

Abstract

We spectrally resolve the conventional clear-sky temperature and water vapor feedbacks in an idealized single-column framework, and show that the well-known partial compensation of these feedbacks is actually due to an almost perfect *cancellation* of the spectral feedbacks at wavenumbers where H₂O is optically thick. This cancellation is a natural consequence of ‘Simpson’s Law’, which says that H₂O emission temperatures do not change with surface warming if RH is fixed. We provide an explicit formulation and validation of Simpson’s Law, and furthermore show that this spectral cancellation of feedbacks is naturally incorporated in the alternative RH-based framework proposed by Held and Shell (2012) and Ingram (2012, 2013), thus bolstering the case for switching from conventional to RH-based feedbacks. We also find a negligible RH-based clear-sky lapse rate feedback, suggesting that the impact of changing lapse rates depends crucially on whether relative or specific humidity is held fixed.

1 Introduction

The climate feedback parameter λ measures the response of net, downward top-of-atmosphere radiation N to a change in surface temperature T_s as

$$\lambda \equiv \frac{dN}{dT_s} \quad (\text{W/m}^2/\text{K}) . \quad (1)$$

Under radiative forcing $\Delta N = F$, then, λ determines the climate response as $\Delta T_s = -F/\lambda$. As such, λ is a central quantity in climate science and has been intensely studied. Typically, λ is decomposed into different terms which aim to isolate the contributions from distinct physical processes. While particular definitions and methodologies have evolved over time, a ‘conventional’ framework has emerged in which λ is decomposed as [e.g. *Sherwood et al.*, 2020], and now writing λ as λ^{tot} :

$$\lambda^{\text{tot}} = \lambda^{\text{planck}} + \lambda^{\text{lapse}} + \lambda^{\text{wv}} + \lambda^{\text{albedo}} + \lambda^{\text{clouds}} . \quad (2)$$

These terms give the radiative response to vertically uniform warming (Planck), deviations from uniform warming (lapse, LR), changes in specific humidity (water vapor, WV), and changes in surface albedo and clouds. Precise definitions for the first three, which typically have the largest magnitude, will be given below.

Although this decomposition has become fairly standard [*Flato et al.*, 2013; *Sherwood et al.*, 2020], it also suffers from various drawbacks. Perhaps the most basic drawback is that the conventional Planck feedback, which gives the ‘reference response’ relative to which the other feedbacks are computed, is not a good null hypothesis for the system response [*Roe*, 2009]: it assumes that specific humidity stays fixed with temperature change, even though we now know from theory, models, and observations that fixed *relative* humidity is a much better null hypothesis [e.g. *Romps*, 2014; *Sherwood et al.*, 2010; *Held and Soden*, 2000; *Soden et al.*, 2005; *Ferraro et al.*, 2015; *Zhang et al.*, 2020]. This inappropriateness of the conventional Planck feedback can even lead to reference responses which are physically unrealizable [*Held and Shell*, 2012].

The conventional decomposition (2) also has practical drawbacks. Across models, λ^{wv} and λ^{lapse} exhibit significant spread but also a strong anti-correlation [*Soden et al.*, 2008; *Soden and Held*, 2006]. This means that the individual spread in λ^{wv} and λ^{lapse} largely cancels in the sum (2), and is thus not indicative of uncertainty in λ^{tot} . Physically, this anti-correlation means that λ^{wv} and λ^{lapse} are not capturing independent physical processes, defeating the purpose of a decomposition such as (2).

Such drawbacks led many studies to consider only the sum $\lambda^{\text{lapse}} + \lambda^{\text{wv}}$ [e.g. *Soden and Held*, 2006; *Soden et al.*, 2008; *Huybers*, 2010; *Ingram*, 2013a; *Sherwood et al.*, 2020]. This defines away the anti-correlation problem, and significantly reduces spread. But, there is a basic physical inconsistency in summing λ^{lapse} and λ^{wv} , in that λ^{wv} is due to the *entire*

specific humidity perturbation, whereas λ^{lapse} is due only to temperature perturbations which are not vertically uniform. Furthermore, the anti-correlation between λ^{wv} and λ^{lapse} does not arise solely from colocated warming and moistening of the tropical upper-troposphere, as previously thought, but also from the nonlocal influence of tropical warming on the extratropical stratification [Po-Chedley *et al.*, 2018]. This further undermines the physical justification for summing λ^{lapse} and λ^{wv} .

This state of affairs led Held and Shell [2012] and Ingram [2012, 2013b] to propose using relative humidity (RH) as the moisture state variable for feedback analyses. This means that the Planck and LR feedbacks are to be computed while holding RH rather than specific humidity (q_v) fixed, and that the WV feedback is now only due to changes in RH rather than q_v . These studies and others [e.g. Caldwell *et al.*, 2016] showed that switching from conventional to RH-based feedbacks not only yields a more physical reference response (Planck feedback), but also greatly reduces the spread in and anti-correlation between the LR and WV feedbacks (spread in the Planck feedback is also reduced).

