



Plane-bed instability at the granular scale

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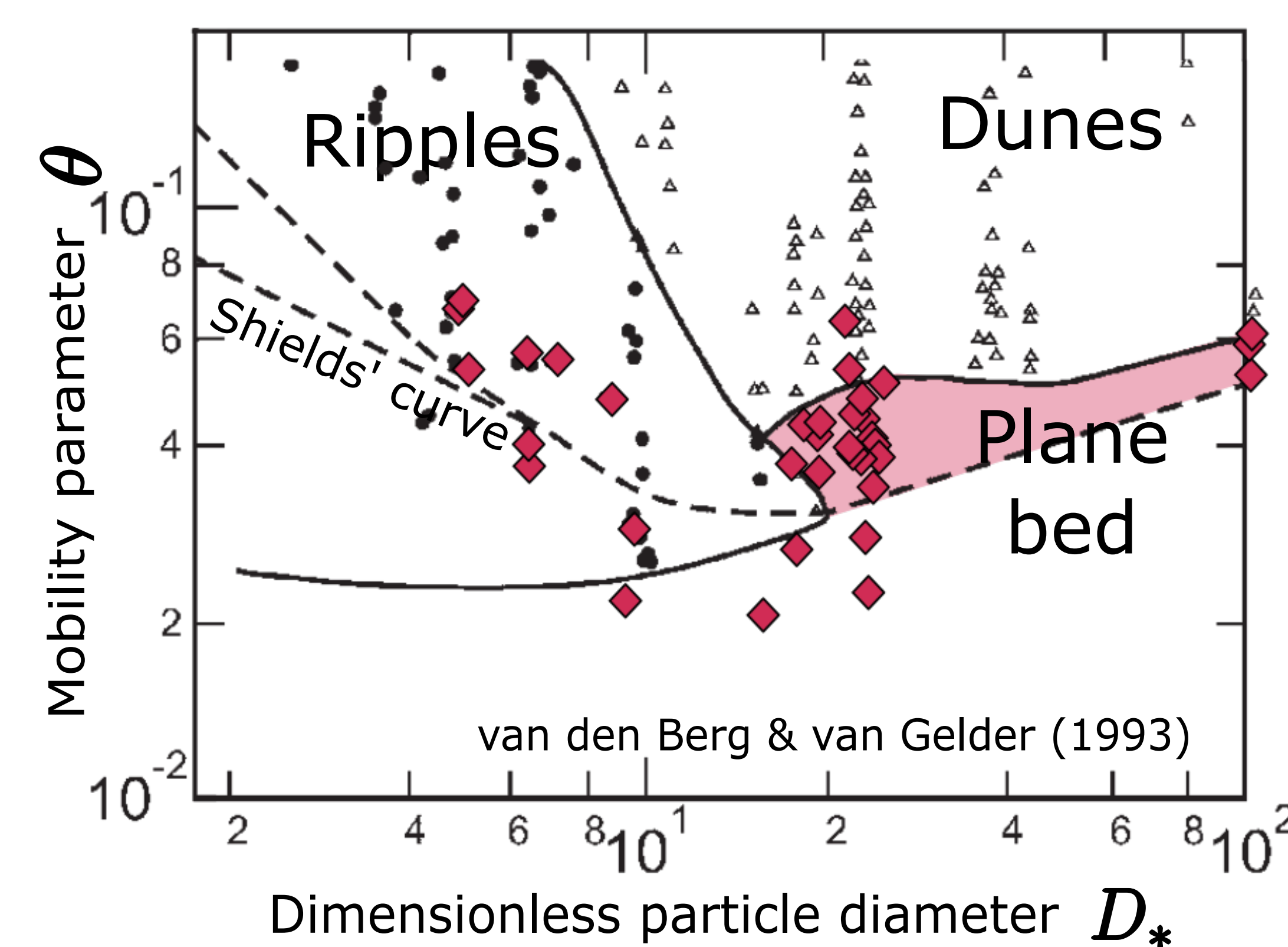
