Fully Distributed Consensus of Fractional Chaotic Multi-agent Systems Based on Combined Event-triggered Mechanism

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Abstract

By designing a novel combined event-triggered control technique, this article analytically studies the distributed leader-following consensus problem of nonlinear fractional chaotic multi-agent systems. First, a novel combined event-triggered mechanism which takes into account both the relative error and the absolute error of the samples is proposed, under which each follower agent executes control update independently at its own event times. Next, a fully distributed event-triggered consensus protocol is designed and the sufficient conditions of consensus are attained. Finally, compared with other event-triggered mechanisms, the simulation experiments illustrate that the event-based consensus protocol proposed in this article can effectively reduce the frequency of actuator data update while ensuring desired consensus performance.

Keywords: Chaotic communication, Distributed control, Event-triggered mechanism, Fractional calculus, Multi-agent systems, Networked control systems.
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1. Introduction

Nowadays, the consensus of **multi-agent systems (MASs)** is becoming more and more attractive, due to its wide applications in networked secure communication, path planing, satellite formation and other fields [1, 2].

It should be stressed that most of the previous works focus on the multi-agent systems described by integer-order dynamics, such as first-order integral dynamics [3, 4], as well as second-order integrator dynamics [5–7] or even high-order integrator dynamics[8, 9]. However, for many multi-agent systems working in the field of macromolecule fluids, porous media or food seeking of microbes, the fractional calculus operator can provide more accurate dynamic system model than the integer calculus operator does. Hence it is necessary to investigate the consistency of fractional multi-agent systems. This topic is first studied by Cao in [10], following which, a series of in-depth related researches have been launched [11–13].

Notice that the establishments of most multi-agent systems are constrained by limited resources, such as on-board resources, actuator capability, processor capacity, and network bandwidth. Therefore, for many complex networks, it is expected that each agent can **minimize the update frequency of the actuator input without compromising the consensus performance**. To achieve this goal, the event triggering control strategy comes into being. Its' basic idea is the sampled-data of actuator will not update until a certain event is triggered, thereby reducing the burden of resource occupancy [14–23].

So far, the consistency problem of integer MASs via a event-triggered mechanism has obtained abundant research results, which make the event-triggered consensus of fractional MASs become more attractive. Although the fractional derivative is a generalization of the integer one, most of the properties and techniques applicable for integer order dynamic system are inapplicable for fractional system. Therefore, the lack of theoretical tools makes the development of fractional dynamic systems is far behind that of integer case,which indicates this subject needs further study.

Extensive literature review shows that the current results of the consistency of fractional MASs based on event triggering control technique are still rare [24–27]. This topic is first discussed in [24] where Xu first extends the **norm event-triggered mechanism (NETM)** dealing with integer system to the fractional system and applies it to solve the consensus problem of fractional MASs. In this scheme, the event-triggering instant is given by

$$t_{k+1} = t_k + \min\{t| \left\|\delta_k(t)\right\|^2 > \sigma \left\|x(t)\right\|^2\},$$

where $t_0 = 0$ and $x(t)$ refers to the current sample output, t_k denotes the nearest event-triggered instant before the current time t, $\sigma \in (0, 1)$ represents the parameter that determines the sampling interval, and $\delta_k(t) = x(t_k) - x(t)$.

Noticing $\left\|\delta_k(t)\right\|^2 > \sigma \left\|x(t)\right\|^2$ is equivalent to $\left\|\frac{\delta_k(t)}{x(t)}\right\|$

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$> \sigma^{\frac{1}{2}}$ as $\|x(t)\| \neq 0$, this mechanism determines whether to update the executor input by measuring the relative error value of the current sample. Shi applied the NETM control method to investigate the exponential consensus protocol of fractional MASs in [25] and Ren extends NETM method to distributed NETM in [26] respectively. However, it is worth pointing out that, since the influence of absolute error is not taken into account in this method, frequent sampling and even Zeno behavior will occur as $\|x(t)\|$ is very close to zero.

In [27], Wang studies the leader-following consensus of fractional MASs via **exponential event-triggered mechanism (EETM)**, in which the sequence of event-triggered instants $\{t_k\}$ is defined as

$$t_{k+1} = t_k + \min\{t | \, \|\delta_k(t)\|^2 > \beta \exp(-\gamma t)\},$$

where the two constants $\beta > 0, \gamma > 0$ are preassigned threshold parameters. Since this triggering mechanism is determined by the absolute error of the current sample, it can avoid Zeno behavior effectively. Unfortunately, it is independent of the relative error, which lead to the shortcoming of poor screening ability for larger sample data.

Hence it is a valuable and challenging topic to combine the above two event-triggered mechanisms NETM and EETM together or to design a better one to handle the consensus control of fractional dynamic systems. This inspires the following work.

In addition, in most of the researches on event-triggered consensus of MASs, the actuators of all agents update their data synchronously generated by a same event-triggered mechanism. Since individual difference is not taken into account, it is impossible to ensure each agent achieves its optimal update frequency. Thus it is necessary to establish a separate triggering mechanism for each agent [28–31] . This is another factor that motivates this article.

In view of the above discussion, this article focuses on designing a novel event-trigger mechanism and applying it to handle the consistency control of nonlinear uncertain fractional MASs with leaders. The framework of the article is arranged as follows. The research background and status are introduces in Section 1. Some mathematical preliminaries as well as the problem statement are formulated in Section 2. In section 3, a novel separate combined event-triggered mechanism is constructed and a consensus control scheme is designed to conserve the limited resources and ensure good consensus performance. In Section 4, a concrete simulation example is given to show the validity and superiority of the proposed consensus scheme. Summary and further work are drawn in Section 5.

The main contributions that make this article more competitive are summarized in the following aspects. Firstly, the separate consensus of uncertain fractional chaotic MASs based on event-triggered mechanism is first investigated. Secondly, the **combined event-triggered mechanism (CETM)** designed in this article skillfully retains the advantages and avoids the disadvantages of the two traditional mechanisms NETM and EETM, so it shows good data-filtering ability throughout the consensus process. Thirdly, the introduction of **separate triggering strategy** further increases the flexibility of the event-driven mechanism and improves the ability of data filtering.

Table 1: Acronyms for Technical terms frequently used in this work

Acronym	Corresponding technical term
MASs	Multi-agent systems
ETC	Event-triggered condition
ETM	Event-triggered mechanism
CETM	Combined event-triggered mechanism
NETM	Norm event-triggered mechanism
EETM	Exponential event-triggered mechanism

Notation index: The notation used in this paper is uniform and standard. For convenience, we apply some acronyms to represent the technical terms which are frequently used in this work. This is shown in Table 1.

2. Preliminaries and Problem Statement

In this section, we will list some relevant preliminaries and introduce the consensus problem.

