Fixed-time projective synchronization of delayed memristive neural networks via aperiodically intermittent switching control

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Abstract. In this paper, the fixed-time projective synchronization issue for a class of delayed memristive neural networks is studied via aperiodically intermittent switching control. Firstly, according to the existing aperiodically intermittent switching strategy, a novel theorem for aperiodically intermittent switching fixed-time stability is proposed and proved through mathematical induction. Subsequently, an aperiodically intermittent switching controller is designed to reach fixed-time projective synchronization for drive-response systems. The power exponent is a function of error system state rather than one or two fixed constants. With the help of the extended differential inclusion framework, the inequality technique and the analysis method, some novel sufficient conditions are derived to ensure fixed-time projective synchronization for the considered systems. The settling time is closely related to the number of neurons and the maximum ratio of the rest width to the aperiodic time span, but independent of the initial value conditions. Furthermore, the fixed-time complete synchronization, fixed-time anti-synchronization and fixed-time stability obtained are special cases of the main theorem. Meanwhile, the conclusions of this paper improve some previous relevant works. Finally, a numerical example is given to verify the effectiveness and feasibility of the obtained results.

Key words: Fixed-time projective synchronization; Memristive neural networks; Mixed time-varying delays; Aperiodically intermittent switching control.

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1 Introduction

In 1971, Professor Chua initially proposed the concept of memristor which reflects the relationship between magnetic flux and charge [1]. The memristor is a kind of nonlinear resistance device meomrizing the amount of charge. Besides, its resistance value changes according to the current and voltage flowing through it. However, it has not attracted much attention until HP engineering researchers successfully manufactured the first physical memristive device nearly forty years later [2]. In addition, memristor similar to the biological synapse, and memory ability is more prominent than resistance [3]. So that, researchers use it to replace the resistance in the artificial neural network to simulate biological neural network, which is called memristor neural network. In recent years, the memristor neural network has attracted increasing attention from researchers due to its wide application in various fields of science and engineering, such as quadratic optimization, bioinspired engineering, associative memory, signal processing, nonvolatile memory, information storage, boolean logic operation function, and so forth[4-9].

In the past few decades, researchers have undertaken extensive studies on the dynamic behaviors of neural networks, especially synchronous ones. Synchronization of neural network means that the dynamic systems achieve identical state behaviors through mutual adjustment, which is common in the real world and has a wide range of applications, such as secure communications[10], image processing[11], optimization problems[13] and intelligent control[12],etc. Yet the synchronization of neural network dynamic systems can only be achieved by adding an appropriate controller to each neuron. So far, many different effective control approaches have been applied [22-29], including feedback control, switching control, adaptive control, quantized control, sliding mode control, intermittent control, impulsive control, event-triggered control, etc. Those methods can be classified as either continuous or discontinuous[22-29]. Compared with the continuous control approaches, the discontinuous control methods have attracted increasing attention from researchers because they are not only more economical and practical but also can reduce the amount of information transmission. Generally, discontinuous control methods include impulsive control [22], intermittent control[31,38,48] and switching control[14,24], etc.

Intermittent control, which was first proposed to control linear econometric systems in [35], has been used in wipers, transportation, ecosystem management, control of hyperuricemia in the treatment of gout, manufacturing. It is well known that intermittent control strategy is divided into the first intermittent subinterval and the second intermittent subinterval [48]. When the control intermittent subinterval time is reduced to a certain point, the intermittent control becomes impulsive ones. In other words, impulsive control is only activated at some instants and effective when the state is observable. If the state of the controlled system is unobservable, it becomes ineffective[47]. Thus, intermittent control is more effective and practical than impulsive control in applications. Periodic intermittent synchronization control has been widely studied recently [30,31,33,34]. For example, in [31], the authors considered the exponential synchronization of delayed memristor-based chaotic neural networks by designing a periodically intermittent controller. In [33], the exponential stabilization and synchronization for the fuzzy model of memristive neural networks by periodically intermittent control was discussed.

However, the requirement of periodic intermittent control has limitations and maybe unreasonable and unnecessary in practice. For instance, the generation of wind power is obviously aperiodically intermittent. Therefore, it is necessary to consider the synchronization problem under aperiodically intermittent control strategy in practical use and theoretical analysis. Fig.1 is a brief description of the aperiodically intermittent control approach to be investigated. The time span $[t_m, t_{m+1})$, $t_0 = 0, m = 0, 1, 2, \cdots$, is divided into the first-intermittent subinterval $[t_m, s_m)$ in which the systems are activated with a controller, and the second $[s_m, t_{m+1})$ in which the systems are activated with another controller [37,39,48]. The control strategy is more general than periodic ones, because when $t_{m+1} - t_m \equiv T, s_m - t_m \equiv \delta$, where T and δ are positive constants, $t_0 = 0, m = 0, 1, 2, \cdots$, the intermittent control type becomes the periodic one. Many achievements have been made in the periodic intermittent control synchronization of neural networks [33,34]. When $t_{m+1} - s_m \equiv 0, m = 0, 1, 2, \cdots$ the intermittent control strategy becomes the continuous type which has been studied in [21,23,36].

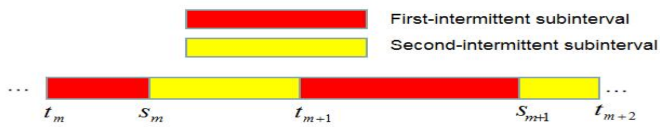

Figure 1: Aperiodically intermittent control strategy.

Compared with periodic intermittent control, aperiodic intermittent control has better practicability. The aperiodic intermittent control of neural network synchronization has been widely studied and some excellent and useful results have been obtained[37-40]. For example, under an aperiodically intermittent controller the synchronization of neural networks with stochastic perturbation was discussed in [37]. In [40], the authors paid attention to pinning synchronization of nonlinear and delayed coupled neural networks with multi-weights by designing an aperiodically intermittent controller. From papers [37-40], one can know that the trajectories of the dynamic error systems about drive-response systems can achieve synchronization when time goes to infinity. However, in actual engineering and theoretical research, to save time and cost, it is expected that the synchronization convergence speed is as high as possible.

Neural network synchronization is segregated into infinite-time synchronization and finite-time synchronization based on the convergence speed. Different from infinite-time synchronization, finite-time synchronization means that the systems can realize synchronization in a finite time. Recently, increasing attention has been paid to the finite-time synchronization[42-45] of neural networks due to faster convergence rate, better robustness against uncertainties and disturbance rejection properties [41]. However, the settling time of finite-time synchronization is a function depending on the initial conditions of considered systems, which may bring inconvenience to practical application. To overcome this problem, the fixed-time stability theory was proposed in [46]. As the fixed-time synchronization of neural networks has important applications in engineering management and theoretical analysis in recent years, it has received more and more attention from scientific researchers and technical workers[10,16,24,26]. For example, under a novel sliding mode controller, the fixed-time synchronization of delayed Cohen-Grossberg neural networks was discussed in [26].

There are many different concepts of neural network synchronization in practical use [14-21], such as exponential synchronization, bipartite synchronization, finite-time synchronization, fixed-time synchronization, prespecified-time synchronization, complete synchronization, projective synchronization, etc. In particular, considering the unpredictability of projective coefficient and the proportional relationship between drive-response systems, the security of communication can be enhanced and fast communication can be realized, respectively [16]. In addition, the projective synchronization is a more general type of synchronization. When the projective coefficients are 1, -1 and 0, complete synchronization, anti-synchronization and stability can be obtained as its special cases, respectively. Therefore, the projective synchronization has been widely concerned about [16,20, 42,43]. For example, in [16], the fixed-time projective synchronization of memristive neural networks with discrete delay was investigated. In [20], under the pinning impulsive control method, the projective synchronization of fuzzy memristive neural networks was studied.

As we all know that due to the finite switching speed of amplifiers and the finite speed of information transmitting, time delays usually exist in in the synchronization of many physical neural network systems, which may lead to neural network instability or poor performance. Generally, mixed time delays can be divided into discrete delays and distributed delays. In order to describe the real memristor-based neural network dynamics more accurately, the mixed-time delays should be taken into consideration. A lot of efforts have been devoted to the synchronization of memristor-based neural networks with mixed-time delays[14,15,28,36,42]. For example, the authors studied the adaptive synchronization of a class of memristor-based BAM neural networks with mixed delays through designing two kinds of adaptive feedback controllers[36]. However, to the best of our knowledge, there has not been any study concerning the fixed-time projective synchronization of delayed memristive neural networks via aperiodically intermittent switching control.

Motivated by the above discuss, we will investigate fixed-time projective synchronization of delayed memristive neural networks via aperiodically intermittent switching control. The paper mainly has four contributions. Firstly, based on the existing aperiodically intermittent switching strategy, a novel and vital theorem for aperiodically intermittent switching fixed-time stability is proposed and proved though mathematical induction. Secondly, a simple aperiodically intermittent switching controller for saving energy and reducing information transmission is designed to realize fixed-time projective synchronization of drive-response systems. Thirdly, some novel sufficient conditions are derived to ensure fixed-time projective synchronization for the considered systems, and the settling time is estimated and closely related to neural network scale and the maximum ratio of the rest width to the aperiodic time span. Besides, fixed-time complete synchronization, fixed-time anti-synchronization and fixed-time stability are special cases of the main theorem in this paper. Meanwhile, the conclusions of this paper improve some previous research results. Finally, a numerical example is given to demonstrate the effectiveness and feasibility of the obtained results.

The rest of This paper is organized as follows. Section 2 introduces model descriptions and preliminaries are given. In section 3, the addressed memristive neural networks are investigated and some fixed-time projective synchronization conditions are obtained by designing an aperiodically intermittent switching controller. The effectiveness of the designed methods is shown by a numerical example in section 4. Conclusions are drawn in the final part.

2 Problem formulation and preliminaries

In this paper, we consider a class of memristive neural networks with mixed time-varying delays described by the following equation in [14]:

$$\begin{aligned}\dot{x}_i(t) &= -a_i(x_i(t))x_i(t) + \sum_{j=1}^{n} b_{ij}(x_i(t))f_j(x_j(t)) + \sum_{j=1}^{n} c_{ij}(x_i(t)) \\ &\quad \times g_j(x_j(t-\tau_j(t))) + \sum_{j=1}^{n} d_{ij}(x_i(t)) \int_{t-\delta_j(t)}^{t} f_j(x_j(s))ds + I_i,\end{aligned} \tag{1}$$

where $i \in I = \{1, 2, \cdots, n\}$, $n \geq 2$ denotes the number of neurons in the neural networks; $x_i(t)$ denotes the state variable of the ith neuron at time t; $f_j(.)$ and $g_j(.)$ are the activation functions; $\tau_j(t)$ and $\delta_j(t)$ are the discrete time-varying delay and the distributed time delay, respectively, and satisfying $\tau_j(t) \leq \tau$, $0 \leq \delta_j(t) \leq \delta$; I_i is the external bias on the ith unit, and $\mid I_i \mid \leq ß$; $a_i(.), b_{ij}(.), c_{ij}(.)$ and $d_{ij}(.)$ represent the memristive connection weights, $i, j \in I$.

According to the property of memristor and current-voltage characteristics, the memristive connection weights are described as follows with mathematical model:

$$a_i(x_i(t)) = \begin{cases} \hat{a}_i, & \mid x_i(t) \mid \leq T_i \\ \check{a}_i, & \mid x_i(t) \mid > T_i \end{cases}, \quad b_{ij}(x_i(t)) = \begin{cases} \hat{b}_{ij} & \mid x_i(t) \mid \leq T_i \\ \check{b}_{ij} & \mid x_i(t) \mid > T_i \end{cases},$$

$$c_{ij}(x_i(t)) = \begin{cases} \hat{c}_{ij}, & \mid x_i(t) \mid \leq T_i \\ \check{c}_{ij}, & \mid x_i(t) \mid > T_i \end{cases}, \quad d_{ij}(x_i(t)) = \begin{cases} \hat{d}_{ij}, & \mid x_i(t) \mid \leq T_i \\ \check{d}_{ij}, & \mid x_i(t) \mid > T_i \end{cases}.$$

for $i, j \in I$, where $\hat{a}_i, \check{a}_i, \hat{b}_{ij}, \check{b}_{ij}, \hat{c}_{ij}, \check{c}_{ij}, \hat{d}_{ij}, \check{d}_{ij}$ are known constants, and $T_i > 0$ it denotes the switching jumps. Interested readers can further understand the structure of memristive neural networks by consulting relevant references[3,14,15,19,22,31,42].

The initial values conditions of system (1) are given by $x_i(s) = \phi_i(s), s \in [-r, 0], i \in I$, where $r = \max\limits_{j \in I}\{\tau_j, \delta_j\}$.

