Supporting Information for

"Using the *aa* index over the last 14 solar cycles to characterize extreme geomagnetic activity."

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1. Figure 1 plots the *aa* index and daily sunspot number over the last 14 solar cycles.
2. Figures 2 and 3 show the intervals selected for the minima, maxima and declining solar cycle phases. Each solar cycle is of a different duration. We take the minima identified from the quietest days of the solar cycle 13-month mean of the International Sunspot Number and then identify intervals for the maximum, declining phase and minimum phase by applying a simple algorithm across the entire dataset. The maximum phase begins 1.5 years after the previous sunspot quietest day minimum and lasts for 4 years at which the declining phase begins. The minimum phase is of 3 years duration centred on the sunspot quietest day minimum.
3. In the main text we used the homogenized *aa* index; the remaining Figures reproduce key Figures using the classic *aa* index.

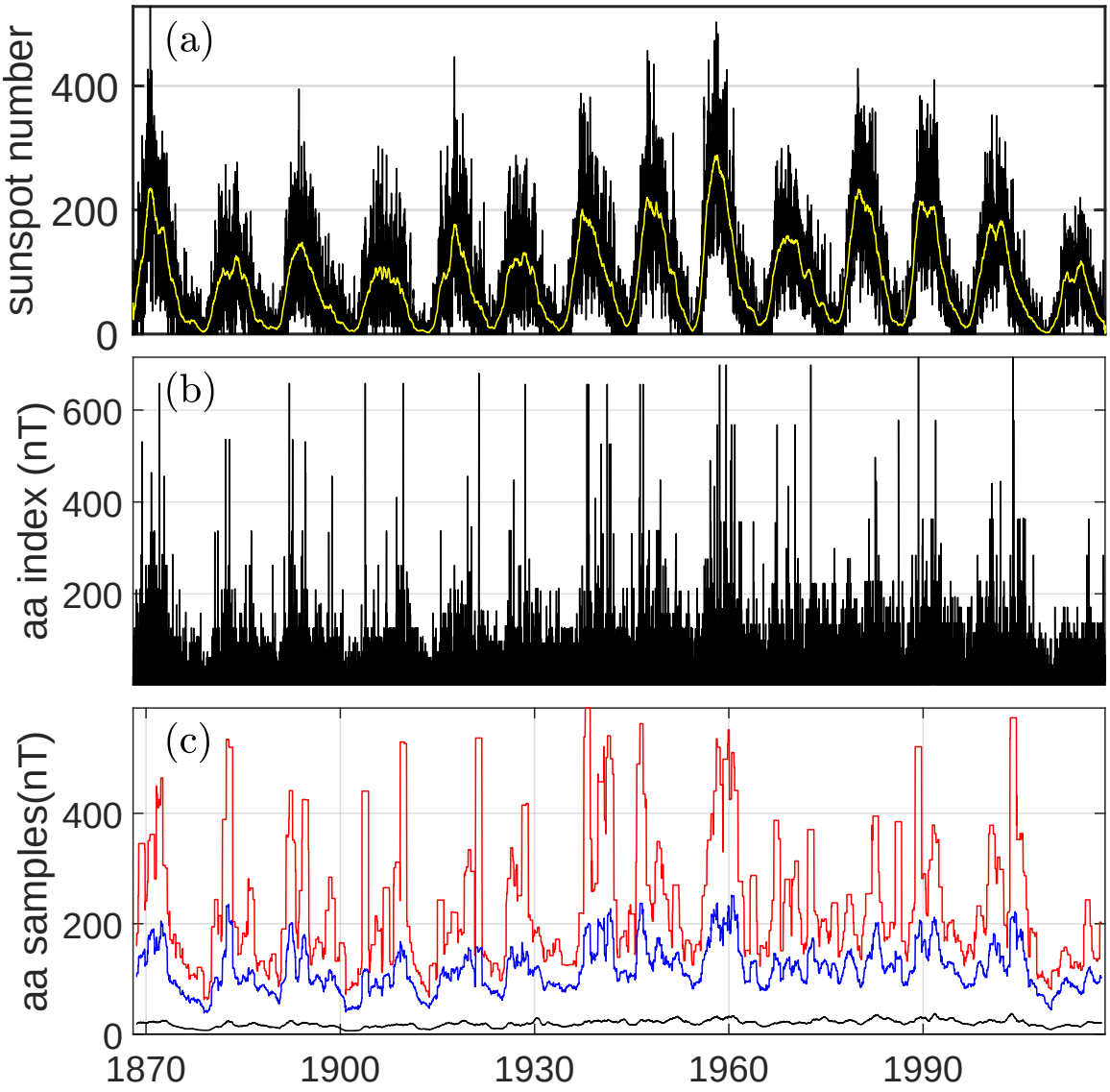

Figure 1. The *aa* index and daily sunspot number over the last 14 solar cycles. From top to bottom the panels plot: (a) the daily sunspot number, with a 1 year running average in yellow; (b) records of the classic 3-hour *aa* index; and (c) a 1 year running sample average of the largest 0.5% (red), largest 5% (blue) and all values (black) of the *aa* index.