2.1. Graph theory

Let $G = (V, E, A)$ represent an undirected graph, in which $V = \{v_1, \cdots, v_N\}$ denotes the node set and $E \subseteq V \times V$ refers to the edge set. $A = [a_{ij}] \in R^{N\times N}$ is the weighted adjacency matrix of G and its element $a_{ij} = a_{ji} > 0$ if there exists a edge between the nodes v_i and v_j, otherwise $a_{ij} = a_{ji} = 0$. Furthermore, we specify $a_{ii} = 0$. $L = D - A \in R^{N\times N}$ denotes the Laplacian matrix of G with $D = diag\{d_1, \cdots, d_N\}$ and $d_i = \sum_{j=1}^{N} a_{ij}$. Moreover, the elements of matrix L satisfies

$$l_{ij} = -a_{ij}, i \neq j,$$
$$l_{ii} = \sum_{j=1, j\neq i}^{N} l_{ij},$$
$$\sum_{j=1}^{N} l_{ij} = 0.$$

This article mainly concerns the leader-following MASs composed of one leader v_0 and N followers $v_i (i = 1, \cdots, N)$. The topology of the follower system is denoted as graph $G = (V, E, A)$, then, the algebraic topological structure of the whole leader-following MASs can be expressed as graph $\bar{G} = (\bar{V}, \bar{E}, \bar{A})$ with $\bar{V} = V \cup \{v_0\}$. The connection weighted matrix between G and v_0 is represented by $B = diag\{b_1, \cdots, b_N\}$. If the information of leader can be obtained by the i-th follower, $b_i > 0$, else $b_i = 0$. $H = L + B$ is a symmetric matrix will be frequently used in subsequent analysis. Assume graph $\bar{G}$ is connected, i.e. at least one follower v_i can obtain the information from v_0.

2.2. Caputo fractional derivative

The dynamic model of the multi-agent system involved in this article is described by the following Caputo fractional derivative.

Definition 1. *[32] For a univariate function $f(t)$, we define its Caputo fractional derivative with order α as*

$$D^{\alpha}_{t_0,t}f(t) = \frac{1}{\Gamma(m-\alpha)}\int_{t_0}^{t}(t-\tau)^{m-\alpha-1}f^{(m)}(\tau)d\tau,$$

in which $\alpha \in (m-1,m)$ and $m \in Z$. $\Gamma(\cdot)$ denotes the Gamma function defined by

$$\Gamma(z) = \int_{0}^{\infty} t^{z-1}\exp(-t)dt$$

and which satisfies $\Gamma(z+1) = z\Gamma(z)$, $\Gamma(1) = 1$.

Specially, let $m = 1$, one obtains $\alpha \in (0,1)$ and

$$D^{\alpha}_{t_0,t}f(t) = \frac{1}{\Gamma(1-\alpha)}\int_{t_0}^{t}(t-\tau)^{-\alpha}f'(\tau)d\tau.$$

For simplicity of notation, we denote$D^{\alpha}_{t_0,t}f(t)$ briefly by $D^{\alpha}f(t)$ in this work.

Definition 2. *[32] For a continuous function $z(\cdot) : R \to R^{m_1\times m_2}$, the Mittag-Leffter function with two parameters $\alpha, \beta > 0$ is defined as*

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{+\infty}\frac{z^k}{\Gamma(\alpha k+\beta)}, \tag{1}$$

Moreover, let $\beta = 1$, (1) will be simplified to the Mittag-Leffter function with single parameter

$$E_{\alpha}(z) = E_{\alpha,1}(z) = \sum_{k=0}^{+\infty}\frac{z^k}{\Gamma(\alpha k+1)}.$$

Lemma 3. *[32] Let $\alpha \in (0,1]$, the inequation below holds*

$$D^{\alpha}(x^T(t)x(t)) \le 2x^T(t)D^{\alpha}x(t)$$

for any derivable vector-valued function $x(t) \in R^n$.

Lemma 4. *[33] Let $\chi(\cdot) : [0,+\infty) \to R$ be a continuous function, and α be a given constant belong to $(0,1]$. If there are two constants $p_1 > 0$ and $p_2 \ge 0$ which comply to*

$$D^{\alpha}\chi(t) \le -p_1\chi(t) + p_2,$$

then

$$\chi(t) \le \chi(0)E_{\alpha}(-p_1t^{\alpha}) + p_2t^{\alpha}E_{\alpha,\alpha+1}(-p_1t^{\alpha}).$$

Lemma 5. *[34] For any constants $\alpha \in (0,2)$ and $\beta > 0$,if there is a constant $\varrho > 0$ satisfies*

$$\frac{\pi\alpha}{2} < \varrho < \min\{\pi, \pi\alpha\},$$

then there exists a positive real constant C complies to

$$|E_{\alpha,\beta}(z)| \le \frac{C}{|z|+1},$$

where $\varrho \le |\arg(z)| \le \pi$, $|z| \ge 0$.

Lemma 6. *[35] Let $\alpha \in (0,1)$ and denote*

$$\Phi_{\alpha\iota} := t^{\iota}E_{\alpha,\iota+1}(\Lambda t^{\alpha}),$$

then there are two finite positive real constants η_1 and η_2 such that

$$\begin{aligned} \|E_{\alpha,\alpha}(\Lambda t^{\alpha})\| &\le \eta_1\|\exp(\Lambda t)\|, \\ \|\Phi_{\alpha\iota}\| &\le \eta_2\|t^{\iota}\exp(\Lambda t)\|. \end{aligned}$$

where $\iota \in \{0,\alpha\}$, Λ is a matrix with appropriate dimension, and $\|\cdot\|$ represents the inductive norm for a vector or matrix.

2.3. Problem Statement

In this work, we focus on the fractional MASs with one leader and N followers that are described as below,

Leader:

$$D^{\alpha}x_0(t) = f(x_0(t),t). \tag{2}$$

The ith follower:

$$D^{\alpha}x_i(t) = f(x_i(t),t) + u_i(t), \tag{3}$$

in which the vectors $x_0(t) = (x_{01}(t),\cdots,x_{0n}(t))^T$ and $x_i(t) = (x_{i1}(t),\cdots,x_{in}(t))^T \in R^n$ denote the states for the agents involved, $f : R\times R^n \to R^n$ represents a continuously differentiable vector-valued function, and $u_i(t)$ $\in R^n$ is the consensus protocol or control input for the ith follower, $i = 1,2,\cdots,N$.

