Simpson's Law and the Spectral Cancellation of Climate Feedbacks

Nadir Jeevanjee[1]**, Daniel D.B. Koll**[2]**, Nicholas Lutsko**[3]

[1]Geophysical Fluid Dynamics Laboratory, Princeton NJ, 08540
[2]Massachusetts Institute of Technology, Cambridge MA
[3]Scripps Institution of Oceanography, La Jolla CA

Key Points:

- Conventional feedbacks exhibit strong spectral cancellation
- This cancellation follows from 'Simpson's Law' for water vapor thermal emission
- RH-based feedbacks do not exhibit this cancellation, and more naturally manifest Simpson's Law

Corresponding author: Nadir Jeevanjee, `nadir.jeevanjee@noaa.gov`

Abstract

We spectrally resolve the conventional clear-sky temperature and water vapor feedbacks in an idealized single-column framework, and show that the well-known partial compensation of these feedbacks is actually due to an almost perfect *cancellation* of the spectral feedbacks at wavenumbers where H2O is optically thick. This cancellation is a natural consequence of 'Simpson's Law', which says that H2O emission temperatures do not change with surface warming if RH is fixed. This cancellation is eliminated for the alternative RH-based feedbacks proposed by Held and Shell (2012). These results bolster the case for switching from conventional to RH-based feedbacks. We also find that the RH-based clear-sky lapse rate feedback is negligible, so the impact of changing lapse rates depends crucially on whether relative or specific humidity is held fixed.

1 Introduction

The climate feedback parameter λ measures the response of net, downward top-of-atmosphere radiation N to a change in surface temperature T_s as

$$\lambda \equiv \frac{dN}{dT_s} \quad (\mathrm{W/m^2/K}) \ . \tag{1}$$

Under radiative forcing $\Delta N = F$, then, λ determines the climate response as $\Delta T_s = -F/\lambda$. As such, λ is a central quantity in climate science and has been intensely studied. Typically, λ is decomposed into different terms which aim to isolate the contributions from distinct physical processes. While particular definitions and methodologies have evolved over time, a 'conventional' framework has emerged in which λ is decomposed as [e.g. *Sherwood et al.*, 2020, and now writing λ as λ^{tot}]:

$$\lambda^{\mathrm{tot}} = \lambda^{\mathrm{planck}} + \lambda^{\mathrm{lapse}} + \lambda^{\mathrm{wv}} + \lambda^{\mathrm{albedo}} + \lambda^{\mathrm{clouds}} \ . \tag{2}$$

These terms give the radiative response to vertically uniform warming (Planck), deviations from uniform warming (lapse, LR), changes in specific humidity (water vapor, WV), and changes in surface albedo and clouds. Precise definitions for the first three, which typically dominate, will be given below.

Although this decomposition has become fairly standard [*Flato et al.*, 2013; *Sherwood et al.*, 2020], it also suffers from various drawbacks. Perhaps the most basic drawback is that the conventional Planck feedback, which gives the 'reference response' relative to which the other feedbacks are computed, is not a good null hypothesis for the system response [*Roe*, 2009]: it assumes that specific humidity stays fixed with temperature change, even though we now know from theory, models, and observations that fixed *relative* humidity is a much better null hypothesis [e.g. *Romps*, 2014; *Sherwood et al.*, 2010; *Held and Soden*, 2000; *Soden et al.*, 2005; *Ferraro et al.*, 2015]. This inappropriateness of the conventional Planck feedback can even lead to reference responses which are physically unrealizable [*Held and Shell*, 2012].

The conventional decomposition (2) also has practical drawbacks. Across models, λ^{wv} and λ^{lapse} exhibit significant spread but also a strong anti-correlation [*Soden et al.*, 2008; *Soden and Held*, 2006]. This means that the individual spread in λ^{wv} and λ^{lapse} largely cancels in the sum (2), and is thus not indicative of uncertainty in λ^{tot}. Physically, this anti-correlation means that λ^{wv} and λ^{lapse} are not capturing independent physical processes, defeating the purpose of a decomposition such as (2).

Such drawbacks led many studies to consider only the sum $\lambda^{\mathrm{lapse}} + \lambda^{\mathrm{wv}}$ [e.g. *Soden and Held*, 2006; *Soden et al.*, 2008; *Huybers*, 2010; *Ingram*, 2013a; *Sherwood et al.*, 2020]. This defines away the anti-correlation problem, and significantly reduces spread. But, there is a basic physical inconsistency in summing λ^{lapse} and λ^{wv}, in that λ^{wv} is due to the *entire* specific humidity perturbation, whereas λ^{lapse} is due only to temperature perturbations which are not vertically uniform. Furthermore, the anti-correlation between λ^{wv} and λ^{lapse} turns out to have a rather subtle origin, complicating the interpretation of $\lambda^{\mathrm{lapse}} + \lambda^{\mathrm{wv}}$ [*Po-Chedley et al.*, 2018].

