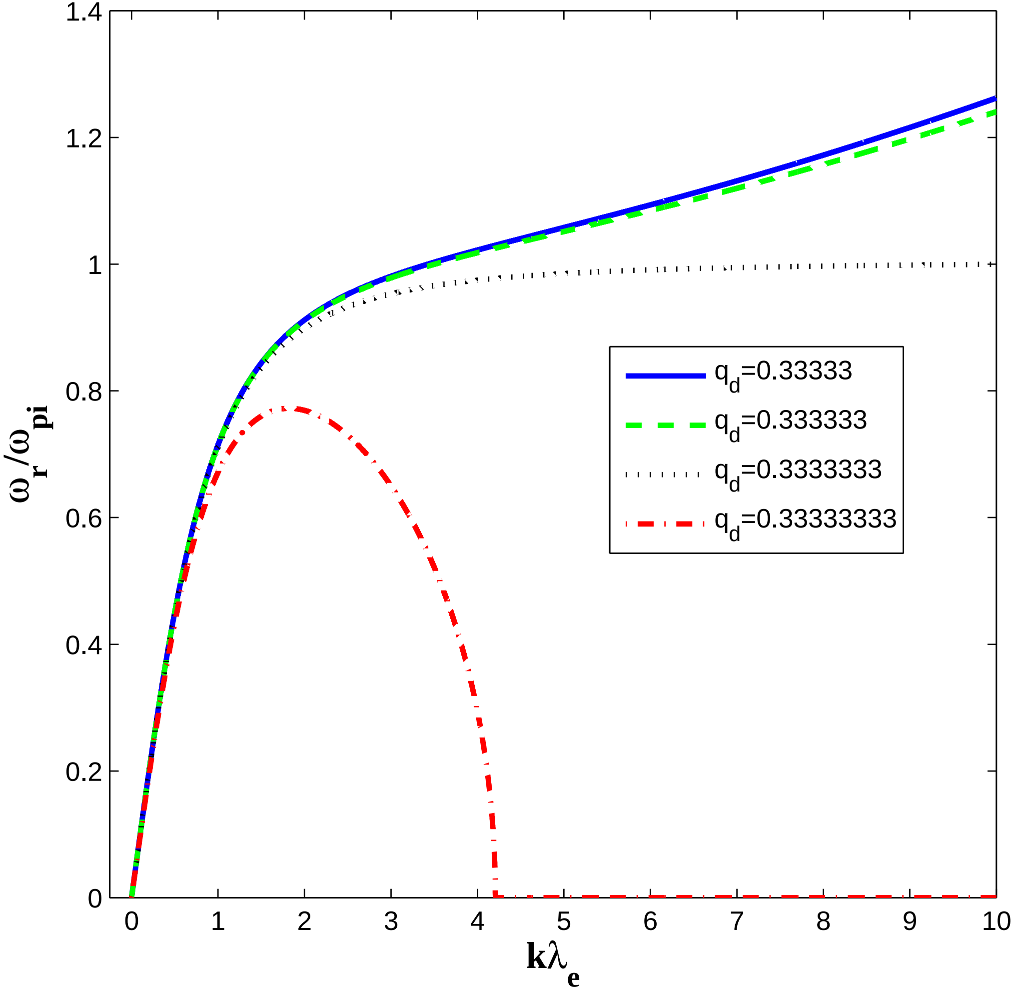
Fig. 3



1.4
1.2
1
0.8
0.6
0.4
0.2
0
0 1 2 3 4 5 6 7 8 9 10
ω_r/ω_{pi}
$k\lambda_e$
q_d=0.6
q_d=0.8
q_d=1
q_d=1.5

Fig4.

Fig. 4



1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
8
9
10
ω_r/ω_{pi}
$k\lambda_e$
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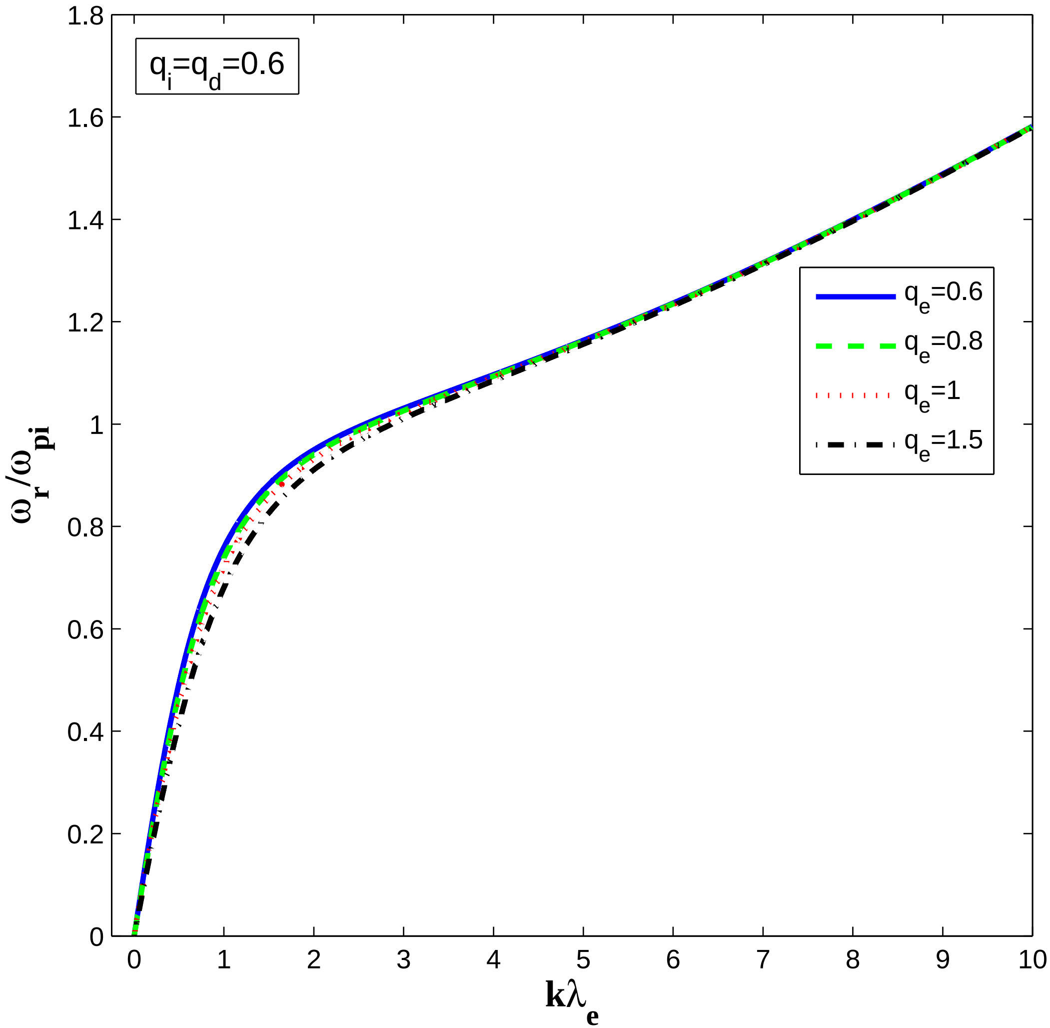
q_d=0.333333
q_d=0.3333333
q_d=0.33333333

Fig5.

Fig. 5



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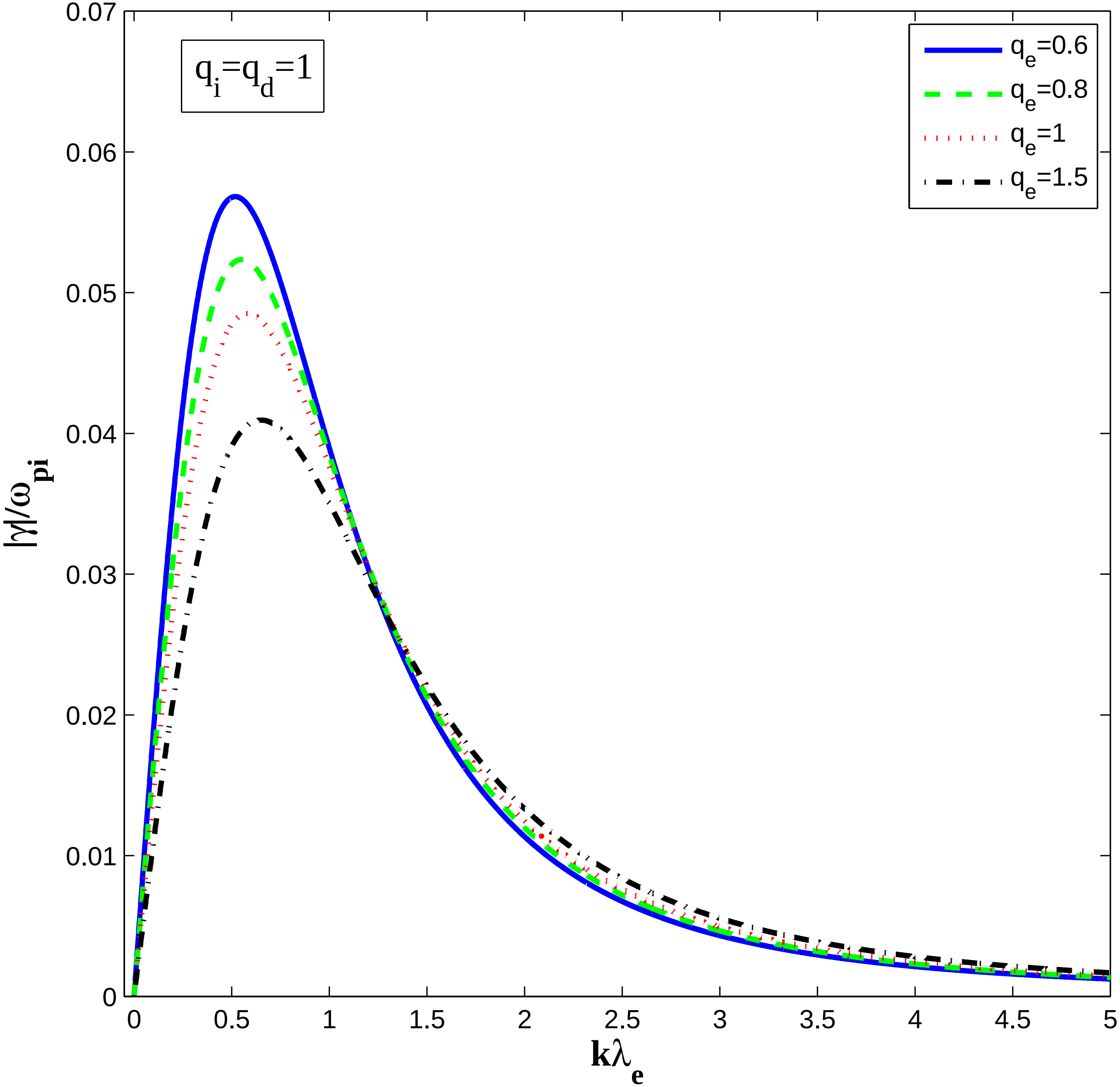
q_e=0.6
q_e=0.8
q_e=1
q_e=1.5
ω_r/ω_{pi}
$k\lambda_e$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0
1
2
3
4
5
6
7
8
9
10

Fig6.