Given these advantages, some recent studies have adopted the RH-based formalism as their primary approach [e.g. Caldwell *et al.*, 2016; Zelinka *et al.*, 2020]. Other influential studies have carried on with the conventional approach, however [e.g. Sherwood *et al.*, 2020], leading to inconsistency in the literature. Furthermore, the underlying radiation physics of these two approaches remains underexplored. Ingram [2010] argued that λ^{wv} must significantly offset λ^{planck} and λ^{lapse} due to what we will call ‘Simpson’s Law’: the fact that to first order, and under fixed RH, the outgoing longwave radiation (OLR) at H₂O-dominated wavenumbers does not change with surface warming [first articulated by Simpson, 1928]. If true, this implies that *at such wavenumbers* the total feedback should be roughly 0, and thus that (at such wavenumbers) the Planck, LR, and WV feedbacks should cancel almost exactly. These implications, however, have not been drawn out in detail or explicitly verified.

Accordingly, our goal in this paper is to highlight Simpson’s Law and then demonstrate that spectrally-resolved conventional feedbacks indeed largely cancel at optically thick, H₂O-dominated wavenumbers. By contrast, we show that the RH-based formalism naturally incorporates this cancellation, providing a clearer view of the clear-sky feedbacks. We hope that this fundamental simplicity of the RH-based approach, and its consistency with the basic physics of Simpson’s Law, will encourage more widespread use of RH-based feedbacks. Our spectrally-resolved results also allow for more detailed interpretations of the various components of the RH-based feedbacks.

We begin in section 2 by reviewing Simpson’s Law and explicitly demonstrating it using line-by-line radiative transfer. After reviewing the definition of the feedbacks in (2) in Section 3, we then apply Simpson’s Law in understanding the spectral cancellation of conventional feedbacks in Section 4. We conclude in Section 5.

2 Simpson’s Law

In this section we briefly review Simpson’s Law, which dates back to Simpson [1928]. Simpson’s Law is the key ingredient in the ‘runaway greenhouse’ effect [e.g. Nakajima *et al.*, 1992; Goldblatt *et al.*, 2013], and has also been used to explain the T_s -dependence of OLR [Koll and Cronin, 2018], the rate of global mean precipitation change [Jeevanjee and Romps, 2018], and the strength of the water vapour feedback [Ingram, 2010]. A pedagogical treatment is given in Jeevanjee [2018]. We emphasize at the outset that Simpson’s ‘Law’ does not hold exactly, but is rather a first-order approximation; we refer to it as a ‘Law’ simply to emphasize the fundamental role it plays in the spectral structure of radiative feedbacks.

To arrive at Simpson’s Law, we first note that if RH is uniform, then the vapor density ρ_v (kg/m³) is a function of temperature only, with no explicit pressure dependence:

$$\rho_v = \rho_v(T) = \frac{\text{RHe}^*(T)}{R_v T} \quad (3)$$

105 where $e^*(T)$ is saturation vapor pressure and all other symbols have their usual meaning.
 106 Viewing T as a vertical coordinate, then, implies that the profile $\rho_v(T)$ should be univer-
 107 sal and independent of surface temperature, i.e. ‘ T_s -invariant’ [cf. Fig. 1 of *Jeevanjee and*
 108 *Romps, 2018*].

109 This then implies that H₂O optical depth at a given wavenumber should also be a T_s -
 110 invariant function of T , at least to first order and under typical circumstance. To see this, we
 111 write H₂O optical depth in temperature coordinates as

$$\tau(T) = \int_{T_{tp}}^T \kappa \rho_v(T') \frac{dT'}{\Gamma} \quad (4)$$

112 where T_{tp} is the tropopause temperature, κ is the mass absorption coefficient (m²/kg), and Γ
 113 the lapse rate. (Such an expression neglects stratospheric water vapor and cannot be used
 114 when tropospheric $T(z)$ is not single-valued, i.e. when there is a temperature inversion.
 115 Future work could investigate the validity of Simpson’s Law under such circumstances.)
 116 Though κ exhibits pressure and temperature dependencies due to collisional broadening
 117 and quantum effects [*Pierrehumbert, 2010*], and moist lapse rates Γ also vary in the vertical,
 118 these variations are expected to be weak compared to the strong exponential T -dependence
 119 of ρ_v . Since ρ_v is T_s -invariant, we expect $\tau(T)$ to be so as well, at least to first order [cf.
 120 Fig. S5 of *Jeevanjee and Romps, 2018*]. Since cooling-to-space can be approximated as
 121 emanating from $\tau \approx 1$ for optically thick wavenumbers ν [e.g. *Petty, 2006; Jeevanjee and*
 122 *Fueglistaler, 2020a*], this suggests that the spectrally-resolved outgoing longwave radiation
 123 OLR _{ν} and corresponding emission temperature T_{em} , defined in terms of the Planck function
 124 $B(\nu, T)$ by