This state of affairs led *Held and Shell* [2012] and *Ingram* [2012, 2013b] to propose using relative humidity (RH) as the moisture state variable for feedback analyses. This means that the Planck and LR feedbacks are to be computed while holding RH rather than specific humidity (q_{v}) fixed, and that the WV feedback is now only due to changes in RH rather than q_{v}. This not only yields a more physical reference response (Planck feedback), but also greatly reduces the spread in and anti-correlation between the LR and WV feedbacks.

Given these advantages, some recent studies have adopted the RH-based formalism as their primary approach [e.g. *Caldwell et al.*, 2016; *Zelinka et al.*, 2020]. Other influential studies have carried on with the conventional approach, however [e.g. *Sherwood et al.*, 2020], leading to inconsistency in the literature. Furthermore, the differing radiation physics of these two approaches remains underexplored. *Ingram* [2010] argued that λ^{wv} must significantly offset $\lambda^{\mathrm{planck}}$ and λ^{lapse} due to what we will call 'Simpson's Law': the fact that to first order, and under fixed RH, the outgoing longwave radiation (OLR) at H_2O-dominated wavenumbers does not change with surface warming [first articulated by *Simpson*, 1928]. If true, this implies that *at such wavenumbers* the total feedback should be roughly 0, and thus that (at such wavenumbers) the Planck, LR, and WV feedbacks should cancel almost exactly. These implications, however, have not been drawn out in detail or explicitly verified.

Accordingly, our goal in this paper is to highlight Simpson's Law and then demonstrate that spectrally-resolved conventional feedbacks indeed largely cancel at optically thick, H_2O-dominated wavenumbers. We show that this cancellation stands in stark contrast to the RH-based formalism, in which no such cancellation exists. We hope that this fundamental simplicity of the RH-based approach, and its consistency with the basic physics of Simpson's Law, will encourage more consistent use of RH-based feedbacks. Our results will also allow for more detailed interpretations of the various components of the RH-based feedbacks.

We begin in section 2 by reviewing Simpson's Law and explicitly verifying it using line-by-line radiative transfer. After reviewing the definition of the feedbacks in (2) in Section 3, we then apply Simpson's Law in understanding the spectral cancellation of conventional feedbacks in Section 4. We conclude in Section 5.

2 Simpson's Law

In this section we briefly review Simpson's Law, which dates back to *Simpson* [1928]. Simpson's Law is the key ingredient in the 'runaway greenhouse' effect [e.g. *Nakajima et al.*, 1992; *Goldblatt et al.*, 2013], and has also been used to explain the T_{s}-dependence of OLR [*Koll and Cronin*, 2018], the rate of global mean precipitation change [*Jeevanjee and Romps*, 2018], and the strength of the water vapour feedback [*Ingram*, 2010]. A pedagogical treatment is given in *Jeevanjee* [2018]. We emphasize at the outset that Simpson's 'Law' does not hold exactly, but is rather a first-order approximation; we refer to it as a 'Law' simply to emphasize the fundamental role it plays in the spectral structure of radiative feedbacks.

To arrive at Simpson's Law, we first note that if RH is uniform, then the vapor density ρ_{v} (kg/m^3) is a function of temperature only:

$$\rho_{\mathrm{v}} \;=\; \rho_{\mathrm{v}}(T) \;=\; \frac{\mathrm{RH}e^*(T)}{R_{\mathrm{v}}T} \tag{3}$$

where $e^*(T)$ is saturation vapor pressure and all other symbols have their usual meaning. Viewing T as a vertical coordinate, then, implies that the profile $\rho_{\mathrm{v}}(T)$ should universal and independent of surface temperature, i.e. 'T_{s}-invariant' [cf. Fig. 1 of *Jeevanjee and Romps*, 2018].

