3D PROBABILISTIC WELL-LOG ANALYSIS WITH UNCERTAIN LOCATION DATA

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ABSTRACT

We developed a method for quantification of uncertainties on spatial data acquired at points with uncertain location coordinates. We applied it to well-log data from wells with uncertain trajectories. The method is fully probabilistic and requires few *a priori* assumptions on the Earth model and the wellbore geometry. We performed simulations of well trace and well-logging showing how the total uncertainty of well-log data vary according to the variability of subsurface properties, and how the reliability of well-log interpretations close to layer boundaries may be compromised.

Keywords: subsurface imaging, uncertainty quantification, natural resources, computational modelling, applied geophysics.

INTRODUCTION

Quantification of uncertainties on model parameters derived from data acquired in the laboratory, in the field or in space, is well understood and well described in geophysics literature (Tarantola, 2005). However, with few exceptions (e.g., Winkler 1917), most applications of this theory to spatial data is based on the assumption that measurement locations are known exactly. I many cases this assumption is satisfactory, but under conditions where it is practically difficult to control where measurements are made, the situation is different. When measuring properties with large spatial variations over small length scales, uncertain measurement locations will be seen in the data as uncertainties in the measured properties, and in some cases the location uncertainty may even overwhelm the uncertainty of the properties we wish to measure.

Practical difficulties with determination of observation coordinates is seen in subsurface drilling operations as described here, but also in remote sensing from space- or aircrafts, and in remotely controlled measurements in planetary exploration.

The uncertainties in these cases and the interference with interpretation and model accuracy have been subject of many investigations. In a study carried out by *Hidalgo-Carrió et all (2017),* Gaussian processes for odometry were used to estimate the errors in location of test rovers, built to explore geological data on Mars. In an environment such as this, deprived from the Global Positioning System, instead using an Inertial Navigation System, the precision of the location in data acquisition turns out to be an immense problem with direct impact on the feasibility of remote interpretations.

Another important example where data is strongly influenced by location accuracy is satellite technologies used in Geographic Information System (GIS). Satellites in the Global Positioning System provide data for geological surveys, geospatial imagery data and machine learning for pattern recognition such as mineral mapping, forestry and agriculture cover, management of resources, natural disasters, among many others. As in *Dixon (1991)*, the uncertainties in satellites orbits are one of the major obstacles in the capability of such technologies and have been a subject of study for the past few decades.

Similarly, surveying and mapping the subsurface offers great challenges. Horizontal wells are drilled by companies exploring natural resources, collecting well-log data at specific locations. These locations are recorded with a depth and horizontal position. As the drilling goes deeper and further, the uncertainties in these locations propagate, increasing the possibility of misinterpretations. The consequences are significant; from increasing the costs of the drilling operation itself, to making wrong decisions and interpretations of the geological structures.

Several numerical modelling studies using well trajectories have been done, aiming to tackle the geosteering problem using a variety of different approaches. As in a study carried out by Eidsvik and Hokstad (2006), seismic data in the form of VSP travel times were used to estimate the trajectory of the well, the geological structures and the seismic velocities. Another work carried out by Kullawan et al. (2014) demonstrated how to measure the distance to bed boundaries near the drill-bit using deep-directional resistivity logs.

Unfortunately, most models do not include these uncertain locations as parameters, so there is a deficiency of knowledge in the quantification of errors in well-location data. Nonetheless, *Winkler (2017)* formulated a probabilistic inverse problem approach using Bayesian networks to include uncertain wellbore locations in the 2D geosteering case. He successfully analyzed the problem in a probabilistic context, thereby providing a quantification of the uncertainties involved when measuring uncertain log-data at an uncertain well trajectory. In his study, errors were considered to be perpendicular to the well trace, however, it is observed in field that the errors behave differently vertical and horizontally.

To have a more complete approach for this problem, we formulated a 3D statistical model considering vertical and lateral errors in relation to the well trajectory, along with errors in well-log measurements. We based our approach on the general inverse problem theory given by Tarantola (1982, 2005). In our formulation, the observed data are the rock properties and their measured positions. The model parameters to be determined are the true well log recordings and their true observation coordinates.