The **global state consensus error** between the ith follower and the leader is defined as

$$e_i(t) = x_i(t) - x_0(t),\ i = 1,2,\cdots,N. \tag{4}$$

The main task of this work is to design a **consensus control protocol** based on the **event-triggered mechanism** so that state trajectory of each follower can be consistent with that of the leader, i.e.

$$\lim_{t\to\infty} e_i(t) = 0, i = 1,2,\cdots,N. \tag{5}$$

For in-depth study, we define the **local consensus error** for the ith follower agent as below

$$\begin{aligned} q_i(t) = \sum_{j=1}^{N} a_{ij}(x_i(t)-x_j(t)) + b_i(x_i(t)-x_0(t)), \\ i = 1,2,\cdots,N. \end{aligned} \tag{6}$$

Denote

$$\begin{aligned} x(t) &= (x_1^T(t),\cdots,x_N^T(t))^T, \\ e(t) &= (e_1^T(t),\cdots,e_N^T(t))^T, \\ q(t) &= (q_1^T(t),\cdots,q_N^T(t))^T, \\ \tilde{f}(x(t),t) &= (f^T(x_1(t),t),\cdots,f^T(x_N(t),t))^T. \end{aligned}$$

and $1_N = (1, 1, \cdots, 1)^T$, then we obtain

$$\begin{aligned} e(t) &= x(t) - 1_N \otimes x_0(t), \\ q(t) &= ((L + B) \otimes I_n)(x(t) - 1_N \otimes x_0(t)) \\ &= (H \otimes I_n)e(t). \end{aligned} \tag{7}$$

where $\otimes$ refers to the Kronecker product.

For function $f(\cdot, \cdot)$, we propose the assumption below.

Assumption 7. *There is a semi-positive matrix $\Upsilon \in R^{n\times n}$ such that the following inequality holds for all $x, y \in R^n$,*

$$(x - y)^T (f(x, t) - f(y, t)) \leq (x - y)^T \Upsilon (x - y).$$

3. Design of event-triggered consensus protocol

Notice the network resource shared by the MASs is limited, we adopt the **event-triggered condition mechanism (ETM)** to cut down the data-broadcasting frequency in the communication network.

It is worth noting that, when adopting the centralized consensus protocol, each follower agent is required to interact directly with the leader, i.e. $b_i > 0, i = 1, \cdots, N$, otherwise, in the process of communication, it is necessary to obtain the consensus error $e_i(t)$ indirectly by using the information of all the nodes on the path between the ith follower and the leader. This will increase the burden of network communication. Thus in this subsection, we discuss the distributed event-triggered consensus protocol, in which each follower updates its control input depending only on its own information and that of its neighbors, without using global information. In this consensus protocol, the series of event-triggered instants for the ith follower is determined by

$$\begin{aligned} t^i_{k+1} = t^i_k + \min\{t \mid \|\bar{\delta}_i(t)\|^2 > \sigma \|q_i(t)\|^2 + \frac{\beta}{N} \exp(-\gamma t)\}, \\ i = 1, 2, \cdots, N, \; k = 0, 1, 2, \cdots, \end{aligned} \tag{8}$$

in which $t^i_0 = 0$ and $\bar{\delta}_i(t) = q_i(t^i_k) - q_i(t)$, $t \in [t^i_k, t^i_{k+1})$.

Accordingly, the **event-triggered condition (ETC)** for the i-th follower is described as

$$\|\bar{\delta}_i(t)\|^2 \leq \sigma \|q_i(t)\|^2 + \frac{\beta}{N} \exp(-\gamma t), \tag{9}$$

and the framework of the **distributed** consensus of leader-following MASs via **ETM** is shown in Fig.1.

Remark 8. *Equation* (8) *shows that, for the ith follower agent, there must be a non-negative integer k, such that*

$$t \in [t^i_k, t^i_{k+1}), \forall t \geq 0.$$

Remark 9. *In the event-triggered mechanism mentioned above, the* ***threshold function*** *on the left side of inequality (9) can be regarded as a linear combination of $\|q_i(t)\|^2$ and $\exp(-\gamma t)$. The norm term determines the system's acceptance of relative error $\|q_i(t) - q_i(t^i_k)\| / \|q_i(t)\|$ while the exponential term describes the tolerance for absolute error $\|q_i(t) - q_i(t^i_k)\|$. The relative error plays the dominant role in the sample screening process as $\|q_i(t)\|$ is large, the absolute error plays the major role to filter the sample and avoid Zeno behavior as $\|q_i(t)\|$ is very small. Therefore, (9) is called the* ***combined event-triggered condition*** *and the corresponding triggering mechanism (8) is called the* ***combined event-triggered mechanism(CETM)****.*

Based on (8) and (9), we design the following control protocol

$$u_i(t) = -K_i q_i(t^i_k), t \in [t^i_k, t^i_{k+1}) \tag{10}$$

in which the real constant $K_i > 0$ denotes the control gain.

Theorem 10. *For the MASs (2) and (3), if the matrix $H = L + B$ is reversible and there exists a control gain matrix $K = diag\{K_1 \otimes I_n, \cdots, K_N \otimes I_n,\}$ such that*

$$\begin{aligned} \Xi = I_N \otimes \Upsilon - H \otimes K + \frac{1}{2\varepsilon^2}(H \otimes K)(H \otimes K)^T \\ + \frac{\sigma\mu\varepsilon^2}{2} \bar{H}^T \bar{H} \\ < 0, \end{aligned} \tag{11}$$

where $\bar{H} = H \otimes I_n$, $\mu = \|\bar{H}^{-1}\|^2$, and $\otimes$ denotes the Kronecker product.

Then the leader-follower consensus will realize a the control protocol (10) and the combined event-triggered mechanism (8) are employed, i.e.

$$\lim_{t\to+\infty} e_i(t) = 0, i = 1, 2, \cdots, N.$$

or

$$\lim_{t\to\infty} e(t) = 0$$

Proof. Constuct the following Lyapunov function

$$V(t) = \frac{1}{2}\|e(t)\|^2 = \frac{1}{2}\sum_{i=1}^{N} \|e_i(t)\|^2.$$

Based on the algebraic topological graph $\bar{G}$, one can derive

$$\begin{aligned} q_i(t^i_k) &= \sum_{j=1}^{N} a_{ij}(x_i(t^i_k) - x_j(t^i_k)) + b_i(x_i(t^i_k) - x_0(t^i_k)) \\ &= \sum_{j=1}^{N} l_{ij} e_j(t^i_k) + b_i e_i(t^i_k) \\ &= \sum_{j=1}^{N} l_{ij}\delta_j(t) + \sum_{j=1}^{N} l_{ij} e_j(t) + b_i \delta_i(t) + b_i e_i(t). \end{aligned} \tag{12}$$

