

# Dual carbonate clumped isotopes ( $\Delta_{47}$ - $\Delta_{48}$ ) constrains kinetic effects and timescales in peridotite-associated springs at The Cedars, Northern California

## Supplementary Material-1

### Modeling Equations and Parameters

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### S.1 IsoDIC Model Equations

The following numerical values have been assigned to the respective isotopologue:  $^{12}\text{C} = 2$ ,  $^{13}\text{C} = 3$ ,  $^{16}\text{O} = 6$ ,  $^{17}\text{O} = 7$ ,  $^{18}\text{O} = 8$ . For example,  $\text{H}^{12}\text{C}^{18}\text{O}^{16}\text{O}^{17}\text{O} = \text{H2867}$ .

1.  $266 + \text{H}_26 \rightleftharpoons \text{H2666} + \text{H}^+$
2.  $267 + \text{H}_26 \rightleftharpoons \text{H2667} + \text{H}^+$
3.  $268 + \text{H}_26 \rightleftharpoons \text{H2668} + \text{H}^+$
4.  $277 + \text{H}_26 \rightleftharpoons \text{H2677} + \text{H}^+$
5.  $278 + \text{H}_26 \rightleftharpoons \text{H2678} + \text{H}^+$
6.  $288 + \text{H}_26 \rightleftharpoons \text{H2688} + \text{H}^+$
7.  $266 + \text{H}_27 \rightleftharpoons \text{H2667} + \text{H}^+$
8.  $267 + \text{H}_27 \rightleftharpoons \text{H2667} + \text{H}^+$
9.  $267 + \text{H}_27 \rightleftharpoons \text{H2678} + \text{H}^+$
10.  $277 + \text{H}_27 \rightleftharpoons \text{H2777} + \text{H}^+$
11.  $278 + \text{H}_27 \rightleftharpoons \text{H2778} + \text{H}^+$
12.  $288 + \text{H}_27 \rightleftharpoons \text{H2788} + \text{H}^+$
13.  $266 + \text{H}_28 \rightleftharpoons \text{H2668} + \text{H}^+$
14.  $267 + \text{H}_28 \rightleftharpoons \text{H2678} + \text{H}^+$
15.  $268 + \text{H}_28 \rightleftharpoons \text{H2688} + \text{H}^+$
16.  $277 + \text{H}_28 \rightleftharpoons \text{H2778} + \text{H}^+$
17.  $278 + \text{H}_28 \rightleftharpoons \text{H2788} + \text{H}^+$
18.  $288 + \text{H}_28 \rightleftharpoons \text{H2888} + \text{H}^+$
19.  $266 + \text{H}_26 \rightleftharpoons \text{H3666} + \text{H}^+$
20.  $366 + \text{H}_26 \rightleftharpoons \text{H3667} + \text{H}^+$

21.  $377 + \text{H}_26 \rightleftharpoons \text{H3668} + \text{H}^+$
22.  $378 + \text{H}_26 \rightleftharpoons \text{H3677} + \text{H}^+$
23.  $378 + \text{H}_26 \rightleftharpoons \text{H3678} + \text{H}^+$
24.  $388 + \text{H}_26 \rightleftharpoons \text{H3688} + \text{H}^+$
25.  $366 + \text{H}_27 \rightleftharpoons \text{H3667} + \text{H}^+$
26.  $367 + \text{H}_27 \rightleftharpoons \text{H3677} + \text{H}^+$
27.  $368 + \text{H}_27 \rightleftharpoons \text{H3678} + \text{H}^+$
28.  $377 + \text{H}_27 \rightleftharpoons \text{H3777} + \text{H}^+$
29.  $378 + \text{H}_27 \rightleftharpoons \text{H3778} + \text{H}^+$
30.  $388 + \text{H}_27 \rightleftharpoons \text{H3788} + \text{H}^+$
31.  $366 + \text{H}_28 \rightleftharpoons \text{H3668} + \text{H}^+$
32.  $367 + \text{H}_28 \rightleftharpoons \text{H3678} + \text{H}^+$
33.  $368 + \text{H}_28 \rightleftharpoons \text{H3688} + \text{H}^+$
34.  $377 + \text{H}_28 \rightleftharpoons \text{H3778} + \text{H}^+$
35.  $378 + \text{H}_28 \rightleftharpoons \text{H3788} + \text{H}^+$
36.  $388 + \text{H}_28 \rightleftharpoons \text{H3888} + \text{H}^+$
37.  $266 + 6\text{H} \rightleftharpoons \text{H2666}$
38.  $267 + 6\text{H} \rightleftharpoons \text{H2667}$
39.  $268 + 6\text{H} \rightleftharpoons \text{H2668}$
40.  $277 + 6\text{H} \rightleftharpoons \text{H2677}$
41.  $278 + 6\text{H} \rightleftharpoons \text{H2678}$
42.  $288 + 6\text{H} \rightleftharpoons \text{H2688}$
43.  $266 + 7\text{H} \rightleftharpoons \text{H2667}$
44.  $267 + 7\text{H} \rightleftharpoons \text{H2677}$
45.  $268 + 7\text{H} \rightleftharpoons \text{H2678}$
46.  $277 + 7\text{H} \rightleftharpoons \text{H2777}$
47.  $278 + 7\text{H} \rightleftharpoons \text{H2778}$
48.  $288 + 8\text{H} \rightleftharpoons \text{H2788}$
49.  $266 + 8\text{H} \rightleftharpoons \text{H2668}$
50.  $267 + 8\text{H} \rightleftharpoons \text{H2678}$
51.  $268 + 8\text{H} \rightleftharpoons \text{H2688}$
52.  $277 + 8\text{H} \rightleftharpoons \text{H2778}$
53.  $278 + 8\text{H} \rightleftharpoons \text{H2788}$
54.  $288 + 8\text{H} \rightleftharpoons \text{H2888}$
55.  $366 + 6\text{H} \rightleftharpoons \text{H3666}$
56.  $367 + 6\text{H} \rightleftharpoons \text{H3667}$
57.  $368 + 6\text{H} \rightleftharpoons \text{H3668}$
58.  $377 + 6\text{H} \rightleftharpoons \text{H3677}$
59.  $378 + 6\text{H} \rightleftharpoons \text{H3678}$
60.  $388 + 6\text{H} \rightleftharpoons \text{H3688}$
61.  $366 + 7\text{H} \rightleftharpoons \text{H3667}$
62.  $367 + 7\text{H} \rightleftharpoons \text{H3677}$
63.  $368 + 7\text{H} \rightleftharpoons \text{H3678}$
64.  $377 + 7\text{H} \rightleftharpoons \text{H3777}$
65.  $378 + 7\text{H} \rightleftharpoons \text{H3778}$
66.  $388 + 7\text{H} \rightleftharpoons \text{H3788}$