This then implies that H_2O optical depth at a given wavenumber should also be a T_s-invariant function of T, at least to first order and under typical circumstance. To see this, we write H_2O optical depth in temperature coordinates as

$$\tau(T) \;=\; \int_{T_{\mathrm{tp}}}^{T} \kappa \rho_{\mathrm{v}}(T') \frac{dT'}{\Gamma} \tag{4}$$

where T_{tp} is the tropopause temperature, κ is the mass absorption coefficient (m^2/kg), and Γ the lapse rate. (Such an expression neglects stratospheric water vapor and cannot be used when tropospheric $T(z)$ is not single-valued, i.e. when there is a temperature inversion. Future work could investigate the validity of Simpson's Law under such circumstances.) Though κ exhibits a pressure-dependence due to pressure broadening [*Pierrehumbert*, 2010], and moist lapse rates Γ also vary in the vertical, these variations are expected to be weak compared to the strong exponential T-dependence of ρ_{v}. Since ρ_{v} is T_s-invariant, we expect $\tau(T)$ to be so as well, at least to first order [cf. Fig. S5 of *Jeevanjee and Romps*, 2018]. Since cooling-to-space can be approximated as emanating from $\tau \approx 1$ for optically thick wavenumbers ν [e.g. *Petty*, 2006; *Jeevanjee and Fueglistaler*, 2020a], this suggests that the spectrally-resolved outgoing longwave radiation OLR_ν and corresponding emission temperature T_{em}, defined in terms of the Planck function $B(\nu, T)$ by

$$\pi B(\nu, T_{\mathrm{em}}) = \mathrm{OLR}_\nu\,, \quad (\mathrm{W/m^2/cm^{-1}}) \tag{5}$$

should also be T_s-invariant (so long as RH is fixed). This then yields Simpson's Law:

> **Simpson's Law**: At fixed RH, and for optically thick wavenumbers dominated by H_2O absorption, emission temperatures and OLR are independent of surface temperature.

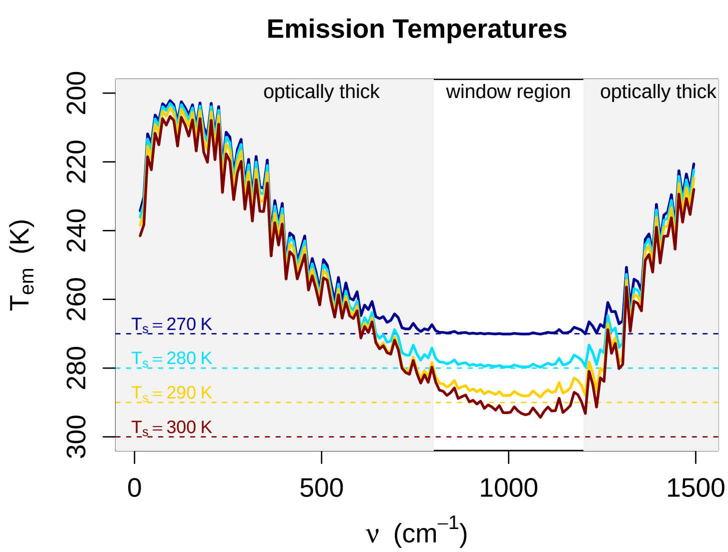

Figure 1. Validation of Simpson's Law. Emission temperatures T_{em} defined by Eq. (5), as calculated with RFM for moist adiabatic atmospheres with varying T_s. Emission temperatures are relatively insensitive to T_s at optically thick wavenumbers (gray shading), but are roughly equal to T_s in the optically thin water vapor 'window' region (800–1200 cm^{-1}, white shading).

We explicitly verify Simpson's Law in Figure 1 by plotting T_{em} (as diagnosed via (5)) as a function of wavenumber for a set of moist adiabatic columns at varying T_s and with

RH $= 0.75$ and no CO_2, using the Reference Forward Model (details of these calculations are given in Section 4). Atmospheric emission emanates from the optically thick sections of the H_2O pure rotational band (0-800 cm^{-1}) and vibration-rotational band (1200-1500 cm^{-1}), while surface emission emanates through the optically thin water vapor 'window' at 800-1200 cm^{-1}. [For further intuition for this structure, see *Jeevanjee and Fueglistaler*, 2020b]. The optically thick wavenumbers show relatively little variation of T_{em} with T_s, validating Simpson's Law. Indeed, the average of dT_{em}/dT_s over $0 - 800$ cm^{-1} at $T_s = 290$ K is 0.2. Of course, the fact that dT_{em}/dT_s is not identically zero shows that Simpson's Law is only approximate, due to our neglect of pressure broadening and lapse-rate changes in deducing Simpson's Law above.

Simpson's Law is nonetheless a useful idealization, as it encapsulates the small changes in optically thick T_{em} relative to the much larger changes in T_{em} in the optically thin water vapor window (in the window, which remains optically thin for $T_s \lesssim 290$ K, we have $T_{em} \approx T_s$ and thus $dT_{em}/dT_s \approx 1$). In particular, differentiating (5) with respect to T_s and invoking Simpson's Law tells us that the total feedback parameter should be roughly zero at H_2O-dominated wavenumbers. This then means that water vapor, planck, and lapse rate feedbacks *must* cancel at those wavenumbers. A primary goal of this paper is to explicitly verify this. A further corollary is that the total feedback is nonzero primarily in the water vapor window, and thus that the window is the main channel through which OLR increase with T_s [as emphasized by *Koll and Cronin*, 2018]. We will sharpen and verify these claims in Section 4.