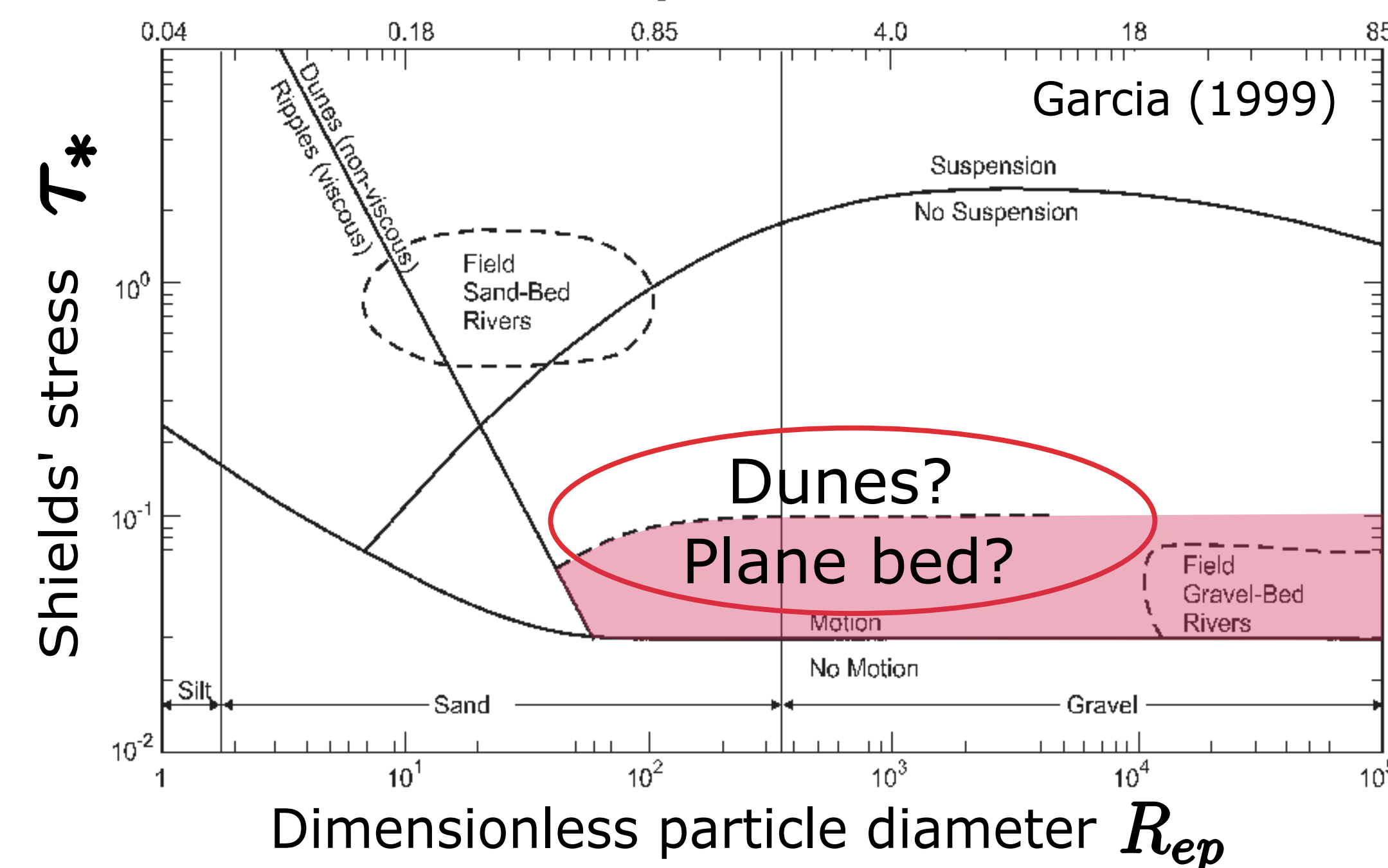
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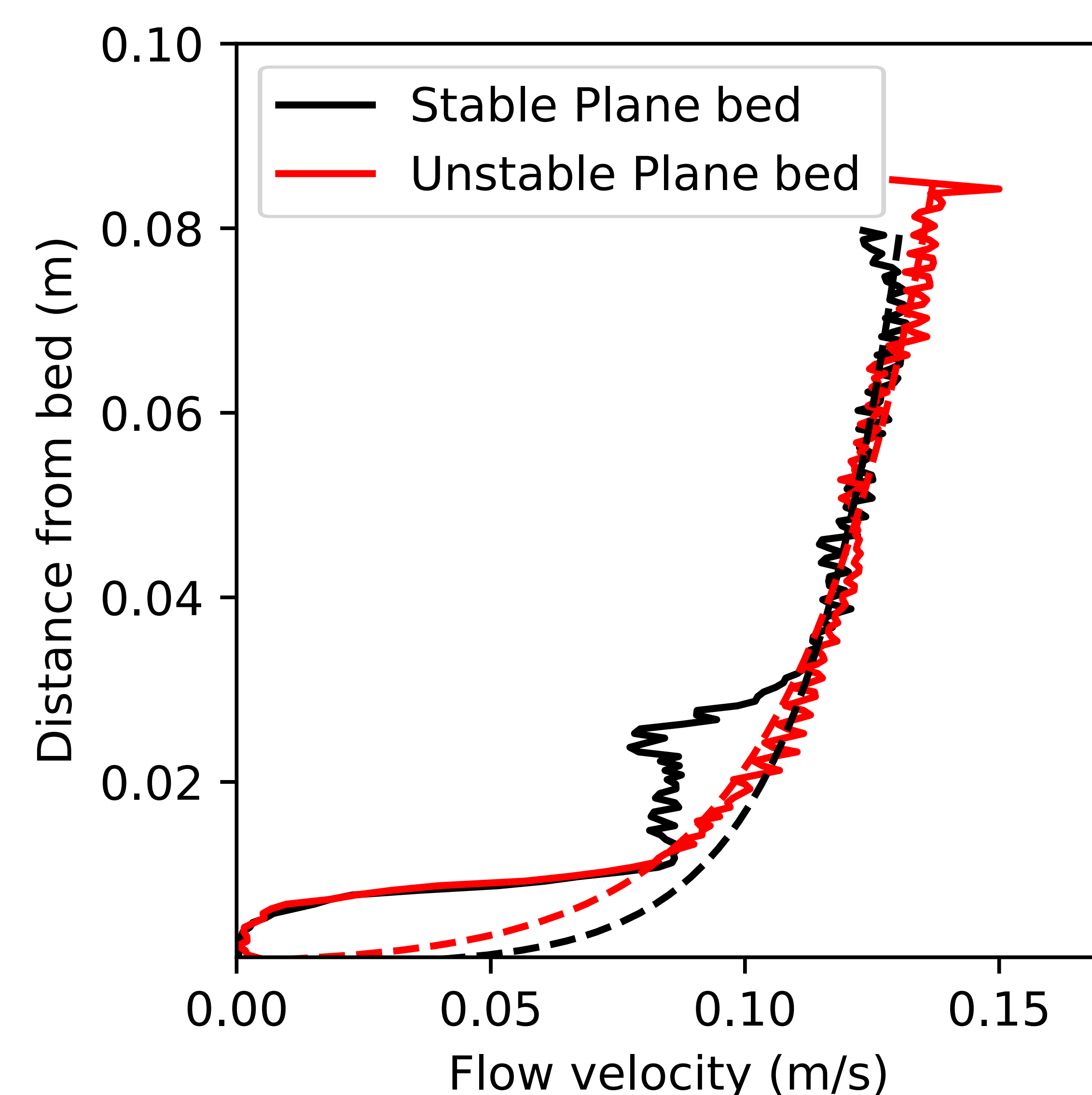
1 MOTIVATION