67.  $366 + 8H \rightleftharpoons H3668$   
68.  $367 + 8H \rightleftharpoons H3678$   
69.  $368 + 8H \rightleftharpoons H3688$   
70.  $377 + 8H \rightleftharpoons H3778$   
71.  $378 + 8H \rightleftharpoons H3788$   
72.  $388 + 8H \rightleftharpoons H3888$   
73 – 152.  $H^iC^jO^kO^lO^- + {}^mOH^- \rightleftharpoons {}^iC^jO^kO^lO^2 + H_2{}^mO$   
where  $i = 12, 13, 16 \leq j \leq k \leq l \leq 18, m = 16, 17, 18$   
153 – 155.  $H_2{}^mO \rightleftharpoons {}^mOH^- + H^+$   
where  $m = 16, 17, 18$

The above reactions and their isotope effects were used by Guo (2020) to calculate intrinsic kinetic clumped isotope fractionation factors (KFFs) associated with CO<sub>2</sub> hydration and hydroxylation and their reverse reactions.

## S.2 COAD Model Equations

### S.2.1 Hydration and hydroxylation reactions

- |   |  |
|---|--|
| 1. $266 + H_26 \rightleftharpoons H2666 + H$  | (forward = $k_{+1}$ ) (reverse = $k_{-1}$ )      |
| 2. $266 + 6H \rightleftharpoons H2666$        | (forward = $k_{+4}$ ) (reverse = $k_{-4}$ )      |
| 3. $366 + H_26 \rightleftharpoons H3666 + H$  | (forward = $c_{+1}$ ) (reverse = $c_{-1}$ )      |
| 4. $366 + 6H \rightleftharpoons H3666$        | (forward = $c_{+4}$ ) (reverse = $c_{-4}$ )      |
| 5. $266 + H_28 \rightleftharpoons H2866 + H$  | (forward = $a_{+1}$ ) (reverse = $1/3a_{-1}$ )   |
| 6. $286 + H_26 \rightleftharpoons H2866 + H$  | (forward = $b_{+1}$ ) (reverse = $2/3b_{-4}$ )   |
| 7. $266 + 8H \rightleftharpoons H2866$        | (forward = $a_{+4}$ ) (reverse = $1/3a_{-4}$ )   |
| 8. $286 + 6H \rightleftharpoons H2866$        | (forward = $b_{+4}$ ) (reverse = $2/3b_{-4}$ )   |
| 9. $366 + H_28 \rightleftharpoons H3866 + H$  | (forward = $p_{+1}$ ) (reverse = $1/3p_{-1}$ )   |
| 10. $386 + H_26 \rightleftharpoons H3866 + H$ | (forward = $s_{+1}$ ) (reverse = $2/3s_{-1}$ )   |
| 11. $366 + 8H \rightleftharpoons H3866$       | (forward = $p_{+4}$ ) (reverse = $1/3p_{-4}$ )   |
| 12. $386 + 6H \rightleftharpoons H3866$       | (forward = $s_{+4}$ ) (reverse = $2/3s_{-1}$ )   |
| 13. $286 + H_28 \rightleftharpoons H2886 + H$ | (forward = $p'_{+1}$ ) (reverse = $2/3p'_{-1}$ ) |
| 14. $288 + H_26 \rightleftharpoons H2886 + H$ | (forward = $s'_{+1}$ ) (reverse = $1/3s'_{-1}$ ) |
| 15. $286 + 8H \rightleftharpoons H2886$       | (forward = $p'_{+4}$ ) (reverse = $2/3p'_{-4}$ ) |
| 16. $288 + 6H \rightleftharpoons H2886$       | (forward = $s'_{+4}$ ) (reverse = $1/3s'_{-4}$ ) |

