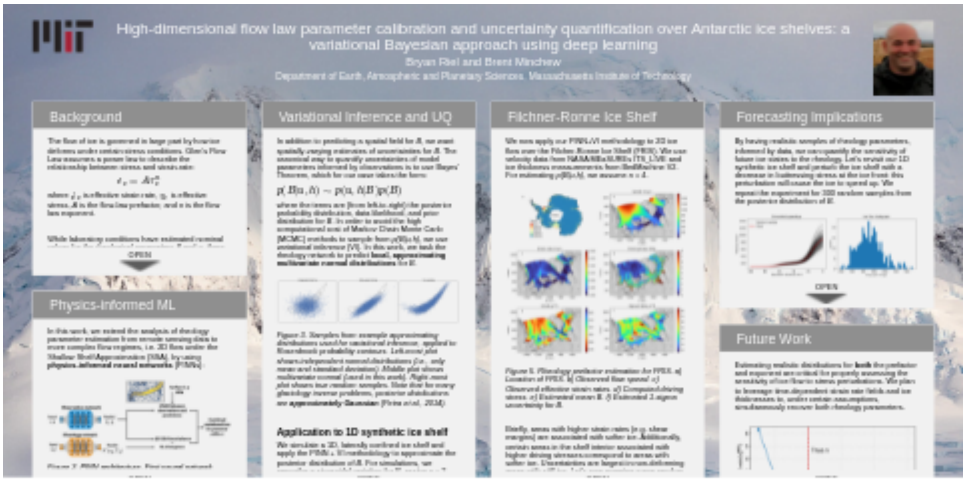
High-dimensional flow law parameter calibration and uncertainty quantification over Antarctic ice shelves: a variational Bayesian approach using deep learning



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BACKGROUND

The flow of ice is governed in large part by how ice deforms under certain stress conditions. Glen's Flow Law assumes a power law to describe the relationship between stress and strain-rate:

$$\dot{\epsilon}_e = A\tau_e^n$$

where $\dot{\epsilon}_e$ is effective strain rate, τ_e is effective stress, A is the flow-law prefactor, and n is the flow law exponent.

While laboratory conditions have estimated nominal values for the rheological parameters A and n, there is still considerable uncertainty in their values in natural ice. Recent work has leveraged continent-scale data sets to estimate n on Antarctic ice shelves flowing in a purely extensional stress regime.

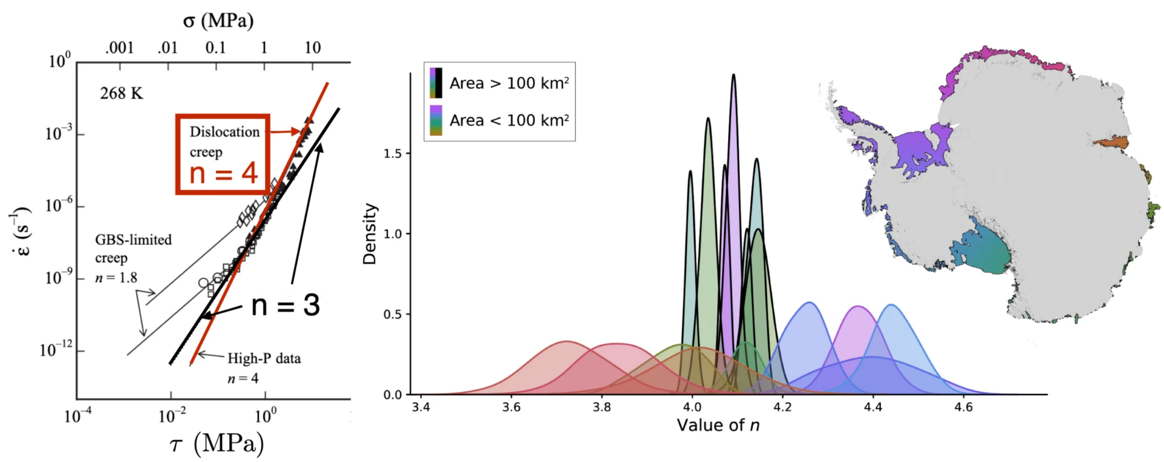

Figure 1. (Left) Uniaxial laboratory experiments of stress vs. strain-rate for estimating n (Goldsby and Kohlstedt, 2001). (Right) Estimated n and uncertainties using remote sensing data for Antarctic ice shelves in extensional stress regime (Millstein et al., in review).