According to the property of fractional derivative, it can be obtained that

$$D^\alpha e_i(t) = f(x_i(t), t) - f(x_0(t), t)$$

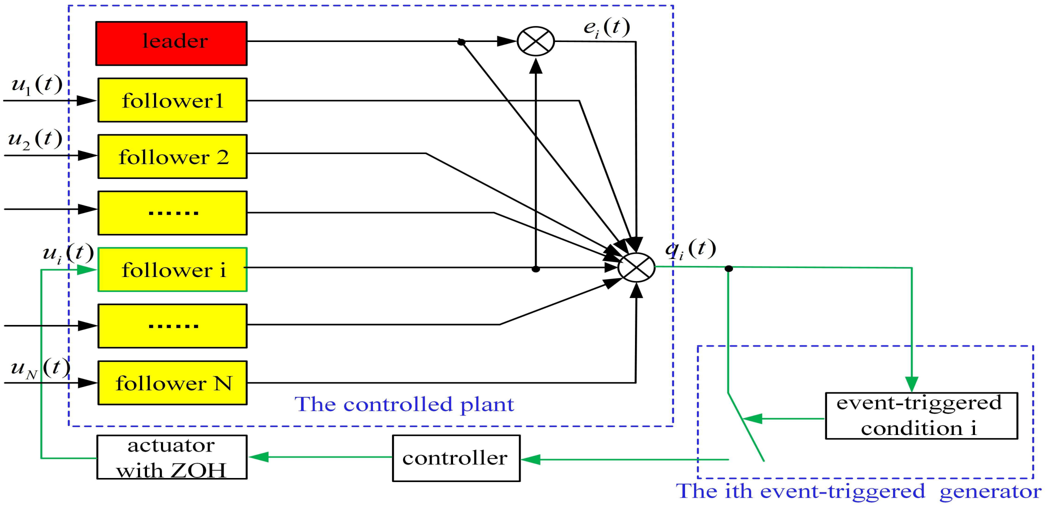

Figure 1: Framework of the distributed consensus scheme via CETM.

$$- \sum_{j=1}^{N} l_{ij} K \delta_j(t) - \sum_{j=1}^{N} l_{ij} K e_j(t) - b_i K \delta_i(t) - b_i K e_i(t). \tag{13}$$

Differentiate the function $V_1(t)$ and use Lemma 3 and Assumption 7, we obtain

$$\begin{aligned} D^{\alpha} V(t) \leq\ & e^T(t) D^{\alpha} e(t) \\ =\ & e^T(t)(\tilde{f}(x(t),t) - 1_N \otimes f(x_0(t),t)) \\ & - e^T(t)(H \otimes K)e(t) - e^T(t)(H \otimes K)\delta(t) \\ \leq\ & e^T(t)(I_N \otimes \Upsilon)e(t) - e^T(t)(H \otimes K)e(t) \\ & - e^T(t)(H \otimes K)\delta(t). \end{aligned} \tag{14}$$

with $\delta(t) = [\delta_1^T(t), \cdots, \delta_N^T(t)]^T$.

Noticing that the following equation holds for any positive constant ε

$$\begin{aligned} & \left\| (H \otimes K)^T e(t) + \varepsilon^2 \delta(t) \right\|^2 \\ = & \left\| (H \otimes K)^T e(t) \right\|^2 + 2\varepsilon^2 e^T(t)(H \otimes K)\delta(t) + \varepsilon^4 \|\delta(t)\|^2, \end{aligned}$$

then, we have

$$\begin{aligned} & e^T(t)(H \otimes K)\delta(t) \\ = & \frac{1}{2\varepsilon^2} \left\| (H \otimes K)^T e(t) + \varepsilon^2 \delta(t) \right\|^2 \\ & - \frac{1}{2\varepsilon^2} \left\| (H \otimes K)^T e(t) \right\|^2 - \frac{\varepsilon^2}{2} \|\delta(t)\|^2 \end{aligned} \tag{15}$$

Combining (14)with (15), we obtain

$$\begin{aligned} & D^{\alpha} V(t) \\ \leq\ & e^T(t)\Omega e(t) - \frac{1}{2\varepsilon^2} \left\| (H \otimes K)^T e(t) + \varepsilon^2 \delta(t) \right\|^2 \\ & + \frac{\varepsilon^2}{2} \|\delta(t)\|^2 \\ \leq\ & e^T(t)\Omega e(t) + \frac{\varepsilon^2}{2} \|\delta(t)\|^2, \end{aligned} \tag{16}$$

where

$$\Omega = I_N \otimes \Upsilon - H \otimes K + \frac{1}{2\varepsilon^2}(H \otimes K)(H \otimes K)^T.$$

Denote $\bar{\delta}(t) = [\bar{\delta}_1^T(t), \cdots, \bar{\delta}_N^T(t)]^T$ and $\bar{H} = H \otimes I_n$, we get $q(t) = \bar{H}e(t), \bar{\delta}(t) = \bar{H}\delta(t)$. Since $H = L + B$ is reversible, further we have $e(t) = \bar{H}^{-1} q(t), \delta(t) = \bar{H}^{-1}\bar{\delta}(t)$.

From the distributed event-triggered condition

$$\left\| \bar{\delta}_i(t) \right\|^2 \leq \sigma \|q_i(t)\|^2 + \frac{\beta}{N} \exp(-\gamma t)$$

we derive it holds for any $t \geq$ that

$$\begin{aligned} \left\| \bar{\delta}(t) \right\|^2 =\ & \sum_{i=1}^{N} \left\| \bar{\delta}_i(t) \right\|^2 \\ \leq\ & \sum_{i=1}^{N} (\sigma \|q_i(t)\|^2 + \frac{\beta}{N} \exp(-\gamma t)) \\ =\ & \sigma \|q(t)\|^2 + \beta \exp(-\gamma t) \\ =\ & \sigma e^T(t) \bar{H}^T \bar{H} e(t) + \beta \exp(-\gamma t), \end{aligned} \tag{17}$$

which yields

$$\begin{aligned} \|\delta(t)\|^2 =\ & \left\| \bar{H}^{-1} \bar{\delta}(t) \right\|^2 \\ \leq\ & \left\| \bar{H}^{-1} \right\|^2 \cdot \left\| \bar{\delta}(t) \right\|^2 \\ \leq\ & \sigma \left\| \bar{H}^{-1} \right\|^2 e^T(t) \bar{H}^T \bar{H} e(t) \\ & + \beta \exp(-\gamma t) \left\| \bar{H}^{-1} \right\|^2. \end{aligned} \tag{18}$$

Applying(16)-(18), we deduce that

$$D^{\alpha}V(t) \leq e^T(t)\Xi e(t) + \frac{\beta\mu\varepsilon^2}{2}\exp(-\gamma t), \qquad (19)$$

where

$$\Xi = \Omega + \frac{\sigma\mu\varepsilon^2}{2}\bar{H}^T\bar{H},$$
$$\mu = \left\|\bar{H}^{-1}\right\|^2.$$