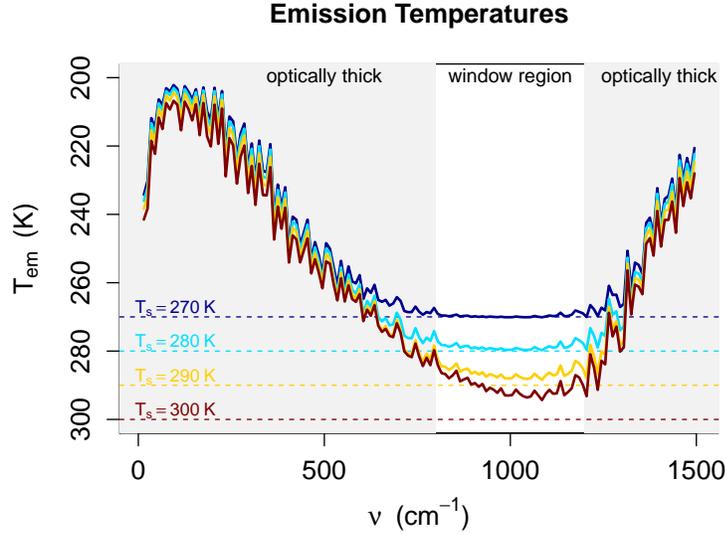
$$\pi B(\nu, T_{em}) = \text{OLR}_\nu, \quad (\text{W/m}^2/\text{cm}^{-1}) \quad (5)$$

125 should also be T_s -invariant (so long as RH is fixed). This then yields Simpson’s ‘Law’:

126 **Simpson’s ‘Law’:** At fixed RH, and for optically thick wavenumbers dominated by
 127 H₂O absorption, emission temperatures and OLR are independent of surface tempera-
 128 ture (to first order).

134 We explicitly verify Simpson’s Law in Figure 1 by plotting T_{em} (as diagnosed via (5))
 135 as a function of wavenumber for a set of moist adiabatic columns at varying T_s and with
 136 RH = 0.75 and no CO₂, using the Reference Forward Model (details of these calculations are
 137 as given in Section 4). Atmospheric emission emanates from the optically thick sections of
 138 the H₂O pure rotational band (0-800 cm⁻¹) and vibration-rotational band (1200-1500 cm⁻¹),
 139 while surface emission emanates through the optically thin water vapor ‘window’ at 800-
 140 1200 cm⁻¹. [For further intuition for this structure, see *Jeevanjee and Fueglistaler, 2020b*].
 141 The optically thick wavenumbers show relatively little variation of T_{em} with T_s , validating
 142 Simpson’s Law. Indeed, the average of dT_{em}/dT_s over 0 – 800 cm⁻¹ at $T_s = 290$ K is 0.2.
 143 Of course, the fact that dT_{em}/dT_s is not identically zero shows that Simpson’s Law is only
 144 approximate, due to our neglect of pressure broadening and lapse-rate changes in deducing
 145 Simpson’s Law above.

146 Simpson’s Law is nonetheless a useful idealization, as it encapsulates the small changes
 147 in optically thick T_{em} relative to the much larger changes in T_{em} in the optically thin water va-
 148 por window (in the window, which remains optically thin for $T_s \lesssim 290$ K, we have $T_{em} \approx T_s$
 149 and thus $dT_{em}/dT_s \approx 1$). In particular, differentiating (5) with respect to T_s and invoking
 150 Simpson’s Law tells us that the total feedback parameter should be roughly zero at H₂O-
 151 dominated wavenumbers. This then means that water vapor, planck, and lapse rate feedbacks
 152 *must* cancel at those wavenumbers. A primary goal of this paper is to explicitly verify this.
 153 A further corollary is that the total feedback is nonzero primarily in the water vapor window,
 154 and thus that the window is the main channel through which OLR increase with T_s [as em-
 155 phasized by *Koll and Cronin, 2018*]. We will sharpen and verify these claims in Section 4.



129 **Figure 1. Demonstration of Simpson’s Law.** Emission temperatures T_{em} defined by Eq. (5), as calculated
 130 with RFM for moist adiabatic atmospheres with varying T_s . Emission temperatures are relatively insensitive
 131 to T_s at optically thick wavenumbers (gray shading), but are roughly equal to T_s in the optically thin water
 132 vapor ‘window’ region (800–1200 cm^{-1} , white shading). Output is smoothed by averaging over bins of width
 133 10 cm^{-1} .

156 3 Feedback formulation

157 With Simpson’s Law in place we now turn to feedbacks. We begin by giving precise
 158 definitions of the Planck, LR, and WV feedbacks, in both the conventional and RH-based
 159 frameworks. Since the choice of moisture variable mostly impacts the clear-sky, longwave
 160 feedbacks, we consider these feedbacks only and do not consider λ^{cloud} , λ^{albedo} , or the short-
 161 wave component of λ^{WV} in our analysis. See *Held and Shell* [2012], however, for a discussion
 162 of how the RH-based framework changes the relative importance of other feedbacks.

