Time-dependent Tomographic Estimation of Global Exospheric Hydrogen Density during Geomagnetic Storms

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ILLINOIS — AGU 100 FALL MEETING, Washington, D.C. | 10–14 Dec 2018

Introduction

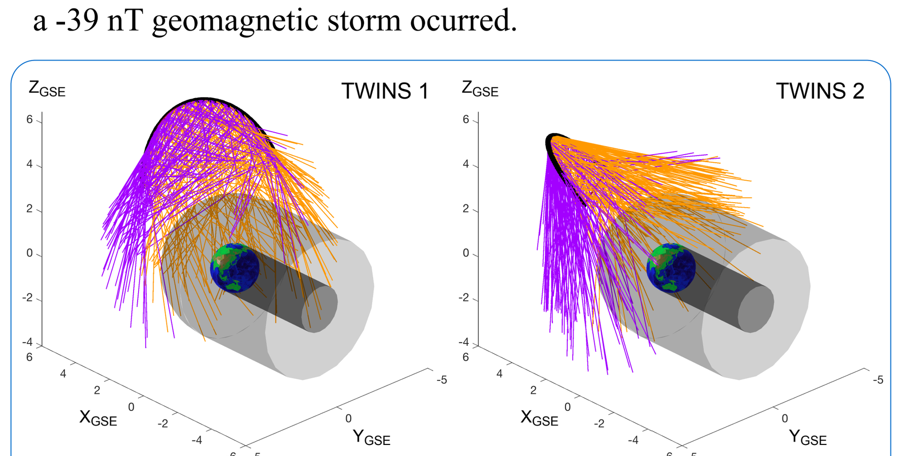
- During geomagnetic storms , charge exchange between neutral hydrogen (H) atoms in the terrestrial exosphere H^+ and O+ in the plasmasphere and the ring current serves to dissipate magnetospheric energy [Ilie, et al. 2013], influence the rate of plasmaspheri refilling [Krall et al., 2018], and enhance the loss of H beyond its quite-time thermal evaporation into space [Hodges et al, 1981].
- Remote sensing of solar Lyman-alpha ("Ly-a", at 121.6nm) photon scattering by exospheric H atoms is the only means available to infer H density over such a vast region and thus quantify the role of ion-neutral coupling in geomagnetic storm recovery and atmospheric evolution.
- At radial distances beyond $4R_E$ exospheric H density is sufficient low that solar photons scatter only once before being detected - this optically thin condition results in a linear relationship between the measured emission radiance (I) and the unknown H density (n_H) integrated along the viewing line-of-sight (LOS)

$$I(\mathbf{r},\hat{\mathbf{n}},t) = \frac{g^*}{10^6}\int_0^{L_{max}} n_H(l,t)\Psi(\beta)dl + I_{IP}(\hat{\mathbf{r}},\hat{\mathbf{n}},t)$$

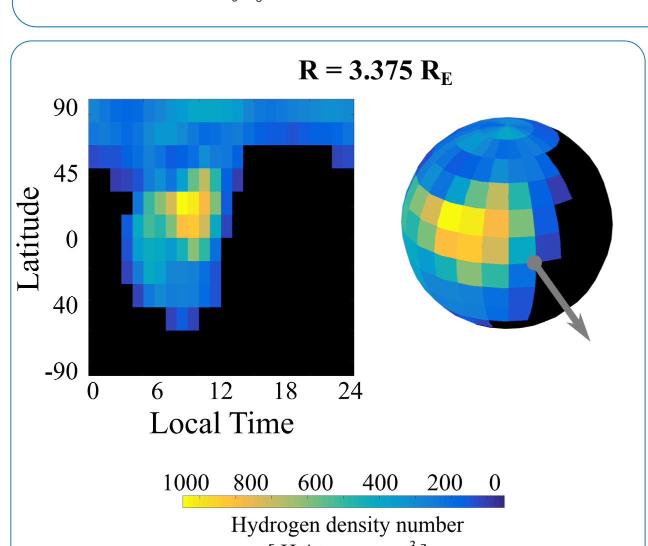
- Conventional parametric estimation of the global, 3D exospheric H density distribution from measurements of its optically-thin Ly-a emission are based on fits to spherical harmonic that adopt ad hoc assumptions regarding its radial decay and require a long time averaging that precludes assessment of storm-time variability [Bailey and Gruntman2011, 2013; Zoeenchen et al., 2011, 2013, 2015, 2017].
- Here, we present a new technique to reconstruct the global, 3D and time-dependent H density distribution beyond 4 Re from optically thin emission data using a robust tomographic inversion algorithm developed for static reconstructions [3] that we have modified to incorporate temporal variability via Kalman filtering.
- This poster describes the first application of this new technique to optically-thin exospheric Ly-a emission data acquired by NASA's TWINS satellites during a geomagnetic storm which occurred on 15 June, 2008.