Denote

$$\bar{p}_1 = 2\lambda_{\min}(-\Xi),$$
$$\bar{p}_2 = \frac{\beta\mu\varepsilon^2}{2}\exp(-\gamma t),$$

then we have

$$D^{\alpha}V(t) \leq -\bar{p}_1V(t) + \bar{p}_2.$$

Applying Lemma 4, we obtain

$$V(t) \leq V(0)E_{\alpha}(-\bar{p}_1t^{\alpha}) + \bar{p}_2t^{\alpha}E_{\alpha,\alpha+1}(-\bar{p}_1t^{\alpha}), t \geq 0. \quad (20)$$

In the next step, we are committed to proving

$$\lim_{t\to+\infty} V(t) = 0.$$

Since

$$\arg(-\bar{p}_1t^{\alpha}) = -\pi, |-\bar{p}_1t^{\alpha}| \geq 0, \forall t \geq 0, \forall\alpha \in (0,1),$$

then, according to Lemma 5, there exists a constant $C > 0$ such that

$$|E_{\alpha}(-\bar{p}_1t^{\alpha})| \leq \frac{C}{1+\bar{p}_1t^{\alpha}} \to 0 \quad (t \to +\infty)$$

which implies

$$\lim_{t\to+\infty} V(0)E_{\alpha}(-\bar{p}_1t^{\alpha}) = 0. \qquad (21)$$

Employing Lemma 6, we deduce that

$$\begin{aligned}\|t^{\alpha}E_{\alpha,\alpha+1}(-\bar{p}_1t^{\alpha})\| &\leq \eta_2\left\|t^{\alpha}\exp(-\bar{p}_1t)\right\| \\ &= \eta_2t^{\alpha}\exp(-\bar{p}_1t) \ \to \ 0,\end{aligned}$$

as $t \to +\infty$, which yields

$$\lim_{t\to+\infty} t^{\alpha}E_{\alpha,\alpha+1}(-\bar{p}_1t^{\alpha}) = 0. \qquad (22)$$

Using (20)-(22), we get

$$\begin{aligned}\lim_{t\to+\infty} V(t) \leq &\lim_{t\to+\infty} V(0)E_{\alpha}(-\bar{p}_1t^{\alpha}) \\ &+ \bar{p}_2\lim_{t\to+\infty} t^{\alpha}E_{\alpha,\alpha+1}(-\bar{p}_1t^{\alpha}) \\ = &\ 0.\end{aligned}$$

It hence appears

$$\lim_{t\to+\infty}\|e(t)\| \leq \lim_{t\to+\infty}\sqrt{2V_1(t)} = 0,$$

thus,

$$\lim_{t\to+\infty}\|e(t)\| = 0.$$

Therefore, MASs (2))−(3) realize the leader-following consensus. □

In the following, we work to prove the Zeno behavior will be excluded when the combined event-triggering consensus strategy is used.

Theorem 11. *Consider the fractional MASs* (2) *and* (3)*, if the distributed consensus protocol* (10) *is driven by the separate combined event-triggered mechanism (CETM)*(8)*, then the Zeno behavior will be excluded.*

Proof. The proof of Theorem 10shows that the fractional derivative of $q_i(t)$ with order $\alpha \in (0,1)$ is norm bounded, i.e., there exists a real constant $M_i \geq 0$, such that

$$\left\|D^{\alpha}_{t^i_k,t}q_i(t)\right\| \leq M_i, \quad \forall t \in [t^i_k, t^i_{k+1}).$$

Since $D^{-\alpha}_{t_0,t}D^{\alpha}_{t_0,t}q_i(t) = q_i(t) - q_i(t_0)$, we obtain

$$\begin{aligned}\left\|\bar{\delta}_i(t)\right\| = \left\|q_i(t^i_k) - q_i(t)\right\| &= \left\|D^{-\alpha}_{t^i_k,t}D^{\alpha}_{t^i_k,t}q_i(t)\right\| \\ &= \left\|\frac{1}{\Gamma(\alpha)}\int_{t^i_k}^{t}(t-\tau)^{\alpha-1}D^{\alpha}_{t^i_k,t}q_i(\tau)d\tau\right\| \\ &= \frac{1}{\Gamma(\alpha)}\int_{t^i_k}^{t}(t-\tau)^{\alpha-1}\left\|Dt^i_k,t^{\alpha}q_i(\tau)\right\|d\tau \\ &\leq \frac{M_i}{\Gamma(\alpha)}\int_{t^i_k}^{t}(t-\tau)^{\alpha-1}d\tau \\ &= \frac{M_i(t-t^i_k)^{\alpha}}{\alpha\Gamma(\alpha)} = \frac{M_i(t-t^i_k)^{\alpha}}{\Gamma(\alpha+1)}. \qquad (23)\end{aligned}$$

For the ith follower agent, assume t^i_k is the latest event-triggering instant before the current instant t. It follows from (8) that, the next event will not be triggered before the instant t^{i*}_k which satisfying

$$\left\|\delta_i(t^{i*}_k)\right\| = [\sigma\left\|e_i(t^{i*}_k)\right\|^2 + \frac{\beta}{N}\exp(-\gamma t^{i*}_k)]^{\frac{1}{2}}. \qquad (24)$$

and $t^{i*}_k > t$.

Let $\Delta t^{i*}_k = t^{i*}_k - t^i_k$, then the right side of inequality (24) can be rewritten as

$$[\sigma\left\|q_i(t^i_k + \Delta t^{i*}_k)\right\|^2 + \frac{\beta}{N}\exp(-\gamma(t^i_k + \Delta t^{i*}_k))]^{\frac{1}{2}}.$$

Combined with (23) and (24), we have

$$\begin{aligned}&[\bar{\sigma}\left\|q_i(t^i_k + \Delta t^{i*}_k)\right\|^2 + \frac{\beta}{N}\exp(-\gamma(t^i_k + \Delta t^{i*}_k))]^{\frac{1}{2}} \\ &\leq \frac{M_i(\Delta t^{i*}_k)^{\alpha}}{\Gamma(\alpha+1)}. \qquad (25)\end{aligned}$$

Notice $\left\|q_i(t^i_k + \Delta t^{i*}_k)\right\|^2$is non-negative and the exponential term $\exp(-\gamma(t^i_k + \Delta t^{i*}_k))$ is strictly positive, therefore, $\frac{M_i(\Delta t^{i*}_k)^{\alpha}}{\Gamma(\alpha+1)} > 0$, which yields

$$\Delta t^{i*}_k > 0.$$

Denote $m_i := \min\{\Delta t^{i*}_k\}$, then, it follows by

$$t^i_{k+1} - t^i_k > \Delta t^{i*}_k \geq m_i > 0,$$

which means that, for each follower agent i, there exists a positive lower bound m_i for the sequence of event intervals. Hence, there is no Zeno behavior. □