163 In a cloud-free atmosphere with H_2O and CO_2 as the only greenhouse gases, the OLR
 164 is determined by their profiles, along with the surface temperature T_s and atmospheric tem-
 165 perature profile T_a (we suppress the vertical coordinate for clarity). A choice must be made,
 166 however, of which state variable to use for specifying H_2O concentrations; we begin with the
 167 conventional choice of specific humidity q_v , and later discuss the modification when using
 168 RH. We specify an atmosphere as an ordered triple (T_s, T_a, q_v) , and the OLR is then a func-
 169 tion of this ordered triple, i.e.

$$\text{OLR} = \text{OLR}(T_s, T_a, q_v). \quad (6)$$

170 We suppress the dependence of OLR on CO_2 concentration since we consider feedbacks
 171 here, not forcings, and feedbacks are always computed with CO_2 concentrations held fixed.
 172 The relevant CO_2 concentrations will be specified in the next section.

173 Consider now an initial atmosphere (T_s^i, T_a^i, q_v^i) and final atmosphere (T_s^f, T_a^f, q_v^f) , and
 174 let $\Delta T_s \equiv T_s^f - T_s^i$. Consistent with the definition (1) and our restriction to clear-sky longwave
 175 radiation only, our total feedback is then minus the change in OLR per unit surface tempera-
 176 ture difference:

$$\lambda^{\text{tot}} \equiv - \frac{\text{OLR}(T_s^f, T_a^f, q_v^f) - \text{OLR}(T_s^i, T_a^i, q_v^i)}{\Delta T_s} \quad (\text{W/m}^2/\text{K}). \quad (7)$$

177 (This simple forward difference, or ‘one-sided’ partial radiative perturbation (PRP), is not
 178 as accurate as the ‘two-sided’ PRP methods or radiative kernel methods employed in com-
 179 prehensive calculations [Colman and Mcavaney, 1997; Soden *et al.*, 2008; Shell *et al.*, 2008;
 180 Yoshimori *et al.*, 2020], but suffices for our purposes here.)

In the conventional (q_v -based) framework, we then define the following individual feedbacks:

$$\lambda^{\text{planck}} \equiv - \frac{\text{OLR}(T_s^i + \Delta T_s, T_a^i + \Delta T_s, q_v^i) - \text{OLR}(T_s^i, T_a^i, q_v^i)}{\Delta T_s} \quad (8a)$$

$$\lambda^{\text{lapse}} \equiv - \frac{\text{OLR}(T_s^f, T_a^f, q_v^i) - \text{OLR}(T_s^i + \Delta T_s, T_a^i + \Delta T_s, q_v^i)}{\Delta T_s} \quad (8b)$$

$$\lambda^{\text{wv}} \equiv - \frac{\text{OLR}(T_s^i, T_a^i, q_v^f) - \text{OLR}(T_s^i, T_a^i, q_v^i)}{\Delta T_s} . \quad (8c)$$

181 The Planck feedback λ^{planck} is minus the (ΔT_s -normalized) OLR response to a uniform
 182 change in surface and atmospheric temperatures, with q_v held fixed at the initial profile. The
 183 lapse-rate feedback λ^{lapse} is minus the OLR response to the difference between the actual
 184 temperature response and the uniform Planck response, still holding q_v fixed. The water va-
 185 por feedback λ^{wv} is then minus the OLR response to the change in q_v , holding temperatures
 186 fixed. Assuming linearity in the finite differences, we then have

$$\lambda^{\text{tot}} = \lambda^{\text{planck}} + \lambda^{\text{lapse}} + \lambda^{\text{wv}} . \quad (9)$$

For RH-based feedbacks, we use the formulae (8) but simply replace q_v with RH, and denote the corresponding RH-based feedbacks with a tilde:

$$\tilde{\lambda}^{\text{planck}} \equiv - \frac{\text{OLR}(T_s^i + \Delta T_s, T_a^i + \Delta T_s, \text{RH}^i) - \text{OLR}(T_s^i, T_a^i, \text{RH}^i)}{\Delta T_s} \quad (10a)$$

$$\tilde{\lambda}^{\text{lapse}} \equiv - \frac{\text{OLR}(T_s^f, T_a^f, \text{RH}^i) - \text{OLR}(T_s^i + \Delta T_s, T_a^i + \Delta T_s, \text{RH}^i)}{\Delta T_s} \quad (10b)$$

$$\tilde{\lambda}^{\text{wv}} \equiv - \frac{\text{OLR}(T_s^i, T_a^i, \text{RH}^f) - \text{OLR}(T_s^i, T_a^i, \text{RH}^i)}{\Delta T_s} . \quad (10c)$$

187 Note that now the water vapor feedback $\tilde{\lambda}^{\text{wv}}$ is due to RH changes, not q_v changes. Our cal-
 188 culations below will be at fixed RH, so $\tilde{\lambda}^{\text{wv}} = 0$ here by definition and we do not consider it
 189 further. This seems permissible because GCM studies find global-mean $\tilde{\lambda}^{\text{wv}}$ to be small, e.g.
 190 $\tilde{\lambda}^{\text{wv}} \approx 0 \pm 0.1 \text{ W/m}^2/\text{K}$ [Held and Shell, 2012; Zelinka *et al.*, 2020].