Throughout this paper, we denote $\underline{a}_i = \min\{\hat{a}_i, \check{a}_i\}, \bar{a}_i = \max\{\hat{a}_i, \check{a}_i\}$,$b^*_{ij} = \min\{\hat{b}_{ij}, \check{b}_{ij}\}$, $b^{**}_{ij} = \max\{\hat{b}_{ij}, \check{b}_{ij}\}, \overline{b}_{ij} = \max\{\mid \hat{b}_{ij} \mid, \mid \check{b}_{ij} \mid\}, c^*_{ij} = \min\{\hat{c}_{ij}, \check{c}_{ij}\}, c^{**}_{ij} = \max\{\hat{c}_{ij}, \check{c}_{ij}\}, \overline{c}_{ij} = \max\{\mid \hat{c}_{ij} \mid, \mid \check{c}_{ij} \mid\}$,$d^*_{ij} = \min\{\hat{d}_{ij}, \check{d}_{ij}\}, d^{**}_{ij} = \max\{\hat{d}_{ij}, \check{d}_{ij}\}, \overline{d}_{ij} = \max\{\mid \hat{d}_{ij} \mid, \mid \check{d}_{ij} \mid\}, i, j \in I$.

Then it is not difficult to obtain that

$$K[a_i(x_i(t))] = \begin{cases} \hat{a}_i, & \mid x_i(t) \mid \leq T_i \\ [\underline{a}_i, \bar{a}_i], & \mid x_i(t) \mid = T_i \\ \check{a}_i, & \mid x_i(t) \mid > T_i \end{cases}, \quad K[b_{ij}(x_i(t))] = \begin{cases} \hat{b}_{ij}, & \mid x_i(t) \mid \leq T_i \\ [b^*_{ij}, b^{**}_{ij}], & \mid x_i(t) \mid = T_i \\ \check{b}_{ij}, & \mid x_i(t) \mid > T_i \end{cases},$$

$$K[c_{ij}(x_i(t))] = \begin{cases} \hat{c}_{ij}, & \mid x_i(t) \mid \leq T_i \\ [c_{ij}^*, c_{ij}^{**}], & \mid x_i(t) \mid = T_i \\ \check{c}_{ij}, & \mid x_i(t) \mid > T_i \end{cases}, \quad K[d_{ij}(x_i(t))] = \begin{cases} \hat{d}_{ij}, & \mid x_i(t) \mid \leq T_i \\ [d_{ij}^*, d_{ij}^{**}], & \mid x_i(t) \mid \leq T_i \\ \check{d}_{ij}, & \mid x_i(t) \mid > T_i \end{cases}.$$

In order to obtain our main results in the next section, for the memristive neural networks system (1), it is necessary to introduce the following assumptions.

(H_1) The activation functions f_j and g_j are bounded. That is, there exist positive constants N_j and M_j satisfy that

$$\mid f_j(u) \mid \leq N_j, \mid g_j(u) \mid \leq M_j, \forall u \in R, j \in I.$$

(H_2) For $\forall u, v \in R$, there exist positive constants L_j^f and L_j^g such that the continuous neuron activation functions $f_j(.)$ and $g_j(.)$ satisfy

$$\mid f_j(v) - f_j(u) \mid \leq L_j^f \mid v - u \mid, \mid g_j(v) - g_j(u) \mid \leq L_j^g \mid v - u \mid, j \in I.$$

Considering memristive neural networks (1) as the drive system, the corresponding controlled response system is expressed by:

$$\begin{aligned} \dot{y}_i(t) &= -a_i(y_i(t))y_i(t) + \sum_{j=1}^{n} b_{ij}(y_i(t))f_j(y_j(t)) + \sum_{j=1}^{n} c_{ij}(y_i(t)) \\ &\quad \times g_j(y_j(t-\tau_j(t))) + \sum_{j=1}^{n} d_{ij}(y_i(t)) \int_{t-\delta_j(t)}^{t} f_j(y_j(s))ds + I_i + K_i(t), \end{aligned} \tag{2}$$

where $K_i(t)$ denotes an aperiodically intermittent switching controller to be designed later. The memristive connection weights of the response system (2) are the same as defined in drive system (1). The initial values conditions of response system (2) are given by $y_i(s) = \varphi_i(s), s \in [-r, 0], i \in I$.

Obviously, the right side of the drive-response systems (1) and (2) are discontinuous. , which makes the classic solution not suitable for drive-response systems (1) and (2). To our knowledge, the Filippov solution is an effective tool to solve the differential equations with discontinuous right side. Thus, based on the solution of the Filippov's sense, similar to that in [14-19,31,42,49,50], the theories of set-valued maps and differential inclusion are used to transform the drive-response systems (1) and (2) into the following differential inclusions:

$$\begin{aligned} \dot{x}_i(t) &\in -K[a_i(x_i(t))]x_i(t) + \sum_{j=1}^{n} K[b_{ij}(x_i(t))]f_j(x_j(t)) + \sum_{j=1}^{n} K[c_{ij}(x_i(t))] \\ &\quad \times g_j(x_j(t-\tau_j(t))) + \sum_{j=1}^{n} d_{ij}(x_i(t)) \int_{t-\delta_j(t)}^{t} f_j(x_j(s))ds + I_i, \end{aligned} \tag{3}$$

and

$$
\begin{aligned}
\dot{y}_i(t) \in & -K[a_i(y_i(t))]y_i(t) + \sum_{j=1}^{n} K[b_{ij}(y_i(t))]f_j(y_j(t)) + \sum_{j=1}^{n} K[c_{ij}(y_i(t))] \\
& \times g_j(y_j(t-\tau_j(t))) + \sum_{j=1}^{n} K[d_{ij}(y_i(t))] \int_{t-\delta_j(t)}^{t} f_j(y_j(s))ds + I_i + K_i(t),
\end{aligned}
\tag{4}
$$

or equivalently, based on the measurable selection theorem[32], for $i, j \in I$ there exist measurable functions $\widetilde{a}_i(t) \in K[a_i(x_i(t))], \widetilde{b}_{ij}(t) \in K[b_{ij}(x_i(t))], \widetilde{c}_{ij}(t) \in K[c_{ij}(x_i(t))], \widetilde{d}_{ij}(t) \in K[d_{ij}(x_i(t))]$, $\grave{a}_i(t) \in K[a_i(y_i(t))], \grave{b}_{ij}(t) \in K[b_{ij}(y_i(t))], \grave{c}_{ij}(t) \in K[c_{ij}(y_i(t))]$ and $\grave{d}_{ij}(t) \in K[d_{ij}(y_i(t))]$, such that

$$
\begin{aligned}
\dot{x}_i(t) = & -\widetilde{a}_i(t)x_i(t) + \sum_{j=1}^{n} \widetilde{b}_{ij}(t)f_j(x_j(t)) + \sum_{j=1}^{n} \widetilde{c}_{ij}(t)g_j(x_j(t-\tau_j(t))) \\
& + \sum_{j=1}^{n} \widetilde{d}_{ij}(t) \int_{t-\delta_j(t)}^{t} f_j(x_j(s))ds + I_i,
\end{aligned}
\tag{5}
$$

and

$$
\begin{aligned}
\dot{y}_i(t) = & -\grave{a}_i(t)y_i(t) + \sum_{j=1}^{n} \grave{b}_{ij}(t)f_j(y_j(t)) + \sum_{j=1}^{n} \grave{c}_{ij}(t)g_j(y_j(t-\tau_j(t))) \\
& + \sum_{j=1}^{n} \grave{d}_{ij}(t) \int_{t-\delta_j(t)}^{t} f_j(y_j(s))ds + I_i + K_i(t).
\end{aligned}
\tag{6}
$$

From [14,49,50], we can know that the drive system (1) and response system (2) have at least one Filippov solution $x(t)$ and $y(t)$ on the interval $[0, +\infty)$, respectively.

Let $e_i(t) = y_i(t) - \zeta x_i(t), i \in I$ is the system of error, and ζ denotes the projective coefficient. From systems (5) and (6), we get the error system as follows:

$$
\begin{aligned}
\dot{e}_i(t) = & -(\grave{a}_i(t)y_i(t) - \zeta\widetilde{a}_i(t)x_i(t)) + \sum_{j=1}^{n}(\grave{b}_{ij}(t)f_j(y_j(t)) - \grave{b}_{ij}(t)f_j(\zeta x_j(t))) \\
& + \sum_{j=1}^{n}(\grave{c}_{ij}(t)g_j(y_j(t-\tau_j(t))) - \grave{c}_{ij}(t)g_j(\zeta x_j(t-\tau_j(t)))) \\
& + \sum_{j=1}^{n}(\grave{d}_{ij}(t) \int_{t-\delta_j(t)}^{t} f_j(y_j(s))ds - \zeta\widetilde{d}_{ij}(t) \int_{t-\delta_j(t)}^{t} f_j(x_j(s))ds) \\
& + \sum_{j=1}^{n}(\grave{b}_{ij}(t)f_j(\zeta x_j(t)) - \zeta\widetilde{b}_{ij}(t)f_j(x_j(t))) + \sum_{j=1}^{n}(\grave{c}_{ij}(t)g_j(\zeta x_j(t \\
& -\tau_j(t))) - \zeta\widetilde{c}_{ij}(t)g_j(x_j(t-\tau_j(t)))) + I_i(1-\zeta) + K_i(t), i \in I.
\end{aligned}
\tag{7}
$$

Definition 1. The drive-response systems (1) and (2) will reach the finite-time projective synchronization if, there exists a constant $T(e_0) > 0$ such that

$$\lim_{t \to T(e_0)} \parallel y(t) - \zeta x(t) \parallel = 0, \parallel y(t) - \zeta x(t) \parallel = 0, t > T(e_0),$$

where $x(t)$ and $y(t)$ are the solutions of drive-response systems (1) and (2) with initial conditions ϕ and φ, respectively.

Definition 2. The drive-response systems (1) and (2) will reach the fixed-time projective synchronization, if for any initial condition, there exist a fixed-time $T_{\max}$ and a settling time function $T(e_0)$ such that

$$\lim_{t \to T(e_0)} \parallel e(t) \parallel = 0, \parallel e(t) \parallel = 0, \forall t \geq T(e_0)$$

where $T(e_0) \leq T_{\max}$, $e(t) = (e_1(t), e_2(t), \cdots, e_n(t))$.

To obtain the main results, we need to introduce the following lemmas.

Lemma 1.(see [48]) For aperiodically intermittent strategy, if $\theta(t) = \dfrac{t - s_m}{t - t_m}$, $t \in (s_m, t_{m+1}]$, and $m \in D$ is a strictly increasing function, one can get that

$$\theta(t) \leq \frac{t_{m+1} - s_m}{t_{m+1} - t_m} \leq \lim_{m \to +\infty} \sup \frac{t_{m+1} - s_m}{t_{m+1} - t_m} = \theta.$$

Then, one can derive $0 \leq \theta \leq 1$.

Lemma 2.(see [49]) If $z_1, z_2, ..., z_n$ are positive constants and $0 < \epsilon < l$, then

$$(\sum_{j=1}^{n} \mid z_j \mid^l)^{\frac{1}{l}} \leq (\sum_{j=1}^{n} \mid z_j \mid^\epsilon)^{\frac{1}{\epsilon}}, (\frac{1}{n}\sum_{j=1}^{n} \mid z_j \mid^l)^{\frac{1}{l}} \geq (\frac{1}{n}\sum_{j=1}^{n} \mid z_j \mid^\epsilon)^{\frac{1}{\epsilon}}.$$

Lemma 3.(see [51]) If there exists a continuous and radially unbounded function $V : R^n \to R_+ \bigcup\{0\}$ such that any solution $x(t)$ of system (1) satisfies the inequality

$$\dot{V}(t) \leq -\mu(V(t))^{\gamma + sign(V(t)-1)}, \tag{8}$$

in which $\mu > 0$, $1 < \gamma < 2$, then the origin of system (1) is globally fixed-time stable. In addition, for any initial state x_0 of system (1), the settling time is described as $T \leq \dfrac{1}{\mu(2-\gamma)} + \dfrac{1}{\mu\gamma}$.

Lemma 4. (see [48]) For a given scalar function $V(t) : R^n \to R$ with $V(0) = 0$ and $V(t) > 0$, $t \in R\backslash\{0\}$, the following inequality holds

$$V^{1-\eta}(t) \leq V^{1-\eta}(0) - \alpha(1-\eta)(1-\theta)t, 0 \leq t \leq T,$$

where T is the settling finite time. If function $V(t), t \in [0, +\infty)$ satisfies that

$$\begin{cases} \dot{V}(t) \leq -\alpha V^{\eta}(t), t_m \leq t < s_m, \\ \dot{V}(t) \leq 0, s_m \leq t < t_{m+1}, \end{cases}$$

where $\alpha > 0, T > 0, 0 < \eta < 1, m \in D$, the settling time $T = \dfrac{V^{1-\eta}(0)}{\alpha(1-\theta)(1-\eta)}$.