Errors were accounted for in both sets of measurements, well data and location data. Position errors are not treated as independent: a physical measurement will be recorded at a position with 3 directional components and the next measured point will also include the previous error. In this model, each wellbore point has an error associated with depth and 2D horizontal location which is inherited by the next neighbouring point in the well. Hence, the error propagates throughout the well trace and we see how the observed well, which is believed to be crossing through a certain layer of interest of the subsurface, could potentially be in another. This mislocation effect introduces errors that can be accurately quantified. In our study, several wells were simulated, and the uncertainties of the measurements and locations were calculated.

Winkler's assumption of parallel - but not necessarily plane - layering introduces a coupling between geology and well trajectory that adds to the posterior information about the system. We avoid the parallel layering assumption but, in that way, relinquish information about the geological structure around the well trajectory. On the other hand, it allows us to more accurately analyse how an uncertain well trajectory influences the uncertainty of well-log information. Our prior assumptions are, in general, more conservative than Winkler's: in addition to allowing any variation in geological structure and properties, we also avoid imposing strong prior information on the well trajectory. Winkler's choice was a reasonable one, namely, to use the planned well trajectory as part of the prior information. However, in order to avoid a too strong bias from this information, we have chosen to rely only on the measured well trajectory.

In this paper we present a revised inverse problem formulation applied to a realistic subsurface property model, and with all sources of uncertainty included in the formulation. In our study, we have used a Monte Carlo approach to evaluate solutions and uncertainties. This is in contrast to Winkler (2017) where an exhaustive evaluation of probabilities was used. Winkler met the computational challenges of his method through an elegant, discrete Bayesian network formulation, but in our case, where a more general geological model is considered, an exhaustive analysis would be infeasible, if not impossible. The Monte Carlo approach is therefore a natural choice, and our implementation is able to analyze real-size problems efficiently, even on present-time PCs. In the theory and methods section, we present the revised mathematical formulation and the numerical approach. In the results and discussion session we present the outcome of the simulations and their interpretations.

THEORETICAL FORMULATION

The theory used in this paper is based on Tarantola and Valette's formulation (1982) and Tarantola (2005).

For unknown parameters $\mathbf{x}$ the general solution of the inverse problem is given by the posterior probability distribution:

$$\sigma(\mathbf{x}) = \rho_x(\mathbf{x}) L_b(\mathbf{x})$$

where $\rho_x(\mathbf{x})$ is the prior probability distribution of the unknown model parameters $\mathbf{x}$, and $L_d(\mathbf{x})$ is the likelihood function describing the degree of fit between observed data $\mathbf{b}_{\mathrm{obs}}$ and the computed data $\mathbf{b}(\mathbf{x})$. Since we have two sources of measured data in this problem: the physical properties (well-log data) $\mathbf{d}$ collected at points in space, and their respective position coordinates $\mathbf{c}$, the forward relation connecting model parameters and data, $\mathbf{b} = g_x(\mathbf{x})$, becomes:

$$\begin{pmatrix}\mathbf{d}\\ \mathbf{c}\end{pmatrix} \equiv \mathbf{b} = g_x(\mathbf{x}) \equiv g_{m,r}(\mathbf{m}, \mathbf{r}) = \begin{pmatrix}\mathbf{d}(\mathbf{m})\\ \mathbf{c}(\mathbf{r})\end{pmatrix} = \begin{pmatrix}\mathbf{m}\\ \mathbf{r}\end{pmatrix}$$

where $\mathbf{m}$ is the true physical parameters (noise-free well-log data to be estimated), and $\mathbf{r}$ are true measurement positions. $g_{m,r}$ is the function that maps true parameters into the observed data.

