# Subsurface Cavity Detection by Improved Reverse Time Migration with Full Waveform Inversion: A Numerical Study


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## Abstract


Old abandoned coal working create major hazards in the form of subsidence of the coalfields. To avoid such hazards, there is need to detect these cavities prior to start of deeper seam mining. In this study, we first develop 2D realistic Air filled and Water Filled Cavity model with a work flow of RTM combined with FWI in a high-frequency Ricker source wavelet as 200 Hz. In order to provide a velocity model with high accuracy for RTM, we apply FWI to estimate the subsurface velocity by considering an initial smooth velocity model with addition of 30 % Gaussian noise.

The conventional RTM fails to image the cavities and yield a large amount of low frequency back scattered noise at shallow depth during the time of cross correlation due to time/space lag. To avoid these situation, we introduced an automatic shift operator at the time of imaging condition that operates automatically both in time or space. It leads to reduce the lag and improve the results by minimizing the noises at shallow subsurface. By comparing both the results it is observed that most of the noises in the migrated section of conventional method were eliminated by the improved form of RTM with the help of FWI velocity model estimation.


## Objectives

- Development of different cavity models to investigate the imaging problems.
- Application of Full Waveform Inversion for model updation in shallow subsurface.
- Imaging of cavity through Improved Reverse Time Migration (RTM) technique.

## Introduction

Geological hazards create a risky situation in the life of coal workers during underground mining such as; abandoned drilling and old workings. As a result cost of mining increases. This situation happens because prior to mining, there is no accurate information available for the voids accumulated in the underground mines during the lack periods. Various geophysical methods have been applied to detect these voids depending upon the environment. Among them, High-Resolution Seismic Survey (HRSS) achieved more success during the last decade but the development is still questionable. In present state, we have not found any evidence for the solidity of the wave equation based formulation to detect these multiple cavities that are observed in field consequence.

To understand these signatures more precisely, in our study, an attempt has been made to image these cavities with the help of Reverse Time Migration (RTM) combined with Full Waveform Inversion (FWI). RTM is mostly used for hydrocarbon exploration targets with low central frequency as source. Application of this method to shallow subsurface exploration is still in research stage. Similarly, for velocity model updation, FWI gives mostly appropriate optimization results as compared to other techniques, but has found limited application in low frequency problems.

So the objective of this study is to use a high resolution based FWI method to optimize the velocity model that will help for further wave form based migration techniques like Reverse Time Migration (RTM). To test the accuracy, we implemented the proposed algorithm over both Air Filled Cavity (AFC) and Water Filled Cavity (WFC).

## Mathematical Background

**FWI**

FWI estimates subsurface material properties (velocity) by minimizing the objective function built it from the error between observed and modelled seismic waveforms.

As we use high frequency for the analysis, in the frequency domain, the objective function is defined with L2- norms as follows;

$$E(m) = \sum_{\omega} \sum_{s} \frac{1}{2} [(\tilde{u} - \tilde{d})^{T} (\tilde{u} - \tilde{d})^{*}] \quad (1)$$

Where, m= model parameter, $\tilde{u}$ and $\tilde{d}$ are model and observed data

So the equation of Conjugate Gradient (CG) is;

$$g = D^{*} C_d^{-1} \Delta d + C_m^{-1} \Delta m \quad (2)$$

In which, $D^{*}$is the adjoint operator, $C_d^{-1}$ is the data covariance, $\Delta d$ is the data misfit, $C_m^{-1}$ is model parameter covariance and $\Delta m$ is model parameter misfit.

The velocity model is updated by a preconditioned Conjugate Gradient (CG) optimization method for the inversion. The advantage of this optimization is that there is no need to store large matrix therefore, it costs only O($N^2$) algebraic operators per iteration. At each iteration, the velocity is updated by smearing the weighted depth along the reflection ray-path. In shallow land data, cycle-skipping problem is normal that create obstacle for data convergence but by applying CG, it increases the convergence chance of the data.