191 Since we expect increases in OLR to emanate from increased surface emission through
 192 the window (as suggested by Fig. 1), we also introduce a ‘surface’ feedback λ^{surf} obtained by
 193 perturbing T_s while holding the atmospheric temperature and water vapor profiles fixed:

$$\lambda^{\text{surf}} \equiv - \frac{\text{OLR}(T_s^i + \Delta T_s, T_a^i, q_v^i) - \text{OLR}(T_s^i, T_a^i, q_v^i)}{\Delta T_s} . \quad (11)$$

194 This feedback is identical in the q_v -based and RH-based frameworks, and is equal to the ‘sur-
 195 face kernel’ of radiative kernel analyses [cf. Fig. 1 of Soden *et al.*, 2008].

196 A key aspect of our analysis will be to consider *spectrally-resolved* OLR and hence
 197 spectrally-resolved versions of the feedbacks in Eqns. (7), (8), (10), and (11). These will be
 198 denoted with a subscript ν , with units $\text{W/m}^2/\text{cm}^{-1}/\text{K}$.

199 4 Spectral cancellation of conventional feedbacks

200 We now turn to spectrally-resolved calculations of the various feedbacks defined above,
 201 for a variety of idealized atmospheric columns. We calculate OLR_ν for these columns using

202 the line-by-line Reference Forward Model [RFM, *Dudhia, 2017*], along with HiTRAN2016
 203 spectroscopic data [*Gordon et al., 2017*] for H₂O from 0 to 1500 cm⁻¹ and CO₂ from 500 to
 204 850 cm⁻¹, using only the most common isotopologue for each gas. We run RFM at a spectral
 205 resolution of 0.1 cm⁻¹ and on 100 evenly-spaced pressure levels between 1000 and 10 hPa.
 206 Lineshapes follow the RFM default of a Voigt profile with 25 cm⁻¹ cutoff, and H₂O contin-
 207 uum effects are parameterized via RFM’s implementation of the MT-CKD2.5 continuum
 208 [*Mlawer et al., 2012*].

209 All atmospheric columns have $T_s^i = 288\text{K}$, $T_s^f = 289\text{K}$, RH=0.75, and an isothermal
 210 stratosphere at $T_{\text{strat}} = 200\text{K}$, with a uniform stratospheric q_v set equal to its tropopause
 211 value. The lapse rates and radiatively active species vary between cases, as described below.

212 4.1 Constant lapse rate, H₂O-only atmosphere

213 We begin by considering atmospheric columns with a constant lapse rate of 7 K/km
 214 and H₂O as the only radiatively active species. This case avoids the complications due to the
 215 LR feedback and due to CO₂, both of which we address below. We calculate the spectrally-
 216 resolved conventional feedbacks λ_ν , according to Eqns. (7) and (8); these are shown in Fig.
 217 2a.

218 The conventional Planck feedback $\lambda_\nu^{\text{planck}}$ is strongly negative, as expected, but is not
 219 a good first approximation to the total feedback λ_ν^{tot} ; there are large cancellations between
 220 $\lambda_\nu^{\text{planck}}$ and the strongly positive water vapor feedback λ_ν^{wv} . Indeed, as expected from Simp-
 221 son’s Law, at optically thick wavenumbers we have

$$\lambda_\nu^{\text{tot}} = \lambda_\nu^{\text{planck}} + \lambda_\nu^{\text{wv}} \approx 0 \quad (\text{optically thick } \nu). \quad (12)$$

222 Thus, at most wavenumbers the conventional feedback decomposition splits the total feed-
 223 back into equal and opposite terms, which are constrained to cancel by basic physics.

224 We now contrast this behavior with that of the RH-based formalism (Fig. 2b). In this
 225 case the picture is markedly simpler: the Planck feedback takes place at constant RH and so
 226 Simpson’s Law is manifest, yielding $\widetilde{\lambda}_\nu^{\text{planck}} \approx 0$ outside the window. In fact, since these ide-
 227 alized columns have no RH or lapse rate perturbations, we find that $\widetilde{\lambda}_\nu^{\text{planck}} = \widetilde{\lambda}_\nu^{\text{tot}}$ identically
 228 (so only one of these curves is visible in Fig. 2b). Thus, when RH is the moisture variable
 229 the reference response is a good null hypothesis (Roe 2009); in fact, for this simple system,
 230 the reference response captures the total system response perfectly.