What process controls the transition from lower-stage plane bed to bedforms?



4 RESULTS

Flow conditions



Ensemble average flow velocity profiles recorded using moving ADV.

2 PARTICLE CONGESTION

Hypothesis: Onset of bedform development reflects a critical condition where particle collisions become frequent enough to influence bulk properties of transport.

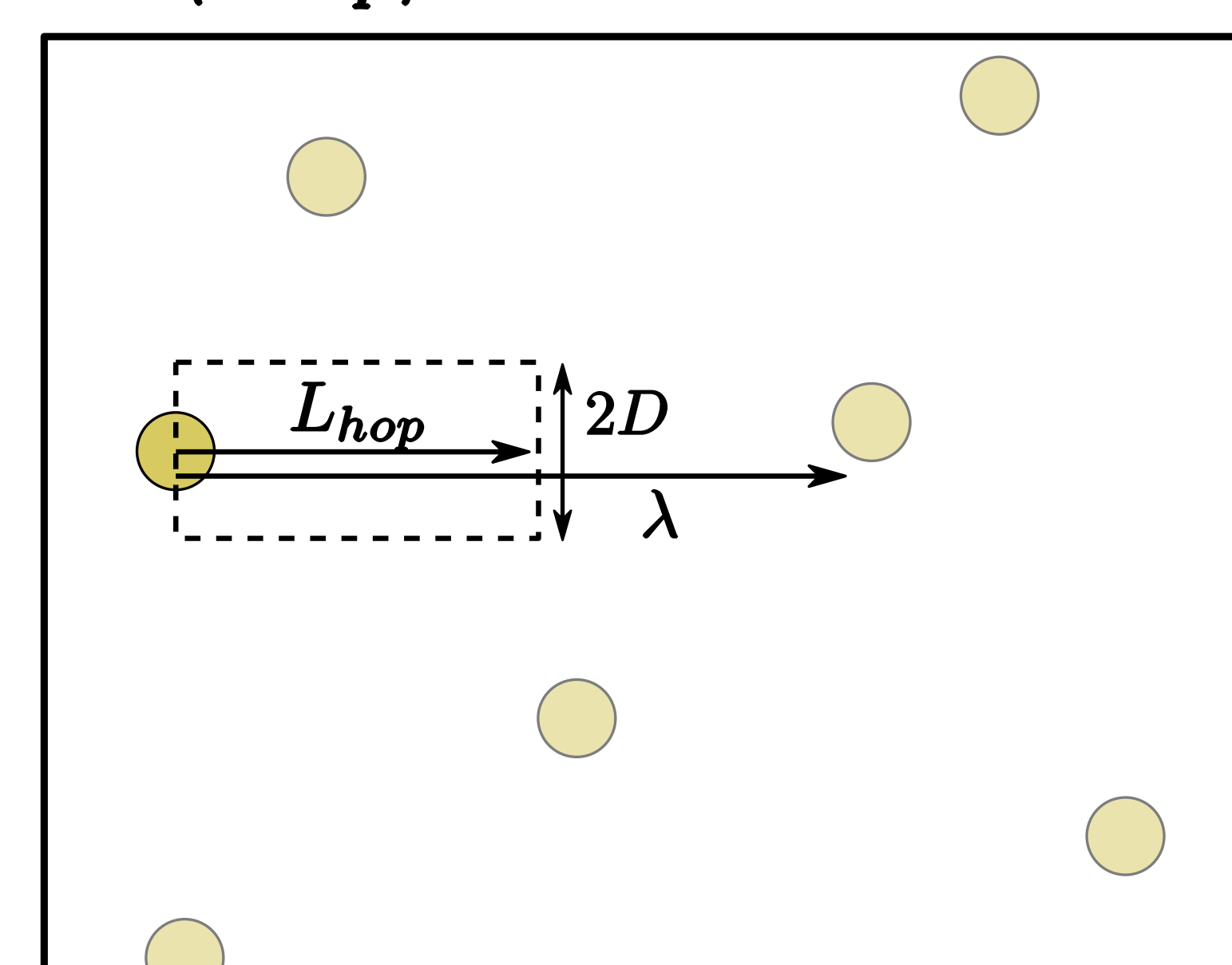
Model for particle congestion:

- Mean free path: $\lambda = \frac{1}{2DC_n}$
- Mean particle hop distance: $\langle L_{hop} \rangle$

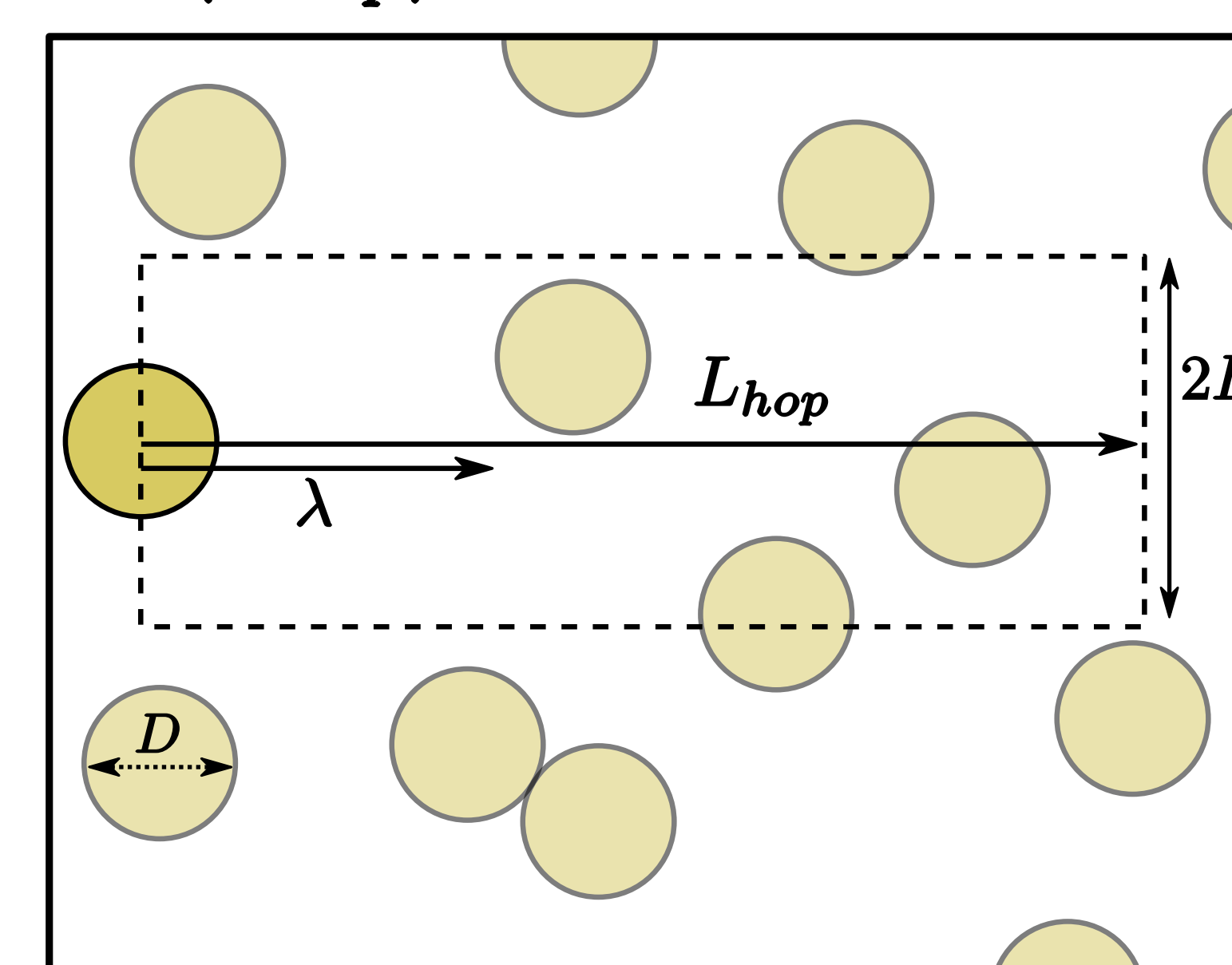
Critical condition for bedform development:

$$\frac{\langle L_{hop} \rangle}{\lambda} \approx 1 \quad (\text{average number of collisions per hop is 1, allowing propagation of collision shockwaves})$$

$\langle L_{hop} \rangle < \lambda$ (Plane bed)



$\langle L_{hop} \rangle > \lambda$ (Bedforms)



Which implies...