3 Main results

In this section, a novel aperiodically intermittent switching fixed-time stability theorem is established. And we will derive some criteria to guarantee the fixed-time projective synchronization for the drive-response systems (1) and (2) by designing an aperiodically intermittent switching controller.

Theorem 1. Suppose that function $V(t)$ is non-negative for $\forall t \in [0, +\infty)$ and satisfies the following conditions:

$$\begin{cases} \dot{V}(t) \leq -\mu(V(t))^{\gamma + sign(V(t)-1)}, t_m \leq t < s_m, \\ \dot{V}(t) \leq 0, s_m \leq t < t_{m+1}, \end{cases} \tag{9}$$

where $\mu > 0$, $1 < \gamma < 2$, $0 < \theta < 1, m \in D$. Then $V(t) \equiv 0$, if $T \geq \dfrac{1}{\mu(2-\gamma)(1-\theta)} + \dfrac{1}{\mu\gamma(1-\theta)}$.

Proof. The following proof process are divided into two cases: Case(I) $0 \leq V(t) < 1$ and Case(II) $V(t) \geq 1$.

First of all, Case (I) is proved as follows

Case (I): When $0 \leq V(t) < 1$, (9) becomes as the following form:

$$\begin{cases} \dot{V}(t) \leq -\mu(V(t))^{(\gamma-1)}, t_m \leq t < s_m, \\ \dot{V}(t) \leq 0, s_m \leq t < t_{m+1}, \end{cases} \tag{10}$$

Supposing $M_0 = (\sup\limits_{s \in [-r,0]} V(s))^{2-\gamma}$, set $F(t) = V^{2-\gamma}(t) + \mu t(2-\gamma)$, and

$$G(t) = F(t) - hM_0,$$

where $h > 1$ is a constant. It is easy to get that, for $\forall t \in [-r, 0]$,

$$G(t) < 0. \tag{11}$$

In the following, we will prove that

$$G(t) < 0, \text{for all} \quad t \in [0, s_0). \tag{12}$$

For $\forall t \in [0, s_0)$, according to the first inequality in (10), we have

$$\begin{aligned} \dot{G}(t) &= (2-\gamma)V^{1-\gamma}(t)\dot{V}(t) + \mu(2-\gamma) \\ &\leq (2-\gamma)V^{1-\gamma}(t)(-\mu V^{\gamma-1}(t)) + \mu(2-\gamma) \\ &= 0. \end{aligned} \tag{13}$$

From (13), one can get that $G(t)$ is a monotonous decreasing function in $[0, s_0)$. Combing (11) , one can derive that $G(t) \leq G(0) < 0$, for $\forall t \in [0, s_0)$.

Let

$$\overline{G}(t) = F(t) - hM_0 - \mu(2-\gamma)(t - s_0), t \in [s_0, t_1). \tag{14}$$

Next, we will prove that $\overline{G}(t) < 0$, for $t \in [s_0, t_1)$.

For $\forall t \in [s_0, t_1)$, on account of the second inequality in (10), we can derive that

$$\begin{aligned} \dot{\overline{G}}(t) &= (2-\gamma)V^{1-\gamma}(t)\dot{V}(t) + \mu(2-\gamma) - \mu(2-\gamma) \\ &\leq (2-\gamma)V^{1-\gamma}(t)(-\mu V^{\gamma-1}(t)) + \mu(2-\gamma) - \mu(2-\gamma) \\ &= -\mu(2-\gamma) \\ &< 0. \end{aligned} \tag{15}$$

Together with (12) and (15), one can obtain that $\overline{G}(t) \leq \overline{G}(s_0) = G(s_0) = F(s_0) - hM_0 < 0$, for $\forall t \in [s_0, t_1)$.

From (11), (12) and (14), for $\forall t \in [-r, t_1)$, one can get that

$$F(t) < hM_0 + \mu(2-\gamma)(t_1 - s_0). \tag{16}$$

For $\forall \in t \in [t_1, s_1)$, supposing

$$G(t) = F(t)) - hM_0 - \mu(2-\gamma)(t_1 - s_0), \tag{17}$$

the derivative of $G(t)$ is calculated as follows

$$\begin{aligned} \dot{G}(t) &= (2-\gamma)V^{1-\gamma}(t)\dot{V}(t) + \mu(2-\gamma) \\ &\leq (2-\gamma)V^{1-\gamma}(t)(-\mu V^{\gamma-1}(t)) + \mu(2-\gamma) \\ &= 0, \end{aligned}$$

and one can get that $G(t)$ is decreasing in $[t_1, s_1)$. According to (14), one can obtain that $G(t) < 0$, for $\forall t \in [t_1, s_1)$. So, for $\forall t \in [t_1, s_1)$, the following formula holds

$$F(t)) < hM_0 + \mu(2-\gamma)(t_1 - s_0). \tag{18}$$

For $\forall t \in [s_1, t_2)$, let

$$\overline{G}(t) = F(t) - hM_0 - \mu(2-\gamma)(t_1 - s_0) - \mu(2-\gamma)(t - s_1). \tag{19}$$

The derivative of $\overline{G}(t)$ is as follows

$$\begin{aligned} \dot{\overline{G}}(t) &= (2-\gamma)V^{1-\gamma}(t)\dot{V}(t) + \mu(2-\gamma) - \mu(2-\gamma) \\ &\leq (2-\gamma)V^{1-\gamma}(t)(-\mu V^{\gamma-1}(t)) \\ &= -\mu(2-\gamma) \\ &< 0. \end{aligned} \tag{20}$$

From (20), one can get that $\overline{G}(t)$ is decreasing in $[s_1, t_2)$. Combing (18), one can easily obtain that $\overline{G}(t) < 0$, for $\forall t \in [s_1, t_2)$. Thus, for $\forall t \in [s_1, t_2)$, the following formula holds

$$F(t)) < hM_0 + \mu(2-\gamma)(t_1 - s_0) + \mu(2-\gamma)(t - s_1). \tag{21}$$

In the following part, we will use mathematical induction method to prove that the following statements are true.

For $\forall t \in [t_m, s_m)$,

$$F(t) \leq hM_0 + \mu(2-\gamma)\sum_{k=1}^{m}(t_k - s_{k-1}), \tag{22}$$

and for $\forall t \in [s_m, t_{m+1})$,

$$F(t) \leq hM_0 + \mu(2-\gamma)\sum_{k=1}^{m}[(t_k - s_{k-1}) + (t - s_m)]. \tag{23}$$

Assuming that inequalities (22) and (23) are true for $m \leq p-1$, where p is a positive integer. Then, for any integer q satisfying $0 \leq q \leq p-1$, and $\forall t \in [t_q, s_q)$

$$\begin{aligned} F(t) &\leq hM_0 + \mu(2-\gamma)\sum_{k=1}^{q}(t_k - s_{k-1}) + \mu(2-\gamma)(t_{q+1} - s_q) \\ &= hM_0 + \mu(2-\gamma)\sum_{k=1}^{q+1}(t_k - s_{k-1}) \\ &= hM_0 + \mu(2-\gamma)\sum_{k=1}^{p}(t_k - s_{k-1}). \end{aligned} \tag{24}$$

For $\forall t \in [t_p, s_p)$, assuming

$$G(t) = F(t) - hM_0 - \mu(2-\gamma)\sum_{k=1}^{p}(t_k - s_{k-1}), \tag{25}$$

similar to the proof of (17), one can prove that inequality $G(t) \leq 0$ holds for $t \in [t_p, s_p)$.

For $\forall t \in [s_q, t_{q+1})$,

$$\begin{aligned} F(t) &\leq hM_0 + \mu(2-\gamma)\sum_{k=1}^{q}[(t_k - s_{k-1}) + (t - s_q)] \\ &\quad +\mu(2-\gamma)[t_{q+1} - s_q) + (t - s_{q+1}]) \\ &= hM_0 + \mu(2-\gamma)\sum_{k=1}^{q+1}[(t_k - s_{k-1}) + (t - s_{q+1})] \\ &= hM_0 + \mu(2-\gamma)\sum_{k=1}^{p}[(t_k - s_{k-1}) + (t - s_p)]. \end{aligned} \tag{26}$$

For $\forall t \in [s_p, t_{p+1})$, let

$$\overline{G}(t) = F(t) - hM_0 - \mu(2-\gamma)\sum_{k=1}^{p}[(t_k - s_{k-1}) + (t - s_p)]. \tag{27}$$

Similar to the proof of (19), one can prove that inequality $\overline{G}(t) \leq 0$ holds for $t \in [s_p, t_{p+1})$.

Therefore, according to the mathematical induction, we get that Eqs (22) and (23) hold for any natural number m and $\forall t \in [0, +\infty)$. For any nonnegative integers m and $\forall t \in [t_m, t_{m+1})$, we can deduce the following estimation of $F(t)$.

For $\forall t \in [t_m, s_m)$, by using Lemma 1

$$\begin{aligned} F(t) &\leq hM_0 + \mu(2-\gamma)\sum_{k=1}^{m}(t_k - s_{k-1}) \\ &= hM_0 + \mu(2-\gamma)\sum_{k=1}^{m}\frac{t_k - s_{k-1}}{t_k - t_{k-1}}(t_k - t_{k-1}) \\ &\leq hM_0 + \mu(2-\gamma)\theta\sum_{k=1}^{m}(t_k - t_{k-1}) \\ &= hM_0 + \mu(2-\gamma)\theta t_m \\ &\leq hM_0 + \mu(2-\gamma)\theta t. \end{aligned} \tag{28}$$

Moreover, for $\forall t \in [s_m, t_{m+1})$, by applying Lemma 1

$$\begin{aligned}
F(t) &\leq hM_0 + \mu(2-\gamma)\sum_{k=1}^{m}[(t_k - s_{k-1} + (t - s_m)] \\
&= hM_0 + \mu(2-\gamma)\sum_{k=1}^{m}[\frac{t_k - s_{k-1}}{t_k - t_{k-1}}(t_k - t_{k-1}) + \frac{t - s_m}{t - t_m}(t - t_m)] \\
&\leq hM_0 + \mu(2-\gamma)\sum_{k=1}^{m}[\theta(t_k - t_{k-1}) + \frac{t_{m+1} - s_m}{t_{m+1} - t_m}(t - t_m)] \\
&\leq hM_0 + \mu(2-\gamma)\theta\sum_{k=1}^{m}[(t_k - t_{k-1}) + (t - t_m)] \\
&= hM_0 + \mu(2-\gamma)\theta t.
\end{aligned} \tag{29}$$

According to the Eqs (28) and (29), we can derive that $F(t) \leq hM_0 + \mu(2-\gamma)\theta t$ holds for $\forall t \in [0, +\infty)$.

From the definition of $F(t)$, one can get that

$$\begin{aligned}
V^{2-\gamma}(t) &\leq hM_0 + \theta\mu(2-\gamma)t - \mu t(2-\gamma) \\
&= hM_0 - \mu(2-\gamma)(1-\theta)t,
\end{aligned} \tag{30}$$

Let $h \to 1$, one can obtain that

$$V^{2-\gamma}(t) \leq 1 - \mu(1-\theta)(2-\gamma)t. \tag{31}$$

Let $\widehat{G}(t) = 1 - \mu(1-\theta)(2-\gamma)t$. It can be found that $\widehat{G}(t)$ is a strictly monotone decreasing function with respect to t. If $\widehat{G}(t) = 0$, we can get that

$$T_1 = \frac{1}{\mu(1-\theta)(2-\gamma)}. \tag{32}$$

It is known that $\lim_{t \to T_1} V^{2-\gamma}(t) = 0$. According to the (31) and (32) and the strict monotone decreasing property of $\widehat{G}(t)$, one can obtain that $\lim_{t \to T_1} V(t) = 0$ and $V(t) \equiv 0$ for all $t \geq T_1$.

Next, we will prove the Case(II) as follows.

Case (II): When $V(t) \geq 1$, one can get the following form:

$$\begin{cases} \dot{V}(t) \leq -\mu(V(t))^{(\gamma+1)}, t_m \leq t < s_m, \\ \dot{V}(t) \leq 0, s_m \leq t < t_{m+1}, \end{cases} \tag{33}$$

If $\widetilde{M_0} = (\sup_{s \in [-r,0]} V(s))^{-\gamma}$ and $Q(t) = V^{-\gamma}(t) - \gamma\mu t$.