PHYSICS-INFORMED ML

In this work, we extend the analysis of rheology parameter estimation from remote sensing data to more complex flow regimes, i.e. 2D flow under the Shallow Shelf Approximation (SSA), by using **physics-informed neural networks** (PINNs):

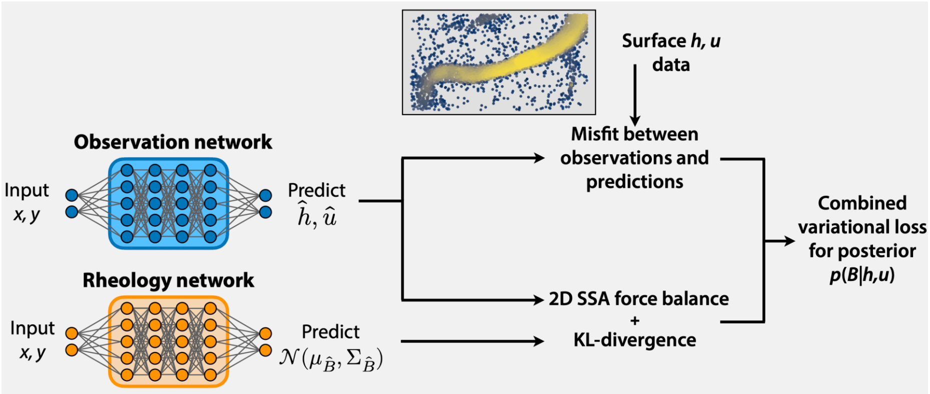

Figure 2. PINN architecture. First neural network (observation network) is tasked with reconstructing ice shelf velocity and thickness data. Second network (rheology network) is tasked with predicting prefactor B. The loss function for training combines a data misfit loss and a probabilistic loss for the posterior distribution p(B|h,u).

Thus, we use PINNs to assimilate large-scale remote sensing datasets while predicting a prefactor field ($B = A^{-1/n}$ for a given *n*) that optimally reconstructs observed data, consistent with the SSA momentum balance. While assimilation of remote sensing data into 2D ice flow models is nothing new (using control methods), our primary goal is to efficiently compute *uncertainties* associated with the estimated parameters for large datasets.

VARIATIONAL INFERENCE AND UQ

In addition to predicting a spatial field for *B*, we want spatially-varying estimates of uncertainties for *B*. The canonical way to quantify uncertainties of model parameters informed by observations is to use Bayes' Theorem, which for our case takes the form:

$$p(B|u,h) \sim p(u,h|B)p(B)$$

where the terms are (from left-to-right) the posterior probability distribution, data likelihood, and prior distribution for *B*. In order to avoid the high computational cost of Markov Chain Monte Carlo (MCMC) methods to sample from *p(B|u,h)*, we use *variational inference* (VI). In this work, we task the rheology network to predict **local, approximating multivariate normal distributions** for *B*.

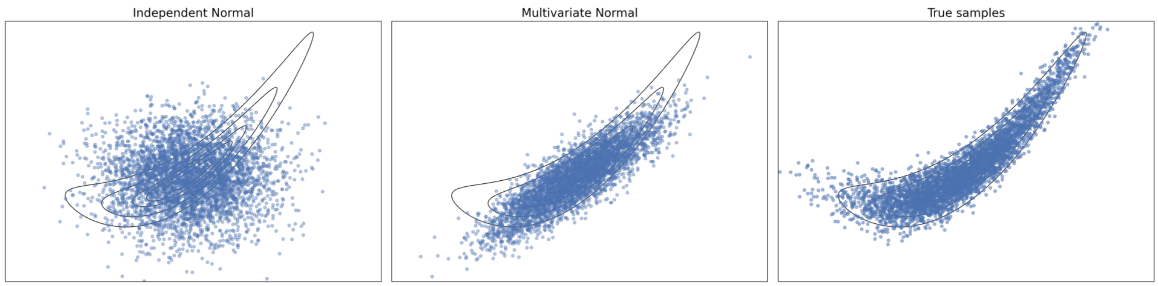

Figure 3. Samples from example approximating distributions used for variational inference, applied to Rosenbrock probability contours. Left-most plot shows independent normal distributions (i.e., only mean and standard deviation). Middle plot shows multivariate normal (used in this work). Right-most plot shows true random samples. Note that for many glaciology inverse problems, posterior distributions are ***approximately Gaussian*** *(Petra et al., 2014).*