231 As mentioned earlier, the dominant contribution to λ^{tot} seen in Fig. 2a,b can be inter-
 232 preted as an increase in surface cooling-to-space through the optically thin water vapor win-
 233 dow. This can be made more precise by invoking the argument of *Koll and Cronin* [2018],
 234 who show that the effects of increasing atmospheric emissivity on surface emission and at-
 235 mospheric emission cancel; the decreased emission-to-space from the surface is compen-
 236 sated for by increased emission from the near-surface atmosphere, which has the same tem-
 237 perature as the surface [*Koll and Cronin, 2018, Eq. S5*]. Hence, the effect of warming on
 238 OLR should be given by the Planck increase in surface emission, with atmospheric emissiv-
 239 ity held *fixed*. This is just the λ^{surf} term of Eq. (11), so this yields the approximation

$$\lambda_\nu^{\text{tot}} \approx \lambda_\nu^{\text{surf}}. \quad (13)$$

240 The surface feedback $\lambda_\nu^{\text{surf}}$ is shown in purple in Fig. 2b, and we find that in this idealized
 241 case Eqn. (13) indeed holds, to an accuracy of about 10% in the spectral integral (errors in
 242 this approximation are due to deviations from Simpson’s Law). Equation (13) thus gives a
 243 straightforward way to interpret the dominant contribution to λ^{tot} .

4.2 Conventional and RH-based feedbacks in a moist-adiabatic, H₂O-only atmosphere

Let us now incorporate the lapse-rate feedback, by replacing our constant lapse-rate temperature profiles with moist pseudo-adiabats based at $T_s^i = 288$ K and $T_s^f = 289$ K. We also introduce the conventional lapse rate feedback $\lambda_\nu^{\text{lapse}}$ as defined in Eqn. (8b). This feedback, and the others calculated as before, are shown in Fig. 2c.

Even though the lapse-rate feedback is present, Simpson's Law still operates: changes in moist adiabatic $\Gamma(T)$ with T_s are relatively small [Ingram, 2010], so by Eq. (4) T_{em} should still be insensitive to T_s . Now, however, Simpson's Law implies that at optically thick ν , there should be a near-complete cancellation of *three* terms:

$$\lambda_\nu^{\text{tot}} = \lambda_\nu^{\text{planck}} + \lambda_\nu^{\text{wv}} + \lambda_\nu^{\text{lapse}} \approx 0. \quad (\text{optically thick } \nu). \quad (14)$$

This means that summing the LR and WV feedbacks only yields a partial cancellation even at optically thick wavenumbers, due to the aforementioned fact that λ^{wv} is due to the entire q_ν perturbation but λ^{lapse} is due only to part of the temperature perturbation [cf. Eqns (8b) and (8c)]. From a spectral point of view, then, little simplification arises from summing only the LR and WV feedbacks.

The RH-based feedbacks for these moist-adiabatic atmospheres are shown in Fig. 2d. As before, the picture simplifies considerably: there is no RH-based water vapor feedback, and the RH-based Planck feedback is a good, if no longer perfect, approximation to the total feedback. A perhaps surprising result is that the RH-based lapse-rate feedback $\tilde{\lambda}_\nu^{\text{lapse}}$ is small, even in a fully moist-adiabatic atmosphere. This is because in the RH-based framework, the lapse-rate temperature perturbation is made at constant RH, and thus Simpson's Law applies. This is consistent with the conclusions of Cess [1975], who finds that changes in lapse rate (at fixed RH) have little impact on global energy balance. Thus, the impact of changing lapse rates depends crucially on whether RH or q_ν is held fixed.