Let

$$P(t) = Q(t) - l\widetilde{M}_0, \tag{34}$$

where $l \in [0,1]$ and it is a constant and small enough. It is easy to see that, for $\forall t \in [-r, 0]$.

$$P(t) > 0. \tag{35}$$

In the following, we will prove that

$$P(t) > 0, \text{which is true for all} \quad t \in [0, s_0).$$

For $\forall t \in [0, s_0)$, calculating the derivative of $P(t)$ with respect to time t, and combining the first inequality in (33), one can derive that

$$\begin{aligned} \dot{P}(t) &= -\gamma V^{-\gamma-1}(t)\dot{V}(t) - \mu\gamma \\ &\geq \gamma V^{-\gamma-1}(t)\mu V^{\gamma+1}(t) - \mu\gamma \\ &= 0. \end{aligned} \tag{36}$$

From (36), we know that $P(t)$ is monotonically increasing in the interval $[0, s_0)$. From (35), one can know that

$$P(t) \geq P(0) > 0, \tag{37}$$

which is true for all $t \in [0, s_0)$.

Next, the following inequality will be proved.

$$\overline{P}(t) = Q(t) - l\widetilde{M}_0 + \mu\gamma(t - s_0) > 0,$$

which is true for $\forall t \in [s_0, t_1)$.

For $\forall t \in [s_0, t_1)$, calculating the derivative of $\overline{P}(t)$ with respect to time t and together with the second inequality in (33), one can obtain that

$$\dot{\overline{P}}(t) = -\gamma V^{-\gamma-1}(t)\dot{V}(t) - \mu\gamma + \mu\gamma \geq \gamma V^{-\gamma-1}(t) \times 0 - \mu\gamma + \mu\gamma = 0. \tag{38}$$

From (37) and (38) and the continuity of $P(t)$, one can get that

$$\overline{P}(t) \geq \overline{P}(s_0) = P(s_0) > 0, \tag{39}$$

which is true for all $t \in [s_0, t_1)$.

And from (39), we have

$$Q(t) > l\widetilde{M}_0 - \mu\gamma(t_1 - s_0). \tag{40}$$

Similar to the proof of (37), one can verify that

$$Q(t)) > l\widetilde{M}_0 - \mu\gamma(t_1 - s_0), \tag{41}$$

which is true for all $t \in [t_1, s_1)$.

Similar to the proof of (40), one can prove that

$$Q(t) > l\widetilde{M}_0 - \mu\gamma(t_1 - s_0) - \mu\gamma(t - s_1), \tag{42}$$

which is true for all $t \in [s_1, t_2)$.

In the following, we will use mathematical induction method to prove that the following statement is true.

For $\forall t \in [t_m, s_m)$,

$$Q(t) > l\widetilde{M}_0 - \mu\gamma \sum_{k=1}^{m} (t_k - s_{k-1}), \tag{43}$$

and for $\forall t \in [s_m, t_{m+1})$,

$$Q(t) > l\widetilde{M}_0 - \mu\gamma \sum_{k=1}^{m} [(t_k - s_{k-1}) + (t - s_m)]. \tag{44}$$

Assuming that inequalities (43) and (44) are true for $m \leq p-1$, where p is a positive integer, then, for any integer q satisfying $0 \leq q \leq p-1$, and $\forall t \in [t_q, s_q)$, we can derive that

$$\begin{aligned} Q(t) &> l\widetilde{M}_0 - \mu\gamma \sum_{k=1}^{q} (t_k - s_{k-1}) \\ &> l\widetilde{M}_0 - \mu\gamma \sum_{k=1}^{q} (t_k - s_{k-1}) - \mu\gamma(t_{q+1} - s_q) \\ &= l\widetilde{M}_0 - \mu\gamma \sum_{k=1}^{q+1} (t_k - s_{k-1}) \\ &= l\widetilde{M}_0 - \mu\gamma \sum_{k=1}^{p} (t_k - s_{k-1}). \end{aligned} \tag{45}$$

Similar to the proof of (37), we can get that $P(t) = Q(t) - l\widetilde{M}_0 + \mu\gamma \sum_{k=1}^{p} (t_k - s_{k-1}) > 0$ in $[t_p, s_p)$.

For $\forall t \in [s_q, t_{q+1})$, we can get

$$Q(t) > l\widetilde{M}_0 - \mu\gamma \sum_{k=1}^{q} [(t_k - s_{k-1}) + (t - s_q)]$$

$$
\begin{aligned}
&> l\widetilde{M_0} - \mu\gamma \sum_{k=1}^{q} [(t_k - s_{k-1}) + (t - s_q)] - [\mu\gamma(t_{q+1} - s_q) + (t - s_{q+1})] \\
&= l\widetilde{M_0} - \mu\gamma \sum_{k=1}^{q+1} [(t_k - s_{k-1}) + (t - s_{q+1})] \qquad (46) \\
&= l\widetilde{M_0} - \mu\gamma \sum_{k=1}^{p} [(t_k - s_{k-1}) + (t - s_p)].
\end{aligned}
$$

Similar to the proof of (40), we can obtain that $\overline{P}(t) = Q(t) - l\widetilde{M_0} + \mu\gamma \sum_{k=1}^{p} [(t_k - s_{k-1}) + (t - s_p)] > 0$ for $t \in [s_p, t_{p+1})$. From the above proof, it can be concluded that inductive hypotheses (43) and (44) hold in the interval $[0, +\infty)$, for any nonnegative integer m. Therefore, we can deduce the following estimate of $Q(t)$ for any nonnegative integer m and $t \in [t_m, t_{m+1})$.

For $\forall t \in [t_m, s_m)$, with the help of Lemma 1 and (43), we can derive

$$
\begin{aligned}
Q(t) &> l\widetilde{M_0} - \mu\gamma \sum_{k=1}^{m} (t_k - s_{k-1}) \\
&= l\widetilde{M_0} - \mu\gamma \sum_{k=1}^{m} \frac{t_k - s_{k-1}}{t_k - t_{k-1}} (t_k - t_{k-1}) \\
&\geq l\widetilde{M_0} - \mu\gamma\theta \sum_{k=1}^{m} (t_k - t_{k-1}) \qquad (47) \\
&= l\widetilde{M_0} - \mu\gamma\theta t_m \\
&\geq l\widetilde{M_0} - \mu\gamma\theta t.
\end{aligned}
$$

For $\forall t \in [s_m, t_{m+1})$, based on Lemma 1 and (44), we can get that

$$
\begin{aligned}
Q(t) &> l\widetilde{M_0} - \mu\gamma \sum_{k=1}^{m} [(t_k - s_{k-1}) + (t - s_m)] \\
&= l\widetilde{M_0} - \mu\gamma \sum_{k=1}^{m} [\frac{t_k - s_{k-1}}{t_k - t_{k-1}} (t_k - t_{k-1}) + \frac{t - s_m}{t - t_m} (t - t_m)] \\
&\geq l\widetilde{M_0} - \mu\gamma \sum_{k=1}^{m} [\frac{t_k - s_{k-1}}{t_k - t_{k-1}} (t_k - t_{k-1}) + \frac{t_{m+1} - s_m}{t_{m+1} - t_m} (t - t_m)] \\
&\geq l\widetilde{M_0} - \mu\gamma \sum_{k=1}^{m} [\theta(t_k - t_{k-1}) + \theta(t - t_m)]
\end{aligned}
$$

$$\begin{aligned} &\geq l\widetilde{M_0} - \mu\theta\gamma \sum_{k=1}^{m} [(t_k - t_{k-1}) + (t - t_m)] \\ &= l\widetilde{M_0} - \mu\theta\gamma t. \end{aligned} \tag{48}$$

From the definition of $Q(t)$, (47) and (48), we can obtain that

$$V^{-\gamma}(t) > l\widetilde{M_0} + \gamma\mu t - \mu\theta\gamma t > \mu(1-\theta)\gamma t. \tag{49}$$

From (49), one can obtain that

$$V^{\gamma}(t) < \frac{1}{\mu\gamma(1-\theta)t}. \tag{50}$$

Let $\overline{G}(t) = \dfrac{1}{\mu\gamma(1-\theta)t}$. It is not difficult to find that $\overline{G}(t)$ is a strictly monotone decreasing function with respect to t. If $\overline{G}(t) = 1$, we can get that

$$T_2 = \frac{1}{\mu(1-\theta)\gamma}. \tag{51}$$

It is known that $\lim\limits_{t \to T_2} V^{\gamma}(t) = 1$. According to (50) and (51) and the strict monotone decreasing property of $\overline{G}(t)$, one can obtain that $\lim\limits_{t \to T_2} V(t) = 1$ and $V(t) \leq 1$ are for all $t \geq T_2$.

Combining Case (I) with Case (II), we can obtain that $\lim\limits_{t \to T_1+T_2} V(t) = 0$ and $V(t) \equiv 0$ hold for all $t \geq T_1 + T_2$.

This completes the proof.

Remark 1. In paper [48], the authors established and proved an aperiodic intermittent adjustment differential inequality within finite-time stability. It's not hard to see that, the settling time for finite-time synchronization depends on the initial values of the systems, which may bring difficulties and inconvenience to solving practical problems, while in Theorem 1 can avoid this problem. Therefore, the conclusion of Theorem 1 is more practical.

Remark 2. Compared with the differential inequalities for fixed-time stability of aperiodic intermittent switching control method in [55], the differential inequality in this paper is simpler for there is only one power exponent which is a function of the error system state rather than two fixed constants.

For convenience, assuming

$$\widetilde{\alpha}_i + \underline{a}_i - 2\sum_{j=1}^{n} \overline{b}_{ji} L_i^f \geq 0, \lambda_i - 2\sum_{j=1}^{n} \overline{c}_{ji} L_i^g \geq 0, i \in I. \tag{52}$$

$$\begin{aligned}&\beta_i - \zeta \mid a_i^{*} - a_i^{**} \mid T_i - \sum_{j=1}^{n}(\bar{d}_{ij} N_j \delta_j (1+\zeta) \\ &+ N_j \mid \hat{b}_{ij} - \zeta \check{b}_{ij} \mid + M_j \mid \hat{c}_{ij} - \zeta \check{c}_{ij} \mid) - \text{ß}(1-\zeta) \geq 0, i \in I.\end{aligned} \tag{53}$$

The aperiodically intermittent switching controller in response system (2) is designed as follows

$$K_i(t) = \begin{cases} -\widetilde{\alpha}_i e_i(t) - sign(e_i(t))(\beta_i + \lambda_i \mid e_i(t-\tau_i(t)) \mid \\ +\omega_i \mid e_i(t) \mid^{\gamma + sign(V(t)-1)}), t_m \leq t < s_m, m \in D, \\ -\widetilde{\alpha}_i e_i(t) - sign(e_i(t))(\beta_i + \lambda_i + \mid e_i(t-\tau_i(t)) \mid), \\ s_m \leq t < t_{m+1}, m \in D, \end{cases} \tag{54}$$

where the positive constants $\widetilde{\alpha}_i, \beta_i, \lambda_i, \omega_i$ are control parameters and $1 < \gamma < 2, i \in I$.
Theorem 2. Suppose that (H_1)-(H_2) hold. If the control parameters $\widetilde{\alpha}_i$, β_i and γ_i satisfy (52) and (53). Then the drive-response systems (1) and (2) can realize fixed-time projective synchronization under the controller (54). More importantly, the settling time for fixed-time projective synchronization is estimated by

$$T \leq \frac{2n^{\gamma}}{\omega\gamma(2-\gamma)(1-\theta)}.$$

Proof. We construct a Lyapunov function as follows

$$V(t) = \sum_{i=1}^{n} \mid e_i(t) \mid .$$

Based on the switching characteristics of aperiodic intermittent switching controller (54), the proof of the above Theorem 1 will be divided into two parts, one is when $t \in [t_m, s_m)$, and the other one is when $t \in [s_m, t_{m+1}), m \in D$.