3 Feedback formulation

With Simpson's Law in place we now turn to feedbacks. We begin by giving precise definitions of the Planck, LR, and WV feedbacks, in both the conventional and RH-based frameworks. Since the choice of moisture variable mostly impacts the clear-sky, longwave feedbacks, we consider these feedbacks only and do not consider λ^{cloud} or λ^{albedo} in our analysis. See *Held and Shell* [2012], however, for a discussion of how the RH-based framework changes the relative importance of other feedbacks.

In a cloud-free atmosphere with H_2O and CO_2 as the only greenhouse gases, the OLR is determined by their profiles, along with the surface temperature T_s and atmospheric temperature profile T_a (we suppress vertical coordinate dependencies for clarity). A choice must be made, however, of which state variable to use for specifying H_2O concentrations; we begin with the conventional choice of specific humidity q_v, and later discuss the modification when using RH. We specify an atmosphere as an ordered triple (T_s, T_a, q_v), and the OLR is then a function of this ordered triple, i.e.

$$\text{OLR} = \text{OLR}(T_s, T_a, q_v) \ . \tag{6}$$

We suppress the dependence of OLR on CO_2 concentration since we consider feedbacks here, not forcings, and feedbacks are always computed with CO_2 concentrations held fixed. The relevant CO_2 concentrations will be specified in the next section.

Consider now an initial atmosphere (T_s^i, T_a^i, q_v^i) and final atmosphere (T_s^f, T_a^f, q_v^f), and let $\Delta T_s \equiv T_s^f - T_s^i$. Consistent with the definition (1) and our restriction to clear-sky longwave radiation only, our total feedback is then minus the change in OLR per unit surface temperature difference:

$$\lambda^{\text{tot}} \equiv - \frac{\text{OLR}(T_s^f, T_a^f, q_v^f) - \text{OLR}(T_s^i, T_a^i, q_v^i)}{\Delta T_s} \quad (\text{W/m}^2/\text{K}). \tag{7}$$

In the conventional (q_v-based) framework, we then define the following individual feedbacks:

$$\lambda^{\mathrm{planck}} \equiv -\frac{\mathrm{OLR}(T_\mathrm{s}^i + \Delta T_\mathrm{s}, T_\mathrm{a}^i + \Delta T_\mathrm{s}, q_\mathrm{v}^i) - \mathrm{OLR}(T_\mathrm{s}^i, T_\mathrm{a}^i, q_\mathrm{v}^i)}{\Delta T_\mathrm{s}} \tag{8a}$$

$$\lambda^{\mathrm{lapse}} \equiv -\frac{\mathrm{OLR}(T_\mathrm{s}^f, T_\mathrm{a}^f, q_\mathrm{v}^i) - \mathrm{OLR}(T_\mathrm{s}^i + \Delta T_\mathrm{s}, T_\mathrm{a}^i + \Delta T_\mathrm{s}, q_\mathrm{v}^i)}{\Delta T_\mathrm{s}} \tag{8b}$$

$$\lambda^{\mathrm{wv}} \equiv -\frac{\mathrm{OLR}(T_\mathrm{s}^i, T_\mathrm{a}^i, q_\mathrm{v}^f) - \mathrm{OLR}(T_\mathrm{s}^i, T_\mathrm{a}^i, q_\mathrm{v}^i)}{\Delta T_\mathrm{s}} . \tag{8c}$$

The Planck feedback $\lambda^{\mathrm{planck}}$ is minus the (ΔT_s-normalized) OLR response to a uniform change in surface and atmospheric temperatures, with q_v held fixed at the initial profile. The lapse-rate feedback λ^{lapse} is minus the OLR response to the difference between the actual temperature response and the uniform Planck response, still holding q_v fixed. The water vapor feedback λ^{wv} is then minus the OLR response to the change in q_v, holding temperatures fixed. Assuming linearity in the finite differences, we then have

$$\lambda^{\mathrm{tot}} = \lambda^{\mathrm{planck}} + \lambda^{\mathrm{lapse}} + \lambda^{\mathrm{wv}} . \tag{9}$$