**RTM**

The proper imaging condition is applied, and the results are summed up for all the shots, which is mathematically written as;

$$I(x) = \int w_s(x,t) * w_r(x,t)\, dt \quad (3)$$

Where $w_s(x,t)$ is shot wave-field and $w_r(x,t)$ is receiver wave-field. Whereas; $I(x)$ is the imaging result of stacked wave-fields.

This shift function developed is a combination of both time and space shift mathematically written as;

$$w(x,\lambda,t) = w_s(x-\lambda, t-\tau) * w_r(x+\lambda, t+\tau) \quad (4)$$

$$I(x,\lambda,\tau) = w(x,\lambda,\tau,t=0) \quad (5)$$

From Eq. (4) and (5) we can generalize the result and can be written as;

$$I(x,\Theta) = \int w_s(x,\Theta) * w_r(x,\Theta)\, dt \quad (6)$$

$$\Theta = f(\lambda,\tau) \quad (7)$$

Where; $\Theta$ is automatic optimized shift function of both space ($\lambda$) and time (t).

## Methodology

At first, the true velocity model is smoothed by adding 30% of Gaussian noise and use it as the initial model for the algorithm to recover all the higher frequency component of velocity model. The source wavelet is estimated. Later, predicted data is computed on recording surface and loop over up-to end. After that, residual is computed on recording surface. At this stage, step-length & C is estimated then slowness is updated. Finally, the predicted data is obtained. The final update model is considered as the input model for RTM.

An end-on type seismic survey is designed for data acquisition. The source-receiver spacing is 1 m. Each shot is implemented with 1 m interval. At the start of the survey, the source is placed at the extreme left and progressed towards the right. The Common Shot Gather (CSG) data generated here are accounted with all seismic events such as primaries, direct waves along with refracted and reflected, ground-rolls and air waves. To image all stratigraphic blocks, CSG is directly migrated by using both conventional RTM method and improved RTM method for comparison purpose without any intermediate processing steps.

*Table 1: List of geological formation with corresponding velocity*

| Formation | Model Parameters | |
|---|---|---|
| | Index | Velocity (m/s) |
| Weathered Shale | WS | 1000 |
| Coal | C | 2500 |
| Void (air-filled) | V | 330 |
| Void (water filled) | V | 1500 |
| Sand Stone | SS | 3000 |
| Shale | SH | 3500 |