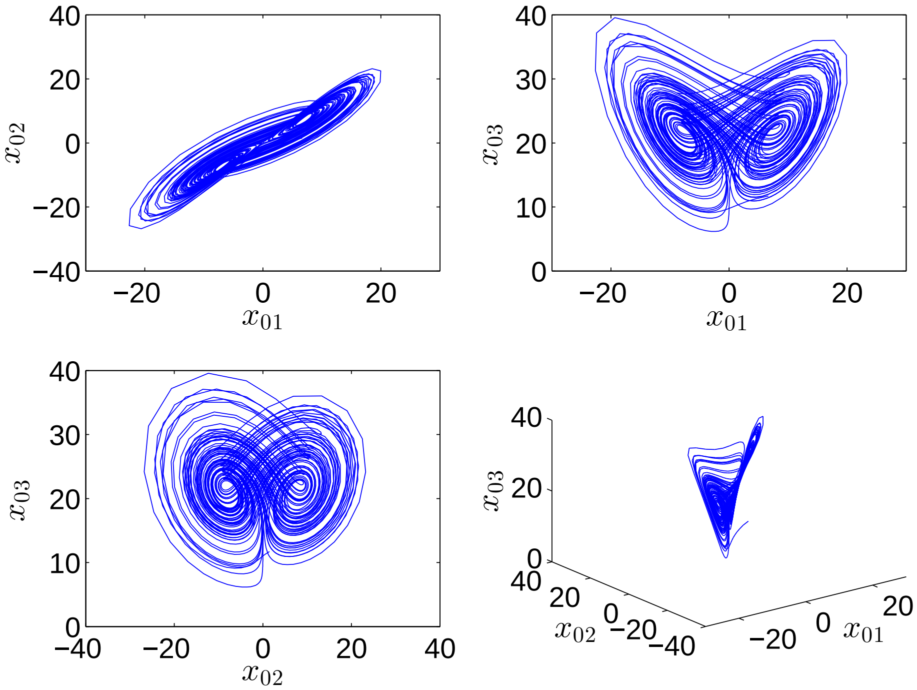

Figure 2: Phase portrait of the leader agent.

Table 2: Special cases of separate combined event-triggered mechanism for distributed consensus protocol

	Parameter selection	Mathematical model	Name
		$\|\bar{\delta}_i(t_k^i)\|^2 \leq \sigma\|q_i(t)\|^2 + \frac{\beta}{N}\exp(-\gamma t)$	Separate CETM (proposed in this work)
(i)	$\beta = 0$	$\|\bar{\delta}_i(t)\|^2 \leq \sigma\|q_i(t)\|^2$	Separate NETM
(ii)	$\sigma = 0$	$\|\bar{\delta}_i(t)\|^2 \leq \frac{\beta}{N}\exp(-\gamma t)$	Separate EETM

Table 3: Comparison of the distributed consensus protocols based on separate CETM and unified CETM.

	Mathematical model	Name
(i)	$\begin{cases} t_{k+1}^i = t_k^i + \min\{t \mid \|\bar{\delta}_i(t)\|^2 > \sigma\|q_i(t)\|^2 + \frac{\beta}{N}\exp(-\gamma t)\}, \\ u_i(t) = -K_i q_i(t_k^i), \quad t \in [t_k^i, t_{k+1}^i), \quad i = 1, 2, \cdots, N. \end{cases}$	Distributed consensus protocol based on separate CETM (proposed inthis work)
(ii)	$\begin{cases} t_{k+1} = t_k + \min\{t \mid \|\bar{\delta}(t)\|^2 > \sigma\|q(t)\|^2 + \frac{\beta}{N}\exp(-\gamma t)\}, \\ u_i(t_k) = -K_i q_i(t_k), \quad t \in [t_k, t_{k+1}), \quad i = 1, 2, \cdots, N. \end{cases}$	Distributed consensus protocol based on unified CETM

4. Simulation Experiment

To verify the feasibility and superiority of our consensus scheme, a simulation experiment is given below.

We focus on the leader-following MASs involving 6 fractional chaotic Chen systems, in which $\alpha = 0.9$,

$$f(t, x_i(t)) = \begin{pmatrix} -35x_{i1} + 35x_{i2} \\ -7x_{i1} + 28x_{i2} - x_{i1}x_{i3} \\ -3x_{i3} + x_{i1}x_{i2} \end{pmatrix},$$

and $i = 0, 1, \cdots, 5$.

Choose the initial state $x_0(0) = (10, 3, 12)^T$, then the attractor of the leader agent system can be depicted in Figure 2.

The connection weighted matrix B and the Laplacian matrix L for the considered MASs are given by

$$B = diag\{15, 15, 15, 0, 0\},$$

$$L = \begin{pmatrix} 2 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{pmatrix}.$$

Taking the simulation time as 3 seconds, we get $|x_1| \leq 17, |x_2| \leq 19, |x_3| \leq 37$. It follows from Assumption 7 that

$$e_i^T(t)(f(x_i(t), t) - f(x_0(t), t))$$

$$\leq e_i^T(t)\begin{pmatrix} 39.5 & 0 & 0 \\ 0 & 44.25 & 0 \\ 0 & 0 & 6.5 \end{pmatrix} e_i(t).$$

In this simulation, the leader agent is initialized with $x_0(0) = (10, 3, 12)^T$ while the follower agents are started from $x_1(0) = (5, 5, 6)^T$, $x_2(0) = (3, 6, 8)^T$, $x_3(0) = (6, -2, 6)^T$, $x_4(0) = (7, -5, 7)^T$, $x_5(0) = (11, 1, 15)^T$.

To ensure inequality (11) in Theorem 10 holds, the parameters in the consensus protocol are taken as $\sigma = 0.5$, $\beta = 0.1$, $\gamma = 0.01$ and $\varepsilon = 100$, the control gains are designed as $K_1 = K_2 = K_3 = 3.5, K_4 = K_5 = 5.5$. Then, applying the control law(10) based on the distributed CETM given by (8), the experiment results are displayed by Tables 2-3 and Figures 3-12.

1) Comparison among CETM and two traditional event-triggered mechanisms

First, we compare the proposed method CETM with two traditional event-triggered mechanisms. The mathematical models of the three mechanisms are given in Table 2. Meanwhile, for the multi-agent systems mentioned above, the simulation results based on the latter two mechanisms are shown in Figures 5-6 and Figures 7-8 respectively.