The likelihood function can be expressed as:

$$L_{d,c}(\mathbf{m}, \mathbf{r}) = \rho_{d,c}(g_{m,r}(\mathbf{m}, \mathbf{r}))$$

(Tarantola and Valette, 1982). Assuming that the physical observation noise is independent of the measured positions and its uncertainty:

$$\rho_{d,c}(\mathbf{d}, \mathbf{c}) = \rho_d(\mathbf{d}) \rho_c(\mathbf{c})$$

we obtain the joint posterior:

$$\begin{aligned}\sigma_{m,r}(\mathbf{m}, \mathbf{r}) &= \rho_{m,r}(\mathbf{m}, \mathbf{r}) L_{d,c}(\mathbf{m}, \mathbf{r}) \\ &= \rho_{m,r}(\mathbf{m}, \mathbf{r}) L_d(\mathbf{m}) L_c(\mathbf{r}) \,.\end{aligned}$$

For the joint positioning and observation problem we typically know the prior of the positions $\rho_r(\mathbf{r})$, but the prior of the observations is only known *conditioned* on the position, $\rho_{m|r}(\mathbf{m}|\mathbf{r})$, and this leads to:

$$\sigma_{m,r}(\mathbf{m}, \mathbf{r}) = L_d(\mathbf{m}) \rho_{m|r}(\mathbf{m}|\mathbf{r}) L_c(\mathbf{r}) \rho_r(\mathbf{r}) \,.$$

NUMERICAL METHOD

We simulate a deviated well in an 7,2 $km \times 8\ km$ geographic area, with a maximum well depth of 2 km (*Figure 1*). In our numerical experiments we consider 50 observation points in the well whose space coordinates have been observed during the drilling process. We assume that our final goal will be to correlate deviated wells and seismic data, so our geological structure model is an acoustic impedance cube, in this case taken from the North Sea F3 Demo 2016 training v6 dataset, Offshore Netherlands (https://terranubis.com/ datainfo/F3-Demo-2016). In our drilling simulations we assume cumulative location errors where each increment (between successive observation points) introduces a Gaussian error.

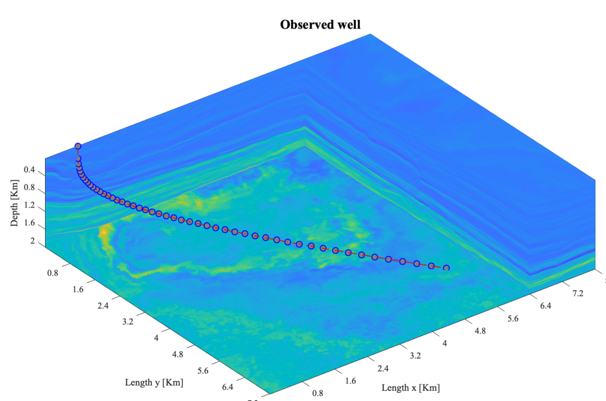

Figure 1 - Observed well positions

The incremental errors are identical at all observation points and calibrated to give a specific cumulative error at the bottom of the well. Standard deviations of the horizontal coordinates at the bottom point are 30 m, whereas the standard deviation of the depth coordinate is 3 m. For the sake of illustration, we have chosen relatively few observation points, and horizontal errors that are in the high end, compared to most practical cases. We assume that sonic- and density logs have been measured in the well, allowing us to compute the acoustic impedance at all observation points. The acoustic impedances thus calculated are assumed to have an uncertainty of 5%.

Our task is to compute the *a posteriori* probability distributions for locations of the observation points, as well as for the acoustic impedances in the well observation points.

The geosteering problem is a joint *tracking* and observation problem. This means that, if the measurement positions are written

$$\mathbf{c} = (\mathbf{c}_1 \ \ldots, \mathbf{c}_K)$$

where $\mathbf{c}_i$ is the position vector of the i'th measurement point, $i = 1$ and $i = K$ indicating the first and the last measurement points respectively. The prior uncertainty of $\mathbf{c}$ then has the form:

$$\rho_c(\mathbf{c}) = \prod_{i=1}^{K-1} \rho_c(\mathbf{c}_{i+1}|\mathbf{c}_i)$$

expressing that the typical situation in tracking is that the position and uncertainty of the $(i+1)$'th point depends on the position and uncertainty of the i'th point. We can now write the tracking likelihood function as

$$L_c(\mathbf{r}) = \prod_{i=1}^{K-1} \rho_c(\mathbf{h}_{i+1}(\mathbf{r}_i))$$

where $\mathbf{h}$ is the function mapping $\mathbf{r}_i$ into $\mathbf{r}_{i+1}$.

