Space-based Remote Sensing Strategies for Tomographic Estimation of Exospheric Hydrogen Density

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I. INTRODUCTION

⊙ Hydrogen (H) atoms are the dominant constituent of the terrestrial exosphere, where they play a signicant role in governing recovery from geomagnetic storms through their charge exchange interactions with ambient H+ and O+ ions. Such reactions can induce ion pressure gradients that drive plasma transport between the ionosphere and plasmasphere as well as dissipate magnetospheric ring current energy through the generation of energetic neutral atoms.

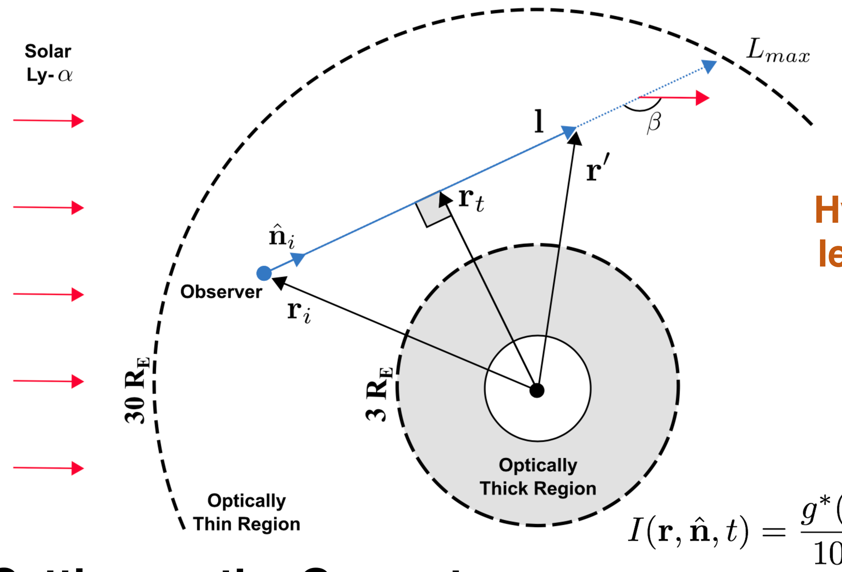
⊙ Remote Sensing of solar Lyman-Alpha ("Ly-α" @ 121.6nm) photon scattering by exospheric H atoms is the only means available to infer the global, time-dependent H density distribution over such a vast region.

⊙ At radial distances beyond 3 R_E, exospheric H density is sufficient low that solar photons scatter only once before being detected. This optically-thin condition results in a linear relationship between the measured emission radiance an the the unknown H density (n_H) integrated along the viewing line-of-sight (LOS)

⊙ This poster presents proof-of-concept tomographic strategies to reconstruct the time-dependent, 2D/3D H density distribution from stereoscopic observations of optically-thin Ly-α emission.

II. THEORY

The Neutral Exosphere

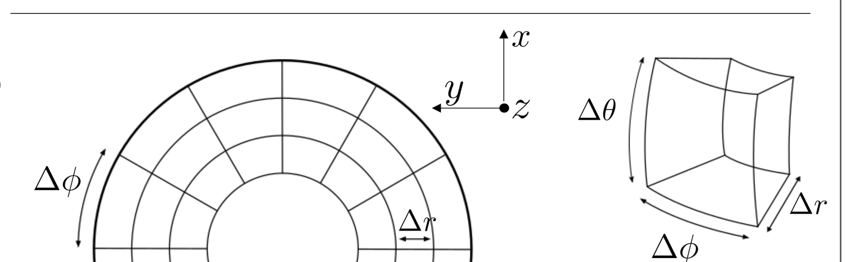


Hydrogen density estimation leverages the linearity of the optically thin emission model (> 3R_E)

$$I(\mathbf{r},\hat{\mathbf{n}},t) = \frac{g^*(t)}{10^6}\int_0^{Lmax} n_H(l)\Psi(\beta)dl + I_{IP}(\hat{\mathbf{n}},t)$$

Setting up the Geometry

⊙ **Step 1:** Discretize region into spherical voxels/polar pixels.