Fig 6



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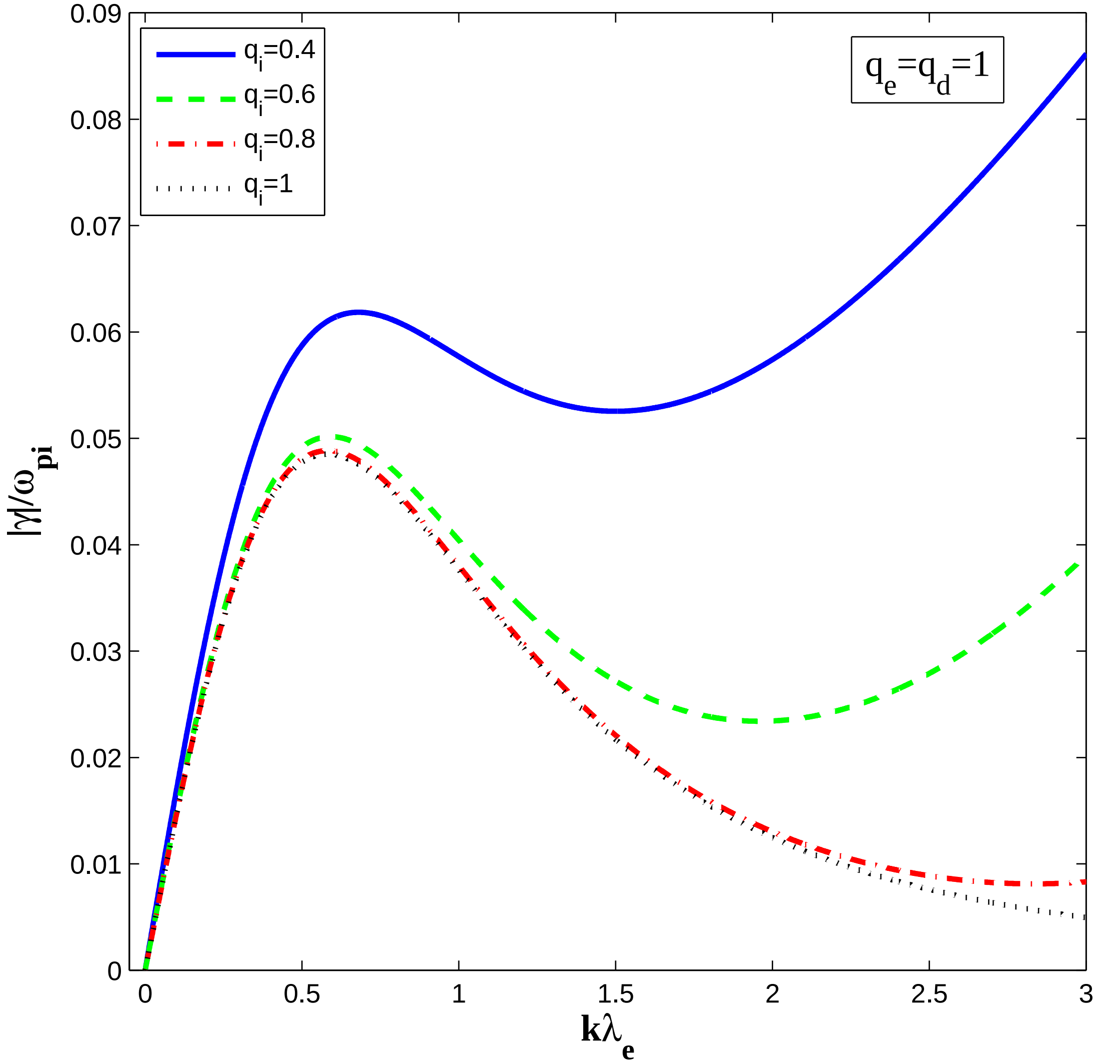
$q_e=0.8$
$q_e=1$
$q_e=1.5$
$|\gamma|/\omega_{pi}$
$k\lambda_e$
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Fig7a.

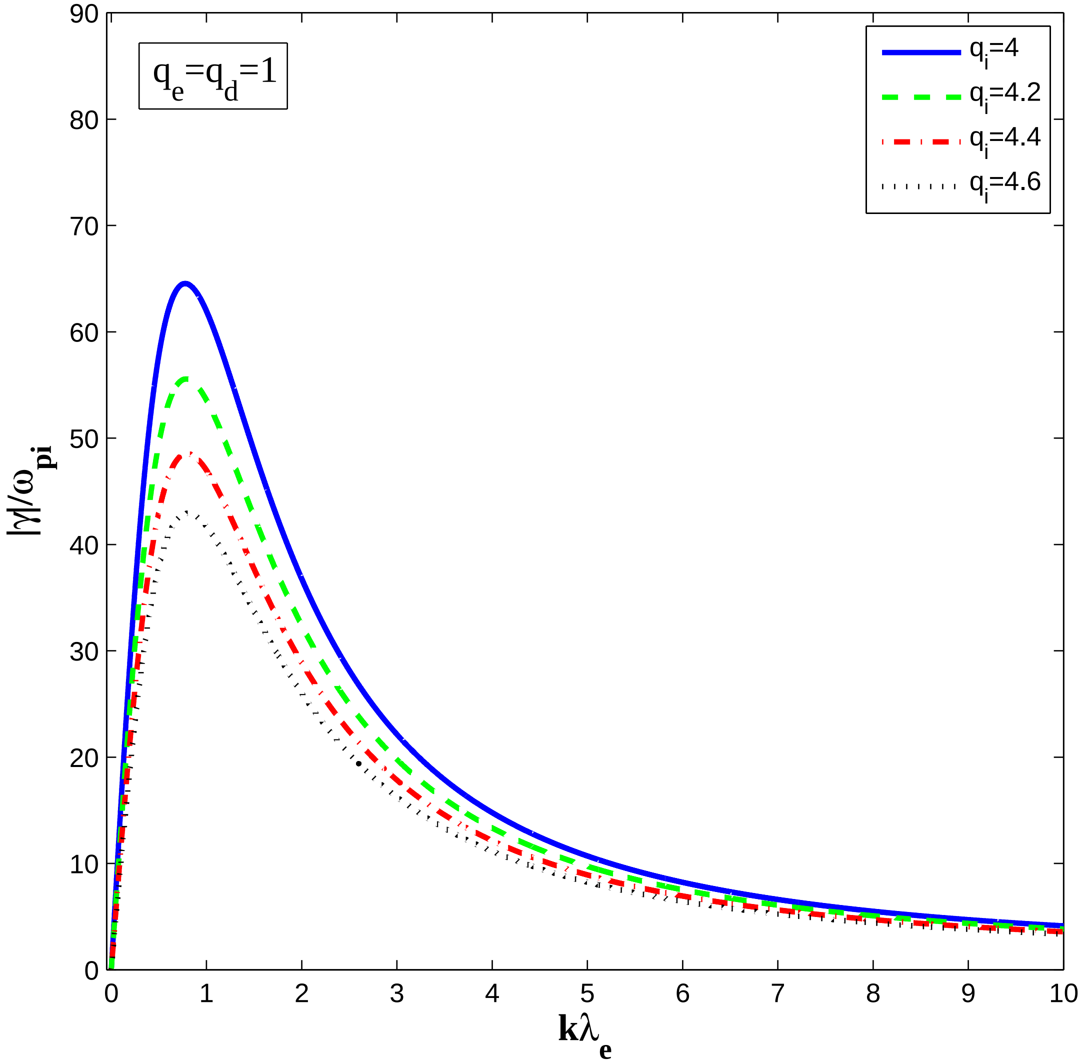
Fig 7 (a)



q_i=0.4
q_i=0.6
q_i=0.8
q_i=1
q_e=q_d=1
|γ|/ω_pi
kλ_e
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0.5
1
1.5
2
2.5
3

Fig7b.

Fig 7 (b)



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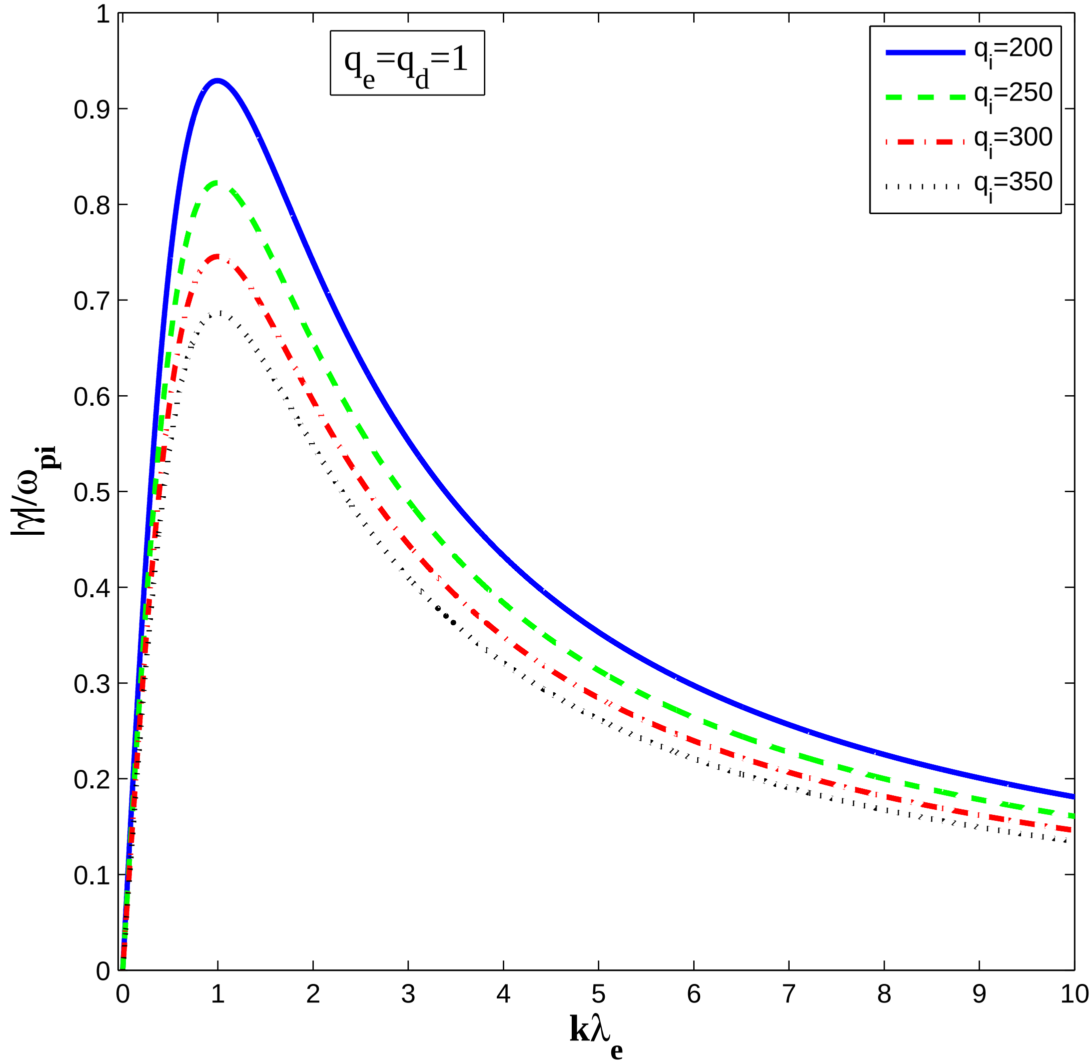
$q_i=4.2$
$q_i=4.4$
$q_i=4.6$
$|\gamma|/\omega_{pi}$
$k\lambda_e$
0
10
20
30
40
50
60
70
80
90
0
1
2
3
4
5
6
7
8
9
10

Fig7c.