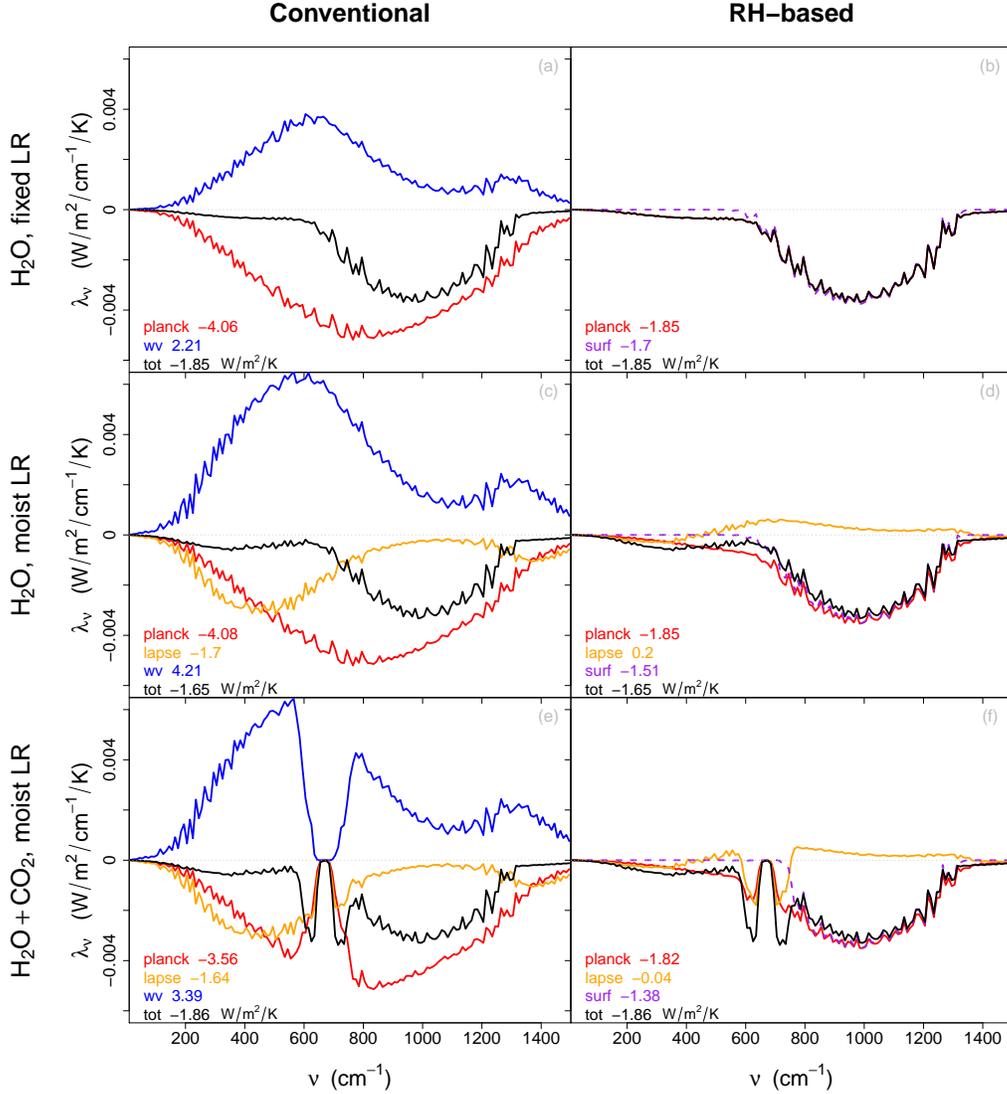
One consequence of $\tilde{\lambda}_\nu^{\text{lapse}}$ being small is that the RH-based Planck feedback $\tilde{\lambda}_\nu^{\text{planck}}$ is still a good null hypothesis for the OLR_ν change. Another, related consequence is that the surface feedback approximation Eqn. (13) continues to hold (Fig. 2d), again to about 10% in the spectral integral.

4.3 Conventional and RH-based feedbacks in a moist-adiabatic atmosphere with H₂O and CO₂

Next we consider the effects of CO₂. We calculate feedbacks for the moist-adiabatic columns of the previous subsection, but now with 280 ppmv of radiatively-active CO₂.

The results are shown in Fig. 2e,f. In the q_ν -based framework, there is still a marked cancellation between $\lambda_\nu^{\text{planck}}$, λ_ν^{wv} , and $\lambda_\nu^{\text{lapse}}$, but it now only occurs for ν which are outside the H₂O window and outside the 575 – 775 cm⁻¹ CO₂ band. Following Seeley and Jeevanjee [2020] we refer to the wings of the CO₂ band as 'CO₂ radiator fins', as they radiate from the upper troposphere and are visible as local extrema in λ_ν^{tot} at roughly 625 and 725 cm⁻¹. These wavenumbers radiate from fixed pressures (at fixed CO₂) rather than fixed temperatures [for CO₂, $\tau \sim p^2$; Pierrehumbert, 2010], and thus do not obey Simpson's Law. They make non-negligible contributions to λ_ν^{tot} , but are overshadowed by other features in $\lambda_\nu^{\text{planck}}$ and $\lambda_\nu^{\text{lapse}}$, due to continued cancellation with λ_ν^{wv} .

In the RH-based framework (Fig. 2f), however, the picture is again much simpler. The non-Simpsonian CO₂ radiator fins remain, but are partially captured by the the RH-based Planck feedback $\tilde{\lambda}_\nu^{\text{planck}}$, which is thus still a reasonable first approximation to λ_ν^{tot} (unlike the conventional $\lambda_\nu^{\text{planck}}$). The rest of the CO₂ radiator fin contribution is due to enhanced upper-tropospheric warming from lapse-rate changes [Seeley and Jeevanjee, 2020], which is indeed the main feature in $\tilde{\lambda}_\nu^{\text{lapse}}$. The advantage of the RH-based formulation is that it



250 **Figure 2. Spectral feedbacks in the conventional and RH-based formalisms.** (a) Conventional feed-
 251 backs with H₂O-only and a fixed lapse-rate. The conventional Planck and WV feedbacks cancel for optically
 252 thick ν (b) As in (a) but in the RH-based formalism. Now the Planck and total feedbacks are equal, and are
 253 well approximated by the surface feedback λ^{surf} (c,d) As in (a,b), but for moist-adiabatic temperature profiles.
 254 We still find $\lambda_{\nu}^{\text{tot}} \approx 0$ for optically thick ν , but now this implies a three-way cancellation of conventional
 255 feedbacks. The picture again simplifies for RH-based feedbacks, with a much smaller LR feedback in the
 256 RH-based formalism (e,f) As in (c,d), but now including 280 ppm of CO₂. Now $\lambda_{\nu}^{\text{tot}}$ exhibits CO₂ ‘radiator
 257 fins’, i.e. local extrema on either side of the CO₂ band. These extrema are overshadowed by other features in
 258 the conventional decomposition, but are highlighted in the RH-based decomposition. Color-coded numbers
 259 give the spectral integrals λ of the corresponding spectrally-resolved feedbacks λ_{ν} . Output is again smoothed
 260 over bins of width 10 cm⁻¹.