⊙ **Step 2:** Project unknown density function onto orthonormal basis functions.

$$y(\mathbf{r}_i,\hat{\mathbf{n}}_i) = \sum_{j=1}^{J}\left[\frac{g^*(\mathbf{r}_i)}{10^6}\Psi(\hat{\mathbf{n}}_i)\int_0^{Lmax}\delta_{H_j}(l)dl\right]x_j$$

⊙ **Step 3:** Rewrite measurement of intensity as a linear equation

$$n_H(r') = \sum_{j=1}^{J} x_j\delta_{H_j}(r'),$$

Discretization of the exospheric volume of interest yields an algebraic linear system

$$\mathbf{y} = L\mathbf{x}$$

Inverse Problem and Regularization: "Static Tomography"

⊙ Observation matrix $L \in \mathbb{R}^{M\times J}$, $M \gg J$ is **not full column rank.**

$$\hat{\mathbf{x}} = \underset{x\geq 0}{\mathrm{argmin}}\,\Phi(\mathbf{x})$$

⊙ Regularization techniques are needed to obtain a solution.

$$\Phi(\mathbf{x}) = ||L\mathbf{x}-\mathbf{y}||_2^2 + \lambda RRPE(\mathbf{x})$$

Cost Function — Data misfit term — Regularization term

⊙ Regularized Robust Positive Estimation (RRPE) method is selected and includes prior knowledge of physical structure of H density.

$$\lambda RRPE(\mathbf{x}) = \lambda_r||\mathbf{x}||_{D_r} + \lambda_\phi||\mathbf{x}||_{D_\phi} + \lambda_\theta||\mathbf{x}||_{D_\theta}$$

$$||\mathbf{x}||_{D_r} = \mathbf{x}^T D_r^T D_r \mathbf{x} \qquad D_r \approx \partial^2/\partial r^2 \quad D_\phi \approx \partial/\partial\phi \quad D_\theta \approx \partial/\partial\theta$$

Discrete matrix form of 1st and 2nd derivatives

Space-state framework approach: "Dynamic Tomography"

Kalman Filter (KF) as an approach to deal with dynamic H density during storm-time

Model: $\mathbf{y}_k = L_k\mathbf{x}_k$

Initial Estimates: $\hat{\mathbf{x}}_{0|-1} = \hat{\mathbf{x}}^s$, $P_{0|-1} = I\sigma^2$

Measurements: $\mathbf{y}_k, L_k, R_k$

Measurement update:

(KG) $K_k = P_{k|k-1}L_k^T(L_kP_{k|k-1}L_k^T + R_k)^{-1}$

(UE) $\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + K_k(\mathbf{y}_k - L_k\hat{\mathbf{x}}_{k|k-1})$

(UC) $P_{k|k} = P_{k|k-1} - K_kL_kP_{k|k-1}$

Time update:

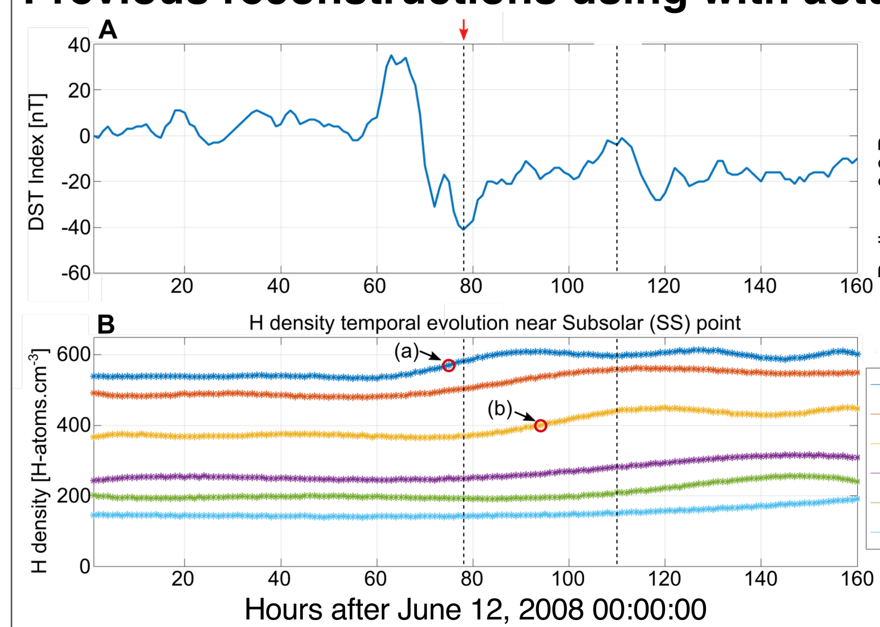
(Prj) $\hat{\mathbf{x}}_{k+1|k} = F_{k-1}\hat{\mathbf{x}}_{k|k}$, $P_{k+1|k} = F_kP_{k|k}F_k^T + Q_k$

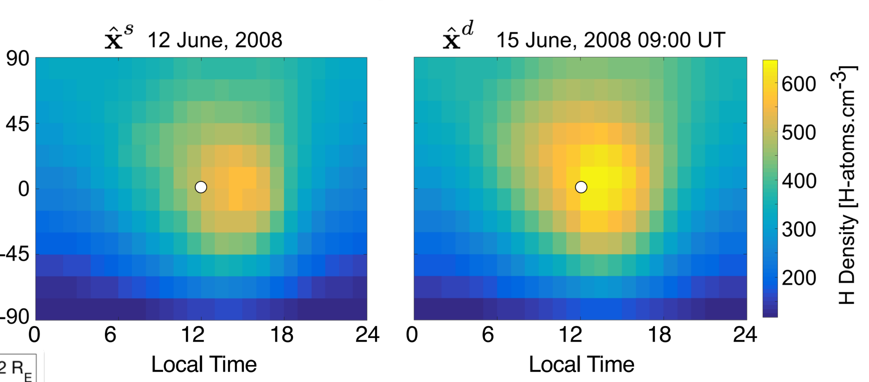
Kalman Gain (KG)
Update Estimate (UE)
Update Covariance (UC)
Project into *k+1* (Prj)

$\hat{\mathbf{x}}_{k+1|k}, P_{k+1|k}$

III. EXPERIMENTS

Previous reconstructions using with actual TWINS data during storm-time





Dynamic Tomography revealed for the first time spatial and time scales of the storm-time H density

Discoveries:

⊙ Global H density at 3 R_E begins to rise promptly by **~15%** after storm onset (June 15, 2008).

⊙ Such a perturbation appears to propagate outward with an effective speed of **~60 m/s**

[Cucho-Padin and Waldrop, GRL, 2019]

Limitations:

⊙ Ly-α emission sensing is not global – viewing geometry strongly favors the dayside northern hemisphere.

⊙ Data is intermittent – daily and seasonal gaps to avoid instrument contamination

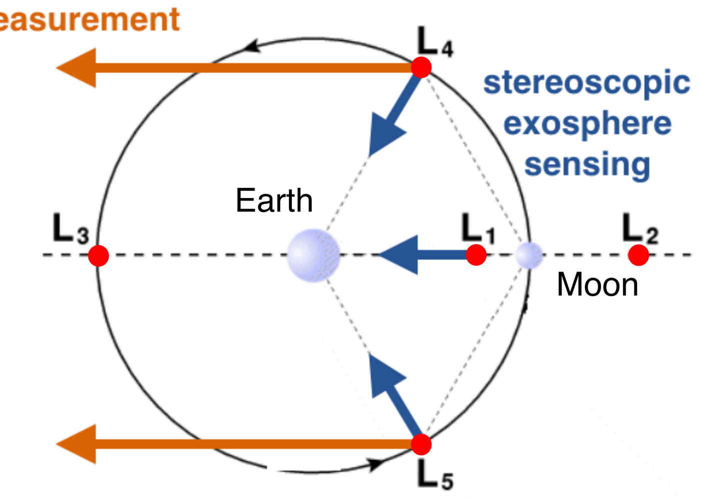
Proof-of-concept reconstructions from lunar orbit