For RH-based feedbacks, we use the formulae (8) but simply replace q_v with RH, and denote the corresponding RH-based feedbacks with a tilde:

$$\widetilde{\lambda}^{\mathrm{planck}} \equiv -\frac{\mathrm{OLR}(T_\mathrm{s}^i + \Delta T_\mathrm{s}, T_\mathrm{a}^i + \Delta T_\mathrm{s}, \mathrm{RH}^i) - \mathrm{OLR}(T_\mathrm{s}^i, T_\mathrm{a}^i, \mathrm{RH}^i)}{\Delta T_\mathrm{s}} \tag{10a}$$

$$\widetilde{\lambda}^{\mathrm{lapse}} \equiv -\frac{\mathrm{OLR}(T_\mathrm{s}^f, T_\mathrm{a}^f, \mathrm{RH}^i) - \mathrm{OLR}(T_\mathrm{s}^i + \Delta T_\mathrm{s}, T_\mathrm{a}^i + \Delta T_\mathrm{s}, \mathrm{RH}^i)}{\Delta T_\mathrm{s}} \tag{10b}$$

$$\widetilde{\lambda}^{\mathrm{wv}} \equiv -\frac{\mathrm{OLR}(T_\mathrm{s}^i, T_\mathrm{a}^i, \mathrm{RH}^f) - \mathrm{OLR}(T_\mathrm{s}^i, T_\mathrm{a}^i, \mathrm{RH}^i)}{\Delta T_\mathrm{s}} . \tag{10c}$$

Note that now the water vapor feedback $\widetilde{\lambda}^{\mathrm{wv}}$ is due to RH changes, not q_v changes. Our calculations below will be at fixed RH so we do not consider $\widetilde{\lambda}^{\mathrm{wv}}$ further. Note that in GCMs, *Held and Shell* [2012] found small global mean values of $|\widetilde{\lambda}^{\mathrm{wv}}| \lesssim 0.1$ W/m^2/K.

Since we expect increases in OLR to emanate from increased surface emission through the window (as suggested by Fig. 1), we also introduce a 'surface' feedback λ^{surf} obtained by perturbing T_s while holding the atmospheric temperature and water vapor profiles fixed:

$$\lambda^{\mathrm{surf}} \equiv -\frac{\mathrm{OLR}(T_\mathrm{s}^i + \Delta T_\mathrm{s}, T_\mathrm{a}^i, q_\mathrm{v}^i) - \mathrm{OLR}(T_\mathrm{s}^i, T_\mathrm{a}^i, q_\mathrm{v}^i)}{\Delta T_\mathrm{s}} . \tag{11}$$

This feedback is identical in the q_v-based and RH-based frameworks, and is equal to the 'surface kernel' of radiative kernel analyses [cf. Fig. 1 of *Soden et al.*, 2008].

A key aspect of our analysis will be to consider *spectrally-resolved* OLR and hence spectrally-resolved versions of the feedbacks in Eqns. (7), (8), (10), and (11). These will be denoted with a subscript ν, with units W/m^2/cm^{-1}/K.

4 Spectral cancellation of conventional feedbacks

We now turn to spectrally-resolved calculations of the various feedbacks defined above, for a variety of idealized atmospheric columns. We calculate OLR_ν for these columns using the line-by-line Reference Forward Model [RFM, *Dudhia*, 2017], along with HiTRAN2016 spectroscopic data [*Gordon et al.*, 2017] for H_2O from 0 to 1500 cm^{-1} and CO_2 from 500 to 850 cm^{-1}, using only the most common isotopologue for each gas. We run RFM at a spectral resolution of 0.1 cm^{-1} and on 100 evenly-spaced pressure levels between 1000 and 10 hPa. We include H_2O continuum effects via RFM's implementation of the MT-CKD continuum [*Mlawer et al.*, 2012].

All atmospheric columns have $T_s^i = 288$K, $T_s^f = 289$ K, RH=0.75, and an isothermal stratosphere at $T_{\rm strat} = 200$K, with a uniform stratospheric q_v set equal to its tropopause value. The lapse rates and radiatively active species vary between cases, as described below.

4.1 Constant lapse rate, H_2O-only atmosphere

We begin by considering atmospheric columns with a constant lapse rate of 7 K/km and H_2O as the only radiatively active species. This case avoids the complications due to the LR feedback and due to CO_2, both of which we address later. We calculate the spectrally-resolved conventional feedbacks λ_ν according to Eqns. (7) and (8); these are shown in Fig. 2a.

The conventional Planck feedback $\lambda_\nu^{\rm planck}$ is strongly negative, as expected, but is not a good first approximation to the total feedback $\lambda_\nu^{\rm tot}$; there are large cancellations with the strongly positive water vapor feedback $\lambda_\nu^{\rm wv}$. Indeed, as expected from Simpson's Law, at optically thick wavenumbers we have

$$\lambda_\nu^{\rm tot} \;=\; \lambda_\nu^{\rm planck} + \lambda_\nu^{\rm wv} \approx 0 \quad \text{(optically thick } \nu). \qquad (12)$$

Thus, at most wavenumbers the conventional feedback decomposition splits the total feedback into equal and opposite terms, which are constrained to cancel by basic physics.