First, when $\forall t \in [t_m, s_m)$, we calculate the upper right derivative of $V(t)$ along with the trajectory of (7), and we can obtain that

$$\begin{aligned} D^{+}V(t) &= \sum_{i=1}^{n} sign(e_i(t))\dot{e}_i(t) \\ &= \sum_{i=1}^{n} sign(e_i(t))[-(\grave{a}_i(t)y_i(t) - \zeta\widetilde{a}_i(t)x_i(t)) \\ &\quad + \sum_{j=1}^{n}(\grave{b}_{ij}(t)f_j(y_j(t)) - \grave{b}_{ij}(t)f_j(\zeta x_j(t))) \\ &\quad + \sum_{j=1}^{n}(\grave{c}_{ij}(t)g_j(y_j(t-\tau_j(t))) - \grave{c}_{ij}(t)g_j(\zeta x_j(t-\tau_j(t)))) \end{aligned}$$

$$
\begin{aligned}
&+\sum_{j=1}^{n}(\grave{d}_{ij}(t)\int_{t-\delta_j(t)}^{t} f_j(y_j(s))ds-\zeta\widetilde{d}_{ij}(t)\int_{t-\delta_j(t)}^{t} f_j(x_j(s))ds)\\
&+\sum_{j=1}^{n}(\grave{b}_{ij}(t)f_j(\zeta x_j(t))-\zeta\widetilde{b}_{ij}(t)f_j(x_j(t)))+\sum_{j=1}^{n}(\grave{c}_{ij}(t)\\
&\times g_j(\zeta x_j(t-\tau_j(t)))-\zeta\widetilde{c}_{ij}(t)g_j(x_j(t-\tau_j(t))))+I_i(1-\zeta)+K_i(t)].
\end{aligned}\tag{55}
$$

Together with the characteristics of switching for memristive connection weights of $a_i(.)$, the following formula can be derived

$$
\begin{aligned}
&-sign(e_i(t))(\grave{a}_i(t)y_i(t)-\zeta\widetilde{a}_i(t)x_i(t))\\
= &-sign(e_i(t))[(\grave{a}_i(t)y_i(t)-\grave{a}_i(t)\zeta x_i(t))+(\grave{a}_i(t)\zeta x_i(t)-\zeta\widetilde{a}_i(t)x_i(t))]\\
\leq &-\underline{a}_i \mid e_i(t)\mid +\zeta \mid a_i^*-a_i^{**}\mid T_i.
\end{aligned}\tag{56}
$$

With the help of (H_2), one can get that

$$
\mid \sum_{j=1}^{n}(\grave{b}_{ij}(t)f_j(y_j(t))-\grave{b}_{ij}(t)f_j(\zeta x_j(t)))\mid \leq \sum_{j=1}^{n}\overline{b}_{ij}L_j^f \mid e_j(t)\mid . \tag{57}
$$

$$
\mid \sum_{j=1}^{n}(\grave{c}_{ij}(t)g_j(y_j(t-\tau_j(t)))-\grave{c}_{ij}(t)g_j(\zeta x_j(t-\tau_j(t))))\mid \leq \sum_{j=1}^{n}\overline{c}_{ij}L_j^g \mid e_j(t-\tau_j(t))\mid . \tag{58}
$$

By applying (H_1), one can derive that

$$
\mid \sum_{j=1}^{n}(\grave{d}_{ij}(t)\int_{t-\delta_j(t)}^{t} f_j(y_j(s))ds-\zeta\widetilde{d}_{ij}(t)\int_{t-\delta_j(t)}^{t} f_j(x_j(s))ds)\mid \leq \sum_{j=1}^{n}\overline{d}_{ij}N_j\delta_j(1+\zeta). \tag{59}
$$

$$
\begin{aligned}
&\mid \sum_{j=1}^{n}(\grave{b}_{ij}(t)f_j(\zeta x_j(t))-\zeta\widetilde{b}_{ij}(t)f_j(x_j(t)))\mid\\
= &\mid \sum_{j=1}^{n}[(\grave{b}_{ij}(t)f_j(\zeta x_j(t))-\grave{b}_{ij}(t)f_j(y_j(t))+\grave{b}_{ij}(t)f_j(y_j(t))-\zeta\widetilde{b}_{ij}(t)f_j(x_j(t)))\mid\\
\leq &\sum_{j=1}^{n}(\overline{b}_{ij}L_j^f \mid e_j(t)\mid +N_j \mid \hat{b}_{ij}-\zeta\check{b}_{ij}\mid).
\end{aligned}\tag{60}
$$

Similar to the proof of (60), it is easy to get that

$$
\begin{aligned}
&\mid \sum_{j=1}^{n}(\grave{c}_{ij}(t)g_j(\zeta x_j(t-\tau_j(t)))-\zeta\widetilde{c}_{ij}(t)g_j(x_j(t-\tau_j(t))))\mid\mid\\
\leq &\sum_{j=1}^{n}(\overline{c}_{ij}L_j^g \mid e_j(t-\tau_j(t))\mid +M_j \mid \hat{c}_{ij}-\zeta\check{c}_{ij}\mid).
\end{aligned}\tag{61}
$$

Substituting (56)-(61) into (55), and with the help of the conditions (52) and (53) of Theorem 1, we can get that

$$\begin{aligned}
D^+V(t) \quad \leq \quad & \sum_{i=1}^{n}[-\underline{a}_i \mid e_i(t) \mid +\zeta \mid a_i^* - a_i^{**} \mid T_i \\
& + \sum_{j=1}^{n} \overline{b}_{ij} L_j^f \mid e_j(t) \mid + \sum_{j=1}^{n} \overline{c}_{ij} L_j^g \mid e_j(t-\tau_j(t)) \mid \\
& + \sum_{j=1}^{n} \overline{d}_{ij} N_j \delta_j (1+\zeta) + \sum_{j=1}^{n} (\overline{b}_{ij} L_j^f \mid e_j(t) \mid +N_j \mid \hat{b}_{ij} - \zeta \check{b}_{ij} \mid) \\
& + \sum_{j=1}^{n} (\overline{c}_{ij} L_j^g \mid e_j(t-\tau_j(t)) \mid +M_j \mid \hat{c}_{ij} - \zeta \check{c}_{ij} \mid) \\
& + ß(1-\zeta) - \widetilde{\alpha}_i \mid e_i(t) \mid -\beta_i - \lambda_i \mid e_i(t-\tau_i(t)) \mid \\
& -\omega_i \mid e_i(t) \mid^{\gamma + sign(V(t)-1)}] \qquad (62) \\
= \quad & \sum_{j=1}^{n} \{\mid e_i(t) \mid (-\underline{a}_i + 2\sum_{j=1}^{n} \overline{b}_{ji} L_i^f - \widetilde{\alpha}_i) + [\zeta \mid a_i^* - a_i^{**} \mid T_i \\
& + \sum_{j=1}^{n} (\overline{d}_{ij} N_j \delta_j (1+\zeta) + N_j \mid \hat{b}_{ij} - \zeta \check{b}_{ij} \mid +M_j \mid \hat{c}_{ij} - \zeta \check{c}_{ij} \mid) \\
& + ß(1-\zeta) - \beta_i] + \mid e_i(t-\tau_i(t)) \mid (2\sum_{j=1}^{n} \overline{c}_{ji} L_i^g - \lambda_i) \\
& -\omega_i \mid e_i(t) \mid^{\gamma + sign(V(t)-1)}\} \\
\leq \quad & -\sum_{j=1}^{n} \omega_i \mid e_i(t) \mid^{\gamma + sign(V(t)-1)} .
\end{aligned}$$

Next, based on the Lemma 2 we will deal with $-\omega \sum_{i=1}^{n} \mid e_i(t) \mid^{\gamma + sign(V(t)-1)}$, and the following inequalities can be derived:
When $0 \leq V(t) < 1$, one can obtain that

$$-\omega \sum_{i=1}^{n} \mid e_i(t) \mid^{\gamma + sign(V(t)-1)} = -\omega \sum_{i=1}^{n} \mid e_i(t) \mid^{\gamma - 1} .$$

As $0 < \gamma - 1 < 1$, according to the first inequality in Lemma 2, we can get

$$-\omega \sum_{i=1}^{n} \mid e_i(t) \mid^{\gamma - 1} \leq -\omega (V(t))^{\gamma - 1}. \qquad (63)$$

When $V(t) \geq 1$, one can have

$$-\omega \sum_{i=1}^{n} \mid e_i(t) \mid^{\gamma+sign(V(t)-1)} = -\omega \sum_{i=1}^{n} \mid e_i(t) \mid^{\gamma+1}.$$

As $\gamma + 1 > 1$, on the basis of the second inequality in Lemma 2, one can obtain

$$-\omega \sum_{i=1}^{n} \mid e_i(t) \mid^{\gamma+1} \leq -\omega n^{-\gamma}(V(t))^{\gamma+1}. \tag{64}$$

From (63) and (64), the following inequality can be derived

$$D^+V(t)) \leq \begin{cases} -\omega(V(t))^{\gamma-1}, & 0 \leq V(t) < 1, \\ -\omega n^{-\gamma}(V(t))^{\gamma+1}, & V(t) \geq 1, \end{cases} \tag{65}$$

It's easy to get that $\omega \geq n^{-\gamma}\omega$. Then, from (65), we can get that

$$D^+V(t) \leq -\omega n^{-\gamma}(V(t))^{\gamma+sign(V(t)-1)}, t \in [t_m, s_m). \tag{66}$$

On the other hand, for $\forall t \in [s_m, t_{m+1})$. Based on (62), it's easy to get that

$$D^+V(t) \leq 0, t \in [s_m, t_{m+1}). \tag{67}$$

So, together with (66) and (67) one can derive the following formula holds, for $\forall t \in [0, +\infty)$.

$$\begin{cases} D^+V(t) \leq -\omega n^{-\gamma}(V(t))^{\gamma+sign(V(t)-1)}, t_m \leq t < s_m, m \in D, \\ D^+V(t) \leq 0, s_m \leq t < t_{m+1}, m \in D, \end{cases}$$

where $\omega = \min_{1 \leq i \leq n} \{\omega_i\}$. By applying Lemma 5, the settling time of fixed-time projective synchronization can be estimated as

$$T \leq \frac{2n^{\gamma}}{\omega\gamma(2-\gamma)(1-\theta)}.$$

According to Definition 2 and Theorem 1, the drive-response systems (1) and (2) can achieve fixed-time projective synchronization under the aperiodically intermittent switching controller (54).

The proof of Theorem 1 is completed. □

Remark 3. In paper [16], the fixed-time projective synchronization for a class of discrete delay memristive neural networks was studied by designing a common feedback controller $u_i(t) = -\alpha_i e_i(t) - sign(e_i(t))(\beta_i + \gamma_i \mid e_i(t) \mid^q), q > 1$ rather than a simple and reducing of the control cost and reducing the amount of information transmission aperiodically intermittent switching controller. In this paper, the influence of mixed time-varying delay on fixed-time projective synchronization is considered, but it was not considered in [16] was not. Thus the control method and neural network mathematical model in this paper is more general.

When the projective coefficient $\zeta = 1, \zeta = -1$ and $\zeta = 0$, we know that the fixed-time complete synchronization, fixed-time anti-synchronization, and fixed-time stability are special cases of fixed-time projective synchronization, respectively. From Theorem 1, we can directly get the following three Corollaries.

Corollary 1. Suppose that (H_1)-(H_2) hold. If the control parameters $\widetilde{\alpha}_i$,λ_i and β_i satisfy (52) and (53), $i \in I$, and the projective coefficient $\zeta = -1$, then the drive-response systems (1) and (2) are anti-synchronized in a fixed-time under controller (54). Moreover, the settling time is defined the same as in Theorem 1.

Corollary 2. Suppose that (H_1)-(H_2) hold. If the control parameters $\widetilde{\alpha}_i$ and λ_i satisfy (52) and $\beta_i - \mid a_i^* - a_i^{**} \mid T_i - \sum_{j=1}^{n}(2\overline{d}_{ij}N_j\delta_j + N_j \mid \hat{b}_{ij} - \check{b}_{ij} \mid + M_j \mid \hat{c}_{ij} - \check{c}_{ij} \mid) \geq 0$, $i \in I$, then the drive-response systems (1) and (2) achieve fixed-time complete synchronization under the controller (54). And the settling time of fixed-time complete synchronization is defined the same as in Theorem 1.

Corollary 3. Suppose that (H_1)-(H_2) hold. If the control parameters $\widetilde{\alpha}_i$ and λ_i satisfy (52) and $\beta_i - \sum_{j=1}^{n}(\overline{d}_{ij}N_j\delta_j + N_j \mid \hat{b}_{ij} \mid + M_j \mid \hat{c}_{ij} \mid) - ß \geq 0$, $i \in I$, then the drive-response systems (1) and (2) realize fixed-time stability under controller (54).

It is not difficult to see that compared with aperiodic intermittent switching control method, periodic intermittent switching control method is a special case of aperiodic intermittent switching control method. That is to say, when each aperiodic intermittent switching control subinterval satisfy $s_m - t_m = \rho$ and $t_{m+1} - t_m = T$, ρ and T are two positive real numbers, $m \in D$, it degenerates into a periodic intermittent switching control subinterval.