Data

- **TWINS mission:** Comprised of two (2) satellites which enable stereoscopic sensing of the magnetosphere. Each satellite has two (2) Lyman-alpha detectors (LADs) that acquired Ly-alpha (121.6 nm) scattered emission from neutral hydrogen. The data used in this study is from June 13, 14, 15 and 16 of 2008 where a -39 nT geomagnetic storm ocurred.



Orbits (in black line) for both TWINS1/2. Line-of-sight LOS from LAD1 in orange and LAD2 in purple. Earth's shadow zone modeled as a cylinder.

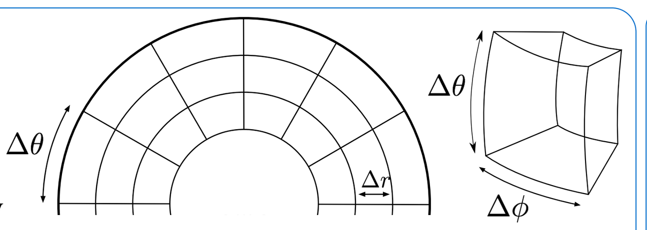


Example of LOS density profile for two radii. These plots are for the total LOSs for both LAD1/2 in TWINS1 for the period 5am-9am on June 15, 2008

Methodology: Tomographic Approach and Kalman Filter for Dynamic Reconstruction

Setting up the geometry

(1) Discretize region into J spherical voxels.
(2) Project unknown density function onto J orthonormal basis functions.
(3) Rewrite i^{th} measurement of intensity and cast measurment ensemble as a matrix equation.



$$y_i(\mathbf{r}_i,\hat{\mathbf{n}}_i) = \frac{g^*(\mathbf{r}_i)}{10^6}\int_0^{L_{max}[\hat{\mathbf{n}}_i]} n_H(\mathbf{r}')\Psi(\hat{\mathbf{n}}_i)dl$$

$$n_H(\mathbf{r}') = \sum_{j=1}^{N} x_j\delta_{H_j}(\mathbf{r}')$$

$$\mathbf{y} = L\mathbf{x}$$

With: $N_r = 12$, $N_\phi = 24$, $N_\theta = 12$

Inverse problem and regularization: *"Static Tomography"*

Since the observation matrix L is not full rank, a regularization technique must be used to solve the system. We have selected the technique known as Robust, regularized, positive estimation (RRPE), defined as follows:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}\geq 0} \Phi(\mathbf{x})$$

$$\Phi(\mathbf{x}) = ||L\mathbf{x} - \mathbf{y}||_2^2 + \lambda RRPE(\mathbf{x})$$