Empirical Predictor for $\langle T_p \rangle$
Lajeunesse et al. (2010)

$$\langle T_p \rangle = 10.6 \sqrt{\frac{D}{Rg}}$$

$$\frac{\langle L_{hop} \rangle}{\lambda} = \frac{12q_b \langle T_p \rangle}{\pi D^2} = \frac{127}{\pi} q_b^*$$

Threshold Criterion:

$$q_b^* = 0.025$$

Variable Definitions:

C_n Numer of active particles per unit streambed area [L^{-2}]

D Particle diameter [L]

T_p Hop duration [T]

$\langle L_{hop} \rangle$ Mean hop distance [L]

Streamwise hop distance [L]:
 $L_x \approx L_{hop}$

Particle activity [L]:

$$\gamma = C_n \frac{\pi}{6} D^3$$

Particle volume

Unit bedload flux [$L^2 T^{-1}$]:

$$q_b = \gamma \frac{\langle L_x \rangle}{\langle T_p \rangle}$$

Avg. particle velocity

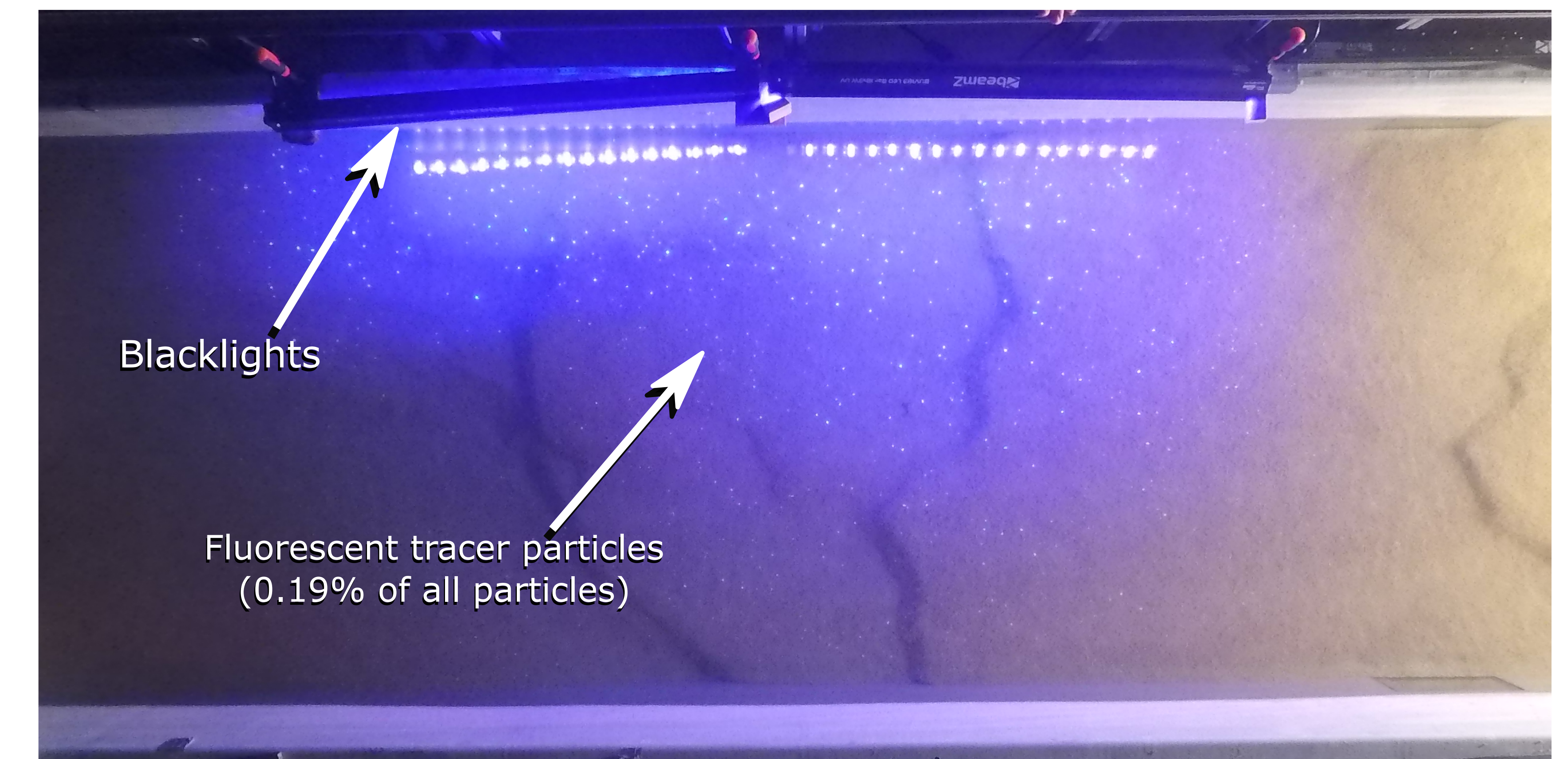
Bedload number:

$$q_b^* = \frac{q_b}{\sqrt{gRD^3}}$$

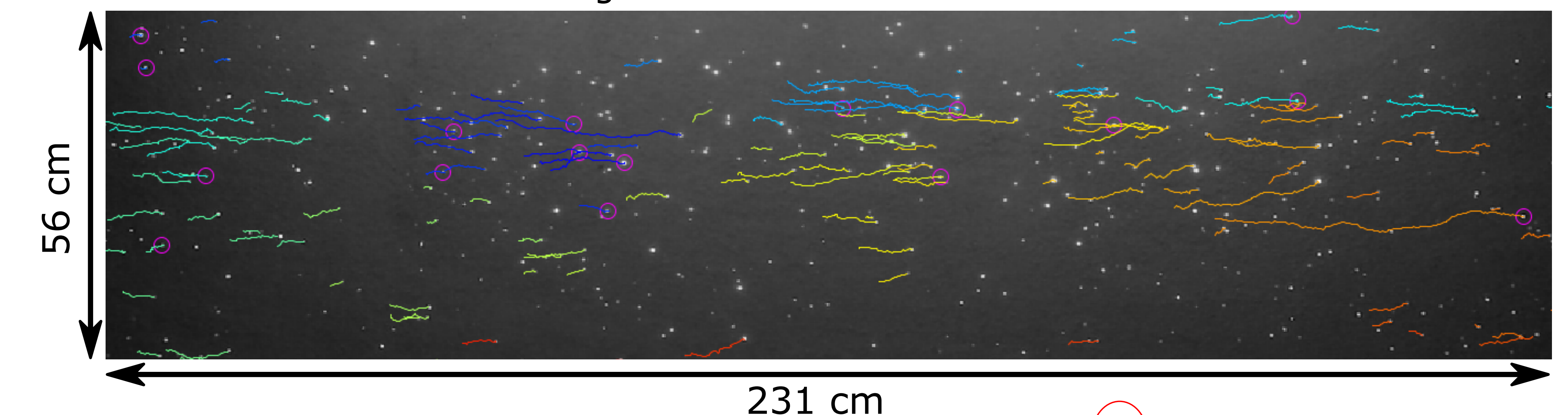
3 EXPERIMENTS

Objective: Measure relevant parameters using tracked tracer particle motions for (a) stable lower-stage plane-bed conditions and (b) unstable plane-bed conditions immediately after an increase in flow strength.