In contrast to Winkler (2017) who used a prior $\rho_r(\mathbf{r})$ centred at the observed well positions, we use a constant prior for $\mathbf{r}$. In this study we also refrain from injecting any prior information about the well-log parameters, based on their measurement coordinates. This means that $\rho_{m|r}(\mathbf{m}|\mathbf{r})$ is also assumed constant in our calculations.

Since the well-log data can be expressed $\mathbf{d} = \mathbf{m} + \mathbf{n}$, where $\mathbf{n}$ is Gaussian noise, the likelihood function $L_d(\mathbf{m})$ is equal to the distribution of $\mathbf{d}$, centred at $\mathbf{m}$ with the same standard deviation as the noise $\mathbf{n}$.

Our algorithm now proceeds by sampling the tracking posterior $\sigma_{m,r}(\mathbf{m}, \mathbf{r})$ through simulation of 100 well traces with cumulative errors, growing along the wells (*figure 2*) and assuming zero error at the surface. Samples of the posterior distribution $\sigma_{m,r}(\mathbf{m}, \mathbf{r})$ of acoustic impedances and true observation coordinates $\mathbf{r}$ are now found by Monte Carlo sampling:

1. Generate well trajectories randomly according to the product distribution $L_c(\mathbf{r})\rho_r(\mathbf{r})$. Since $\rho_r(\mathbf{r})$ is assumed constant, this amounts to producing samples from $L_c(\mathbf{r})$ through a simple, sequential generation of well points where each increment of the well trajectory introduces an error (here Gaussian).
2. Conditioned on the location coordinate samples $\mathbf{r}$, generate well-log data randomly according to the product distribution. $L_d(\mathbf{m})\rho_{m|r}(\mathbf{m}|\mathbf{r})$. The combined samples $(\mathbf{m}, \mathbf{r})$ generated in this way are samples from the posterior $\sigma_{m,r}(\mathbf{m}, \mathbf{r})$.

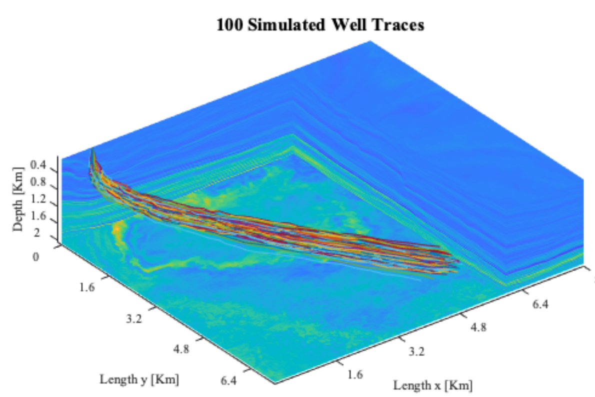

Figure 2 - Simulations of 100 wells with cumulative errors.

RESULTS

On *figure 1*, the observed well is shown in a cross section of the acoustic impedance model. In each of the drilled observation points well data have been measured, along with their positions.