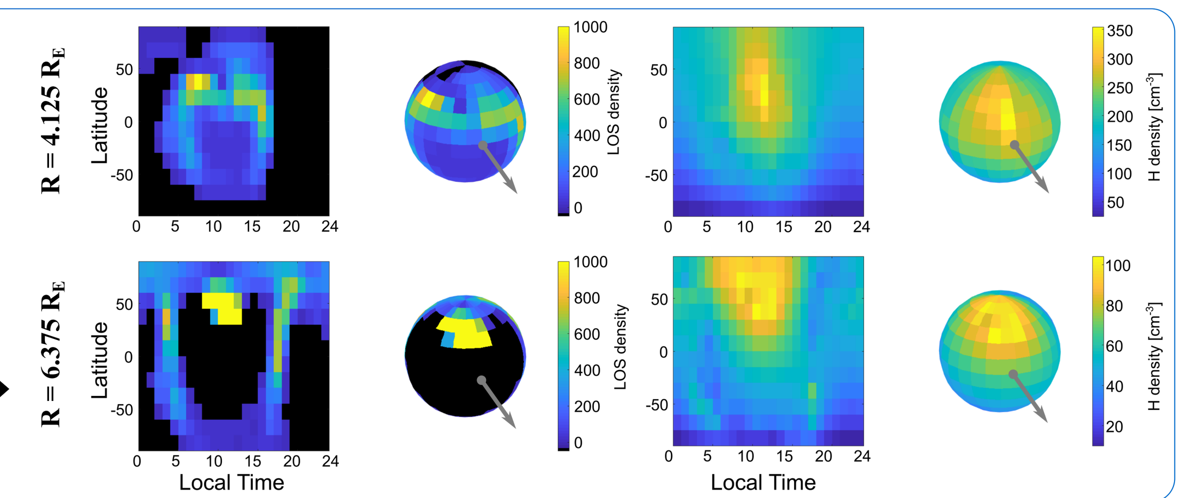
$$\lambda RRPE(x) = \lambda_r||x||_{D_r} + \lambda_\phi||x||_{D_\phi} + \lambda_\theta||x||_{D_\theta}$$

Where: $D_r \rightarrow \partial^2/\partial r^2$, $D_\phi \rightarrow \partial/\partial\phi$, $D_\theta \rightarrow \partial/\partial\theta$

Previous related work

(1) Analysis of TWINS data in [2] has depicted Hydrogen density variations during geomagnetic storms, however, the analysis is based only on several LOS directions.
(2) Our previous work [3] demostrated the feasibility of a tomographic approach for exospheric atomic hydrogen estimation based on optically thin emission data from TWINS.

The 1-day averaged tomographic reconstruction from our previous work. The first column presents the line-of-sight LOS density per voxel for two different radii. The second column shows the reconstructed hydrogen densities for the same radial shells



Space-state framework approach: *"Dynamic Tomography"*

As exospheric H densities are prone to be dynamic during storm-time, we use the state-space model as a means for time-varying estimation:

Measurement equation:

$$\mathbf{y}_i = H_i\mathbf{x}_i + \mathbf{v}_i$$

Model evolution equation:

$$\mathbf{x}_{i+1} = F_i\mathbf{x}_i + \mathbf{u}_i$$

Kalman Filter as solver

Measurement update:

$$K_i = P_{i|i-1}H_i^T(H_iP_{i|i-1}H_i^T + R_i)^{-1}$$
$$\hat{\mathbf{x}}_{i|i} = \hat{\mathbf{x}}_{i|i-1} + K_i(\mathbf{y}_i - H_i\hat{\mathbf{x}}_{i|i-1})$$
$$P_{i+1|i} = F_iP_{i|i}F_i^T + Q_i$$

Time update:

$$\hat{\mathbf{x}}_{i+1|i} = F_i\hat{\mathbf{x}}_{i|i}$$
$$P_{i|i} = P_{i|i-1} - K_iH_iP_{i|i-1}$$

Inclusion of regularization terms

$$\begin{bmatrix}\mathbf{y}_i\\0\end{bmatrix} = \begin{bmatrix}H_i\\D_i\end{bmatrix}\mathbf{x}_i + \begin{bmatrix}\mathbf{v}_i\\\mathbf{w}_i\end{bmatrix}$$

$$\mathbf{y}_i' = H_i'\mathbf{x}_i + v_i'$$

$$R_i' \triangleq \mathbb{E}[\mathbf{v}_i'(\mathbf{v}_i')^T] = \begin{bmatrix}R_i & 0\\0 & \lambda_i^{-1}I\end{bmatrix}$$