We now contrast this behavior with that of the RH-based formalism (Fig. 2b). In this case the picture is markedly simpler: the Planck feedback takes place at constant RH and so Simpson's Law is manifest, yielding $\widetilde{\lambda}_\nu^{\rm planck} \approx 0$ outside the window. In fact, since these idealized columns have no RH or lapse rate perturbations, we find that $\widetilde{\lambda}_\nu^{\rm planck} = \widetilde{\lambda}_\nu^{\rm tot}$ identically (so only one of these curves is visible in Fig. 2b). Thus, when RH is the moisture variable the reference response is a good null hypothesis (Roe 2009); in fact, for this simple system, the reference response captures the total system response perfectly.

As mentioned earlier, the dominant contribution to $\lambda^{\rm tot}$ seen in Fig. 2 can be interpreted as an increase in surface cooling-to-space through the optically thin water vapor window. This can be made more precise by invoking the argument of *Koll and Cronin* [2018], who show that this should be given by the increase in surface emission while holding atmospheric variables fixed. This is just the $\lambda^{\rm surf}$ term of Eq. (11), so this yields the approximation

$$\lambda_\nu^{\rm tot} \;\approx\; \lambda_\nu^{\rm surf}\,. \qquad (13)$$

The surface feedback $\lambda_\nu^{\rm surf}$ is shown in purple in Fig. 2b, and we find that in this idealized case Eqn. (13) indeed holds, to an accuracy of about 10% in the spectral integral. Equation (13) thus gives a straightforward way to interpret the dominant contribution to $\lambda^{\rm tot}$.

4.2 Conventional and RH-based feedbacks in a moist-adiabatic, H_2O-only atmosphere

Let us now incorporate the lapse-rate feedback, by replacing our constant lapse-rate temperature profiles with moist pseudo-adiabats based at $T_s^i = 288$ K and $T_s^f = 289$ K. We also introduce the conventional lapse rate feedback $\lambda_\nu^{\rm lapse}$ as defined in Eqn. (8b). This feedback, and the others calculated as before, are shown in Fig. 2c.

Even though the lapse-rate feedback is present Simpson's Law still operates, only now it implies that at optically thick ν, there should be a near-complete cancellation of *three* terms:

$$\lambda_\nu^{\rm tot} \;=\; \lambda_\nu^{\rm planck} + \lambda_\nu^{\rm wv} + \lambda_\nu^{\rm lapse} \;\approx 0. \quad \text{(optically thick } \nu). \qquad (14)$$

This means that summing the LR and WV feedbacks only yields a partial cancellation even at optically thick wavenumbers, due to the aforementioned fact that $\lambda^{\rm wv}$ is due to the entire q_v perturbation but $\lambda^{\rm lapse}$ is due only to part of the temperature perturbation [cf. Eqns (8b)