Experimental Setup

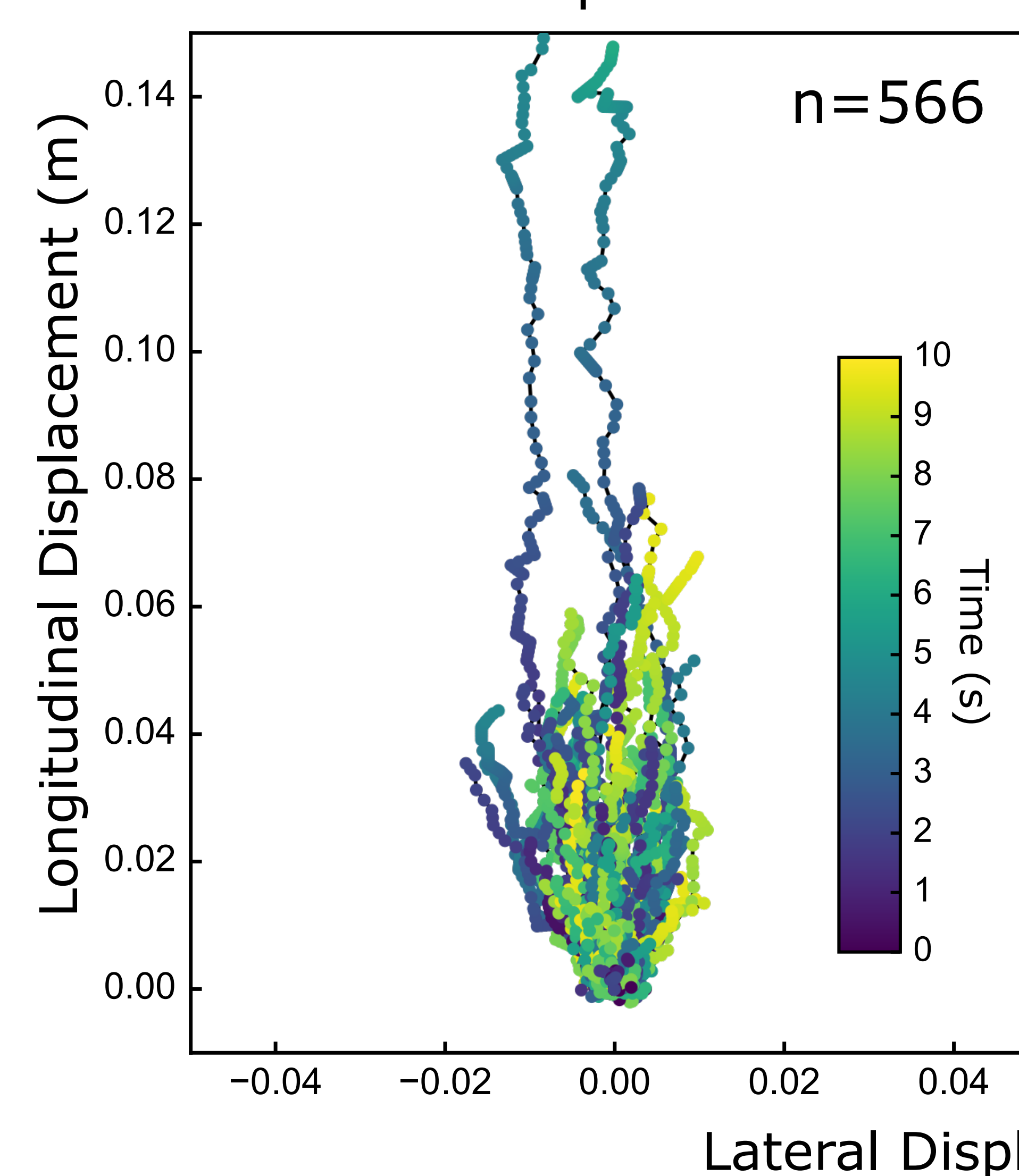


Digitized Tracer Particle Paths

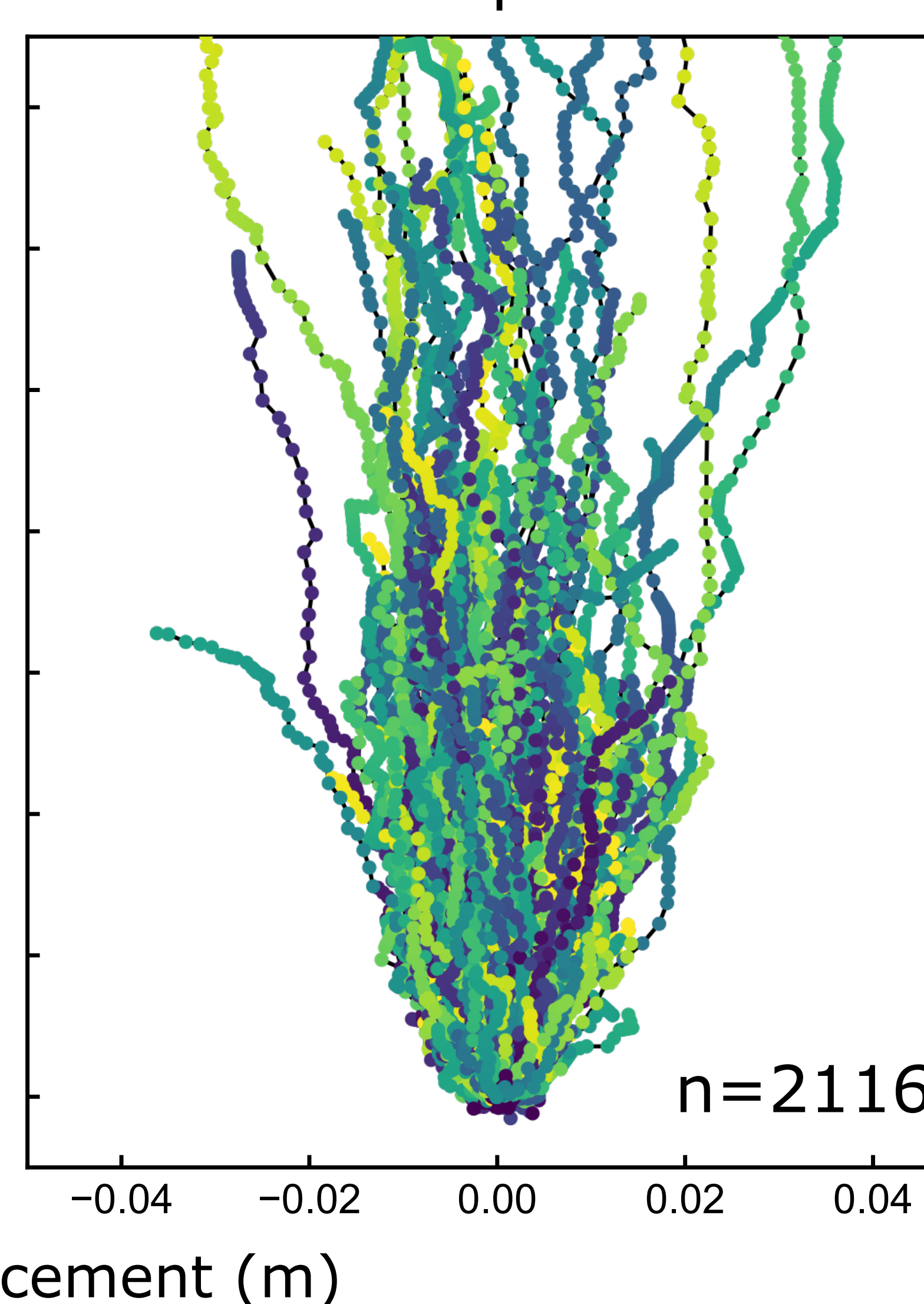


Particle motions

Stable plane bed



Unstable plane bed

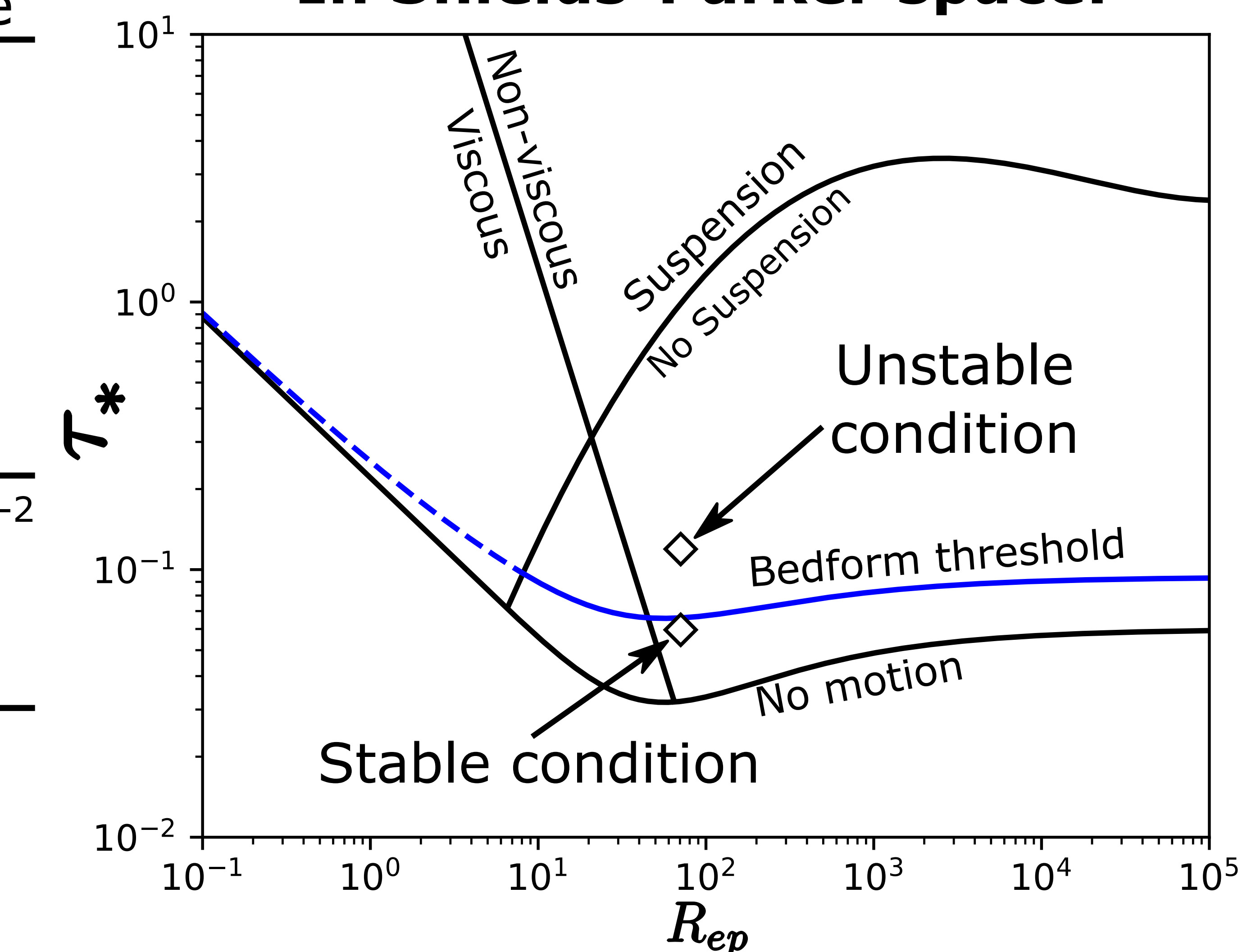


Visualization of tracer particle paths. Each plot shows all paths recorded over two ten second intervals (20 seconds total time represented for each condition).

Summary

	Stable	Unstable
Depth	11 cm	11 cm
Discharge	20 ls ⁻¹	25 ls ⁻¹
Width	1.2 m	1.2 m
D	2.1 mm	2.1 mm
R	0.055	0.055
Re_p	71	71
C_n	2962 m ⁻²	14300 m ⁻²
q_b^*	0.0051	0.029
$\tau_* - \tau_{*c}$	0.012	0.037
$\langle L_{hop} \rangle$	0.9 cm	1.4 cm
λ	8.0 cm	1.6 cm
$\langle L_{hop} \rangle / \lambda$	0.11	0.85

In Shields-Parker space:



Bedform threshold computed using Wong & Parker bedload equation as $\tau_* = \tau_{*c} + 0.034$.

5 CONCLUSIONS

- Plane-bed topography is stable when the mean free path significantly exceeds the mean particle hop distance. This suggests that congested transport conditions destabilize plane bed topography leading to bedform development.
- The ratio of particle hop distance to mean free path varies in proportion to the Einstein bedload number near the threshold of sediment motion. The resulting bedform threshold is consistent with observations across a wide range of conditions.