Fig 7 (c)



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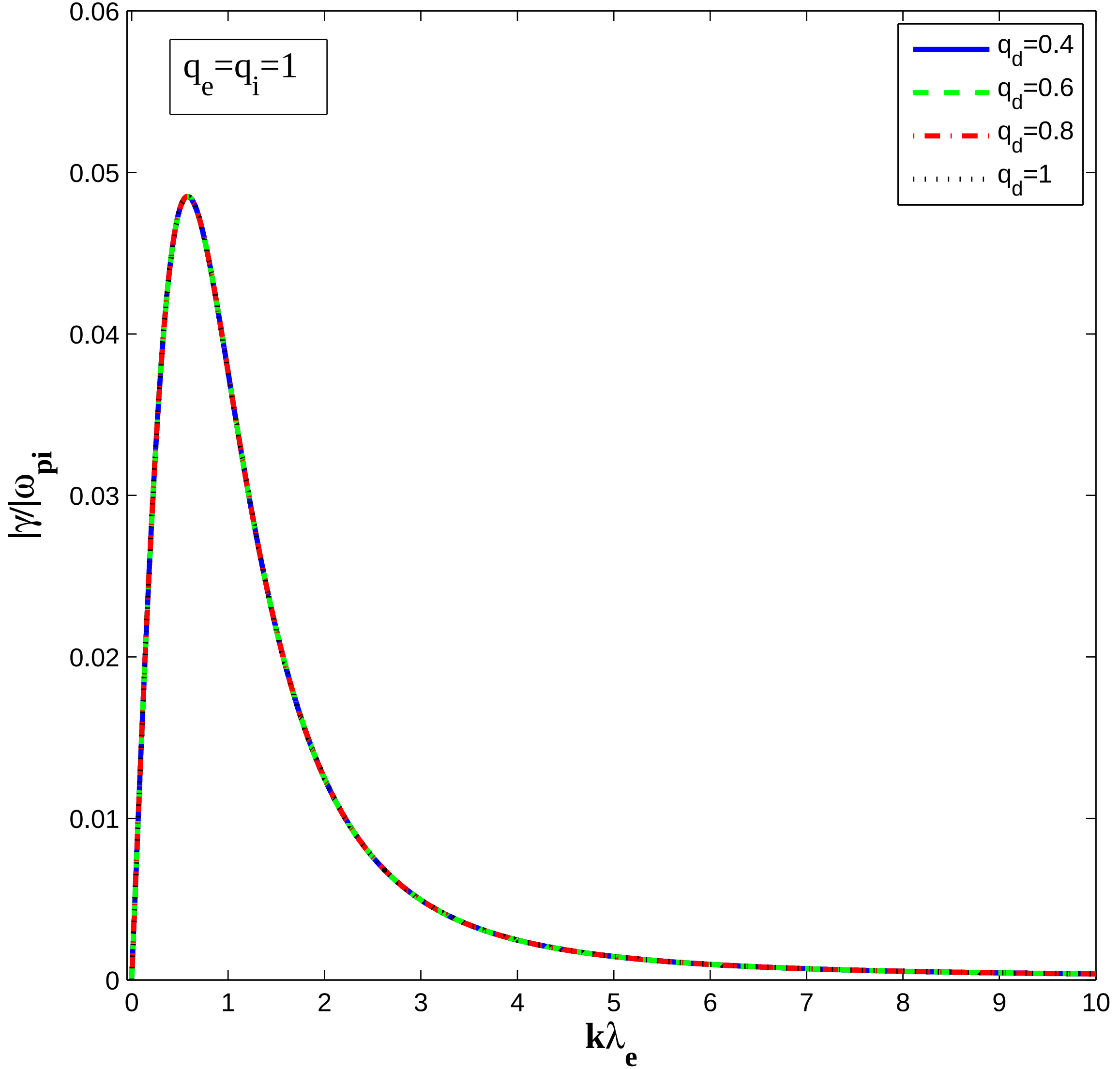
$q_i=350$
$|\gamma|/\omega_{pi}$
$k\lambda_e$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5
6
7
8
9
10

Fig8a.

Fig 8 (a)



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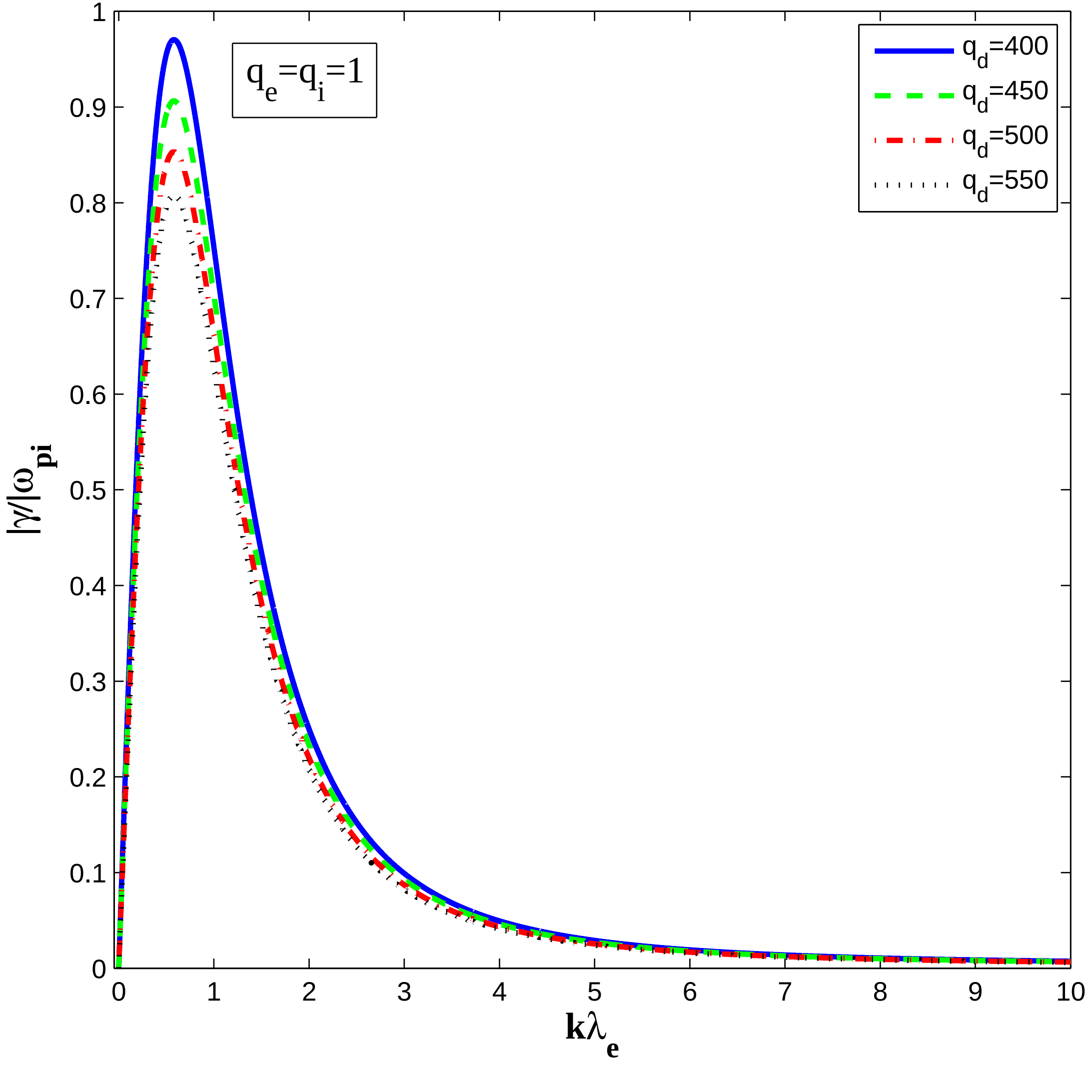
$q_d=1$
0.06
0.05
0.04
0.03
0.02
0.01
0
$|\gamma|/\omega_{pi}$
0 1 2 3 4 5 6 7 8 9 10
$k\lambda_e$

Fig8b.

Fig 8 (b)



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q_d=500
q_d=550
$|\gamma/|\omega_{pi}$
$k\lambda_e$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5
6
7
8
9
10

Fig9.