Application to 1D synthetic ice shelf

We simulate a 1D, laterally confined ice shelf and apply the PINN + VI methodology to approximate the posterior distribution of *B*. For simulations, we prescribe a sinusoidal variation for *B*, assign *n* = 3, and add 5-10% correlated noise to the simulation outputs.

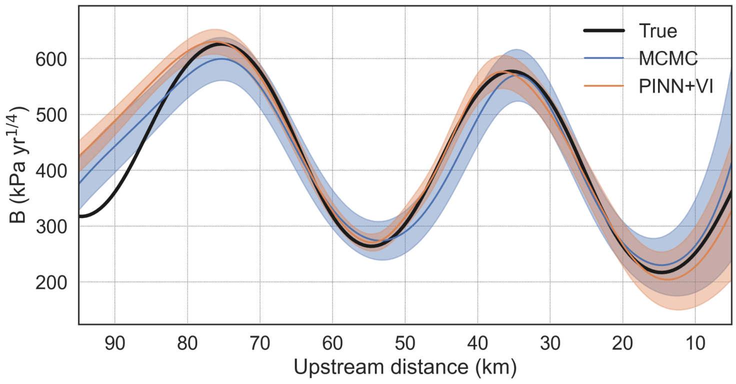

Figure 4. True (black) and estimated B using MCMC (blue) and PINN+VI (orange). Shaded regions indicate 1-sigma uncertainties. Both methods recover the true B profile while indicating larger uncertainties near the ice front.