Theorem 3. Suppose that function $V(t)$ is non-negative for $\forall t \in [0, +\infty)$ and satisfies the following conditions:

$$\begin{cases} \dot{V}(t) \leq -\mu(V(t))^{\gamma + sign(V(t)-1)}, mT \leq t < (m+\vartheta)T, m \in D, \\ \dot{V}(t) \leq 0, (m+\vartheta)T \leq t < (m+1)T, m \in D, \end{cases}$$

where $\mu > 0$, $1 < \gamma < 2$, $0 < \vartheta < 1, m \in D$. Then $V(t) \equiv 0$, if $T \geq \frac{1}{\mu(2-\gamma)(1-\vartheta)} + \frac{1}{\mu\gamma(1-\vartheta)}$.

Remark 4. In the literature [53], the authors established a differential inequality about the periodic intermittent stability of the dynamic system in a finite time. It is worth noting that in the literature [53], the finite time T obtained by the authors depends on the initial value conditions of the system, but not in the above Theorem 3. For this reason, Theorem 3 improves the results in [53].

The aperiodic intermittent switching controller (54) is modified as follows periodic intermittent switching controller,

$$\widetilde{K}_i(t) = \begin{cases} -\widetilde{\alpha}_i e_i(t) - sign(e_i(t))(\beta_i + \lambda_i \mid e_i(t-\tau_i(t)) \mid \\ +\omega_i \mid e_i(t) \mid^{\gamma+sign(V(t)-1)}, mT \le t < (m+\vartheta)T, m \in D, \\ -\widetilde{\alpha}_i e_i(t) - sign(e_i(t))(\beta_i + \lambda_i + \mid e_i(t-\tau_i(t)) \mid), \\ (m+\vartheta)T \le t < (m+1)T, m \in D, \end{cases} \tag{68}$$

where the control parameters are defined as the same as in (54), and $1 < \gamma < 2$, $0 < \vartheta < 1$.

Proof. The proof process is similar to Theorem 1, so it is omitted here.

From Theorem 3, the following conclusion can be obtained.

Corollary 4. Suppose that (H_1)-(H_2) hold. If the control parameters $\widetilde{\alpha}_i$, λ_i and β_i satisfy (52) and (53), $i \in I$. Then the drive-response systems (1) and (2) realize fixed-time projective synchronization under the controller (68). Moreover, the settling time is estimated the same as in Theorem 1.

Proof. Based on Theorem 1, the proof process is similar to Theorem 2, it is omitted here.

Let $\curlywedge = \gamma + sign(V(t) - 1)$, through simple calculation, one can get that $0 < \curlywedge < 3$. Based on controller (54), then one can derive the following aperiodic intermittent switching controller

$$K_i(t) = \begin{cases} -\widetilde{\alpha}_i e_i(t) - sign(e_i(t))(\beta_i + \lambda_i \mid e_i(t-\tau_i(t)) \mid \\ +\omega_i \mid e_i(t) \mid^{\curlywedge}, t_m \le t < s_m, m \in D, \\ -\widetilde{\alpha}_i e_i(t) - sign(e_i(t))(\beta_i + \lambda_i + \mid e_i(t-\tau_i(t)) \mid), \\ s_m \le t < t_{m+1}, m \in D, \end{cases} \tag{69}$$

where the control parameters $\widetilde{\alpha}_i, \beta_i, \lambda_i$ and ω_i are defined as the same as in (54), $\curlywedge \in (0, 1)$.

Corollary 5. Suppose that (H_1)-(H_2) hold. If the control parameters $\widetilde{\alpha}_i$, λ_i and β_i satisfy (52) and (53), $i \in I$. Then the drive-response systems (1) and (2) realize finite-time projective synchronization under controller (69). In addition, the settling time of finite-time projective synchronization is estimated by

$$T = \frac{V^{1-\curlywedge}(0)}{\omega(1-\theta)(1-\curlywedge)}.$$

Proof. Based on Lemma 4 and the proof process is similar to the (63) and (67) in proving Theorem 2. Thus, it is omitted here.

Remark 5. In paper [42], the finite-time projective synchronization of memristor-based neural networks with leakage and time-varying delays was discussed through continuous control that keeps the control intensity constant. However, in practice, the continuous controller with constant control strength does not reduce control cost. In order to improve the control efficiency, the aperiodic intermittent switching controller (69) for finite-time projective synchronization is constructed. The controlled intensity is divided into two parts: the strong control stage and the weak control stage. Compared with the controller in [42], the aperiodic intermittent switching controller (69) proposed in this paper can save more control costs and achieve better results.

Remark 6. Due to the sign function that exists in the controllers (54),(68),(69) and (70), as a hard switcher, it may cause an unexpected chattering phenomenon. In order to avoid the chattering phenomenon, the sign function in the controllers of [52] were replaced by continuous $tanh(.)$. Based on the method in reference [52], the sign function in controller (54),(68),(69) and (70) are replaced by continuous $tanh(.)$ to eliminate chattering phenomenon. For example, the aperiodically intermittent switching controller (54) can be modified as follows

$$K_i(t) = \begin{cases} -\widetilde{\alpha}_i e_i(t) - tanh(e_i(t))(\beta_i + \lambda_i \mid e_i(t-\tau_i(t)) \mid \\ +\omega_i \mid e_i(t) \mid^{\gamma+sign(V(t)-1)}), t_m \le t < s_m, m \in D, \\ -\widetilde{\alpha}_i e_i(t) - tanh(e_i(t))(\beta_i + \lambda_i + \mid e_i(t-\tau_i(t)) \mid), \\ s_m \le t < t_{m+1}, m \in D, \end{cases} \tag{70}$$

where the control parameters $\overline{\alpha}_i, \beta_i, \lambda_i$ and ω_i are same as in (54), for $i \in I$.

Corollary 6. Suppose that (H_1)-(H_2) hold. If the control parameters $\widetilde{\alpha}_i$, λ_i and β_i satisfy (52) and (53), $i \in I$. Then the drive-response systems (1) and (2) achieve fixed-time projective synchronization under the controller (70). In addition, the settling time for fixed-time projective synchronization is estimated the same as in Theorem 1.

If $\theta = 0$, controller (54) is modified as following feedback controller

$$\hat{K}_i(t) = -\widetilde{\alpha}_i e_i(t) - sign(e_i(t))(\beta_i + \lambda_i + \mid e_i(t-\tau_i(t)) \mid +\omega_i \mid e_i(t) \mid^{\gamma+sign(V(t)-1)}). \tag{71}$$

In addition, when the projective coefficient $\zeta = 1$, from Theorem 1, we can obtain the following Corollary.

Corollary 7. Suppose that (H_1)-(H_2) hold. If the control parameters $\widetilde{\alpha}_i$ and λ_i satisfy (52) and $\beta_i - \mid a_i^* - a_i^{**} \mid T_i - \sum_{j=1}^{n}(2\overline{d}_{ij}N_j\delta_j + N_j \mid \hat{b}_{ij} - \check{b}_{ij} \mid + M_j \mid \hat{c}_{ij} - \check{c}_{ij} \mid) \geq 0$, $i \in I$. Then the drive-response systems (1) and (2) are reached fixed-time synchronization under controller (71). What is more, the settling time for fixed-time synchronization is estimated as

$$T \leq \frac{2n^{\gamma}}{\omega\gamma(2-\gamma)}.$$

Proof. The proof process of Corollary 7 is similar to formula (66) in Theorem 2. So, it is omitted here.

Remark 7. In the paper [54], the authors studied the finite-time synchronization of memristor-based neural networks with mixed delays. Compared with the settling time of finite-time synchronization in [54], the settling time T in Corollary 7 is independent of the system's initial value. Therefore, Corollary 7 has improved some of the previous related conclusions.

Now, we consider a special case of the drive system (1) and the response system (2). If the distributed time delays memristive connection weights $d_{ij}(.) = 0, i, j \in I$ the drive system (1) is reduced to the following form:

$$\dot{x}_i(t) = -a_i(x_i(t))x_i(t) + \sum_{j=1}^{n} b_{ij}(x_i(t))f_j(x_j(t)) + \sum_{j=1}^{n} c_{ij}(x_i(t))g_j(x_j(t-\tau_j(t))) + I_i. \quad (72)$$

Accordingly,the response system (2) is degenerated to the following form:

$$\dot{y}_i(t) = -a_i(y_i(t))y_i(t) + \sum_{j=1}^{n} b_{ij}(y_i(t))f_j(y_j(t)) + \sum_{j=1}^{n} c_{ij}(y_i(t))g_j(y_j(t-\tau_j(t))) + I_i + K_i(t). \quad (73)$$

According to Theorem 1, the following conclusion can be obtained

Corollary 8. Suppose that (H_1)-(H_2) hold. If the control parameters $\widetilde{\alpha}_i$ and λ_i satisfy (52) and $\beta_i - \zeta \mid a_i^* - a_i^{**} \mid T_i - \sum_{j=1}^{n}(N_j \mid \hat{b}_{ij} - \zeta\check{b}_{ij} \mid + M_j \mid \hat{c}_{ij} - \zeta\check{c}_{ij} \mid) - ß(1-\zeta) \geq 0$, $i \in I$. Then the drive-response systems (72) and (73) achieve fixed-time projective synchronization under controller (70). What is more, the settling time for fixed-time projective synchronization is defined the same as in Theorem 1.

Remark 8. In [24] and [45], the anti-synchronization of neural networks with time-varying delays by designing a feedback controller and an intermittent adjustment controller was investigated, respectively. In this paper, we consider the fixed-time projective synchronization of memristive neural networks with mixed time-varying delays. So, the conclusions of [24] and [45] are the special cases of this Theorem 2 in this paper.

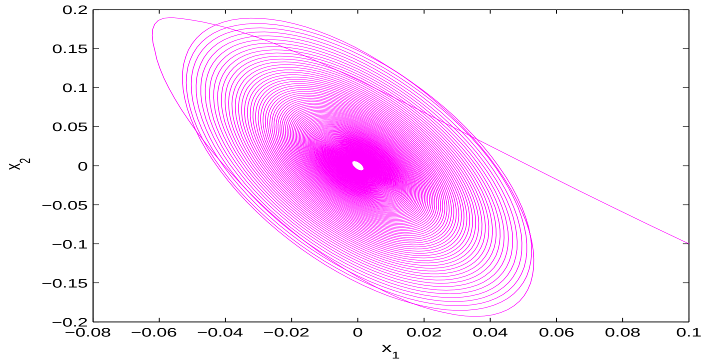

Figure 2: The chaotic attractor of system (74) with the initial values $x_1(s) = 0.1$, $x_2(s) = -0.1$ for $s \in [-1, 0]$.

Remark 9. Compared with the periodic intermittent control method in [25,28,30], the aperiodic intermittent switching control method of Theorem 1 in this paper can set the control time to some sub intervals instead of a fixed time sub intervals, according to the control needs of practical problems. In addition, periodic intermittent control can be regarded as a special form of aperiodic intermittent control. And the aperiodic intermittent switching control method proposed in this paper not only shortens the control synchronization time, saves the control cost and reduces the amount of information transmission, but also has better practical significance.

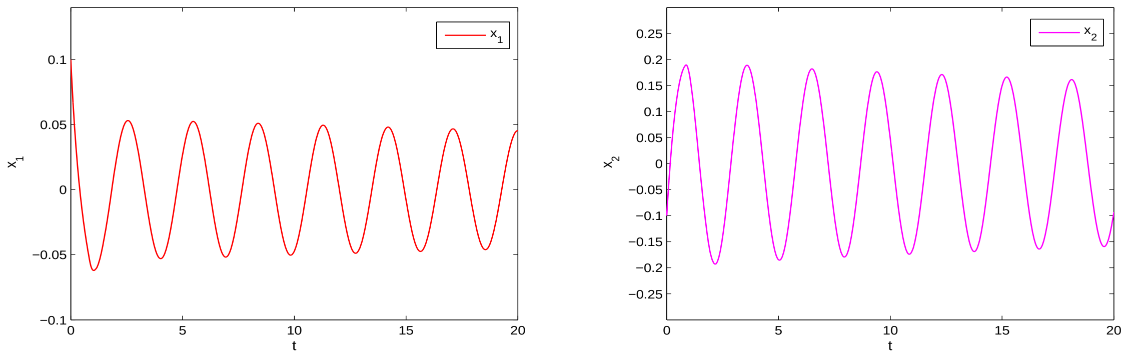

Figure 3: The state curves of $x_1(t)$ and $x_2(t)$ with $x_1(s) = 0.1$ and $x_2(s) = -0.1$, respectively, for $s \in [-1, 0]$.

4 Numerical simulations

In this section, an example is given to illustrate the effectiveness of the results obtained in this paper.