As shown in Table 2, CETM will degenerate to NETM or EETM respectively if we set $\beta = 0$ or $\sigma = 0$.

Comparing Figure 3 with Figures 5 and Figures 7 one can see, a satisfactory consensus performance can be achieved no matter applying which event-triggered mechanism mentioned above.

It follows from Figure 6 that, for each follower agent, if the consensus protocol is based on NETM, Zeno behavior will occur when $\|e_i(t)\|$ is close to zero. On the other hand, as shown in Figure 8, if it is based on EETM , events will be triggered very frequently as $\|e_i(t)\|$ is large. However, when we employ CETM, both of the two adverse performance mentioned above will be excluded, which can be shown by Figure 6.

Furthermore, It can be perceived from Figure9 that, no matter comparing with CETM or CETM, there is a significant reduction in the number of samples that need to be transmitted to the actuator via the network as CETM is applied, which verifies the advancement of the event-based consensus strategy designed in this work.

2) Comparison between the separate CETM and the unified CETM

The event-triggered mechanism CETM designed in this work is **separate**. To further show the superiority of this mechanism, we compare it with the **unified** CETM. The comparison between the mathematical models of this two mechanisms are described in Table 3 and the simulation results based on the unified CETM are given by Figures 10-12.

As shown in Table 3, for the unified CETM, all the follower agents update their actuator inputs at the same time according to a same triggering condition. But in contrast, for the separate CETM, each follower agent updates its actuator input independently according to its own triggering condition, so it is more flexible.

Comparing Figures 3-4 with Figures10-**??**, we can see that both this two triggering mechanism can achieve good consensus performance, and the event-update number of each follower agent based on the the separate CETM is far less than the unified CETM. And this conclusion can be further proved by Figure 12.

To sum up, the fully distributed CETM designed in this work has significant advantages in data filtering whether it is compared with the traditional distributed NETM and EETM, or compared with the unified-distributed CETM.

5. Conclusion

This work proposes a novel combined event-triggered control technique to realize the consensus of nonlinear fractional chaotic leader-following MASs. Applying the interaction graph theory and the fractional Lyapunov stability theory, some sufficient conditions are presented to achieve the event-triggered consensus for the fractional MASs. The simulation examples illustrate the superiority of the proposed consensus protocol from two aspects.

It is believed that the proposed CETM has a wide application prospect in dealing with other control or synchronization problems of fractional dynamic systems. In addition, the event trigger mechanism CETM has the same advantages for integer order systems. Our further work is to extend the results in this work to the case of $\alpha > 1$ by combining the integer differential theory with the fractional differential theory.

Disclosure statement

No potential conflict of interest was reported by the authors.

Author¡¯s contributions

All authors contributed equally to this work.

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Data Availability

The data that supports the findings of this study are available within the article.

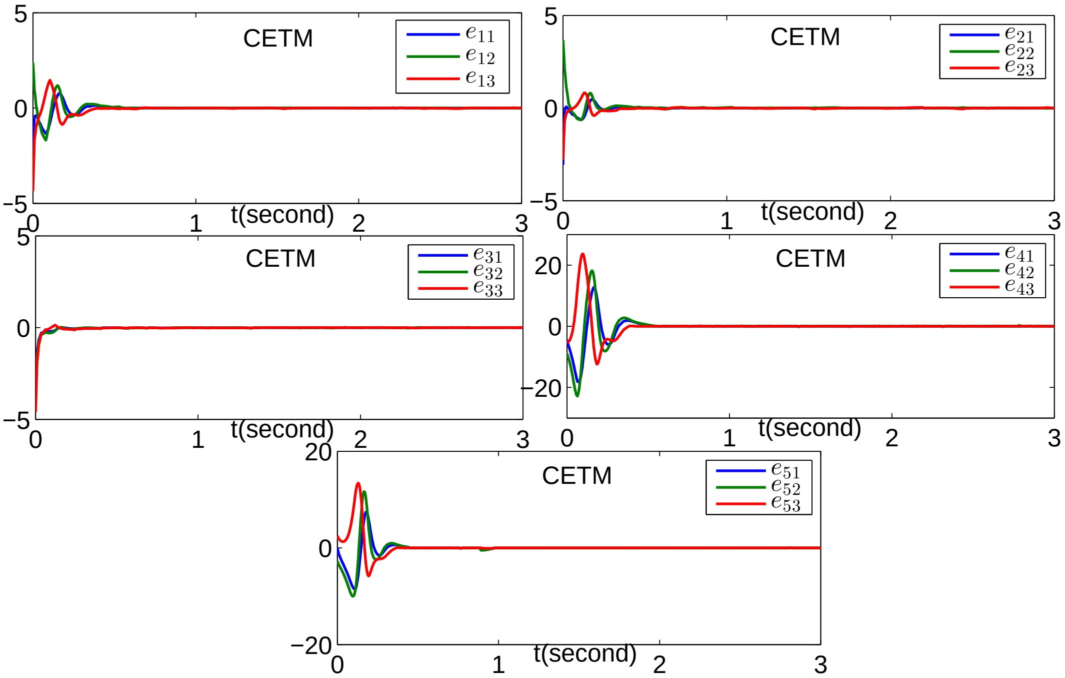

Figure 3: Consensus error of five follower agents based on distributed consensus protocol with separate CETM.

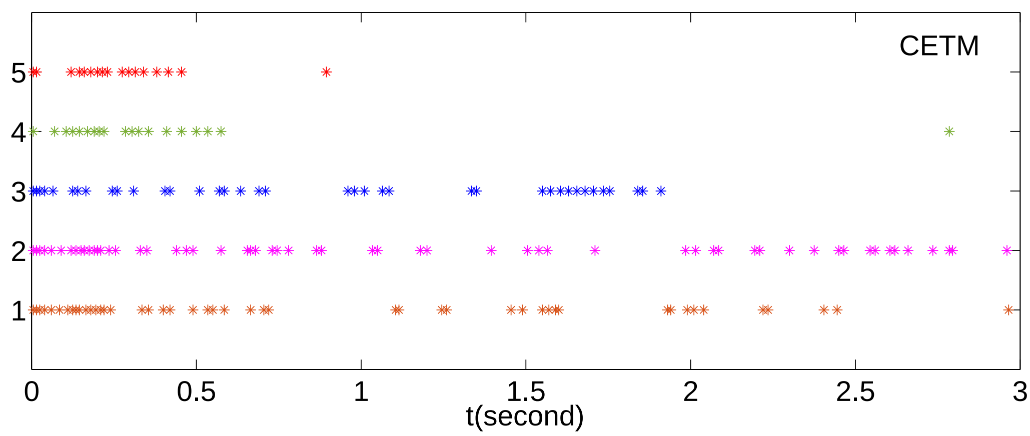

Figure 4: Event-triggered instants based on distributed consensus protocol with separate CETM.