⊙ Constellation sensor deployment to the Earth-Moon L1/L4/L5 points enables global, stereoscopic sensing through 28-day precession.

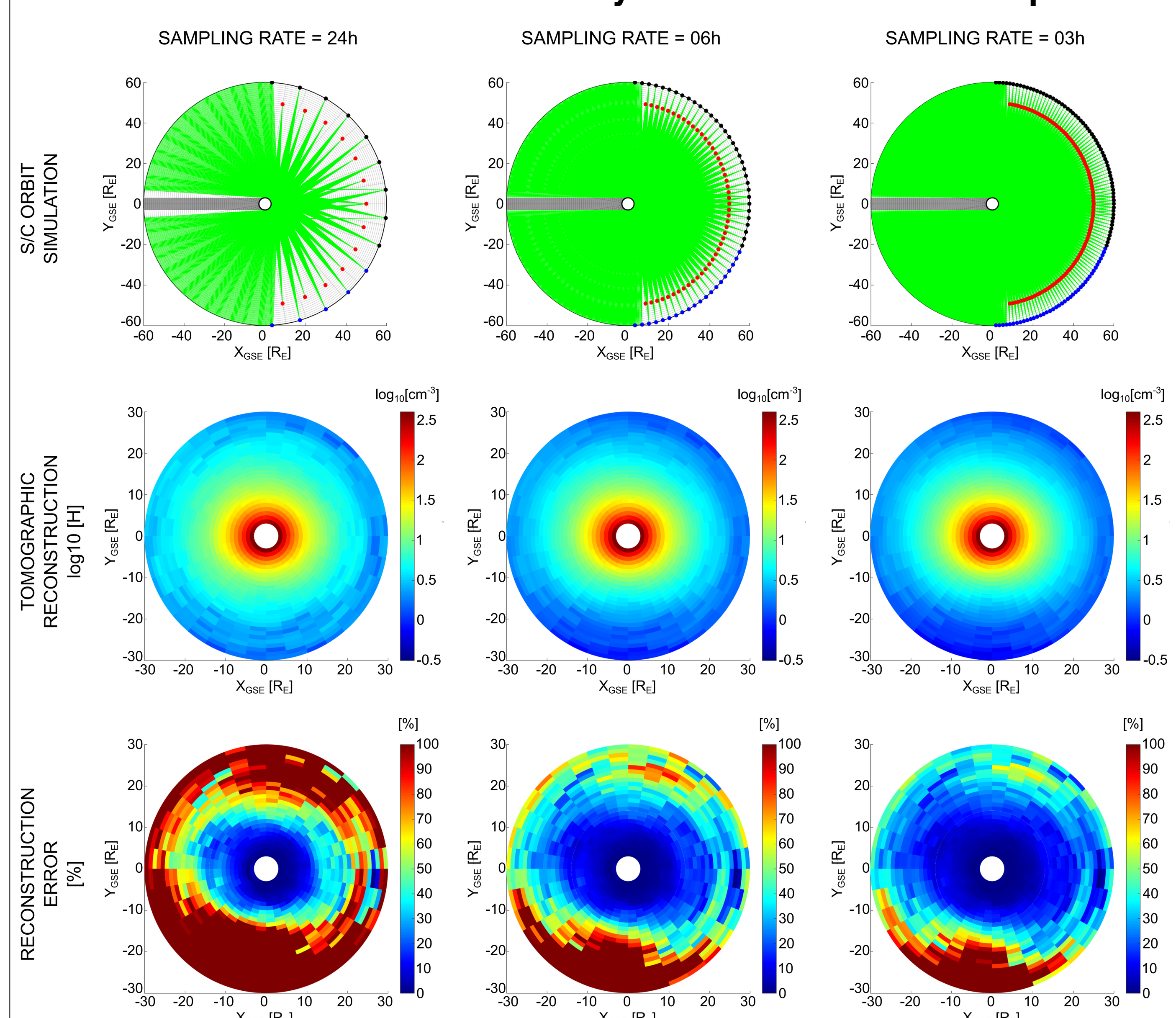
⊙ Line-of-sight (LOS) diversity from each platform can be achieved by mechanical scanning of a narrow FOV or by wide-field imaging (UV imager used below).