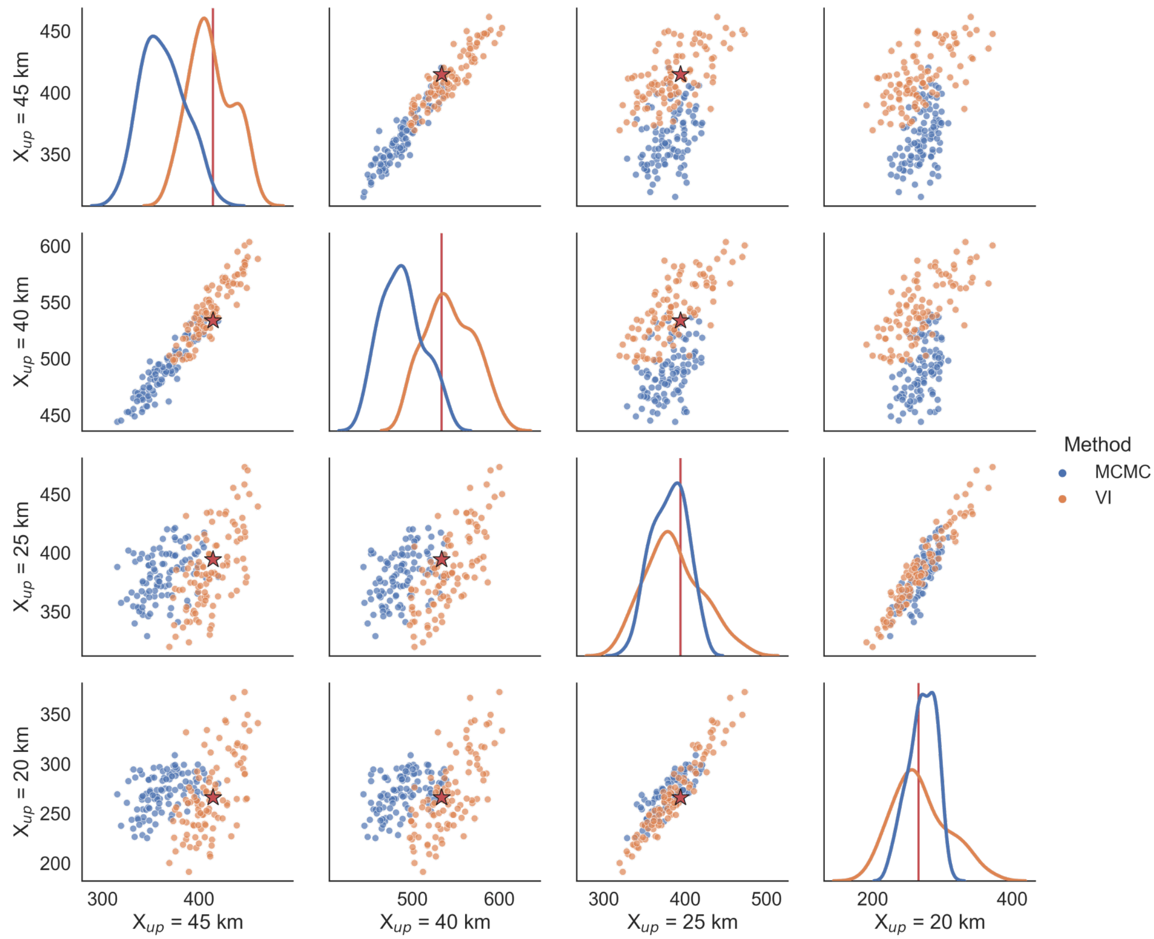

Figure 5. Joint plot of select points within modeling domain showing samples from the posterior distribution of B (blue = MCMC; orange = VI). Diagonal plots show KDE-smoothed 1D marginals (true values shown in red) while off-diagonal plots show random samples. Overall, the PINN+VI approach accurately recovers distance-dependent covariance structure.

FILCHNER-RONNE ICE SHELF

We now apply our PINN+VI methodology to 2D ice flow over the Filcher-Ronne Ice Shelf (FRIS). We use velocity data from NASA MEaSUREs ITS_LIVE and ice thickness measurements from BedMachine V2. For estimating $p(B|u,h)$, we assume $n = 4$.