Dynamic tomographic estimation connected to the LMMSE estimation

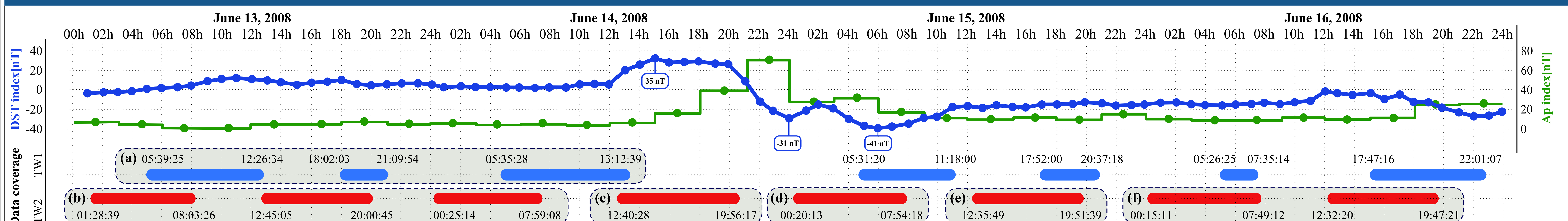
$$\hat{\mathbf{x}}_{i|i}^d = \underset{\mathbf{x}_i}{\text{argmin}}\,||\mathbf{y}_i' - H_i'\mathbf{x}_i||_{R_i'^{-1}}^2 + ||\mathbf{x}_i - \hat{\mathbf{x}}_{i|i-1}||_{P_{i|i-1}^{-1}}^2 + \lambda_\phi||D_\phi\mathbf{x}_i||_2^2 + \lambda_\theta||D_\theta\mathbf{x}_i||_2^2$$

Main Objective: We propose to analyze a storm-time event through the generation of dynamic tomographic reconstructions and focus on the hydrogen structure in the magnetospheric ring current region