### S.2.2 Governing equations for the reaction-diffusion model

17.  $\frac{d[266]}{dt} = D_{CO_2} * \frac{d^2[266]}{dz^2} - k_{+1}[266] + k_{-1}[E2666]\chi[H] - k_{+4}[266][6H] + k_{-4}[E2666]\chi$   
18.  $\frac{d[366]}{dt} = D_{CO_2} * \frac{d^2[366]}{dz^2} - c_{+1}[366] + c_{-1}[E3666]^{13}\chi[H] - c_{+4}[366][6H] + c_{-4}[E3666]^{13}\chi$   
19.  $\frac{d[286]}{dt} = D_{CO_2} * \frac{d^2[286]}{dz^2} - b_{+1}[286] + \frac{2}{3}b_{-1}[E2866]^{18}\chi[H] - b_{+4}[286][6H] + \frac{2}{3}b_{-4}[E2866]^{18}\chi$   
20.  $\frac{d[386]}{dt} = D_{CO_2} * \frac{d^2[386]}{dz^2} - p_{+1}[386] + \frac{1}{3}p_{-1}[E3866]^{63}\chi[H] - s_{+4}[386][6H] + \frac{2}{3}s_{-4}[E3866]^{63}\chi$

21.  $\frac{d[288]}{dt} = D_{CO_2} * \frac{d^2[288]}{dz^2} - s'_{+1}[288] + \frac{1}{3}s'_{-1}[E2886]^{64}X[H] - s'_{+4}[288][6H] + \frac{1}{3}s'_{-4}[E2886]^{64}X$
22.  $\frac{d[E2666]}{dt} = D_{CO_2} * \frac{d^2[E2666]}{dz^2} + k_{+1}[266] - k_{-1}[E2666]X[H]k_{+4}[266][6H] - k_{-4}[E2666]X - Sp * J_{CaCO_3}$
23.  $\frac{d[E3666]}{dt} = D_{CO_2} * \frac{d^2[E3666]}{dz^2} + c_{+1}[366] - c_{-1}[E3666]^{13}X[H]c_{+4}[366][6H] - c_{-4}[E3666]^{13}X - Sp * J_{CaCO_3} * \frac{[E3666]}{[E2666]} *^{13} \alpha_{CaCO_3-EIC}$
24.  $\frac{d[E2866]}{dt} = D_{CO_2} * \frac{d^2[E2866]}{dz^2} + a_{+1}[266]r_w - \frac{1}{3}a_{-1}[E2866]^{18}X[H] + a_{+4}[266][8H] - \frac{1}{3}a_{-4}[E2866]^{18}X + b_{+1}[286] - \frac{2}{3}b_{-1}[E2866]^{18}X[H] + b_{+4}[286][6H] - \frac{2}{3}b_{-4}[E2866]^{18}X - Sp * J_{CaCO_3} * \frac{[E2866]}{[E2666]} *^{18} \alpha_{CaCO_3-EIC}$
25.  $\frac{d[E3866]}{dt} = D_{CO_2} * \frac{d^2[E3866]}{dz^2} + p_{+1}[366]r_w - \frac{1}{3}p_{-1}[E3866]^{63}X[H] + p_{+4}[266][8H] - \frac{1}{3}p_{-4}[E3866]^{63}X + s_{+1}[386] - \frac{2}{3}s_{-1}[E3866]^{63}X[H] + s_{+4}[386][6H] - \frac{2}{3}s_{-4}[E3866]^{63}X - Sp * J_{CaCO_3} * \frac{[E3866]}{[E2666]} *^{63} \alpha_{CaCO_3-EIC} *^{13} \alpha_{CaCO_3-EIC} *^{18} \alpha_{CaCO_3-EIC}$
26.  $\frac{d[E2886]}{dt} = D_{CO_2} * \frac{d^2[E2886]}{dz^2} + p'_{+1}[286]r_w - \frac{2}{3}p'_{-1}[E2886]^{64}X[H] + p'_{+4}[286][8H] - \frac{2}{3}p'_{-4}[E2886]^{64}X + s'_{+1}[288] - \frac{1}{3}s'_{-1}[E2886]^{64}X[H] + s'_{+4}[288][6H] - \frac{1}{3}s'_{-4}[E2886]^{64}X - Sp * J_{CaCO_3} * \frac{[E2886]}{[E2666]} *^{64} \alpha_{CaCO_3-EIC} *^{18} \alpha_{CaCO_3-EIC} *^{18} \alpha_{CaCO_3-EIC}$
27.  $\frac{d[Alk]}{dt} = D_{Alk} * \frac{d^2[Alk]}{dz^2} - 2 * Sp * J_{CaCO_3}$
28.  $\frac{d[Ca^{2+}]}{dt} = D_{Ca^{2+}} * \frac{d^2[Ca^{2+}]}{dz^2} - Sp * J_{CaCO_3}$