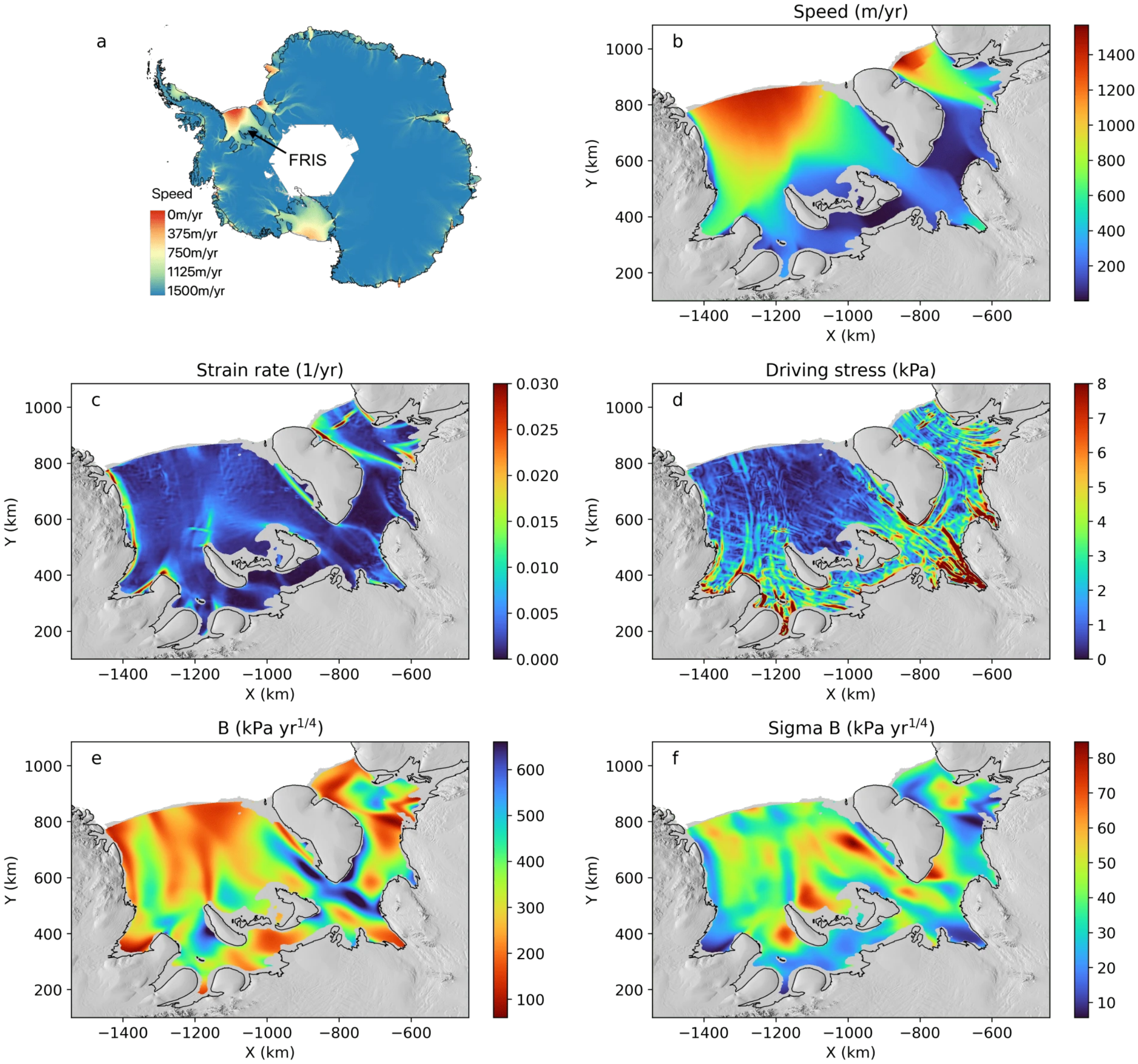

Figure 5. Rheology prefactor estimation for FRIS. a) Location of FRIS. b) Observed flow speed. c) Observed effective strain rates. d) Computed driving stress. e) Estimated mean B. f) Estimated 1-sigma uncertainty for B.

Briefly, areas with higher strain rates (e.g. shear margins) are associated with softer ice. Additionally, certain areas in the shelf interior associated with higher driving stresses correspond to areas with softer ice. Uncertainties are largest in non-deforming areas with stiff ice. Let's now examine some random realizations of the prefactor field (all consistent with SSA momentum balance):