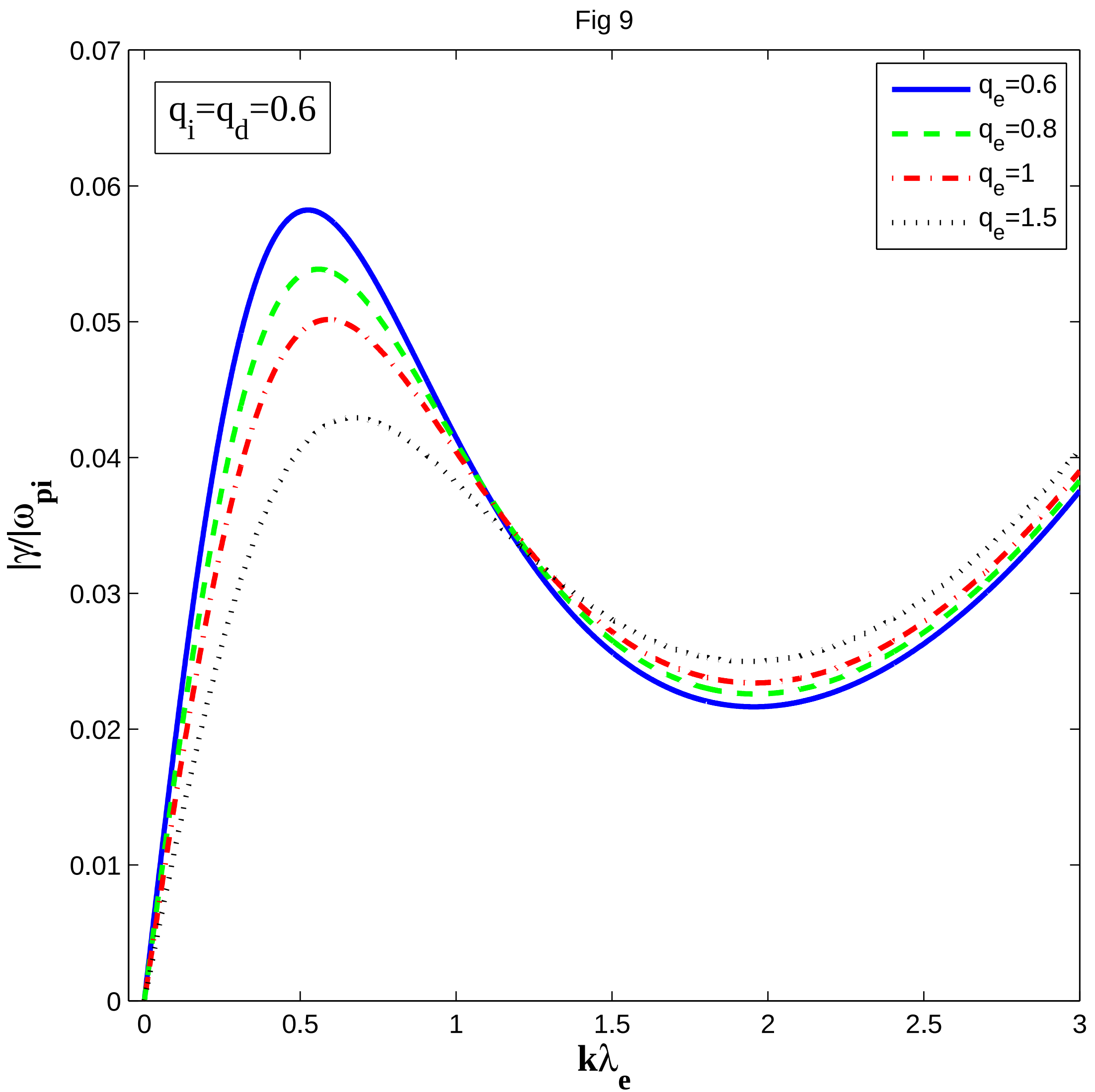
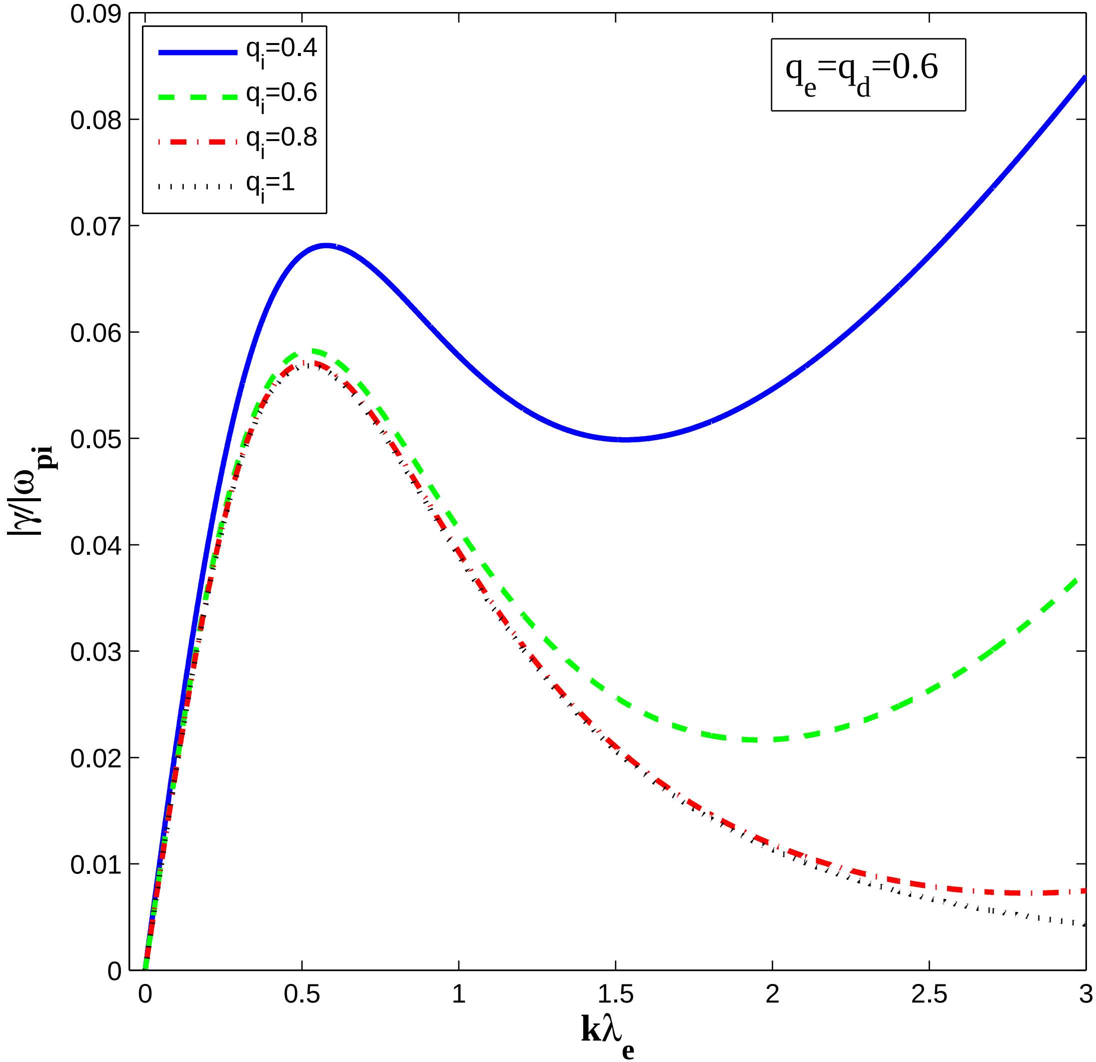


Fig 9
q_i=q_d=0.6
q_e=0.6
q_e=0.8
q_e=1
q_e=1.5
$|\gamma|/\omega_{pi}$
$k\lambda_e$
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0.5
1
1.5
2
2.5
3

Fig10a.

Fig 10 (a)



0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.5
1
1.5
2
2.5
3
$|\gamma/|\omega_{pi}$
$k\lambda_e$
q_i=0.4
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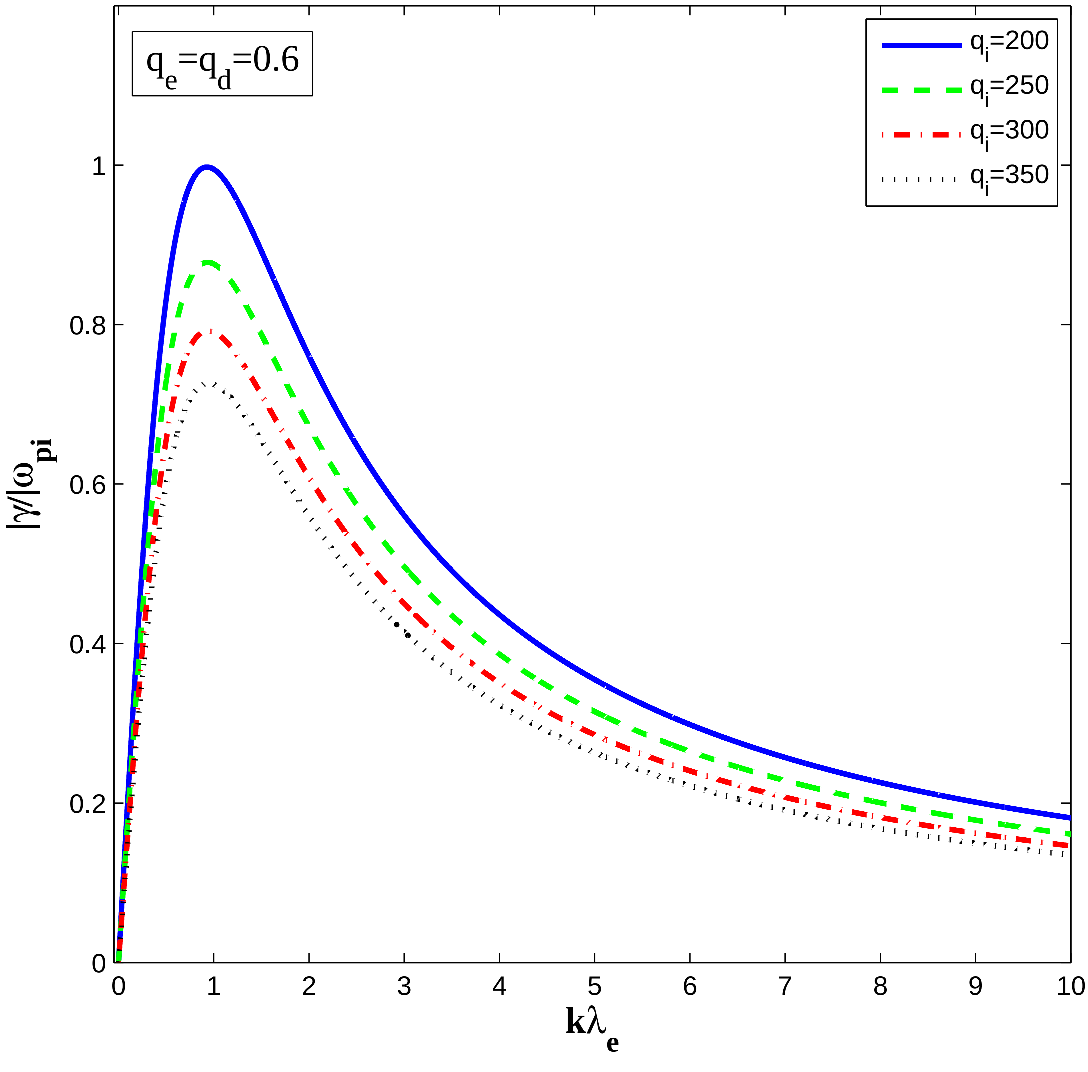
q_i=0.8
q_i=1
q_e=q_d=0.6

Fig10b.

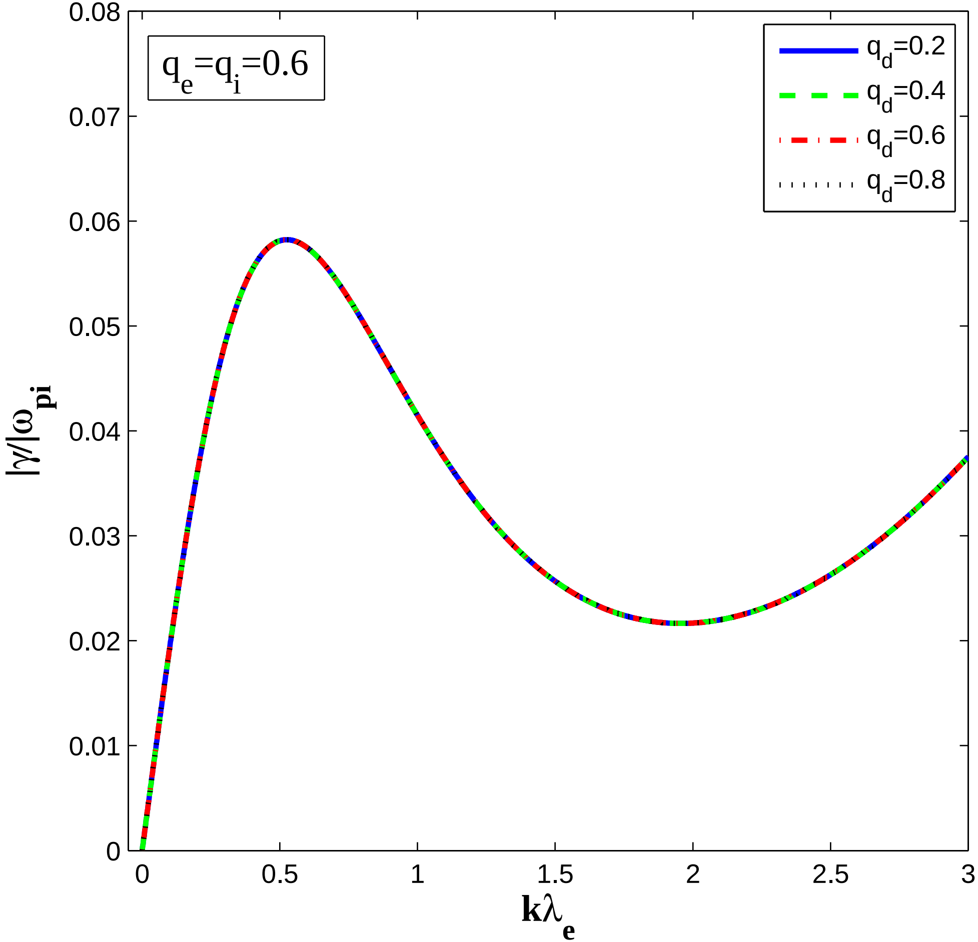
Fig 10 (b)



$q_e=q_d=0.6$
$q_i=200$
$q_i=250$
$q_i=300$
$q_i=350$
$|\gamma/|\omega_{pi}$
$k\lambda_e$
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5
6
7
8
9
10

Fig11a.