⊙ Avoidance of solar contamination introduces ~10-day data gap per month.

⊙ Interplanetary background sensing requires off-nadir pointing.

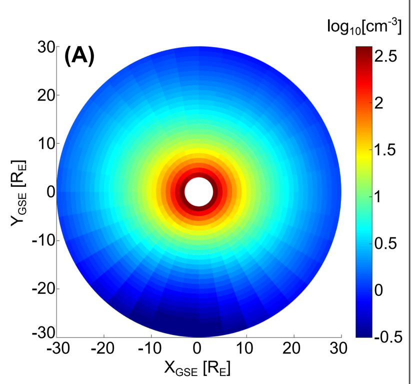


Static Reconstructions for ~14-day ensemble from L1/4/5 platforms



⊙ Assumed sensor specifications:
- 1024 pixels (2-D)
- Sensitivity $S=5.10^{-2}$ cts/R/sec
- FOV=15 deg.

● L1 ● L4 ● L5



Ground Truth

Experiment configuration:

- Target region: 3-30 R_E.
- Δr = 1R_E, $\Delta\phi$ = 10°.
- Simulated measured intensities **y'** are based on assumed H densities from [Zoennchen et al., 2015]

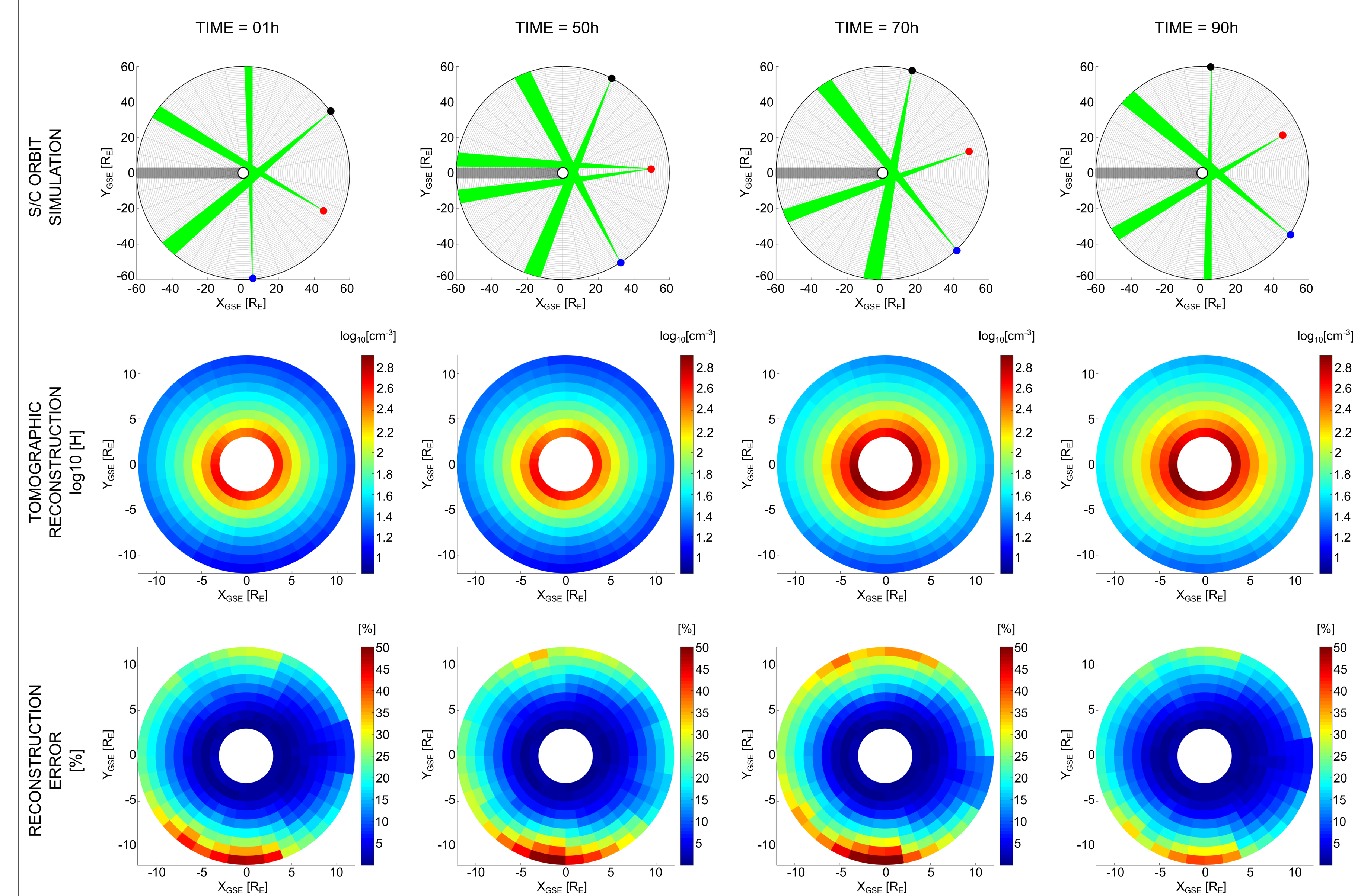
⊙ Daily imaging (24h cadence) yields <10% reconstruction error for exospheric region ≤ 10R_E (**magnetopause**)

⊙ Higher cadence (3h) acquisition yields <10% reconstruction error for exospheric region out to 20 R_E.

⊙ Assumption of time-invariance of H density over 14 days

⊙ Static reconstruction yields one image per 28 day precession period.

Dynamic Reconstructions using L1/4/5 platforms (1h cadence)



Experiment configuration:

- Target region: 3-12 R_E.
- Δr = 1R_E, $\Delta\phi$ = 10°.
- Simulated measured intensities **y'** are based on assumed H densities from [Cucho and Waldrop., 2019]
- Total duration of simulations is 90h ~ 3.75 days. Storm's commencement occurs ~50h.

⊙ Use of three satellites enables **hourly** reconstruction with errors < 10% in the exospheric region < 8 R_E.

⊙ This analysis is restricted to 5~7 days when all satellites are located at the Earth's day-side.

IV. SUMMARY

Conclusions

⊙ Stereoscopic imaging of Ly-α emission from lunar orbit is a promising new technique for exospheric density estimation.

⊙ < 6-hour sampling yields monthly static reconstructions (< 5% error from ~3-10 R_E), while 1-hour sampling enables dynamic reconstructions (< 5% error from ~3-5 R_E).

Caveats of lunar orbit deployment

⊙ 28-day precession around Earth introduces long (10-day) data gaps each month.

⊙ Background sensing requires off-nadir pointing and is not continuously available.

Mission opportunities

⊙ NASA's Lunar Orbital Platform – Gateway could offer hosted payload deployment to L1.

⊙ **Wide-field Ly-α imaging from the Earth-Sun L1 point by the NASA GLIDE Mission of Opportunity is the ideal means for truly routine, global exospheric sensing.**

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