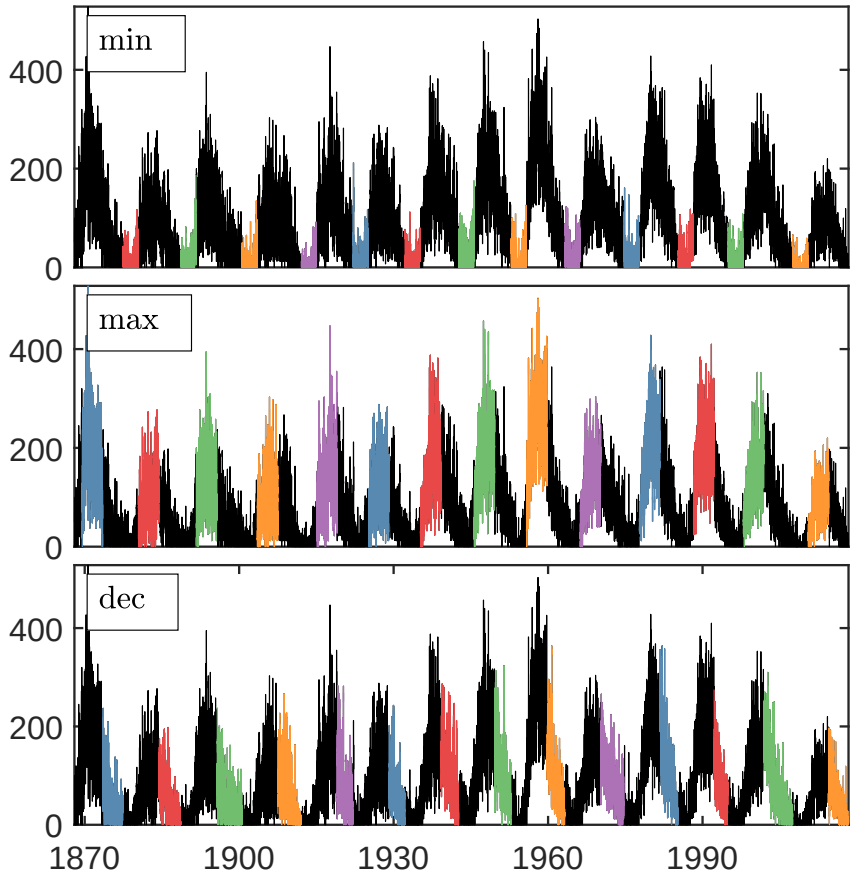

Figure 2. The panels plot from top to bottom: daily sunspot number time-series overplotted to show the intervals selected for the minima, maxima and declining solar cycle phases.