### S.2.3 Governing equations for the box model

29.  $\frac{d[266]}{dt} = -k_{+1}[266] + k_{-1}[E2666]X[H] - k_{+4}[266][6H] + k_{-4}[E2666]X + \frac{F_{spr}}{V_{pool}}([266]_{spr} - [266]) + \frac{F_{cr}}{V_{pool}}([266]_{cr} - [266])$
30.  $\frac{d[366]}{dt} = -c_{+1}[366] + c_{-1}[E3666]^{13}X[H] - c_{+4}[366][6H] + c_{-4}[E3666]^{13}X + \frac{F_{spr}}{V_{pool}}([366]_{spr} - [366]) + \frac{F_{cr}}{V_{pool}}([366]_{cr} - [266])$
31.  $\frac{d[286]}{dt} = -b_{+1}[286] + \frac{2}{3}b_{-1}[E2866]^{18}X[H] - b_{+4}[286][6H] + \frac{2}{3}b_{-4}[E2866]^{18}X + \frac{F_{spr}}{V_{pool}}([286]_{spr} - [286]) + \frac{F_{cr}}{V_{pool}}([286]_{cr} - [266])$
32.  $\frac{d[386]}{dt} = -p_{+1}[386] + \frac{1}{3}p_{-1}[E3866]^{63}X[H] - s_{+4}[386][6H] + \frac{2}{3}s_{-4}[E3866]^{63}X + \frac{F_{spr}}{V_{pool}}([386]_{spr} - [386]) + \frac{F_{cr}}{V_{pool}}([386]_{cr} - [266])$
33.  $\frac{d[288]}{dt} = -s'_{+1}[288] + \frac{1}{3}s'_{-1}[E2886]^{64}X[H] - s'_{+4}[288][6H] + \frac{1}{3}s'_{-4}[E2886]^{64}X + \frac{F_{spr}}{V_{pool}}([288]_{spr} - [288]) + \frac{F_{cr}}{V_{pool}}([288]_{cr} - [266])$
34.  $\frac{d[E2666]}{dt} = k_{+1}[266] - k_{-1}[E2666]X[H]k_{+4}[266][6H] - k_{-4}[E2666]X + \frac{J_{atm} * S_{A_{pool}}}{V_{pool}} + \frac{F_{spr}}{V_{pool}}([E2666]_{spr} - [E2666]) + \frac{F_{cr}}{V_{pool}}([E2666]_{cr} - [E2666]) - Sp * J_{CaCO_3}$