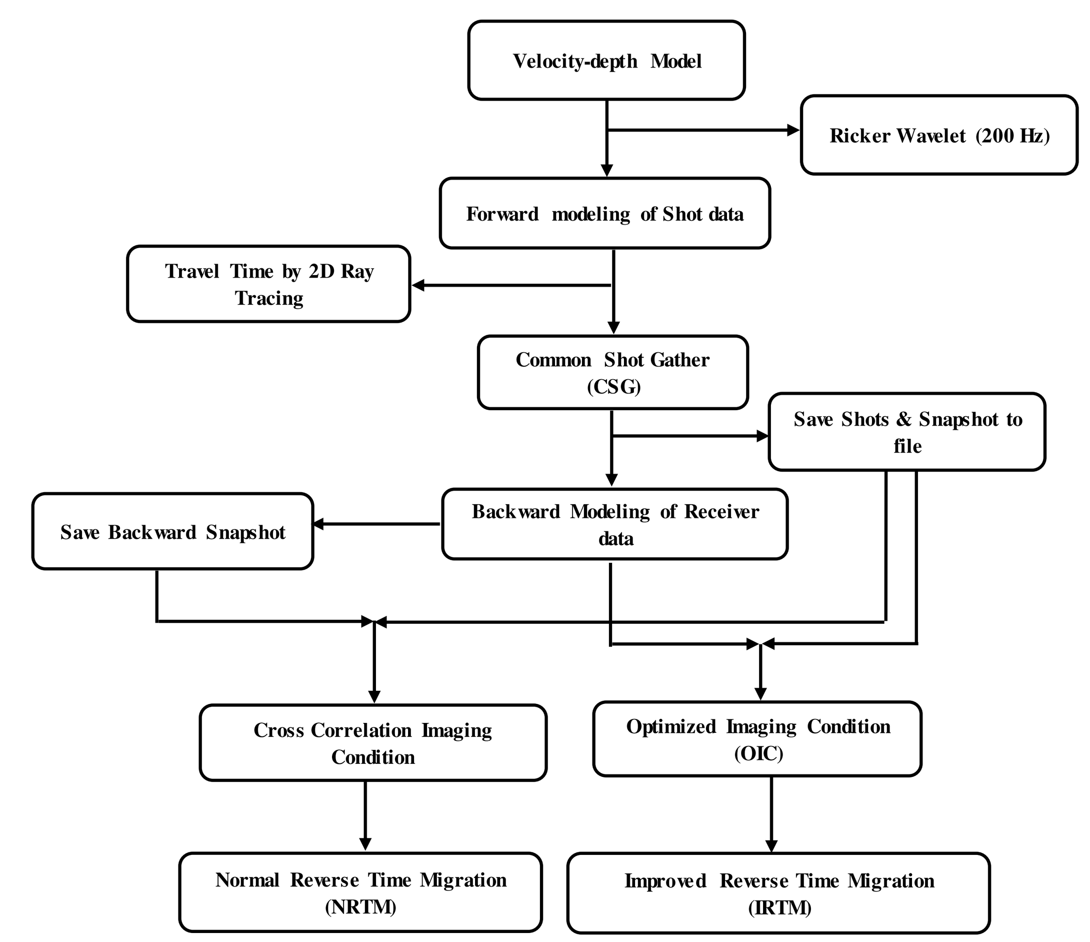


**Figure 1:** Flowchart for detail processing steps for Improved Reverse Time Migration

## Result & Discussion

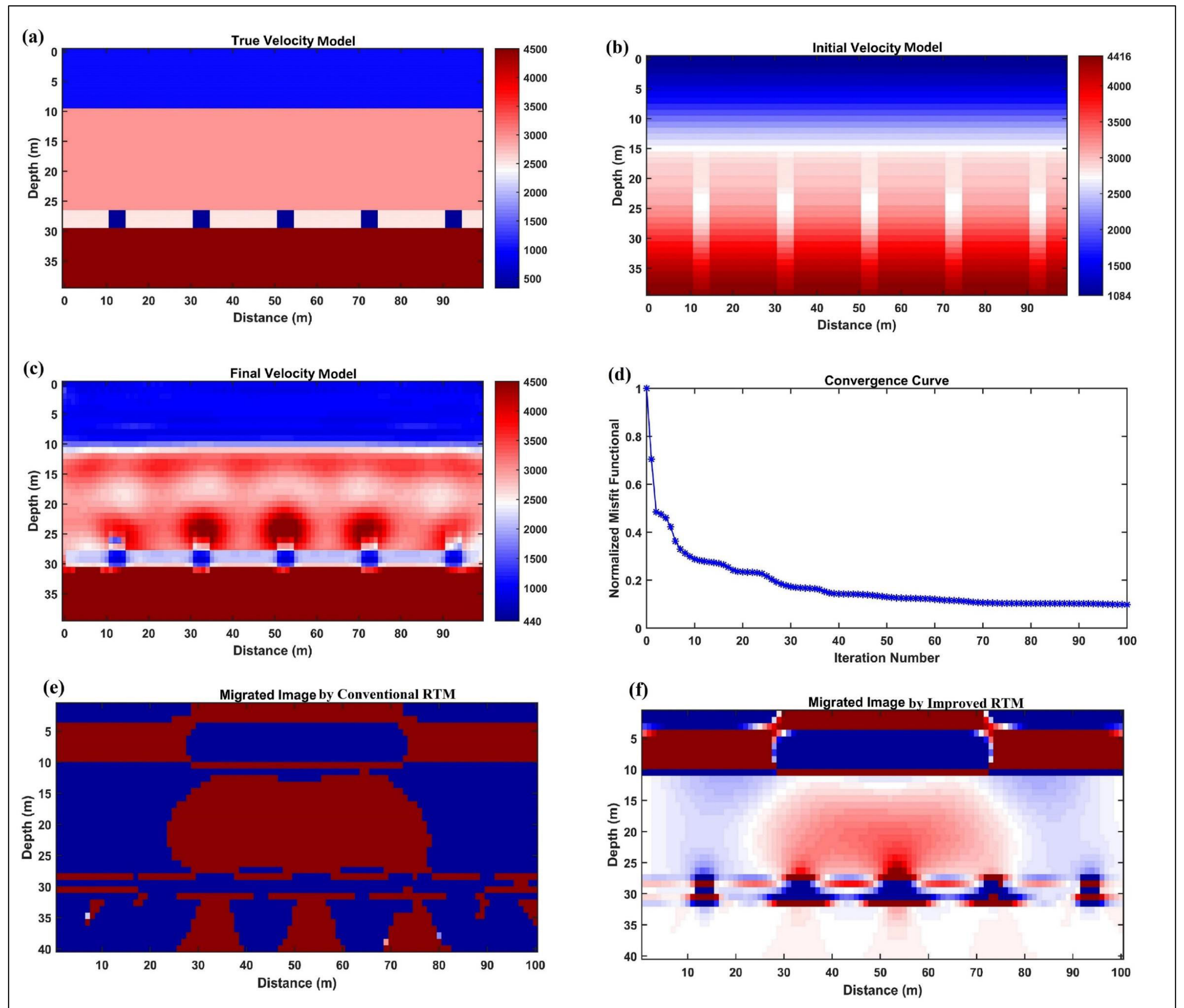


**Figure 2:** Illustration for showing the FWI along with improved RTM for Air Filled Cavity (AFC) Model

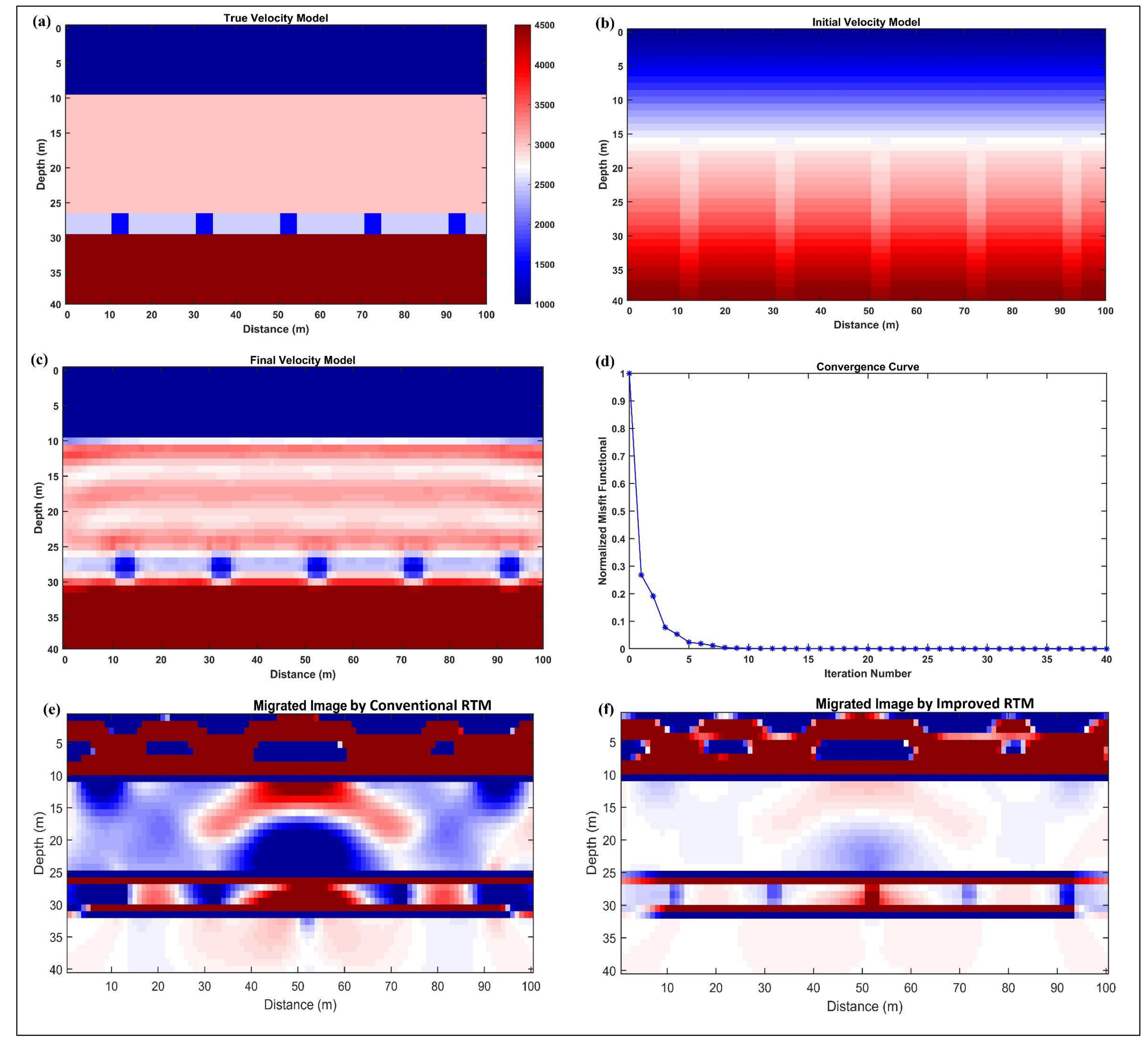


**Figure 3:** Illustration for showing the FWI along with improved RTM for Water Filled Cavity (WFC) Model

A key interest of this research is related to alternate multiple cavity imaging under RTM technique along with FWI. The velocity model is updated by a preconditioned Conjugate Gradient (CG) optimization method for the inversion. The advantage of this optimization is that there is no need to store large matrix therefore, it costs only O($N^2$) algebraic operators per iteration. At each iteration, the velocity is updated by smearing the weighted depth along the