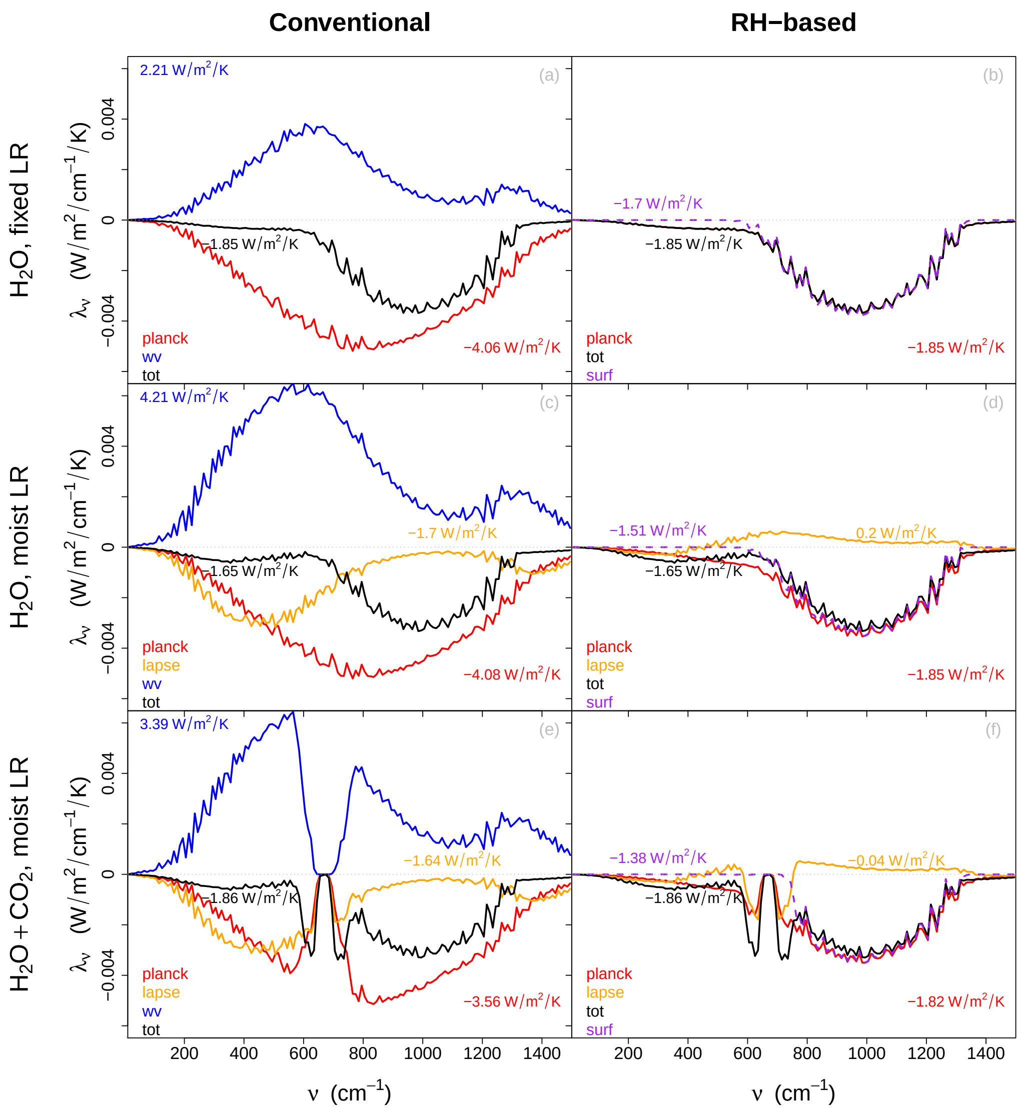

Figure 2. Spectral feedbacks in the conventional and RH-based formalisms. (a) Conventional feedbacks with H_2O-only and a fixed lapse-rate. The conventional Planck and WV feedbacks cancel for optically thick ν **(b)** As in (a) but in the RH-based formalism. Now the Planck and total feedbacks are equal, and are well approximated by the surface feedback λ^{surf} **(c,d)** As in (a,b), but for moist-adiabatic temperature profiles. We still find $\lambda_\nu^{\mathrm{tot}} \approx 0$ for optically thick ν, but now this implies a three-way cancellation of conventional feedbacks. The picture again simplifies for RH-based feedbacks, with a much smaller LR feedback in the RH-based formalism **(e,f)** As in (c,d), but now including 280 ppm of CO_2. Now $\lambda_\nu^{\mathrm{tot}}$ exhibits CO_2 'radiator fins', i.e. local extrema on either side of the CO_2 band. These extrema are overshadowed by other features in the conventional decomposition, but are highlighted in the RH-based decomposition. Color-coded numbers give the spectral integrals λ of the corresponding spectral feedbacks λ_ν.

and (8c)]. From a spectral point of view, then, little simplification arises from summing only the LR and WV feedbacks.

The RH-based feedbacks for these moist-adiabatic atmospheres are shown in Fig. 2d. As before, the picture simplifies considerably: there is no water vapor feedback, and the Planck feedback is a good, if no longer perfect, approximation to the total feedback. A perhaps surprising result is that the RH-based lapse-rate feedback $\widetilde{\lambda}_\nu^{\text{lapse}}$ is small, even in a fully moist-adiabatic atmosphere. This is because in the RH-based framework, the lapse-rate temperature perturbation is made at constant RH, and thus Simpson's Law applies. This is consistent with the conclusions of *Cess* [1975], who finds that changes in lapse rate (at fixed RH) have little impact on global energy balance. Thus, the impact of changing lapse rates depends crucially on whether RH or q_v is held fixed.

One consequence of $\widetilde{\lambda}_\nu^{\text{lapse}}$ being small is that the RH-based Planck feedback $\widetilde{\lambda}_\nu^{\text{planck}}$ is still a good null hypothesis for the OLR_ν change. Another, related consequence is that the surface feedback approximation Eqn. (13) continues to hold (Fig. 2d), again to about 10% in the spectral integral.

4.3 Conventional and RH-based feedbacks in a moist-adiabatic atmosphere with H_2O and CO_2

Next we consider the effects of CO_2. We calculate feedbacks for the moist-adiabatic columns of the previous subsection, but now with 280 ppmv of radiatively-active CO_2.