$$\begin{aligned}
35. \quad \frac{d[E3666]}{dt} &= c_{+1}[366] - c_{-1}[E3666]^{13}X[H]c_{+4}[366][6H] - c_{-4}[E3666]^{13}X \frac{J_{atm} * SA_{pool}}{V_{pool}} * \\
&\quad \left( \frac{[E3666]}{[E2666]} \right)_{hydrox} + \frac{F_{spr}}{V_{pool}} ({}^{13}R_{EIC(spr)}[EIC]_{spr} - [E3666]) + \frac{F_{cr}}{V_{pool}} ([E3666]_{cr} - \\
&\quad [E3666]) - Sp * J_{CaCO_3} \frac{[E3666]}{[E2666]} * {}^{13}\alpha_{CaCO_3-EIC} \\
36. \quad \frac{d[E2866]}{dt} &= a_{+1}[266]r_w - \frac{1}{3}a_{-1}[E2866]^{18}X[H] + a_{+4}[266][8H] - \frac{1}{3}a_{-4}[E2866]^{18}X + \\
&\quad b_{+1}[286] - \frac{2}{3}b_{-1}[E2866]^{18}X[H] + b_{+4}[286][6H] - \frac{2}{3}b_{-4}[E2866]^{18}X + \frac{J_{atm} * SA_{pool}}{V_{pool}} * \\
&\quad \left( \frac{[E2866]}{[E2666]} \right)_{hydrox} + \frac{F_{spr}}{V_{pool}} ([E2866]_{spr} - [E2866]) + \frac{F_{cr}}{V_{pool}} ([E2866]_{cr} - [E2866]) - Sp * \\
&\quad J_{CaCO_3} \frac{[E2866]}{[E2666]} * {}^{18}\alpha_{CaCO_3-EIC} \\
37. \quad \frac{d[E3866]}{dt} &= p_{+1}[366]r_w - \frac{1}{3}p_{-1}[E3866]^{63}X[H] + p_{+4}[266][8H] - \frac{1}{3}p_{-4}[E3866]^{63}X + \\
&\quad s_{+1}[386] - \frac{2}{3}s_{-1}[E3866]^{63}X[H] + s_{+4}[386][6H] - \frac{2}{3}s_{-4}[E3866]^{63}X + \frac{J_{atm} * SA_{pool}}{V_{pool}} * \\
&\quad \left( \frac{[E3866]}{[E2666]} \right)_{hydrox} + \frac{F_{spr}}{V_{pool}} ([E3866]_{spr} - [E3866]) + \frac{F_{cr}}{V_{pool}} ([E3866]_{cr} - [E3866]) - Sp * \\
&\quad J_{CaCO_3} \frac{[E3866]}{[E2666]} * {}^{63}\alpha_{CaCO_3-EIC} * {}^{13}\alpha_{CaCO_3-EIC} * {}^{18}\alpha_{CaCO_3-EIC} \\
38. \quad \frac{d[E2886]}{dt} &= p'_{+1}[286]r_w - \frac{2}{3}p'_{-1}[E2886]^{64}X[H] + p'_{+4}[286][8H] - \\
&\quad \frac{2}{3}p'_{-4}[E2886]^{64}X + s'_{+1}[288] - \frac{1}{3}s'_{-1}[E2886]^{64}X[H] + s'_{+4}[288][6H] - \\
&\quad \frac{1}{3}s'_{-4}[E2886]^{64}X + \frac{J_{atm} * SA_{pool}}{V_{pool}} * \left( \frac{[E2886]}{[E2666]} \right)_{hydrox} + \frac{F_{spr}}{V_{pool}} ([E2886]_{spr} - [E2886]) + \\
&\quad \frac{F_{cr}}{V_{pool}} ([E2886]_{cr} - [E2886]) - Sp * \\
&\quad J_{CaCO_3} \frac{[E2886]}{[E2666]} * {}^{64}\alpha_{CaCO_3-EIC} * {}^{18}\alpha_{CaCO_3-EIC} * {}^{13}\alpha_{CaCO_3-EIC} \\
39. \quad \frac{d[Alk]}{dt} &= \frac{F_{spr}}{V_{pool}} ([Alk]_{spr} - [Alk]) + \frac{F_{cr}}{V_{pool}} ([Alk]_{cr} - [Alk]) - 2 * Sp * J_{CaCO_3} \\
40. \quad \frac{d[Ca^{2+}]}{dt} &= \frac{F_{spr}}{V_{pool}} ([Ca^{2+}]_{spr} - [Ca^{2+}]) + \frac{F_{cr}}{V_{pool}} ([Ca^{2+}]_{cr} - [Ca^{2+}]) - Sp * J_{CaCO_3}
\end{aligned}$$

#### S.2.4 Post-processing: Isotopologue concentrations to isotope ratios

$$\begin{aligned}
41. \quad \delta^{13}C(\text{‰}) &= \left( \frac{{}^{13}R}{0.0118} - 1 \right) * 1000, \\
42. \quad {}^{13}R &= \frac{[3666]}{[2666]}, \\
43. \quad \delta^{18}O(\text{‰}) &= \left( \frac{{}^{3*18}R}{0.0020052} - 1 \right) * 1000, \\
44. \quad {}^{18}R &= \frac{[2866]}{[2666]}, \\
45. \quad \Delta_{47} &= \left( \frac{{}^{47}R}{{}^{47}R^*} - 1 \right) * 1000, \\
46. \quad \frac{{}^{47}R}{{}^{47}R^*} &= \frac{[386][266]}{[366][286]}, \\
47. \quad \Delta_{63} &= \left( \frac{{}^{63}R}{{}^{63}R^*} - 1 \right) * 1000, \\
48. \quad \frac{{}^{63}R}{{}^{63}R^*} &= \frac{[3866][2666]}{[3666][2866]}, \\
49. \quad \Delta_{48} &= \left( \frac{{}^{48}R}{{}^{48}R^*} - 1 \right) * 1000, \\
50. \quad \frac{{}^{48}R}{{}^{48}R^*} &= 4 * \frac{[288][266]}{[286][286]},
\end{aligned}$$

$$51. \Delta_{64} = \left( \frac{{}^{64}R}{{}^{64}R^*} - 1 \right) * 1000,$$

$$52. \frac{{}^{64}R}{{}^{64}R^*} = 3 * \frac{[2886][2666]}{[2866][2866]}$$

### S.2.5 Adding $\Delta_{48}$ to the model

Homogeneous reactions and clumped isotope definitions:

1.  $286 + 286 \rightleftharpoons 288 + 266$
2.  ${}^{48}K_{CO_2} = \frac{[288][266]}{[286][286]}$
3.  $\Delta_{48} = \left( \frac{{}^{48}R}{{}^{48}R^*} - 1 \right) * 1000,$
4.  ${}^{48}R = \frac{[288]}{[266]}$
5.  ${}^{48}K_{CO_2} = \frac{[288][266][266]}{[266][286][286]} = {}^{48}R * (2 * {}^{18}r_{CO_2})^{-1} = \frac{1}{4} \left( \frac{{}^{48}R}{{}^{48}R^*} \right)^{eq}_{CO_2}$
6.  $\frac{[288]}{[266]} = {}^{48}R^* = {}^{18}r_{CO_2}^2.$
7.  ${}^{48}K_{CO_2} = \frac{[288][266]}{[286][286]} = \frac{1}{4} \left( \frac{{}^{48}R}{{}^{48}R^*} \right)^{eq}_{CO_2} = \frac{1}{4} \left( \frac{\Delta_{48,CO_2}^{eq}}{1000} + 1 \right)$
8.  $H2866 + H2866 \rightleftharpoons H2886 + H2666$
9.  $2866 + 2866 \rightleftharpoons 2886 + 2666,$
10.  ${}^{64}K_{HCO_3^-} = \frac{[H2886][H2666]}{[H2866][H2866]} = \frac{1}{3} \left( \frac{{}^{64}R}{{}^{64}R^*} \right)^{eq}_{HCO_3^-} = \frac{1}{3} \left( \frac{\Delta_{64,HCO_3^-}^{eq}}{1000} + 1 \right)$
11.  ${}^{64}K_{CO_3^{2-}} = \frac{[2886][2666]}{[2866][2866]} = \frac{1}{3} \left( \frac{{}^{64}R}{{}^{64}R^*} \right)^{eq}_{CO_3^{2-}} = \frac{1}{3} \left( \frac{\Delta_{64,CO_3^{2-}}^{eq}}{1000} + 1 \right).$

Heterogeneous reactions involving  $\Delta_{48}$  isotopologues:

12.  $286 + H_28 \xrightleftharpoons[2/3p'_{-1}]{p'_{+1}} H2886^- + H^+$
13.  $288 + H_26 \xrightleftharpoons[1/3s'_{-1}]{s'_{+1}} H2886^- + H^+$
14.  $286 + 8H^- \xrightleftharpoons[2/3p'_{-4}]{p'_{+4}} H2886^-$
15.  $288 + 6H^- \xrightleftharpoons[1/3s'_{-4}]{s'_{+4}} H2886^-$
16.  $\frac{d[288]}{dt} = -s'_{+1}[288] + \frac{1}{3}s'_{-1}[E2886]{}^{64}X[H] - s'_{+4}[288][6H] + \frac{1}{3}s'_{-4}[E2886]{}^{64}X$   
 $\frac{d[E2886]}{dt} = p'_{+1}[286]r_w - \frac{2}{3}p'_{-1}[E2886]{}^{64}X[H] + p'_{+4}[286][8H] - \frac{2}{3}p'_{-4}[E2886]{}^{64}X$   
 $+ s'_{+1}[288] - \frac{1}{3}s'_{-1}[E2886]{}^{64}X[H] + s'_{+4}[288][6H] - \frac{1}{3}s'_{-4}[E2886]{}^{64}X$

### S.3 Model Parameters

Symbol	Expression or value at 25°C	Reference
$K_I$	$\frac{[H][H2666]}{[266]}$	-

$K_2$	$\frac{[H][H2666]}{[H2666]}$	-
$K_w$	$\frac{[6H][H]}{[H]}$	-
$k_{+1}$	$\ln k_{+1} = 1246.98 - \frac{61900}{T_K} - 183.0 \ln T_k$	Uchikawa and Zeebe (2012)
$k_{-1}$	$k_{-1} = k_{+1}/K_1$	-
$k_{+4}$	$\ln k_{+4} = 17.67 - \frac{2790.47}{T_K}$	Uchikawa and Zeebe (2012)
$k_{-4}$	$k_{-4} = k_{+4} \frac{K_w}{K_1}$	-
$\chi$	$\chi = \frac{1}{1 + \frac{K_2}{[H^+]}}$	-
<b>Carbon isotope parameters</b>		
$^{13}\alpha_{HCO_3^- - CO_2}^{eq}$	$-9.866 T^{-1} + 1.02412$	Zhang et al. (1995)
$^{13}\alpha_{HCO_3^- - CO_2}^{eq}$	$-0.867 T^{-1} + 1.00252$	Zhang et al. (1995)
$^{13}\alpha_{c+1}^{KFF}$	0.9872	Yumol et al. (2020)
$^{13}\alpha_{c+4}^{KFF}$	0.9814	Christensen et al. (2021)
$c_{+1}$	$c_{+1} = ^{13}\alpha_{c+1}^{KFF} \cdot k_{+1}$	-
$c_{-1}$	$c_{-1} = c_{+1} / \left( K_1 \cdot ^{13}\alpha_{HCO_3^- - CO_2}^{eq} \right)$	-
$c_{+4}$	$c_{+4} = ^{13}\alpha_{c+4}^{KFF} \cdot k_{+4}$	-
$c_{-4}$	$c_{-4} = c_{+4} / \left( \frac{K_1}{K_w} \cdot ^{13}\alpha_{HCO_3^- - CO_2}^{eq} \right)$	-
$^{13}\chi$	$^{13}\chi = \frac{1}{1 + \frac{K_2 \cdot ^{13}\alpha_{HCO_3^- - CO_3}^{eq}}{[H^+]}}$	-
<b>Oxygen isotope parameters</b>		
$^{18}\alpha_{CO_2 - H_2O}^{eq}$	$\exp(2520 T_K^{-2} + 0.01212)$	Beck et al. (2005)
$^{18}\alpha_{HCO_3^- - H_2O}^{eq}$	$\exp(2590 T_K^{-2} + 0.00189)$	Beck et al. (2005)
$^{18}\alpha_{CO_3^{2-} - H_2O}^{eq}$	$\exp(2390 T_K^{-2} - 0.00270)$	Beck et al. (2005)
$^{18}\alpha_{OH^- - H_2O}^{eq}$	$5.6676 \times 10^{-5} T_K + 0.9622$	Zeebe (2020)
$^{18}\alpha_{a+1}^{KFF}$	1.0000	Yumol et al. (2020)
$^{18}\alpha_{b+1}^{KFF}$	0.9812	Yumol et al. (2020)
$^{18}\alpha_{a+4}^{KFF}$	0.971 at 5 °C 0.978 at 25 °C	Watkins and Devriendt (2021)
$^{18}\alpha_{b+4}^{KFF}$	1.0000	Christensen et al. (2021)
$a_{+1}$	$a_{+1} = ^{18}\alpha_{a+1}^{KFF} \cdot k_{+1}$	-