Example 1. Consider the 2-dimensional memristive neural networks with mixed time-varying delays given by

$$\begin{aligned}\dot{x}_i(t) &= -a_i(x_i(t))x_i(t) + \sum_{j=1}^{2} b_{ij}(x_i(t))f_j(x_j(t)) + \sum_{j=1}^{2} c_{ij}(x_i(t)) \\ &\quad \times g_j(x_j(t-\tau_j(t))) + \sum_{j=1}^{2} d_{ij}(x_i(t)) \int_{t-\delta_j(t)}^{t} f_j(x_j(s))ds + I_i, i = 1, 2\end{aligned} \tag{74}$$

where the values of the memristors are as follows

$$a_1(x_1(t)) = \begin{cases} 0.2, & \mid x_1(t) \mid \leq 1.1 \\ -0.42, & \mid x_1(t) \mid > 1.1 \end{cases}, \qquad a_2(x_2(t)) = \begin{cases} -0.3, & \mid x_2(t) \mid \leq 1.1 \\ 0.6, & \mid x_2(t) \mid > 1.1 \end{cases},$$

$$b_{11}(x_1(t)) = \begin{cases} -2.1, & \mid x_1(t) \mid \leq 1.1 \\ -0.2, & \mid x_1(t) \mid > 1.1 \end{cases}, \qquad b_{12}(x_1(t)) = \begin{cases} 0.9, & \mid x_1(t) \mid \leq 1.1 \\ -1.3, & \mid x_1(t) \mid > 1.1 \end{cases},$$

$$b_{21}(x_2(t)) = \begin{cases} 1.3, & \mid x_2(t) \mid \leq 1.1 \\ -1.9, & \mid x_2(t) \mid > 1.1 \end{cases}, \qquad b_{22}(x_2(t)) = \begin{cases} -1.4, & \mid x_2(t) \mid \leq 1.1 \\ 0.7, & \mid x_2(t) \mid > 1.1 \end{cases},$$

$$c_{11}(x_1(t)) = \begin{cases} -1.2, & \mid x_1(t) \mid \leq 1.1 \\ 0.1, & \mid x_1(t) \mid > 1.1 \end{cases}, \quad c_{12}(x_1(t)) = \begin{cases} 1.15, & \mid x_1(t) \mid \leq 1.1 \\ -1.22, & \mid x_1(t) \mid > 1.1 \end{cases},$$

$$c_{21}(x_2(t)) = \begin{cases} -1.35, & \mid x_1(t) \mid \leq 1.1 \\ 1.4, & \mid x_1(t) \mid > 1.1 \end{cases}, \quad c_{22}(x_2(t)) = \begin{cases} 0.7, & \mid x_2(t) \mid \leq 1.1 \\ -0.5, & \mid x_2(t) \mid > 1.1 \end{cases},$$

$$d_{11}(x_1(t)) = \begin{cases} -0.2, & \mid x_1(t) \mid \leq 1.1 \\ -0.5, & \mid x_1(t) \mid > 1.1 \end{cases}, \qquad d_{12}(x_1(t)) = \begin{cases} 1.7, & \mid x_1(t) \mid \leq 1.1 \\ -0.3, & \mid x_1(t) \mid > 1.1 \end{cases},$$

$$d_{21}(x_2(t)) = \begin{cases} 1.12, & \mid x_2(t) \mid \leq 1.1 \\ 0.4, & \mid x_2(t) \mid > 1.1 \end{cases}, \qquad d_{22}(x_2(t)) = \begin{cases} -1.1, & \mid x_2(t) \mid \leq 1.1 \\ -0.6, & \mid x_2(t) \mid > 1.1 \end{cases}.$$

We consider system (74) to be the drive system. And $\tau_i(t) = \dfrac{e^t}{1+e^t}, \delta_i(t) = 1$, $I_i = 0$, and take activation function as $f_i(u) = g_i(u) = 1.15\tanh(u), i = 1, 2$. Obviously,$\mid f_i(u) \mid = \mid g_i(u) \mid \leq 1.15, \mid f_j(v) - f_i(u) \mid \leq 1.15 \mid v - u \mid, \mid g_j(v) - g_i(u) \mid \leq 1.15 \mid v - u \mid, i, j = 1, 2$. Therefore, $(H_1) - (H_2)$ hold for drive system (74).

The model (74) has a chaotic attractor with the initial values $x_1(s) = 0.1$, $x_2(s) = -0.15$ for $s \in [-1, 0]$ can be seen in Fig. 2. And the state trajectories of system (74) with initial conditions $x_1(s) = 0.1, x_2 = -0.1$ are described in Fig. 3.

The response system is given by

$$
\begin{aligned}
\dot{y}_i(t) \;\; = \;\; & -a_i(y_i(t))y_i(t) + \sum_{j=1}^{n} b_{ij}(y_i(t))f_j(y_j(t)) + \sum_{j=1}^{n} c_{ij}(y_i(t)) \\
& \times g_j(y_j(t-\tau_j(t))) + \sum_{j=1}^{n} d_{ij}(y_i(t)) \int_{t-\delta_j(t)}^{t} f_j(y_j(s))ds + I_i + K_i(t),
\end{aligned} \tag{75}
$$

where the values of the memristors, $f_j, g_j, \tau_j(t), \delta_j(t)$ and I_i, $i, j = 1, 2$ are defined as the same as in system (74), and the fixed-time aperiodically intermittent switching controller $K_i(t)$ is designed as follows

$$
K_i(t) = \begin{cases} -\widetilde{\alpha}_i e_i(t) - tanh(e_i(t))(\beta_i + \lambda_i \mid e_i(t-\tau_i(t)) \mid \\ +\omega_i \mid e_i(t) \mid^{\gamma + sign(V(t)-1)}), t_m \leq t < s_m, m \in D, \\ -\widetilde{\alpha}_i e_i(t) - tanh(e_i(t))(\beta_i + \lambda_i + \mid e_i(t-\tau_i(t)) \mid), \\ s_m \leq t < t_{m+1}, m \in D, \end{cases} \tag{76}
$$

The aperiodically intermittent switching control exists on time span $[0, 0.1) \cup [0.1, 0.35) \cup [0.35, 0.55) \cup [0.55, 1) \cup [1, 1.3) \cup [1.3, 1.95) \cup [1.95, 2.35) \cup [2.35, 3.2) \cup [3.2, 3.7) \cup [3.7, 4.75) \cup [4.75, 5.35) \cup [5.35, 6.6) \cup [6.6, 7.3) \cup [7.3, 8.78) \cup [8.78, 9.88) \cup [9.88, 11.2) \cup [11.2, 12.1] \cup [12.1, 13.95) \cup [13.95, 14.95) \cup [14.95, 17) \cup [17, 18.1) \cup \cdots$, by plain calculation, we can get $\theta = \frac{1}{3}$.

It is not difficult to check that $\underline{a}_1 = -0.42, \underline{a}_2 = -0.3, \overline{b}_{11} = 2.1, \overline{b}_{12} = 1.9, \overline{b}_{21} = 1.9, \overline{b}_{22} = 1.4$, $\overline{c}_{11} = 1.2, \overline{c}_{12} = 1.22, \overline{c}_{21} = 1.4, \overline{c}_{22} = 0.7$, $\overline{d}_{11} = 0.5, \overline{d}_{12} = 1.7, \overline{d}_{21} = 1.12, \overline{d}_{22} = 1.1, T_1 = T_2 = 1.1$, $\delta_1 = \delta_2 = 1$, ß $= 0$. And we take $\gamma = 1.6$.

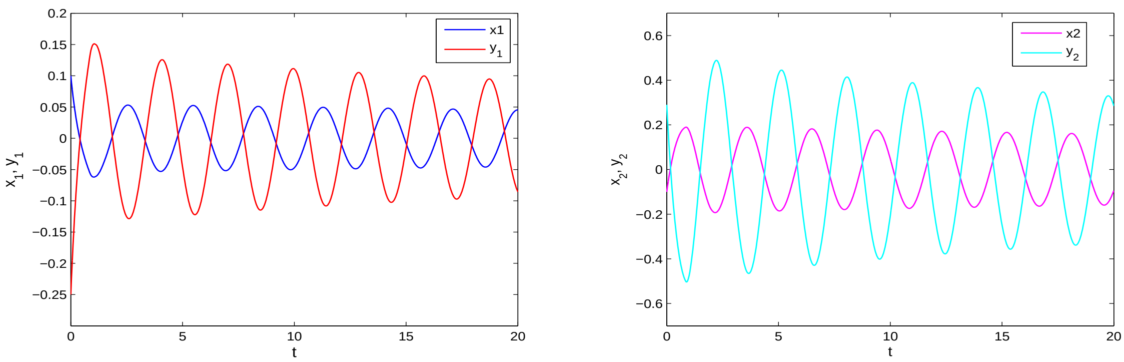

Figure 4: The state curves of $x_i(t)$ and $y_i(t)$ without control input.

We choose the control parameters $\widetilde{\alpha}_1 = 10$, $\widetilde{\alpha}_2 = 7$, $\lambda_1 = 6.5$ and $\lambda_2 = 5$. Through simple calculation, we can obtain that $\widetilde{\alpha}_1 + \underline{a}_1 - 2\sum_{j=1}^{2} \overline{b}_{j1} L_1^f = 0.38 \geq 0, \widetilde{\alpha}_2 + \underline{a}_2 -$

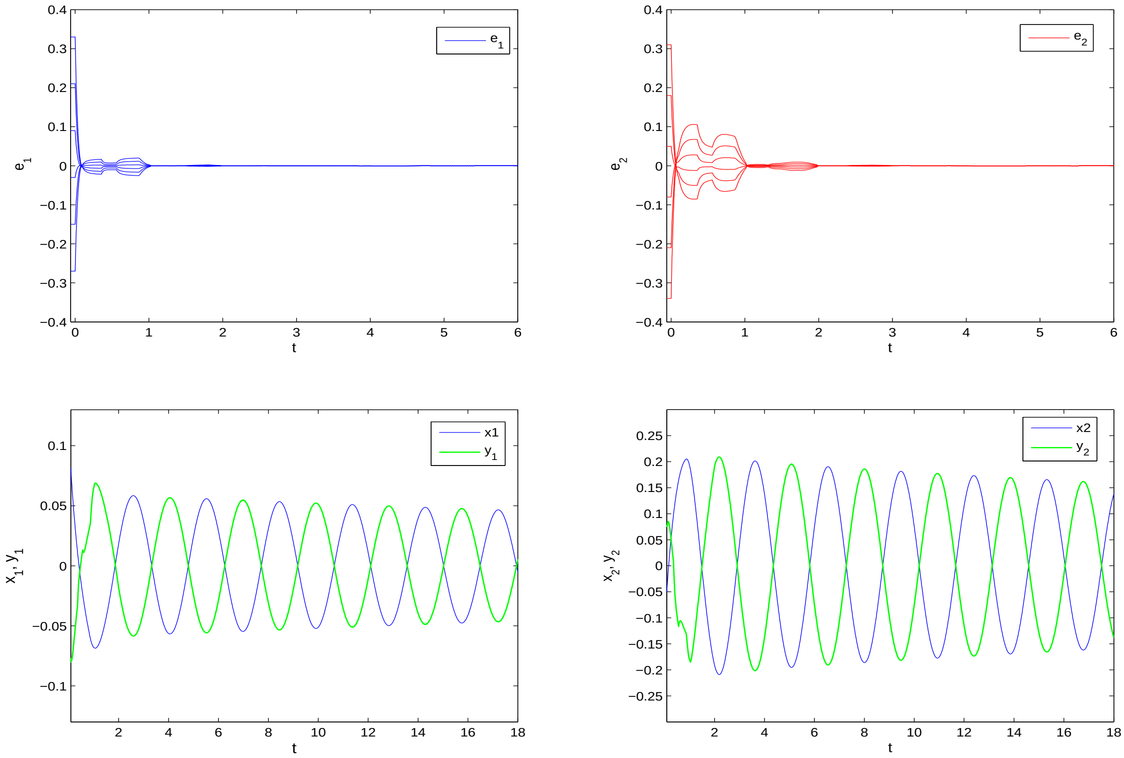

Figure 5: The evaluation of synchronization errors and curves under the controller (76) for projective coefficient $\zeta = -1.1$.

$2\sum_{j=1}^{2}\sum_{j=1}^{2}\bar{b}_{j2}L_2^f = 0.49 \geq 0$, and $\lambda_1 - 2\sum_{j=1}^{2}\bar{c}_{j1}L_1^g = 0.52 \geq 0$, $\lambda_2 - 2\sum_{j=1}^{2}\bar{c}_{j2}L_2^g = 0.5840 \geq 0$.