Fig 11 (a)



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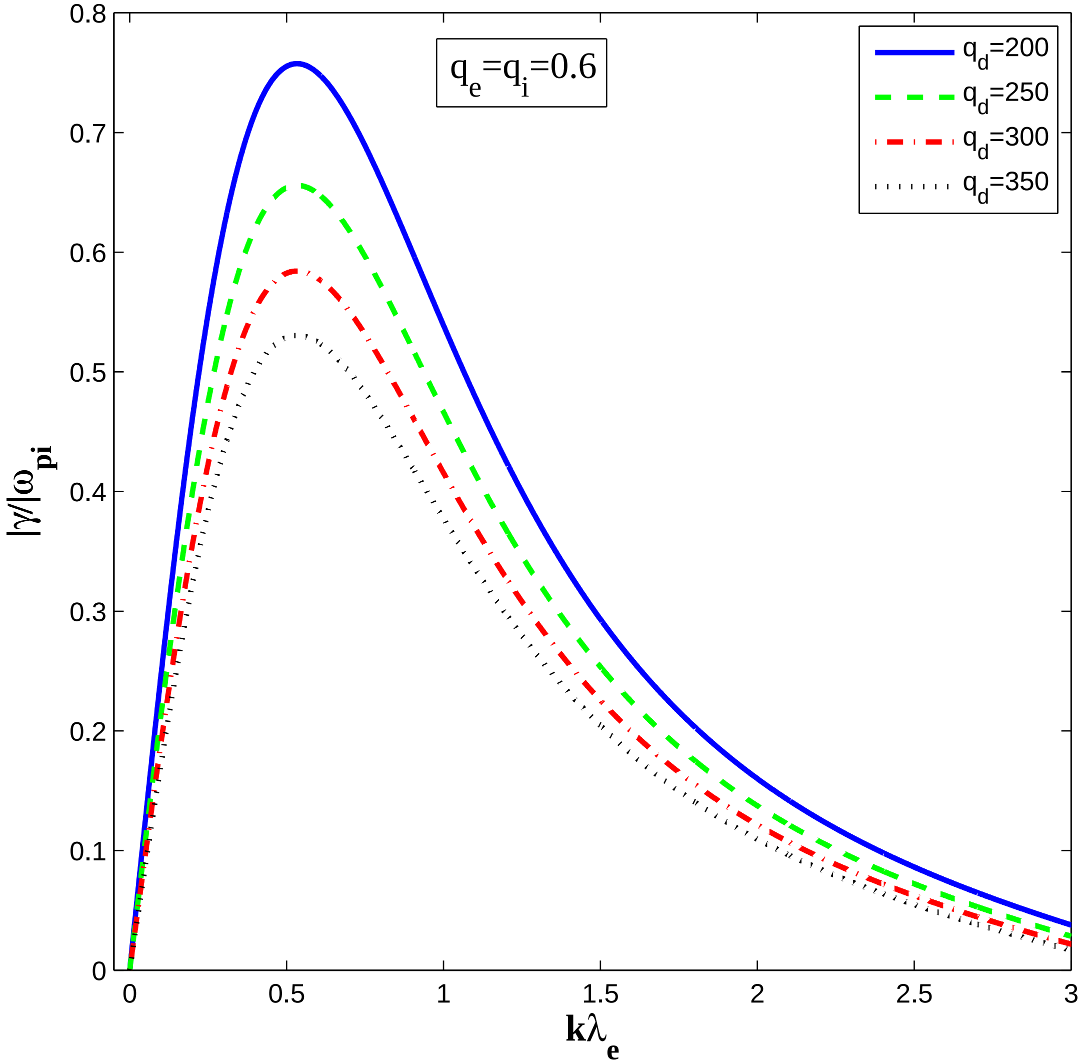
$q_d=0.4$
$q_d=0.6$
$q_d=0.8$
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.5
1
1.5
2
2.5
3
$|\gamma/|\omega_{pi}$
$k\lambda_e$

Fig11b.

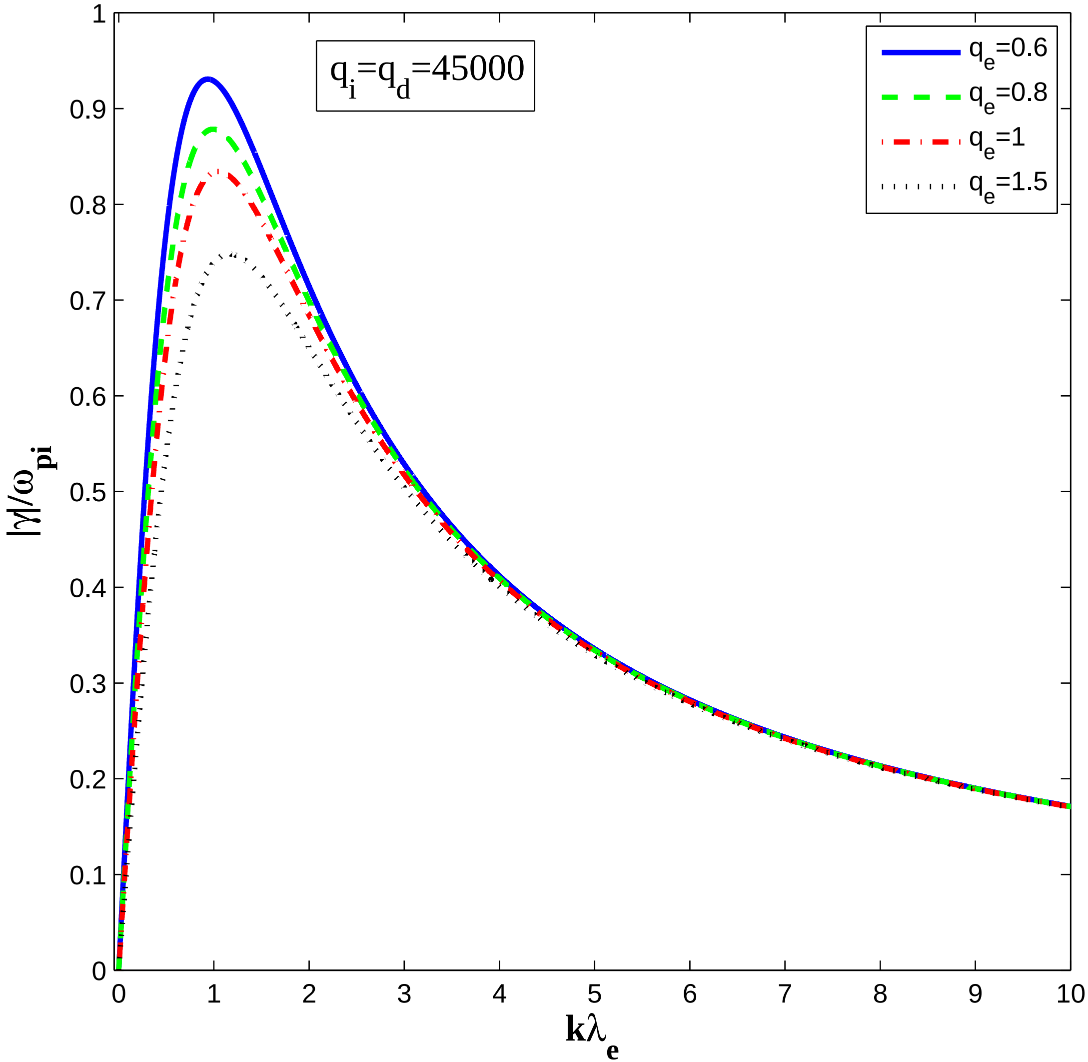
Fig 11 (b)



0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
|γ/|ω_pi
0
0.5
1
1.5
2
2.5
3
kλ_e
q_e=q_i=0.6
q_d=200
q_d=250
q_d=300
q_d=350

Fig12.

Fig 12



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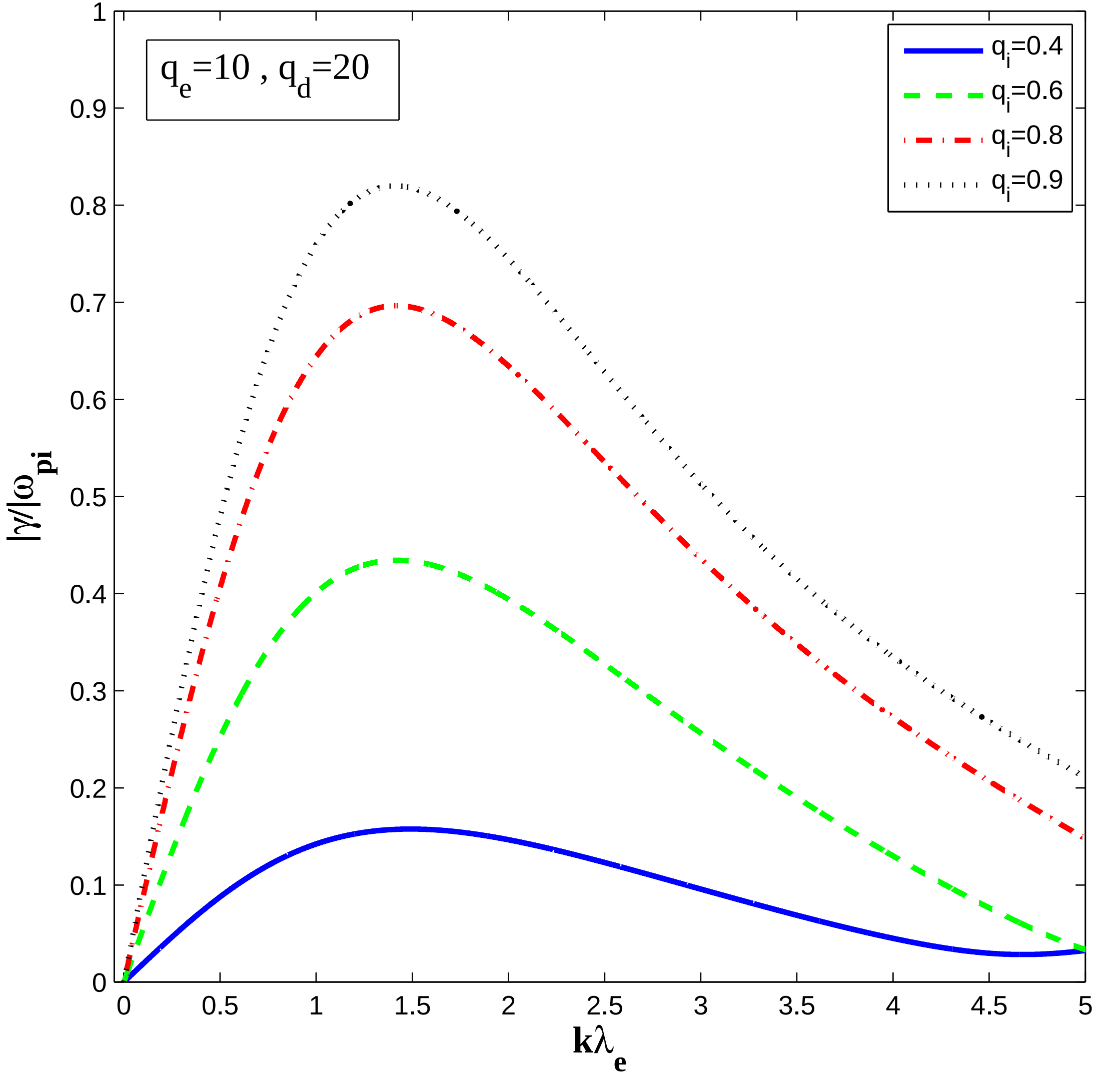
$q_e=1$
$q_e=1.5$
$|\gamma|/\omega_{pi}$
$k\lambda_e$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5
6
7
8
9
10