reflection ray-path. In shallow land data, cycle- skipping problem is normal that create obstacle for data convergence but by applying CG, it increases the convergence chance of the data.

There was the use of high central frequency and high-velocity contrast between void and surrounding rocks, with an excellent reflecting interface for each and every Bord & Pillar with one-to-one depth location. In case of normal RTM, the existence of coal pillars in shallow depth may be interpreted partly from the diffraction events, but that fails to acquire the information regarding exact position and size of coal pillar as well as voids/cavities. Our approach i.e. improved RTM, accurately imaged and distinguished all the interfaces between coal and voids, coal and sandstone, sandstone and shale.

## Conclusions

- It has been observed that the FWI+RTM approach was able to handle strong velocity variations both in vertical as well as horizontal direction.
- First of all, both AFC and WFC are delineated. However, the AFC contaminated with numerical artifacts. In both the cavity model, dimension of the cavities delineated properly with true depth location.
- However, our implementation converges fairly and fast which is observed from the convergence curve (Fig1 (d) & 2(d). We are able to demonstrate that the conjugate-gradient FWI is able to produce acceptable data fit and generate realistic velocity model and with combination of improved RTM it is able to detect subsurface shallow cavity.
- By combining these two approaches, it accurately detects the location of the source (Bord & Pillar) even for shallow surface and low thickness beds and can be interpreted for the dangerous level of air/water voids by which proper production safety can be ensured before to mining.


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