302 highlights these features, rather than lumping them in with the larger $\lambda_{\nu}^{\text{planck}}$ and $\lambda_{\nu}^{\text{lapse}}$ which
 303 then cancel with λ^{wv} .

304 Note that the negative contribution of the CO₂ radiator fins to spectrally-resolved $\tilde{\lambda}_v^{\text{lapse}}$
 305 offsets the positive contribution from the window (which results from fixed-RH upper-tropospheric
 306 moistening helping to close the window), leading to an even smaller spectrally-integrated
 307 $\tilde{\lambda}^{\text{lapse}}$. Thus, the conclusion from the previous H₂O-only calculation – that the strength of
 308 the lapse rate feedback is highly dependent on the choice of moisture variable – is only re-
 309 inforced by the addition of CO₂. The presence of the CO₂ radiator fins also means that the
 310 surface approximation (13) breaks down in the CO₂ band. The surface feedback (11) still,
 311 however, gives a precise way of interpreting and accounting for the increase in surface emis-
 312 sion through the window, which is a dominant contribution to λ^{tot} in the present-day climate
 313 [*Slingo and Webb, 1997; Raghuraman et al., 2019; Seeley and Jeevanjee, 2020*].

314 5 Summary

315 This paper has shown that:

- 316 1. The well-known compensation of conventional, q_v -based feedbacks is actually due to
 317 a near-perfect *cancellation* of these feedbacks at wavenumbers where H₂O is optically
 318 thick, as dictated by Simpson’s Law
- 319 2. This cancellation is incorporated more naturally in RH-based feedbacks, which more
 320 naturally manifest Simpson’s Law.

321 Furthermore, because constant RH is our null hypothesis under surface warming, the RH-
 322 based Planck feedback $\tilde{\lambda}^{\text{planck}}$ is a much better reference response (i.e. is closer to λ^{tot}) than
 323 the conventional Planck feedback. We also explicitly demonstrated that the increase in sur-
 324 face emission through the window is accurately captured by the surface feedback term (11),
 325 in line with the argument of *Koll and Cronin [2018]*.

326 Our findings also add nuance to the interpretation of the lapse-rate feedback. The con-
 327 ventional view that λ^{lapse} and λ^{wv} should be summed is called in to question by the three-
 328 way cancellation of λ^{planck} , λ^{lapse} , and λ^{wv} found here. Furthermore, we find [similar to *Held*
 329 *and Shell, 2012; Zelinka et al., 2020*] that $\tilde{\lambda}^{\text{lapse}}$ can be an order of magnitude smaller than
 330 λ^{lapse} , raising questions about the notion of a single, well-defined LR feedback.

331 One limitation of this study is its single-column framework with idealized tempera-
 332 ture and moisture profiles. While these were chosen to represent global mean conditions, this
 333 framework assumed tight surface-troposphere coupling and thus cannot account for decou-
 334 pled conditions with temperature inversions. Indeed, the zonal mean analyses of *Po-Chedley*
 335 *et al. [2018]* found relatively *large* values of $\tilde{\lambda}^{\text{lapse}}$ over the Southern Ocean, in contrast to
 336 our findings here. Future work could apply spectral feedback analyses to such conditions, to
 337 better understand these results. Future work could also consider how other greenhouse gases
 338 (e.g. ozone, methane) modulate the results shown here.

339 More broadly, however, our results suggest that RH-based feedbacks are not only more
 340 physical than conventional feedbacks from a thermodynamic point of view, as argued by
 341 *Held and Shell [2012]*, but are also simpler from a *radiative* point of view. Notably, a similar
 342 tension between RH and q_v -based points of view manifests in remote sensing applications,
 343 where it is long known that satellite-measured H₂O brightness temperatures are more sen-
 344 sitive to RH than q_v [*Möller, 1961; Soden and Bretherton, 1993*], but some more recent ob-
 345 servational studies nonetheless focus on q_v [e.g. *Dessler et al., 2008*], perhaps because of its
 346 use in conventional feedback analyses. We hope that our explicit formulation and validation
 347 of Simpson’s Law fosters a better appreciation of the emergent simplicity of H₂O radiative
 348 transfer, and encourages the use of RH as moisture variable in such applications where this
 349 simplicity manifests, as is the case here.

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