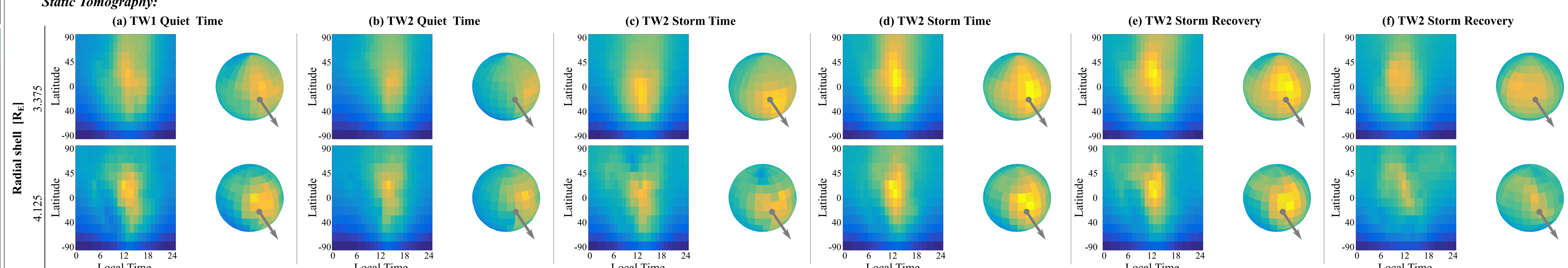
Conclusions

- TWINS observes exospheric H density increase abruptly in response to the geomagnetic storm on 15 June, 2008, with the onset of the increase, its rate and its magnitude varying with distance from Earth. In general, density increases begin soonest in the innermost exospheric region in the reconstruction (3.325 Re) and reach a peak density fastest there. Overall density enhancements of ~10% are observed at 3.325 Re, while much larger ~30% enhancements are seen at 4.125 Re and beyond (not shown). Recovery to pre-storm values is very slow, on timescales of days.
- TWINS' preferential sampling of the dayside Northern hemisphere, its temporal intermittency, and the likely cross-calibration error between TW1 and TW2, all serve to severely limit the LOS coverage available for accurate tomographic density reconstruction to highly localized and globally sparse regions.
- Measurement of truly global exospheric variability in response to geomagnetic storms, as well as identification of its physical drivers, requires more complete sampling of both the optically thin and thick regimes of scattered Ly-a emission. Wide-field, high-resolution, and high-sensitivity imaging from a distant vantage beyond the exosphere itself is the ideal means to obtain the data needed to understand the role of ion-neutral coupling in mediating storm recovery and to interpret ENA images acquired during storms

Results: Time-dependent Tomographic Hydrogen Density Estimation



Static Tomography:



Legend

- Disturbance index (DST), 1-hour averaged measurement.
- Ap index, 3-hour averaged measurement.
- Data used for Static Time-dependent reconstruction.
- Sunward pointer in Geocentered Solar Ecliptic (x_{GSE}) coordinates.

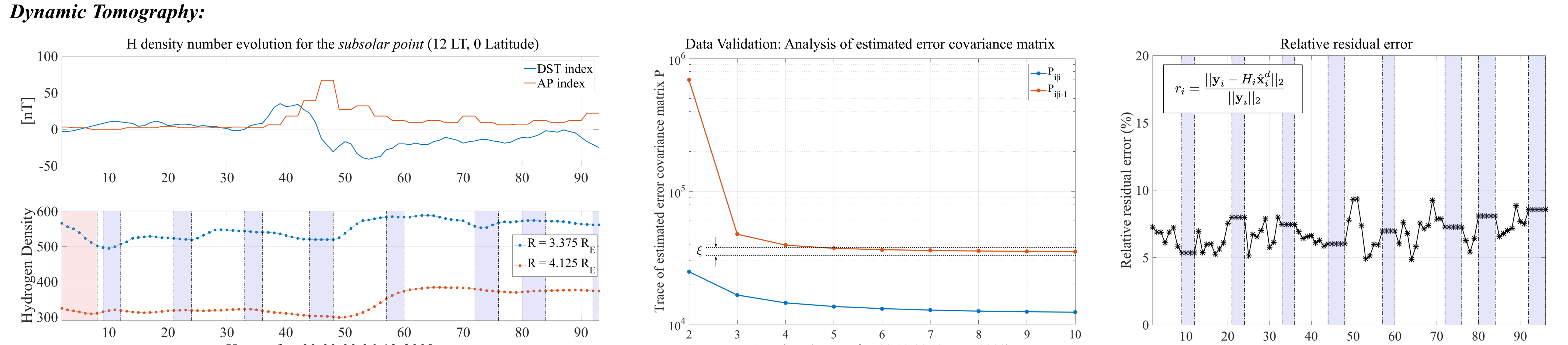
Colorbar for tomographic reconstructions:

R = 3.325 R_E: 600 500 400 300 200 100

R = 4.125 R_E: 350 300 250 200 150 100 50

Hydrogen density number [H Atoms . cm^{-3}]

Dynamic Tomography:



References

[1] Butala et al. (2009), "Tomographic Imaging of Dynamic Objects with the Ensemble Kalman Filter", *IEEE Trans. on Image Processing, 18, 1573 - 1587.*

[2] Zoennchen et al. (2017), The Response of the H Geocorona between 3 and 8 Re to Geomagnetic Disturbances studied using TWINS stereo Lyman-alpha data, *Annales Geophysicae, 35, 171 - 179.*

[3] Cucho-Padin G. and Waldrop L. (2018), "Tomographic Estimation of Exospheric Hydrogen Density Distributions". *Journal of Geophysical Research: Space Physics, 123.*

Acknowledgment

This work was supported by NASA HGIP (NNX16AF77G).