$b_{+1}$	$b_{+1} = {}^{18}\alpha_{b_{+1}}^{KFF} \cdot k_{+1}$	-
$a_{-1}$	$a_{-1} = a_{+1}/(K_1 \cdot {}^{18}\alpha_{HCO_3^- - H_2O})$	-
$b_{-1}$	$b_{-1} = b_{+1}/(K_1 \cdot {}^{18}\alpha_{HCO_3^- - CO_2})$	-
$a_{+4}$	$a_{+4} = {}^{18}\alpha_{a_{+4}}^{KFF} \cdot k_{+4}$	-
$b_{+4}$	$b_{+4} = {}^{18}\alpha_{b_{+4}}^{KFF} \cdot k_{+4}$	-
$a_{-4}$	$a_{-4} = a_{+4}/\left(\frac{K_1}{K_w} \cdot \frac{{}^{18}\alpha_{HCO_3^- - H_2O}}{{}^{18}\alpha_{OH^- - H_2O}^{eq}}\right)$	-
$b_{-4}$	$b_{-4} = b_{+4}/\left(\frac{K_1}{K_w} \cdot {}^{18}\alpha_{HCO_3^- - CO_2}\right)$	-
${}^{18}\chi$	${}^{18}\chi = \frac{1}{K_2 \cdot {}^{13}\alpha_{CO_3^{2-} - HCO_3^-}^{eq} + [H^+]}$	-
<b><math>\Delta_{47}</math> parameters</b>		
$\left(\frac{{}^{47}R}{{}^{47}R^*}\right)^{eq}_{CO_2}$	$(\Delta_{47,CO_2}^{eq}/1000) + 1$	Hill et al. (2020)
$\left(\frac{{}^{63}R}{{}^{63}R^*}\right)^{eq}_{HCO_3^-}$	$(\Delta_{63,HCO_3^-}^{eq}/1000) + 1$	Hill et al. (2020); Tripathi et al. (2015)
$\left(\frac{{}^{63}R}{{}^{63}R^*}\right)^{eq}_{CO_3^{2-}}$	$(\Delta_{63,CO_3^{2-}}^{eq}/1000) + 1$	Hill et al. (2020); Tripathi et al. (2015)
${}^{63}\alpha_{CO_3^{2-} - HCO_3^-}^{eq}$	$\left(\frac{{}^{63}R}{{}^{63}R^*}\right)^{eq}_{CO_3^{2-}} / \left(\frac{{}^{63}R}{{}^{63}R^*}\right)^{eq}_{HCO_3^-}$	-
${}^{63}K_2$	${}^{63}\alpha_{CO_3^{2-} - HCO_3^-}^{KFF} \cdot {}^{13}\alpha_{HCO_3^- - CO_3^{2-}}^{eq} \cdot {}^{18}\alpha_{CO_3^{2-} - HCO_3^-}^{eq} \cdot K_2$	-
${}^{63}\chi$	$\frac{1}{1 + \frac{{}^{63}K_2 \cdot {}^{63}\alpha_{CO_3^{2-} - HCO_3^-}^{eq}}{[H^+]}}$	-
${}^{13-18}KIE_{p+1}$	1-0.146/1000	Guo (2020)
${}^{13-18}KIE_{s+1}$	1-0.219/1000	Guo (2020)
${}^{13-18}KIE_{p+4}$	1-0.016/1000	Guo (2020)
${}^{13-18}KIE_{s+1}$	1-0.175/1000	Guo (2020)
$p_{+1}$	$\frac{c_{+1} \cdot a_{+1}}{k_{+1} \cdot {}^{47}KIE_{p_1}}$	Uchikawa et al. (2021)
$s_{+1}$	$\frac{c_{+1} \cdot b_{+1}}{k_{+1} \cdot {}^{47}KIE_{s_1}}$	Uchikawa et al. (2021)
$p_{-1}$		-
$s_{-1}$		-
$p_{+4}$	$\frac{c_{+4} \cdot a_{+4}}{k_{+4} \cdot {}^{13-18}KIE_{p_4}}$	Uchikawa et al. (2021)