The results are shown in Fig. 2e,f. In the q_v-based framework, there is still a marked cancellation between $\lambda_\nu^{\text{planck}}$, λ_ν^{wv}, and $\lambda_\nu^{\text{lapse}}$, but it now only occurs for ν which are outside the H_2O window *and* outside the $575 - 775$ cm^{-1} CO_2 band. Following *Seeley and Jeevanjee* [2020] we refer to the wings of the CO_2 band as 'CO_2 radiator fins', as they radiate from the upper troposphere and are visible as local extrema in λ_ν^{tot} at roughly 625 and 725 cm^{-1}. These wavenumbers radiate from fixed pressures (at fixed CO_2) rather than fixed temperatures [for CO_2, $\tau \sim p^2$; *Pierrehumbert*, 2010], and thus do not obey Simpson's Law. They make non-negligible contributions to λ_ν^{tot}, but are overshadowed by other features in $\lambda_\nu^{\text{planck}}$ and $\lambda_\nu^{\text{lapse}}$, due to continued cancellation with λ_ν^{wv}.

In the RH-based framework (Fig. 2f), however, the picture is again much simpler. The non-Simpsonian CO_2 radiator fins remain, but are partially captured by the the RH-based Planck feedback $\widetilde{\lambda}_\nu^{\text{planck}}$, which is thus still a reasonable first approximation to λ_ν^{tot} (unlike the conventional $\lambda_\nu^{\text{planck}}$). The rest of the CO_2 radiator fin contribution is due to enhanced upper-tropospheric warming from lapse-rate changes [*Seeley and Jeevanjee*, 2020], which is indeed the main feature in $\widetilde{\lambda}_\nu^{\text{lapse}}$. The advantage of the RH-based formulation is that it highlights these features, rather than lumping them in with the larger $\lambda_\nu^{\text{planck}}$ and $\lambda_\nu^{\text{lapse}}$ which then cancel with λ^{wv}.

Note that the negative contribution of the CO_2 radiator fins to $\lambda_\nu^{\text{lapse}}$ offsets the positive contribution from the window (which results from fixed-RH upper-tropospheric moistening helping to close the window), leading to an even smaller $\widetilde{\lambda}^{\text{lapse}}$. Thus, the conclusion from the previous H_2O-only calculation – that the strength of the lapse rate feedback is highly dependent on the choice of moisture variable – is only strengthened by the addition of CO_2. The presence of the CO_2 radiator fins also means that the surface approximation (13) breaks down in the CO_2 band. The surface feedback (11) still, however, gives a precise way of interpreting and accounting for the non-Simpsonian increase in surface emission through the window, which is the dominant contribution to λ^{tot} in the present-day climate [*Slingo and Webb*, 1997; *Raghuraman et al.*, 2019; *Seeley and Jeevanjee*, 2020].

5 Summary

This paper has shown that:

1. The well-known compensation of conventional, q_v-based feedbacks is actually due to a near-perfect *cancellation* of these feedbacks at wavenumbers where H_2O is optically thick, as dictated by Simpson's Law
2. This cancellation does not occur for RH-based feedbacks, which more naturally manifest Simpson's Law.

Furthermore, because constant RH is our null hypothesis under surface warming, the RH-based Planck feedback $\widetilde{\lambda}^{\text{planck}}$ is a much better reference response (i.e. is closer to λ^{tot}) than the conventional Planck feedback. We also explicitly demonstrated that the increase in surface emission through the window is accurately captured by the surface feedback term (11), in line with the argument of *Koll and Cronin* [2018].

Our findings also add nuance to the interpretation of the lapse-rate feedback. The conventional view that λ^{lapse} and λ^{wv} should be summed is called in to question by the three-way cancellation of λ^{planck}, λ^{lapse}, and λ^{wv} found here. Furthermore, we find [similar to *Held and Shell*, 2012] that the fixed-RH LR feedback can be an order of magnitude smaller than the conventional LR feedback, raising questions about the notion of a single, well-defined LR feedback. Thus, attribution of phenomena (such as polar amplification) to LR feedbacks should come with caveats about the choice of feedback formalism.

More broadly, these results show that RH-based feedbacks are not only more physical from a thermodynamic point of view, as argued by *Held and Shell* [2012], but are also simpler from a *radiative* point of view. We hope that our explicit formulation and validation of Simpson's Law fosters a better appreciation of this simplicity.

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https://www.dropbox.com/s/0c2p312fbo7bf4v/21h2o_feedback_data.tgz?dl=0 ; finalized data will be available on Zenodo upon acceptance.

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