Fig13a.

Fig 13 (a)



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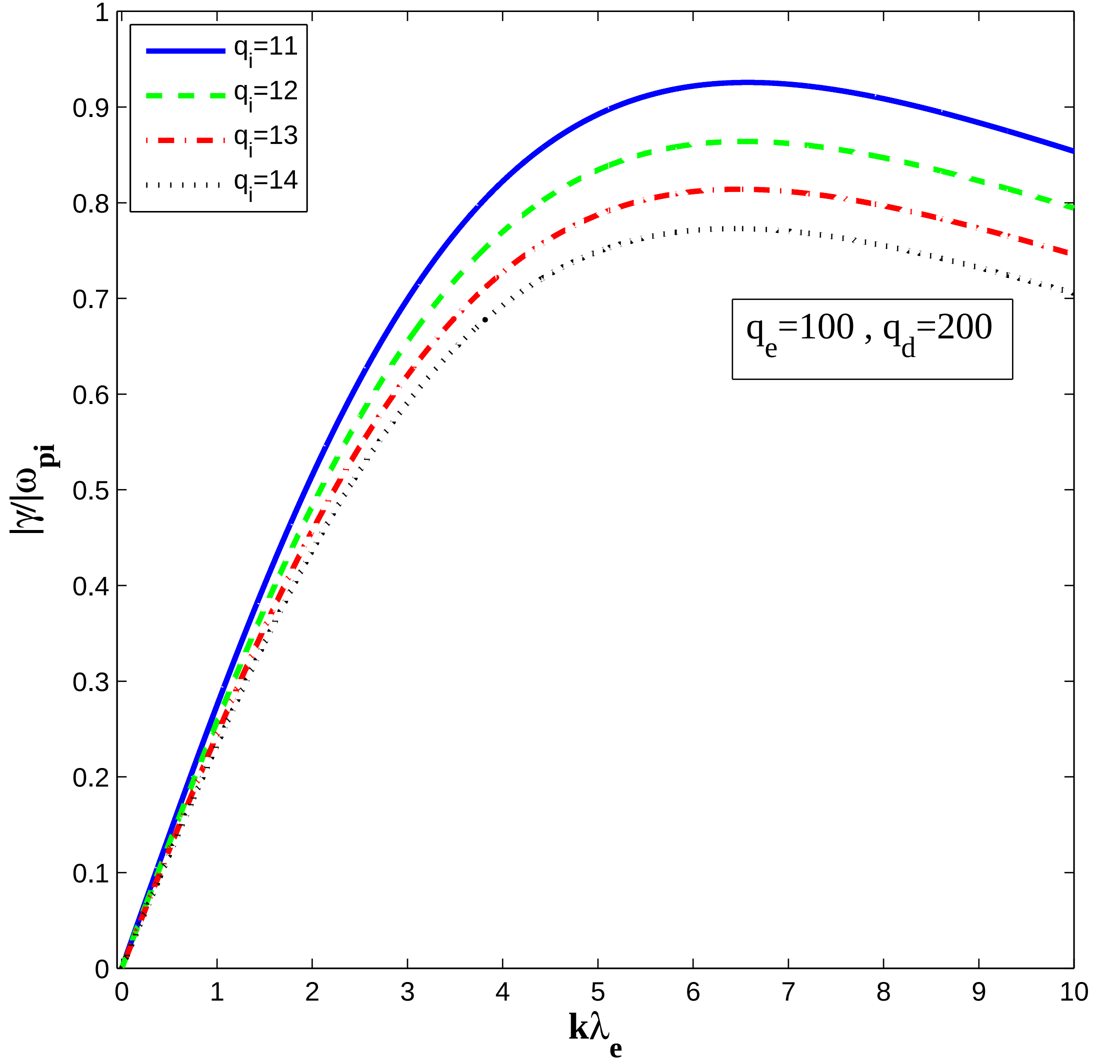
q_i=0.4
q_i=0.6
q_i=0.8
q_i=0.9
$|\gamma|/\omega_{pi}$
$k\lambda_e$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.5
1.5
2
2.5
3
3.5
4
4.5
5

Fig13b.

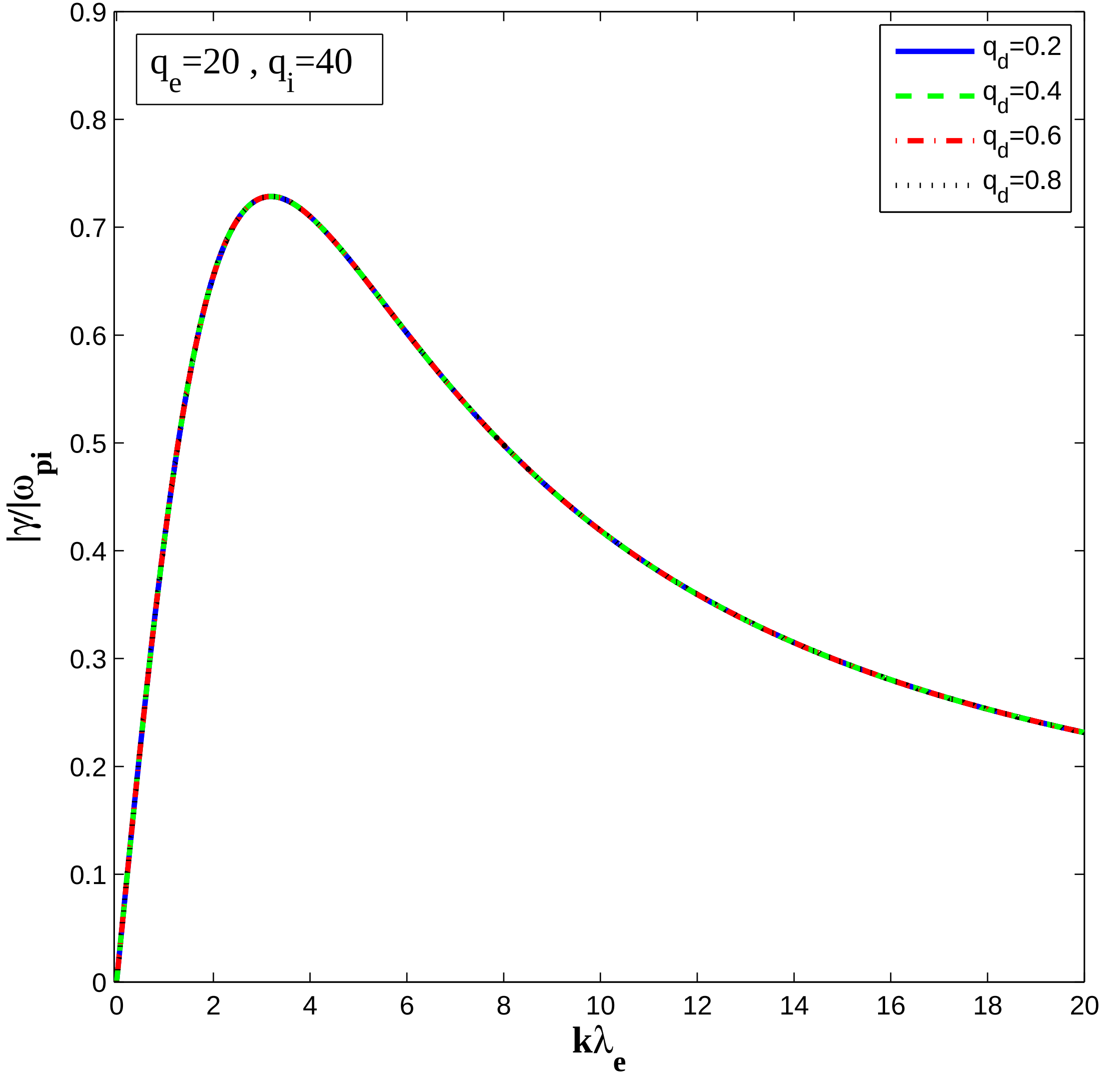
Fig 13 (b)



q_i=11
q_i=12
q_i=13
q_i=14
q_e=100 , q_d=200
$|\gamma/|\omega_{pi}$
$k\lambda_e$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0 1 2 3 4 5 6 7 8 9 10

Fig14a.

Fig 14 (a)



q_e=20 , q_i=40
q_d=0.2
q_d=0.4
q_d=0.6
q_d=0.8
$|\gamma|/\omega_{pi}$
$k\lambda_e$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0
2
4
6
8
10
12
14
16
18
20

Fig14b.