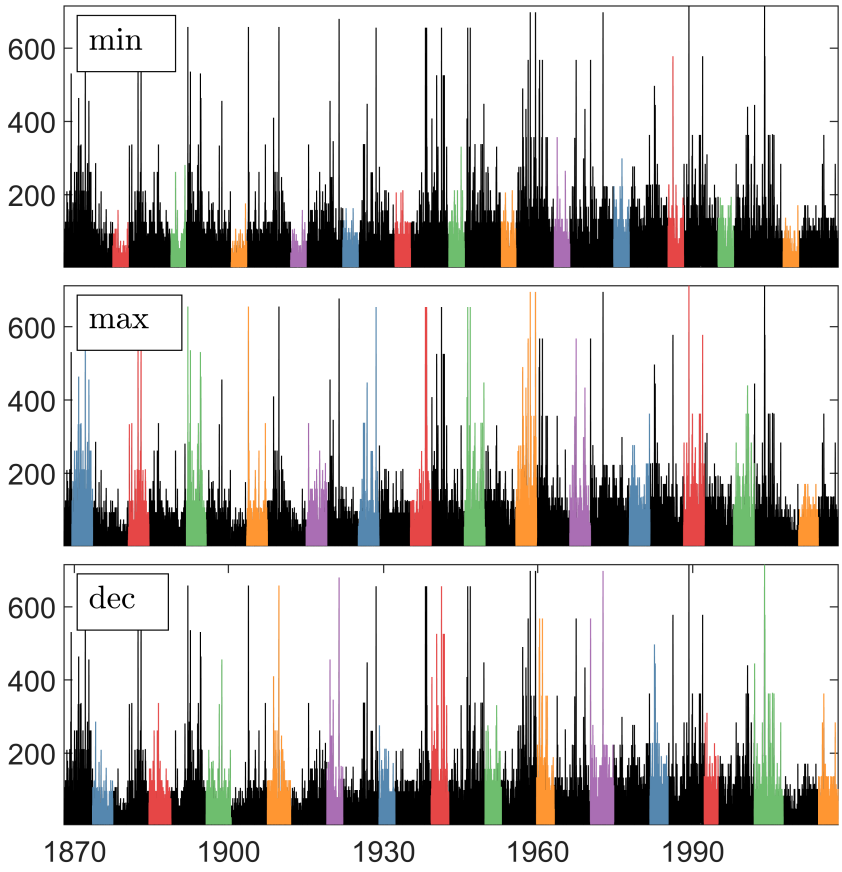

Figure 3. The panels plot from top to bottom: the 3 hourly classic *aa* index time-series overplotted to show the intervals selected for the minima, maxima and declining solar cycle phases.

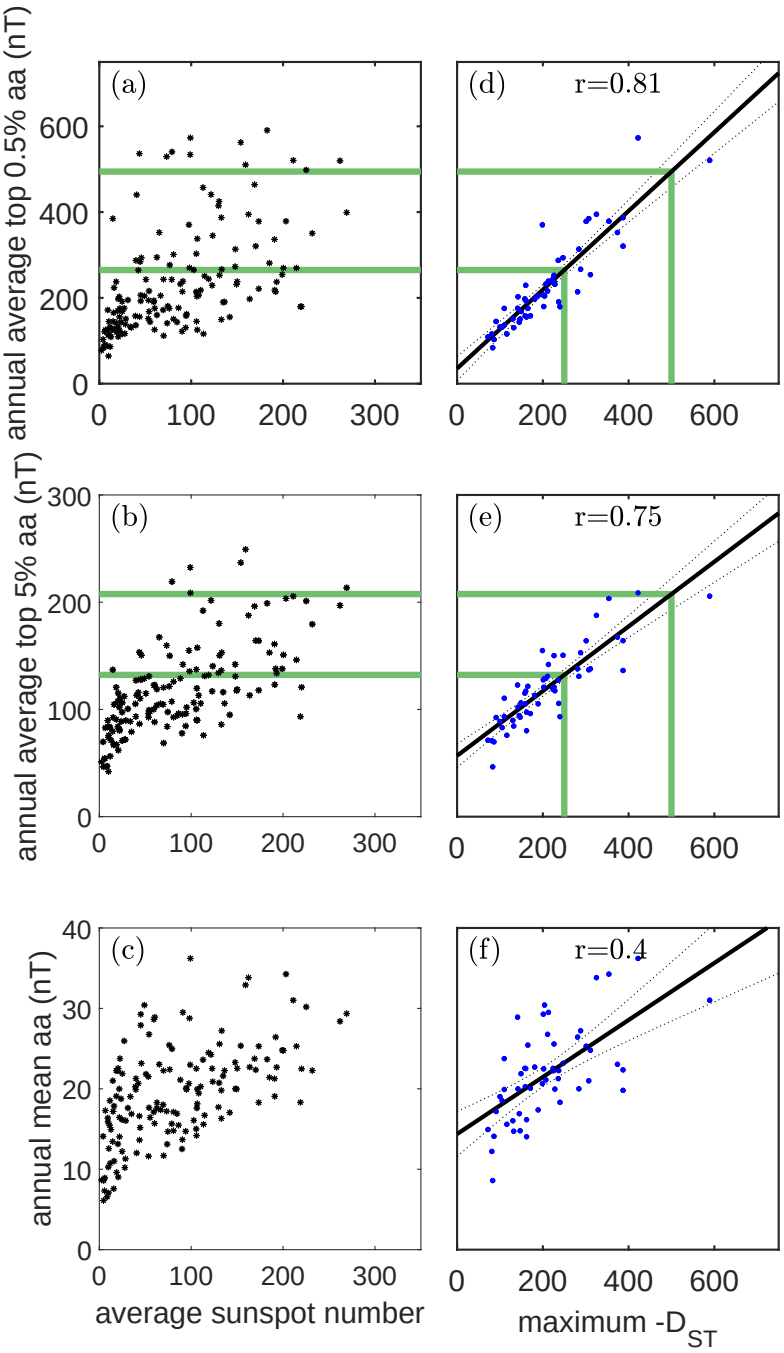

Figure 4. Panels (a-c) plot each value (black *) of the average of the largest 0.5 %, largest 5 % and all classic *aa* index records in each calendar year, versus average sunspot number, for all observations 1868-2017 inclusive. The annual (calendar year) intervals are non-overlapping. Panels (d-f) plot (blue dots) the subset of the non-overlapping calendar year *aa* averages versus the maximum value of $-D_{ST}$ that occurred in the same year-long window, taken over the last five solar cycles. In each panel the solid black line plots the least squares fit and the dotted lines, the 0.95 confidence level of the fit. The green lines use this fit to map across D_{ST} thresholds of $-250nT$ and $-500nT$ to corresponding *aa* values.