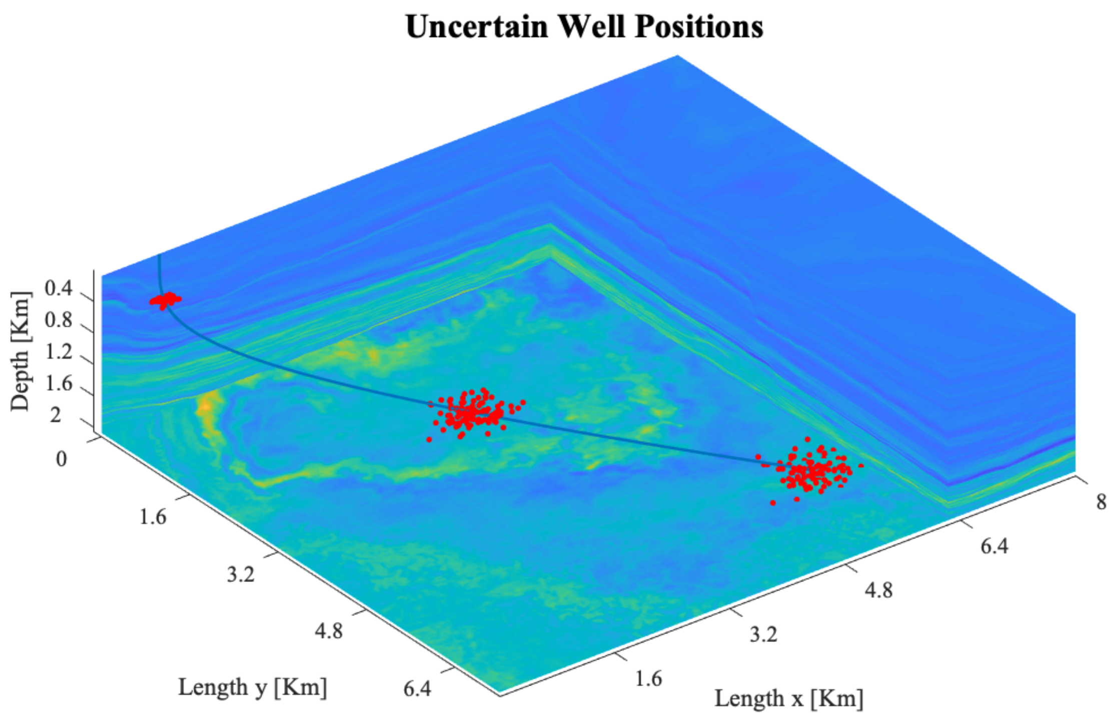

Figure 3 - Observed well with the simulated positions showing cumulative errors.

We simulated 100 well trajectories with cumulative errors growing along the well (*figure 2*). The errors were zero at the top of the well, and it was assumed that the depth errors were 10 times smaller than the horizontal errors.

On *figure 3* is shown the cloud of points representing the possible positions of 3 points at from the 100 simulated well traces, and *figure 4* shows how the errors in depths of the well points grow along the well. In figure 3 it is observed that, because the locations become more uncertain at the bottom of the well, they coincide with different layers of the acoustic impedance model.

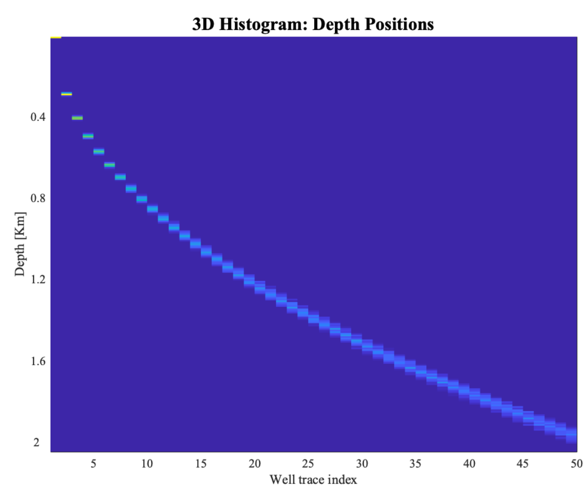

Figure 4 - Histograms of the sampled depths of the 50 well points.

The uncertainty of positions adds to the total uncertainty on measured rock properties. *Figure 5* shows the distribution of the x-coordinate of measurement locations collected at the cloud of points represented on *figure 3*. The general trend is that the measured coordinates vary significantly with depth.

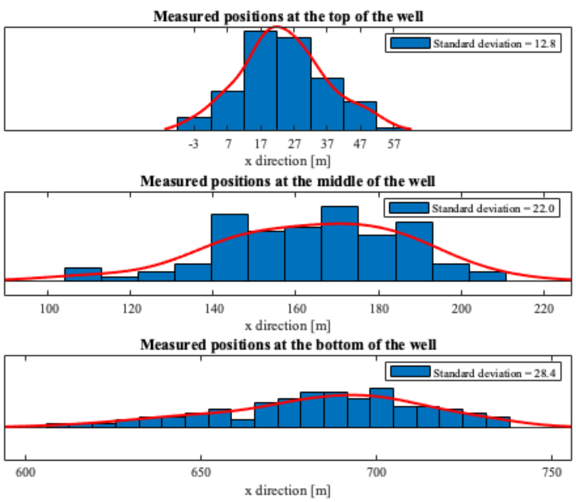

Figure 5 - Histograms of the measured positions on x direction at three different locations.