We choose the control parameters $\beta_1 = 3$, $\beta_2 = 1$, and projective coefficient $\zeta = -1.1$, by simple calculation, we derive that $\beta_1 - \zeta \mid a_1^* - a_1^{**} \mid T_1 - \sum_{j=1}^{2}(\bar{d}_{1j}N_j\delta_j(1+\zeta) + N_j \mid \hat{b}_{1j} - \zeta\check{b}_{1j} \mid + M_j \mid \hat{c}_{1j} - \zeta\check{c}_{1j} \mid) - \text{ß}(1-\zeta) = 0.4704 \geq 0$, and $\beta_2 - \zeta \mid a_2^* - a_2^{**} \mid T_2 - \sum_{j=1}^{2}(\bar{d}_{2j}N_j\delta_j(1+\zeta) + N_j \mid \hat{b}_{2j} - \zeta\check{b}_{2j} \mid + M_j \mid \hat{c}_{2j} - \zeta\check{c}_{2j} \mid) - \text{ß}(1-\zeta) = 0.6653 \geq 0$.

When $\zeta = -1.1$, according to the previous calculations, all the conditions of Corollary 6 are satisfied, so the drive-response systems (74) and (75) realize the fixed-time projective synchronization under the aperiodically intermittent switching controller (76). And six classes of different initial values are chosen for systems (74) and (75) by $x_1^k(s) = 0.1$, $x_2^k(s) = -0.1$, $y_1^k(s) = 0.1 + 0.12k$ and $y_2^k(s) = -0.1 - 0.13k$, for $s \in [-1, 0]$ and $k \in \{-4, -3, -2, -1, 0, 1\}$. The Fig. 5 shows that, when the projection coefficient $\zeta = -1.1$, the driver-response systems (74) and (75) realize the fixed-time projective synchronization under the controller (76), the settling time of fixed-time

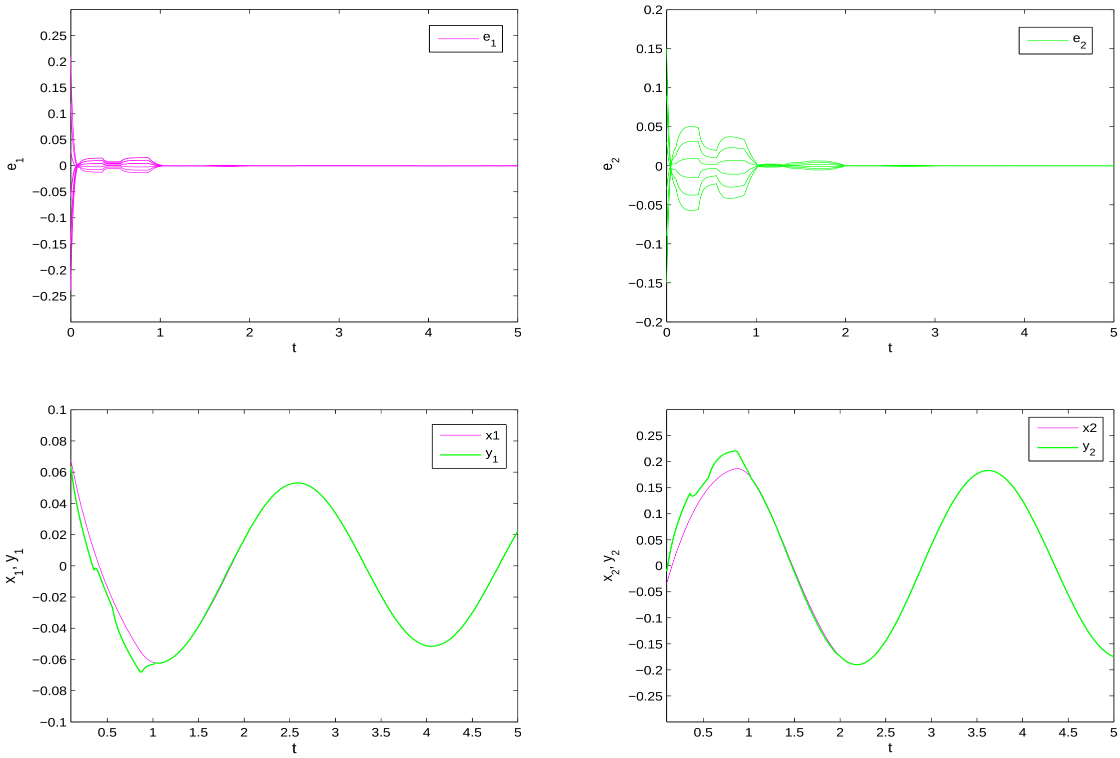

Figure 6: The evaluation of synchronization errors and curves under the controller (76) for projective coefficient $\zeta = 1$.

projective synchronization is estimated as $T \leq 3.6435$, which verifies the correctness of the Corollary 6 in this paper.

We take the control parameters $\beta_1 = 17.5$, $\beta_2 = 17.3$, and projective coefficient $\zeta = 1$, by simple calculation, we get that $\beta_1 - \zeta \mid a_1^* - a_1^{**} \mid T_1 - \sum_{j=1}^{2}(\bar{d}_{1j} N_j \delta_j (1+\zeta) + N_j \mid \hat{b}_{1j} - \zeta \check{b}_{1j} \mid + M_j \mid \hat{c}_{1j} - \zeta \check{c}_{1j} \mid) - ß(1-\zeta) = 0.6725 \geq 0$, and $\beta_2 - \zeta \mid a_2^* - a_2^{**} \mid T_2 - \sum_{j=1}^{2}(\bar{d}_{2j} N_j \delta_j (1+\zeta) + N_j \mid \hat{b}_{2j} - \zeta \check{b}_{2j} \mid + M_j \mid \hat{c}_{2j} - \zeta \check{c}_{2j} \mid) - ß(1-\zeta) = 0.5665 \geq 0$.

When $\zeta = 1$, according to the previous calculations, all the conditions of Corollary 6 are satisfied, so the drive-response systems (74) and (75) realize the fixed-time projective synchronization under the aperiodically intermittent switching controller (76), that is to achieve fixed-time complete synchronization. And six classes of different initial values are chosen for systems (74) and (75) by $x_1^k(s) = 0.1$, $x_2^k(s) = -0.1$, $y_1^k(s) = 0.22 + 0.09k$ and $y_2^k(s) = -0.19 - 0.06k$, for $s \in [-1, 0]$ and $k \in \{-4, -3, -2, -1, 0, 1\}$. The Fig.6 shows that, when the projection coefficient $\zeta = 1$, the driver-response systems (74) and (75) realize the fixed-time projective synchronization under the controller (76), that is to achieve fixed-time complete synchronization,

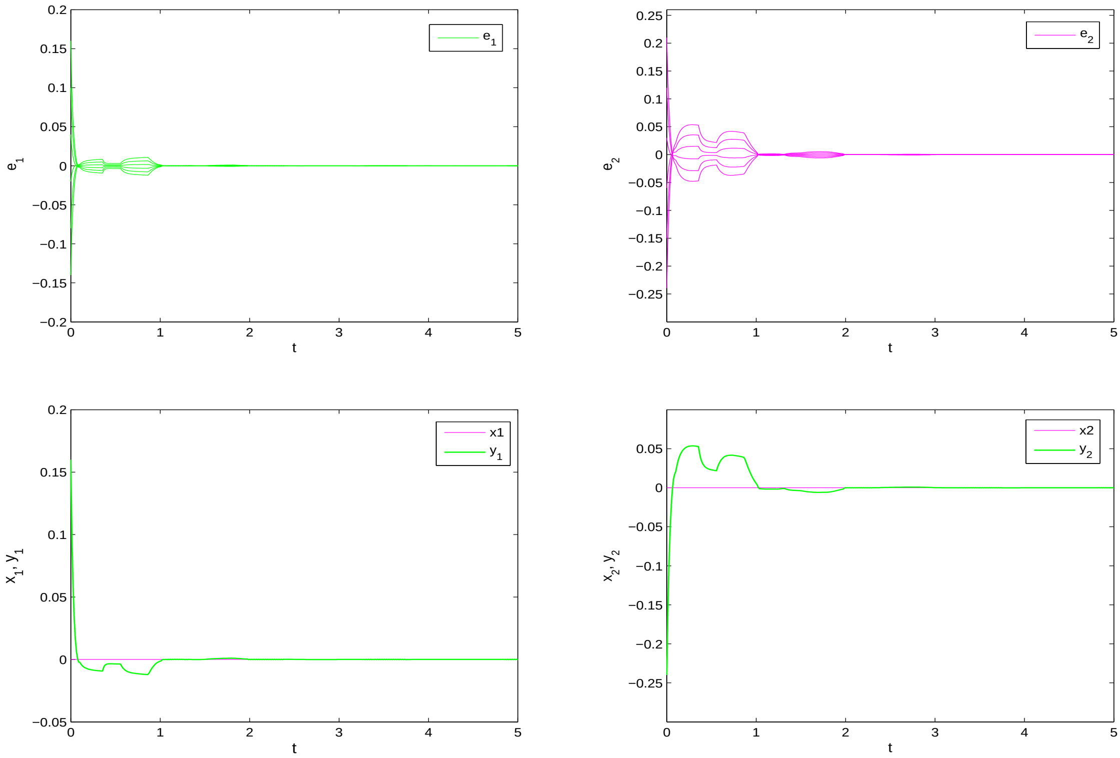

Figure 7: The evaluation of synchronization errors and curves under the controller (76) for projective coefficient $\zeta = 0$.

the settling time is estimated as $T \leq 3.6435$, which verifies the correctness of the Corollary 2 and Corollary 6 in this paper.

We choose the control parameters $\beta_1 = 9$, $\beta_2 = 9.0155$, and projective coefficient $\zeta = 0$, by simple calculation, we derive that $\beta_1 - \sum_{j=1}^{2}(\bar{d}_{1j}N_j\delta_j + N_j \mid \hat{b}_{1j} \mid +M_j \mid \hat{c}_{1j} \mid) - ß = 0.3175 \geq 0$, and $\beta_2 - \sum_{j=1}^{2}(\bar{d}_{2j}N_j\delta_j + N_j \mid \hat{b}_{2j} \mid +M_j \mid \hat{c}_{2j} \mid) - ß(1-\zeta) = 1 \geq 0$.

When $\zeta = 0$, according to the previous calculations, all the conditions of Corollary 6 are satisfied, so the drive-response systems (74) and (75) realize the fixed-time projective synchronization under the aperiodically intermittent switching controller (76), that is to say, it achieves fixed-time stability. And six classes of different initial values are chosen for systems (74) and (75) by $x_1^k(s) = 0.1$, $x_2^k(s) = -0.1$, $y_1^k(s) = 0.1 + 0.06k$ and $y_2^k(s) = -0.15 - 0.09k$, for $s \in [-1, 0]$ and $k \in \{-4, -3, -2, -1, 0, 1\}$. The Fig.7 shows that, when the projection coefficient $\zeta = 0$, the driver-response systems (74) and (75) realize the fixed-time projective synchronization under the controller (76), that is to say, it achieves fixed-time stability, the settling time of fixed-time stability is estimated as $T \leq 3.6435$, which verifies the correctness of the Corollary 3 and Corollary 6 in this paper.

5 Conclusions

In this paper, the fixed-time projective synchronization problem for a class of memristive neural networks with mixed time-varying delays has been investigated. Based on the existing aperiodically intermittent strategy, a novel Theorem for aperiodically intermittent switching fixed-time stability is proposed and proved through mathematical induction. Some new sufficient conditions ensuring the fixed-time projective synchronization for the considered systems are obtained via aperiodically intermittent control and feedback control. In particular, the power exponent of the controller designed in this paper is a function of the error system state rather than one or two fixed constants, which makes the controller more general. Compared with the continuous controller, the controller in this article can save energy and control cost, reduce information transmission, and has better practical significance. In addition, the settling time is estimated and closely related to neural network scale and the maximum ratio of the rest width to the aperiodic time span, and independent of systems initial value conditions. Especially, as special cases, some corollaries on the proposed fixed-time projective synchronization theorem are obtained. Meanwhile, the conclusions of this paper improve some previous research relevant results. Finally, a numerical example is presented to verify the effectiveness and feasibility of the obtained results. In the future, we will study the predefined-time aperiodic intermittent control synchronization of neural networks.

Acknowledgements: This work was supported by the National Natural Science Foundation of People's Republic of China (Grant No. 12061055); Ningxia Natural Science Foundation (Grant No. 2020AAC03030, 2021AAC03175); The first class discipline construction project in Ningxia Universities: mathematics; Youth Science and Technology Talents Development Project of Guizhou Education Department(Qian Jiao he KY zi [2018] 313), and Zunyi Normal college 2017 academic new talent cultivation and innovation exploration project(Qiankehe platform talent [2017] Grant No. 5727-24).

Conflict of Interest: The authors declare that they have no conflict of interest.

Data Availability Statements: The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

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