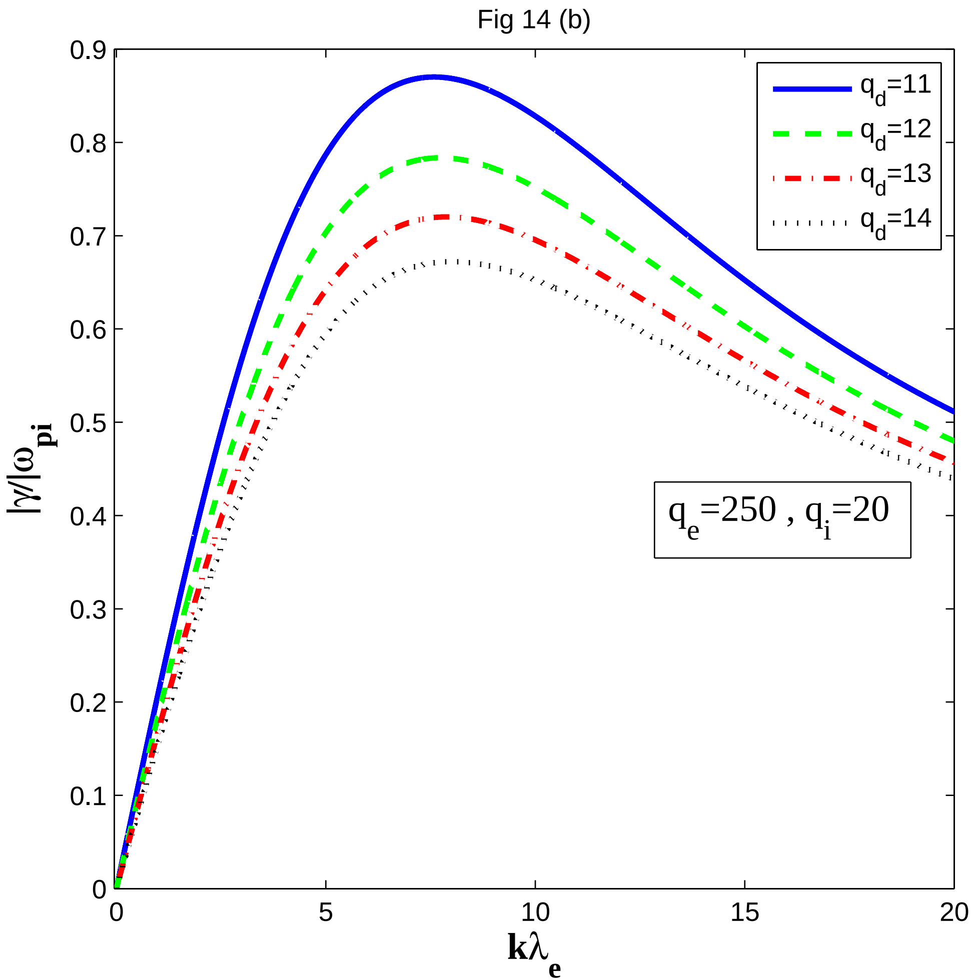


Fig 14 (b)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
|γ/|ω_pi
kλ_e
q_d=11
q_d=12
q_d=13
q_d=14
q_e=250 , q_i=20

Fig15a.

Fig 15 (a)

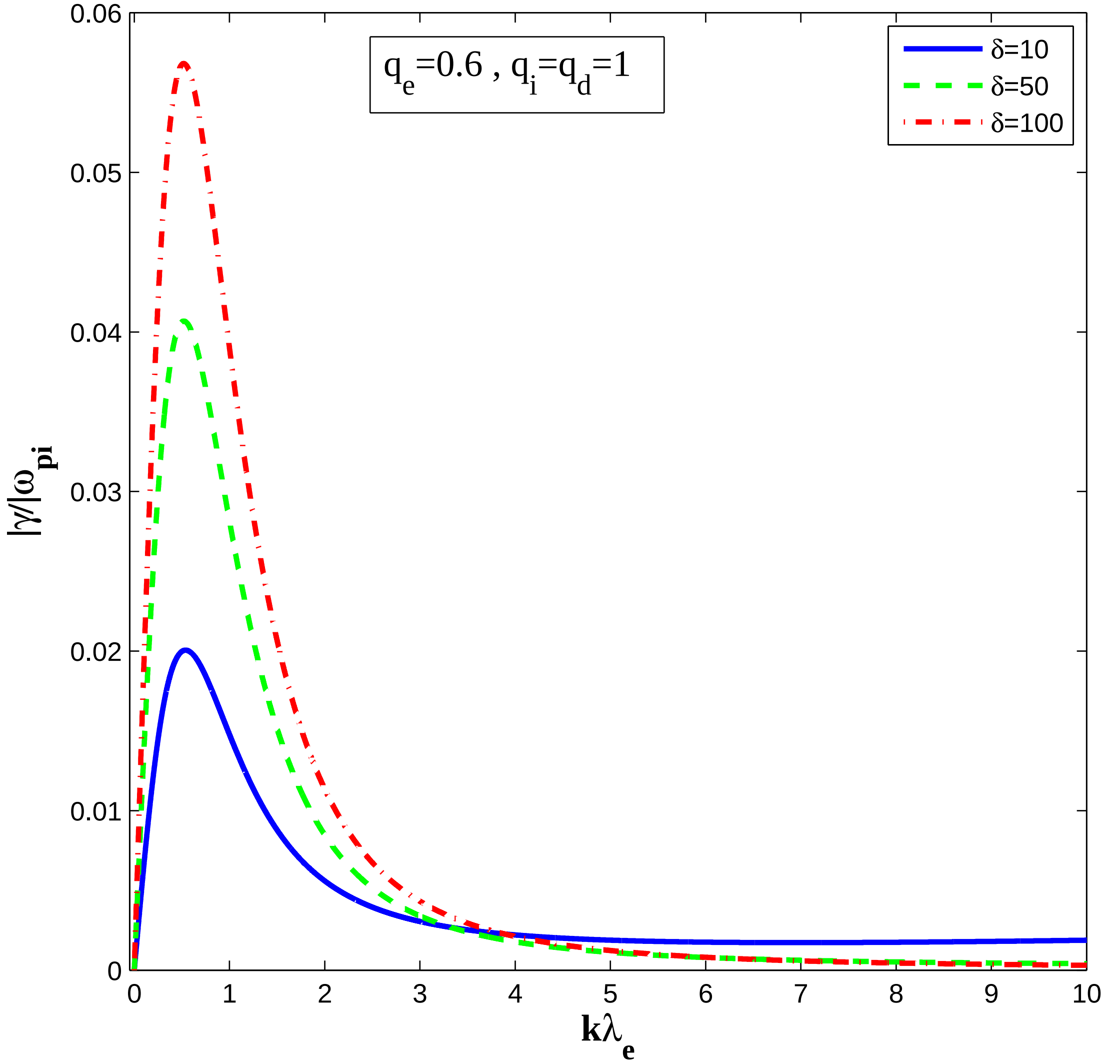
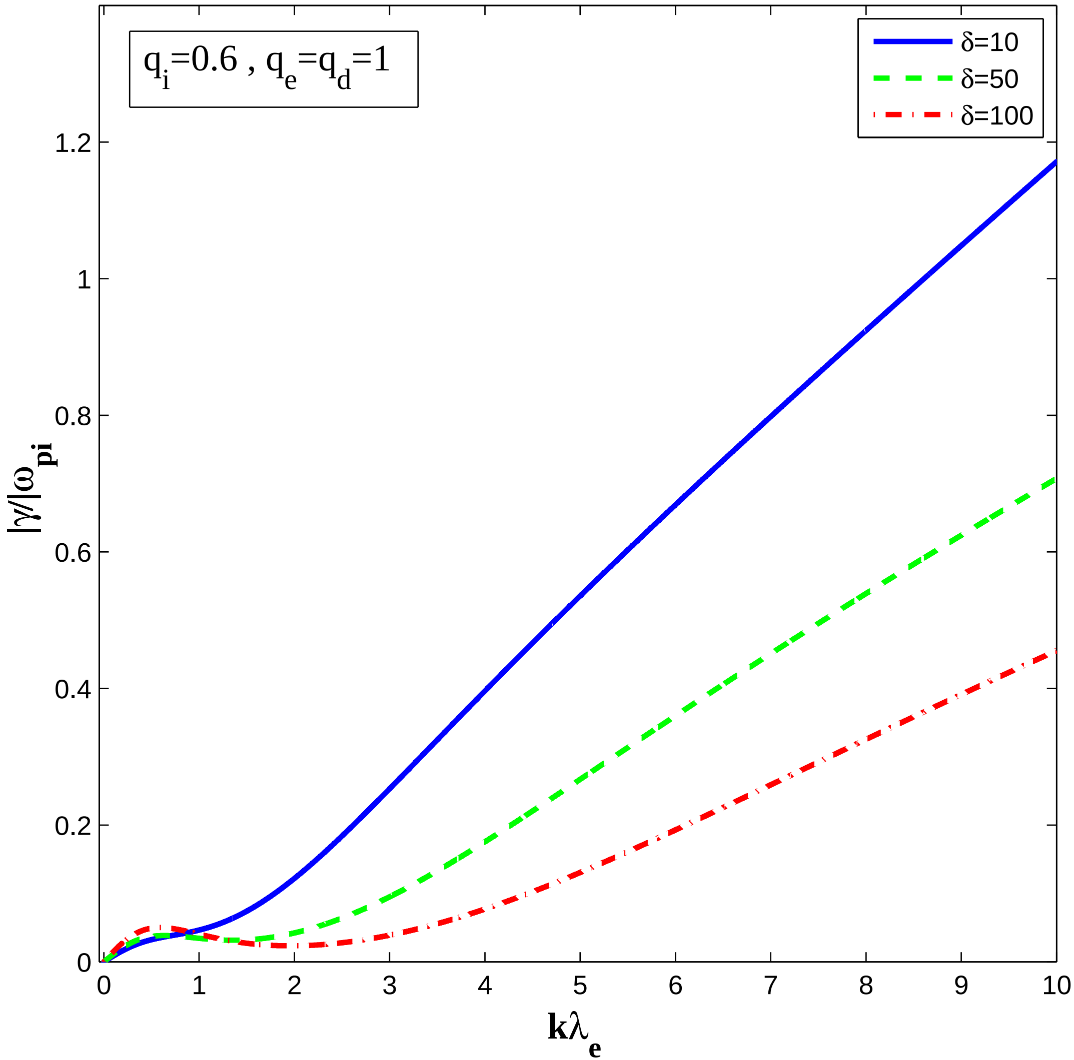

q_e=0.6 , q_i=q_d=1
δ=10
δ=50
δ=100
|γ/|ω_{pi}
$k\lambda_e$
0
0.01
0.02
0.03
0.04
0.05
0.06
0
1
2
3
4
5
6
7
8
9
10

Fig15b.