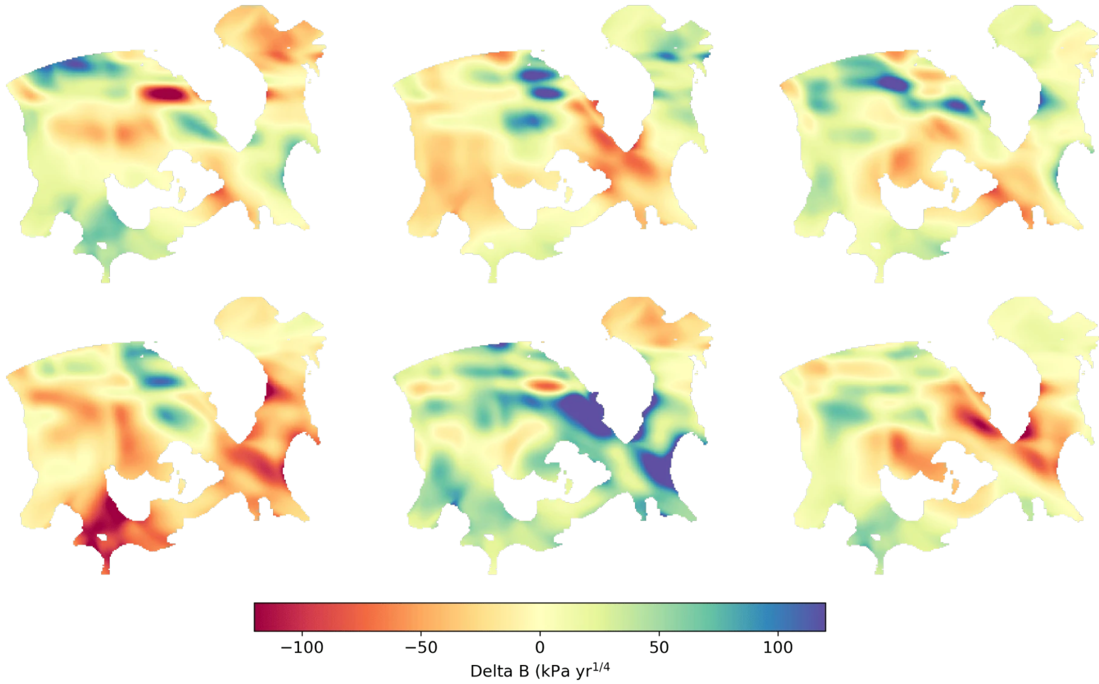

Figure 6. Random realizations of B relative to the mean. Warmer colors correspond to softer ice while cooler colors correspond to stiffer ice. For several samples, we can observe a trade-off between the soft and stiff ice (in the mean field), but other shamples show bulk, shelf-wide variations in the rheology are also probable. These larger-scale variations may potentially be constrained by independent measurements of ice temperature.

FORECASTING IMPLICATIONS

By having realistic *samples* of rheology parameters, informed by data, we can quantify the sensitivity of future ice states to the rheology. Let's revisit our 1D synthetic ice shelf and perturb the ice shelf with a decrease in buttressing stress at the ice front: this perturbation will cause the ice to speed up. We repeat the experiment for 300 random samples from the posterior distribution of *B*.

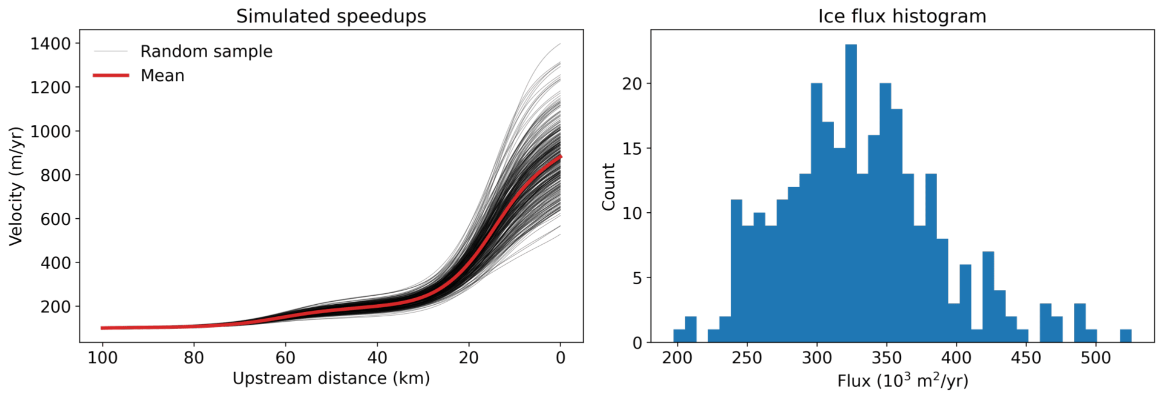

Figure 7. Results of prognostic simulations of ice shelf with decreased buttressing stress (2-year simulation). (Left) Black lines correspond to final shelf velocities for different realizations of B while the red line shows the mean velocity. (Right) Histogram of ice front flux at end of simulation.

From this simple test, we can see a large range in simulated velocity and flux values, even for a constant value of the exponent, $n = 3$. Moreover, the distribution of flux values is skewed, showing a longer tail in higher-flux simulations, which ultimately corresponds to a greater contribution to sea level rise. We expect the sensitivity of these prognostic runs to the exponent n to be even larger, underscoring the need to properly constrain both the prefactor and exponent from data.

FUTURE WORK

Estimating realistic distributions for **both** the prefactor and exponent are critical for properly assessing the sensitivity of ice flow to stress perturbations. We plan to leverage time-dependent strain rate fields and ice thicknesses to, under certain assumptions, simultaneously recover both rheology parameters.