This is also seen in the growth of the standard deviation of the well depth coordinate in *figure 6*. This curve shows that the error grows as the square root of the depth (as expected from cumulated Gaussian errors).

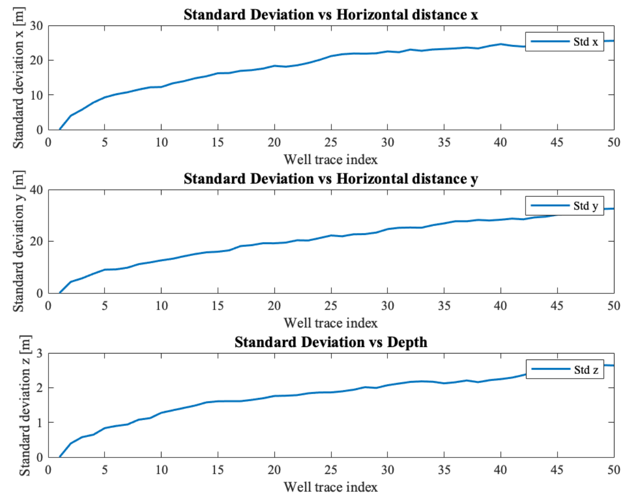

Figure 6 - Standard deviation of the positions in x, y and z directions growing with the depth.

However, a practically very significant effect of well location uncertainties is seen in *figure 7*. This figure shows the posterior distribution of rock properties at each well point. It is seen that the total uncertainties of these parameters are, in general, significantly higher at locations with large spatial variations in rock properties. The total error in this example is up to 2.5 times larger than the error associated with the well-log measurement itself. The errors are locally so large that correlation with seismic data may be compromised.