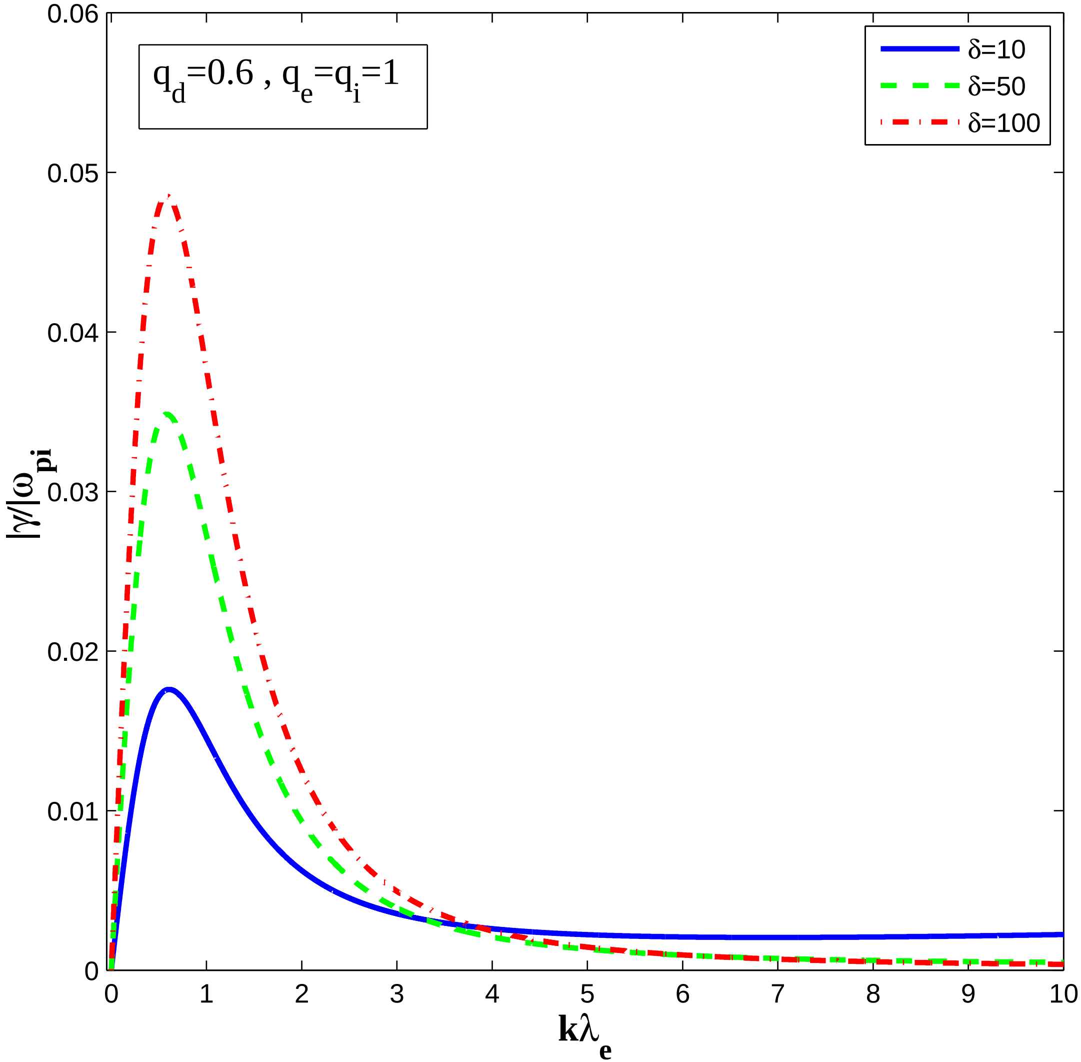
Fig 15 (b)



$q_i=0.6$, $q_e=q_d=1$
δ=10
δ=50
δ=100
$|\gamma|/\omega_{pi}$
$k\lambda_e$
0
0.2
0.4
0.6
0.8
1
1.2
0
1
2
3
4
5
6
7
8
9
10

Fig15c.

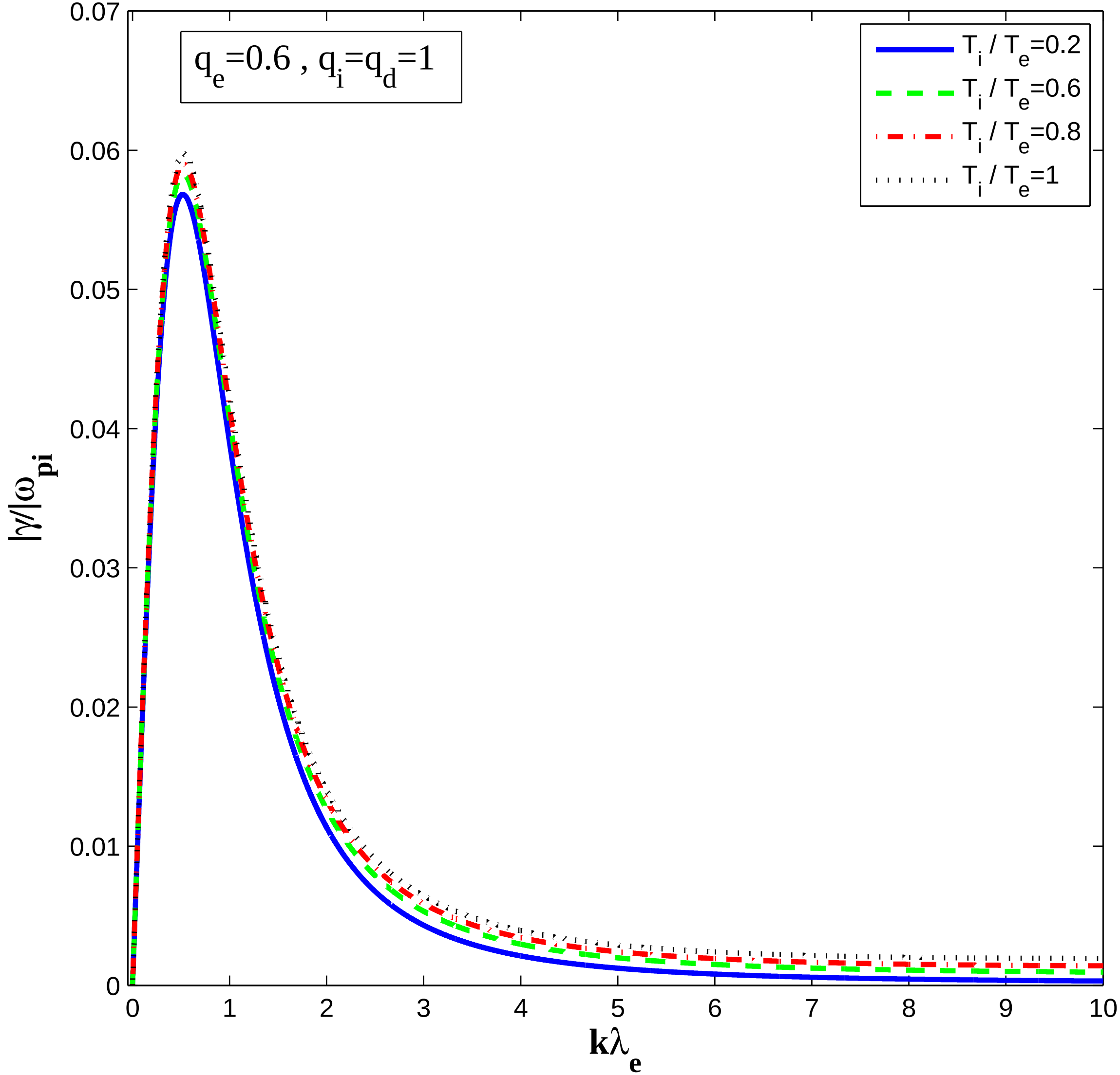
Fig 15 (c)



q_d=0.6 , q_e=q_i=1
δ=10
δ=50
δ=100
0.06
0.05
0.04
0.03
0.02
0.01
0
0 1 2 3 4 5 6 7 8 9 10
$|\gamma|/|\omega_{pi}$
$k\lambda_e$

Fig16.

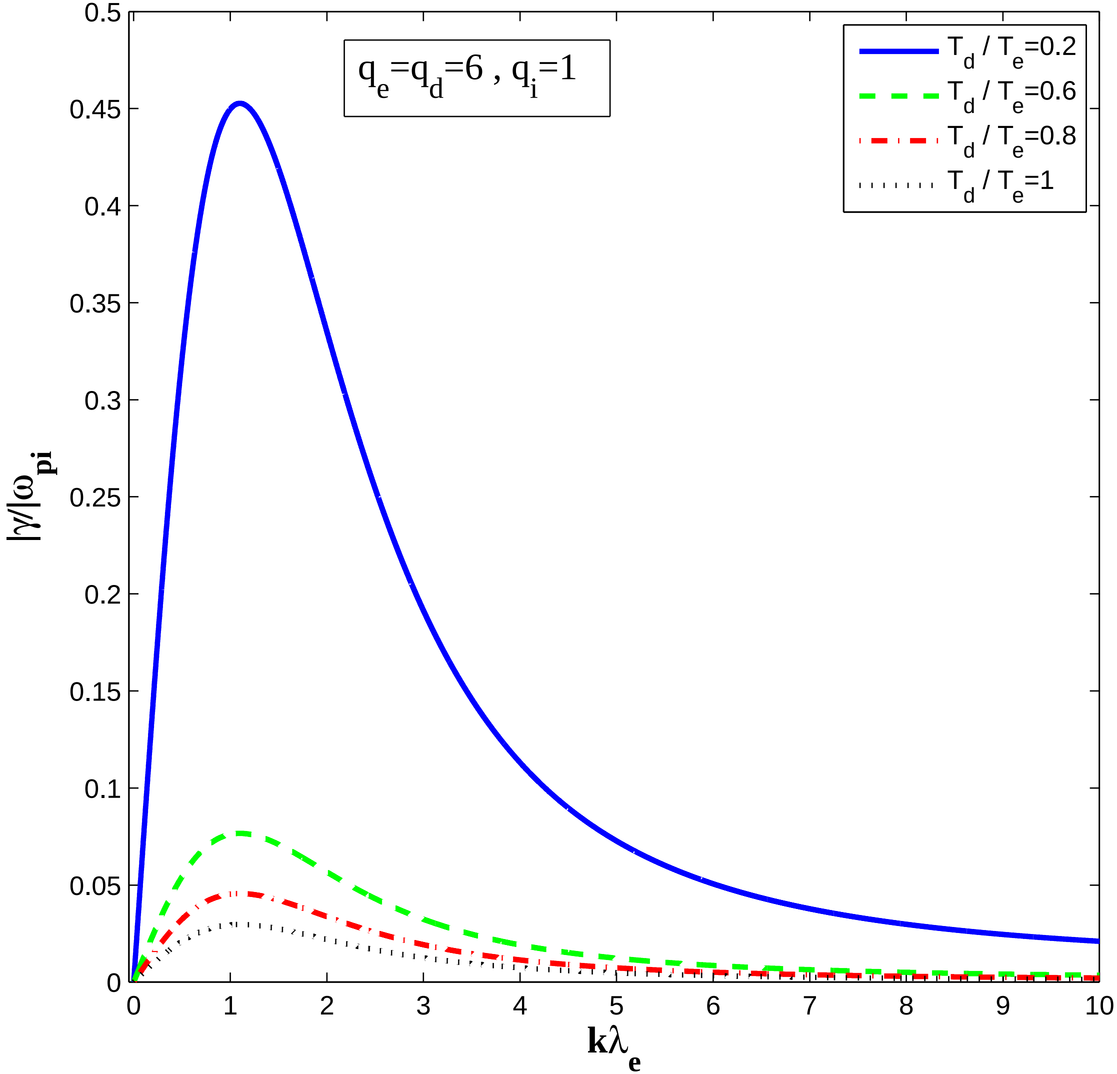
Fig 16



q_e=0.6 , q_i=q_d=1
T_i / T_e=0.2
T_i / T_e=0.6
T_i / T_e=0.8
T_i / T_e=1
$|\gamma|/\omega_{pi}$
$k\lambda_e$
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0
1
2
3
4
5
6
7
8
9
10

Fig17.

Fig 17



$q_e=q_d=6$, $q_i=1$
T_d / T_e=0.2
T_d / T_e=0.6
T_d / T_e=0.8
T_d / T_e=1
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1
2
3
4
5
6
7
8
9
10
$|\gamma|/\omega_{pi}$
$k\lambda_e$