$s_{+4}$	$\frac{c_{+4} \cdot b_{+4}}{k_{+4} \cdot {}^{47}KIE_{s4}}$	Uchikawa et al. (2021)
$p_{-4}$	$p_{+4} / \left[ \left( \frac{{}^{63}R}{{}^{63}R^*} \right)_{HCO_3^-}^{eq} \cdot \frac{K_1}{K_w} \cdot {}^{13}\alpha_{HCO_3^- - CO_2}^{eq} \cdot {}^{18}\alpha_{HCO_3^- - OH^-}^{eq} \right]$	-
$s_{-4}$	$s_{+4} \cdot \left[ \left( \frac{{}^{47}R}{{}^{47}R^*} \right)_{CO_2}^{eq} / \left( \frac{{}^{63}R}{{}^{63}R^*} \right)_{HCO_3^-}^{eq} \cdot \frac{K_1}{K_w} \cdot {}^{13}\alpha_{HCO_3^- - CO_2}^{eq} \cdot {}^{18}\alpha_{HCO_3^- - CO_2}^{eq} \right]$	-
<b><math>\Delta_{48}</math> parameters</b>		
$\left( \frac{{}^{64}R}{{}^{64}R^*} \right)_{CO_2}^{eq}$	$(\Delta_{48,CO_2}^{eq}/1000) + 1$	Hill et al. (2020)
$\left( \frac{{}^{64}R}{{}^{64}R^*} \right)_{HCO_3^-}^{eq}$	$(\Delta_{64,HCO_3^-}^{eq}/1000) + 1$	Hill et al. (2020)
$\left( \frac{{}^{64}R}{{}^{64}R^*} \right)_{CO_3^{2-}}^{eq}$	$(\Delta_{64,CO_3^{2-}}^{eq}/1000) + 1$	Hill et al. (2020)
${}^{64}\alpha_{CO_3^{2-} - HCO_3^-}^{eq}$	$\left( \frac{{}^{64}R}{{}^{64}R^*} \right)_{CO_3^{2-}}^{eq} / \left( \frac{{}^{64}R}{{}^{64}R^*} \right)_{HCO_3^-}^{eq}$	-
${}^{64}K_2$	${}^{64}\alpha_{CO_3^{2-} - HCO_3^-}^{KFF} \cdot {}^{18}\alpha_{CO_3^{2-} - HCO_3^-}^{eq} \cdot {}^{18}\alpha_{CO_3^{2-} - HCO_3^-}^{eq} \cdot K_2$	-
${}^{64}\chi$	$\frac{1}{1 + \frac{{}^{64}K_2 \cdot {}^{64}\alpha_{CO_3 - HCO_3^-}^{eq}}{[H^+]}}$	-
${}^{18-18}KIE_{p'_{+1}}$	$1+0.049/1000$	Guo (2020)
${}^{18-18}KIE_{s'_{+1}}$	$1-0.146/1000$	Guo (2020)
${}^{18-18}KIE_{p'_{+4}}$	$1-0.144/1000$	Guo (2020)
${}^{18-18}KIE_{s'_{+4}}$	$1-0.086/1000$	Guo (2020)
$p'_{+1}$	$\frac{a_{+1} \cdot b_{+1}}{{}^{48}KIE_{p'_1} \cdot k_{+1}}$	-
$s'_{+1}$	$\frac{b_{+1} \cdot b_{+1}}{{}^{48}KIE_{s'_1} \cdot k_{+1}}$	-
$p'_{-1}$		-
$s'_{-1}$		-
$p'_{+4}$	$\frac{a_{+4} \cdot b_{+4}}{{}^{48}KIE_{p'_4} \cdot k_{+4}}$	-
$s'_{+4}$	$\frac{b_{+4} \cdot b_{+4}}{{}^{48}KIE_{s'_4} \cdot k_{+4}}$	-
$p'_{-4}$	$p'_{+4} / \left[ \left( \frac{{}^{64}R}{{}^{64}R^*} \right)_{HCO_3^-}^{eq} \cdot \frac{K_1}{K_w} \cdot {}^{18}\alpha_{HCO_3^- - CO_2}^{eq} \cdot {}^{18}\alpha_{HCO_3^- - OH^-}^{eq} \right]$	-

$S'_{-4}$	$S'_{+4} \cdot \left[ \left( \frac{{}^{48}R}{{}^{48}R^*} \right)_{CO_2}^{eq} / \left( \frac{{}^{64}R}{{}^{64}R^*} \right)_{HCO_3^-}^{eq} \cdot \frac{K_1}{K_w} \right. \\ \left. \cdot {}^{18}\alpha_{HCO_3^- - CO_2}^{eq} \cdot {}^{18}\alpha_{HCO_3^- - CO_2}^{eq} \right]$	-
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