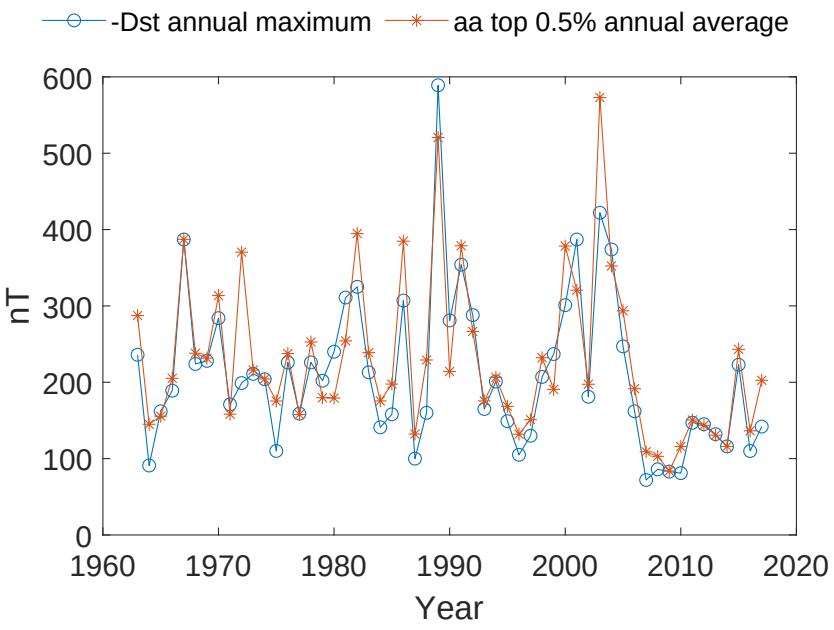

Figure 5. Comparison between $(-)D_{ST}$ and classic *aa* across the last 5 solar cycles. The average of the largest 0.5 % classic *aa* index records in each calendar year (*) is plotted alongside the maximum $(-)D_{ST}$ (o) record that occurred in that year. The calendar year samples are non-overlapping.

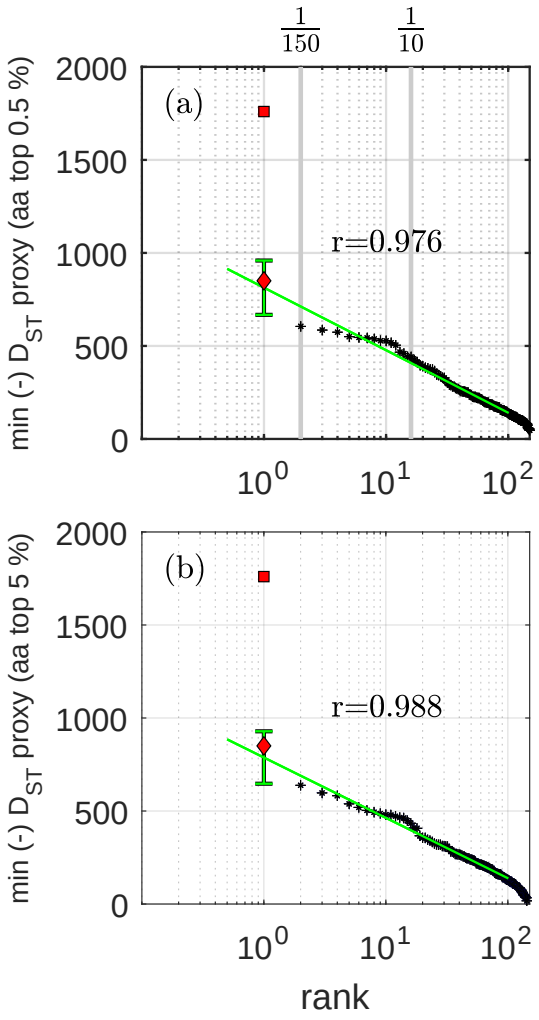

Figure 6. Rank order plots of parametrized classic *aa* shown in Figure 4. The panels show rank order plots of non-overlapping annual estimates of the (-) D_{ST} proxy based on averages of: (a) largest 0.5 % and (b) the largest 5 % of homogenized *aa*. The largest of these samples is plotted as rank 2, the next largest as rank 3 and so on. We plot as rank 1 two estimates of the Carrington event $D_{ST} = -850nT$ (red diamond) and $D_{ST} = -1760nT$ (red square). The uncertainty in the first ranked sample is estimated for an underlying exponential distribution. The green line indicates an exponential fit to the largest 100 values.