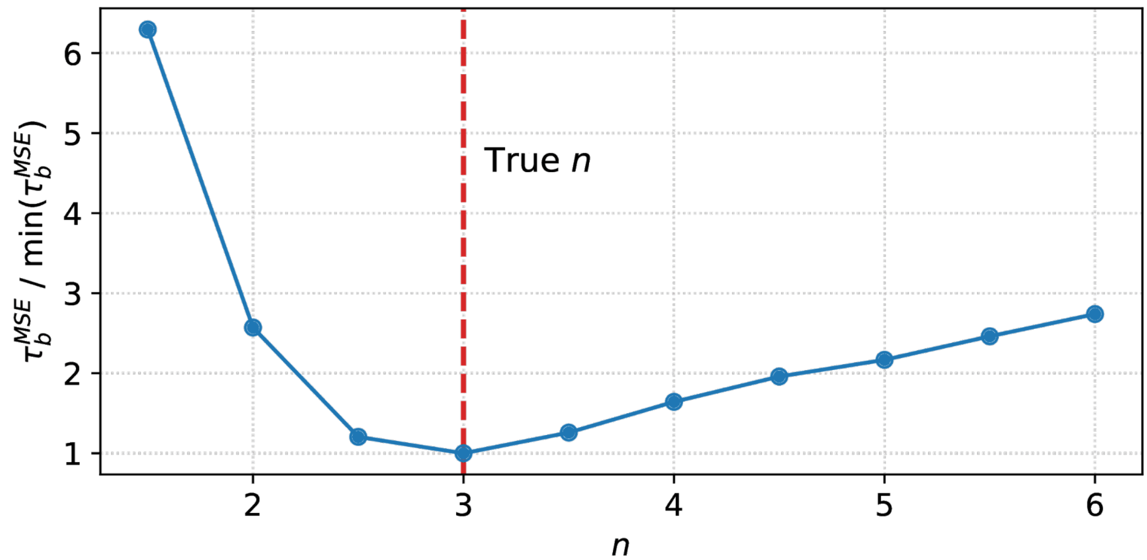

Figure 8. Momentum balance residuals for synthetic ice shelf for different values of n. For each n, we estimate a B profile. Then, holding B fixed, we re-compute the momentum balance on a post-perturbation velocity field. The true value of n is where the ***change in momentum balance residual*** *is minimal.*

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ABSTRACT

The flow and deformation of glacier ice in response to stress is often described using Glen's Flow Law, a power-law relation that compactly represents ice rheology with a prefactor, A, and stress exponent, n. For natural ice, these parameters (and the parameters subsumed within them) come with large uncertainties that have not been robustly quantified with observations. Modern remote sensing technologies that collect data with finer resolutions and broader coverage provide us with an opportunity to robustly calibrate these rheological parameters for certain environments. Here, we utilize publicly available observations of ice sheet surface velocity and elevation acquired with remote sensing platforms to calibrate the flow law parameters over select Antarctic ice shelves. We build upon recent work that used remote sensing observations to quantify the relationship between ice stress and strain rate in extensional flow to infer an exponent of n = 4.1 +/- 0.4 for Antarctica. Here, we model two-dimensional flow and perform parameter calibration by constructing and training physics-informed neural networks (PINNs) to learn spatially-varying A and uniform n for each ice shelf. We cast the parameter estimation problem as a neural network optimization problem through minimization of a cost function that includes both data reconstruction errors and momentum balance residuals derived from the 2D shallow-shelf approximation. Additionally, we formulate the networks to predict spatially-varying uncertainties for A by using variational inference techniques, which approximate Bayesian inference (traditionally a computationally-intensive procedure) as an additional optimization objective. Finally, we demonstrate the use of time-dependent surface velocities, which are becoming increasingly more available over the ice sheets, to independently constrain the stress exponent n, confirming the appropriateness of n = 4 derived from previous work. Overall, calibration of these parameters with robust uncertainties are critical for placing observational constraints on prognostic ice flow model parameters and to improve our understanding of flow and fracture processes on ice shelves in Antarctica.

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