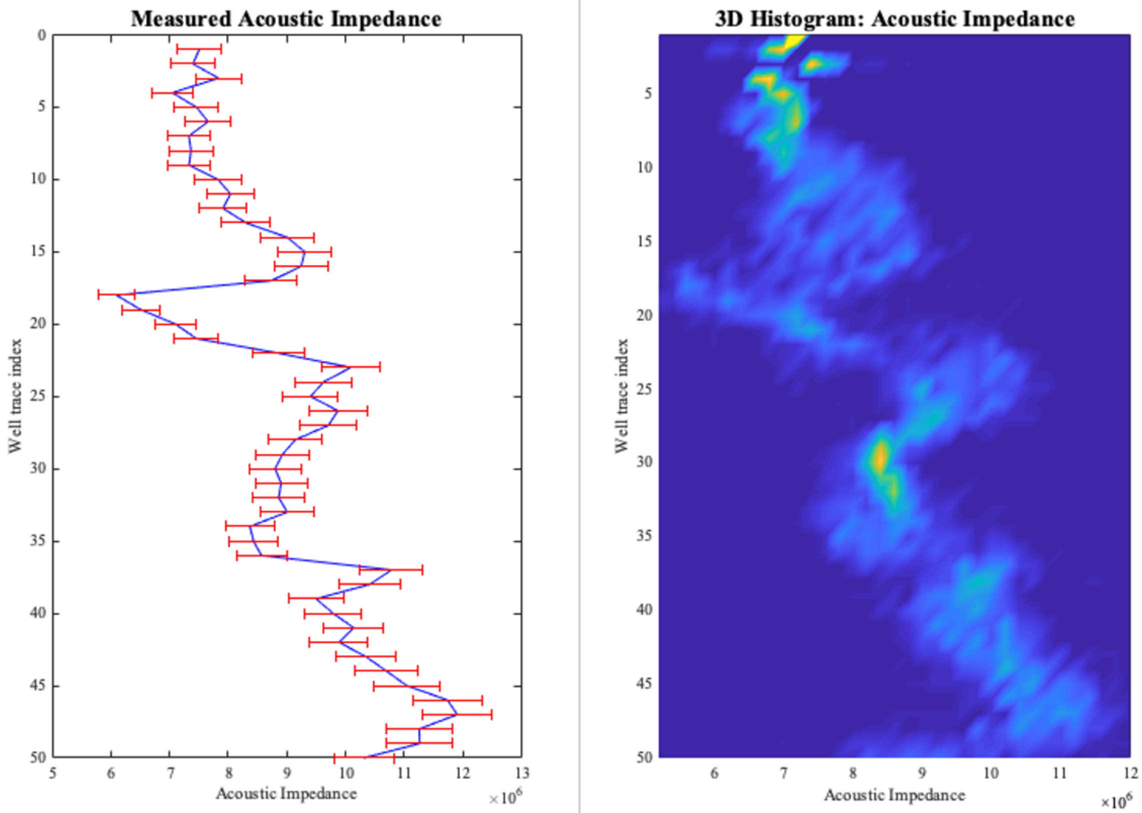

Figure 7 – Left: Simulated well-log measurements with instrumental errors. Right: Posterior distribution of the measured rock property. The total (location + measurement) is locally large compared to the measurement error itself.

DISCUSSION

Knowing measurements in a well, together with the associated instrumental errors, is not sufficient to quantify the total error in the measurements. In this paper we have shown that measurement location errors together with large variations in rock properties can generate significant total errors. The amount of error will be highly dependent on the specific conditions, including drilling technique and geological structure and properties. For clarity of presentation we have looked at drilling errors in the high range, but our rock property model is, on the other hand rather smooth (an acoustic impedance model produced by seismic inversion). For highly varying rock properties, the effect may be stronger than seen in this study.

The theory developed in this paper is fully Bayesian/probabilistic, allowing all uncertainties to be considered in the analysis. We made a conservative choice in the selection of a priori information about the location of the well trajectory, assuming a 'non-informative', constant prior distribution. The same strategy was followed in the selection of prior distribution of the dependence of rock properties on spatial position. Also, here we chose a constant distribution, but in some cases, it could be meaningful to use this conditional prior to express previous experience about the spatial distribution of rock properties in a given area.

CONCLUSION

We have presented a method to quantify how errors in geosteering influences analysis of well log data. We showed that the total uncertainties on rock properties measured in well logs may be significantly higher than the basic instrumental errors, when uncertainties in measuring positions are taken into account. In some cases, the errors may be so large that correlation with seismic data may be compromised.

REFERENCES

Dixon, T. H. (1991), An introduction to the global positioning system and some geological applications, *Rev.Geophys.*, 29(2), 249– 276, doi:10.1029/91RG00152.

Eidsvik, J., and K. Hokstad, 2006, Positioning drill-bit and look-ahead events using seismic traveltime data: Geophysics, 71, no. 4, F79–F90, doi: 10.1190/1.2212268.

J. Hidalgo-Carrió, D. Hennes, J. Schwendner and F. Kirchner, "Gaussian process estimation of odometry errors for localization and mapping," *2017 IEEE International Conference on Robotics and Automation (ICRA)*, Singapore, 2017, pp. 5696-5701, doi: 10.1109/ICRA.2017.7989670.

Jamieson, A., 2012, Introduction to wellbore positioning, v01.05.14 ed.: University of the Highlands and Islands.

Kullawan, K., R. Bratvold, and J. E. Bickel, 2014, A decision analytic approach to geosteering operations: SPE Drilling & Completion, 29, 36–46, doi: 10.2118/167433-PA.

Mosegaard, K., and Hansen, T. M., 2016. Inverse Methods: Problem formulation and Probabilistic Solution. Chapter 2, Integrated Imaging of the Earth, Theory and Applications. AGU, ISBN: 978-1-118-92905-6.

Mosegaard, K., and Tarantola, A., 1995. Monte Carlo sampling of solutions to inverse problems: Journal of Geophysical Research 100, B7, pp. 12431-12447.

Tarantola, A., and B. Valette, 1982, Inverse problems = quest for information: Journal of Geophysics Research, 50, 150-170.

Tarantola, A., Inverse Problem Theory and Methods for Model Parameter Estimation. Siam, Philadelphia, 2005.

Williamson, H. S., 1999, Accuracy prediction for directional MWD: Pre- sented at the SPE Annual Technical Conference and Exhibition.

Winkler, H., Geosteering by exact inference on a Bayesian network, 2017, Geophysics, Vol. 82, No. 5 (September-October 2017); P. D279–D291.