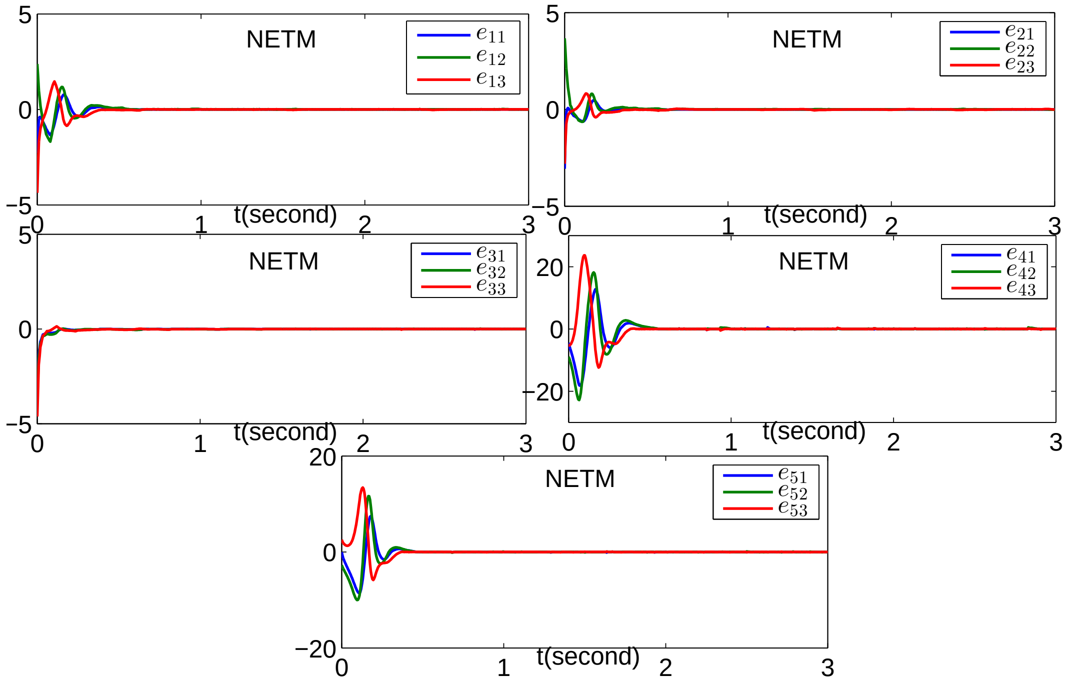

Figure 5: Consensus error of five follower agents based on distributed consensus protocol with separate NETM.

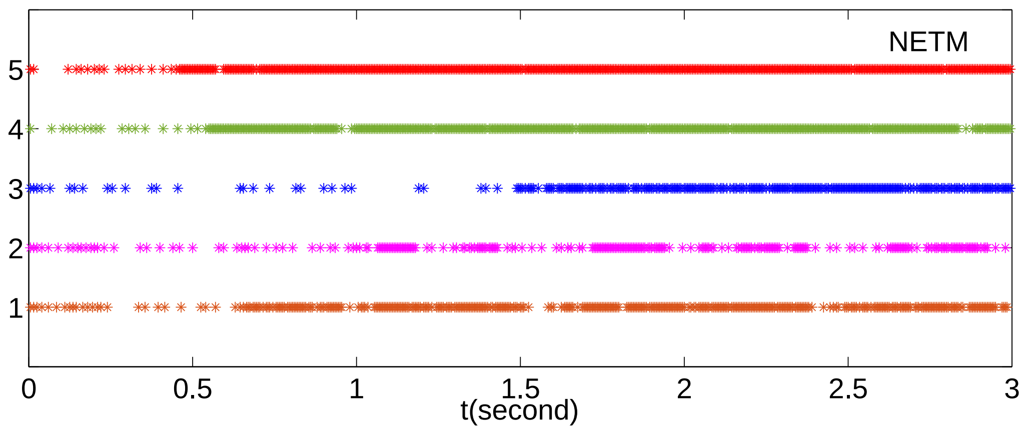

Figure 6: Event-triggered instants based on distributed consensus protocol with separate NETM.

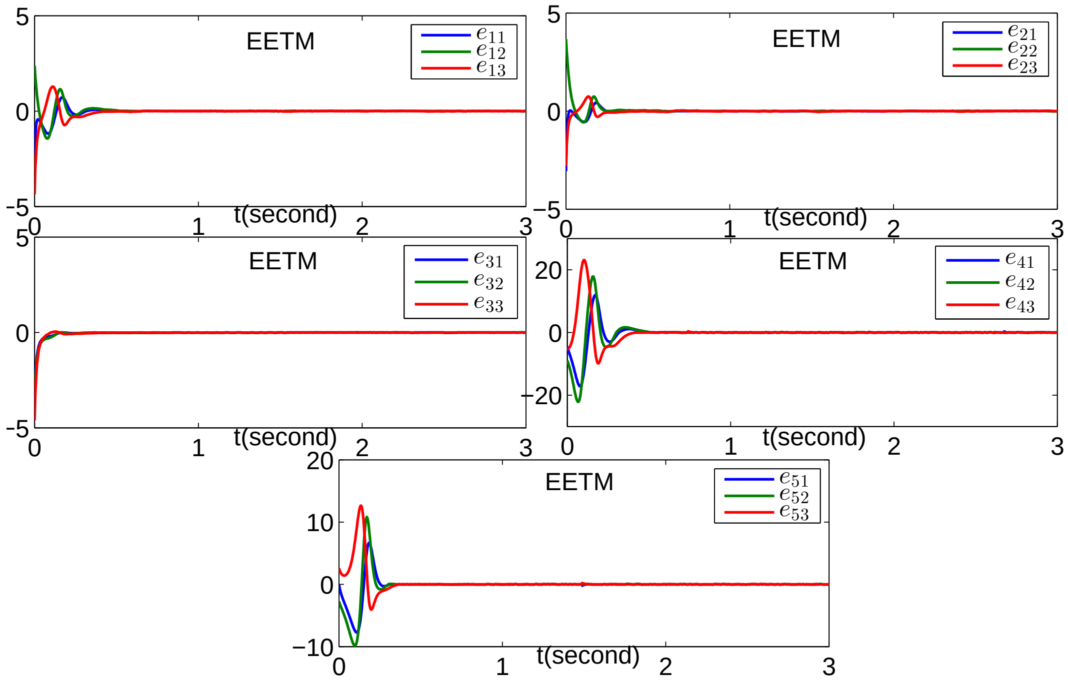

Figure 7: Consensus error of five follower agents based on distributed consensus protocol with separate EETM.

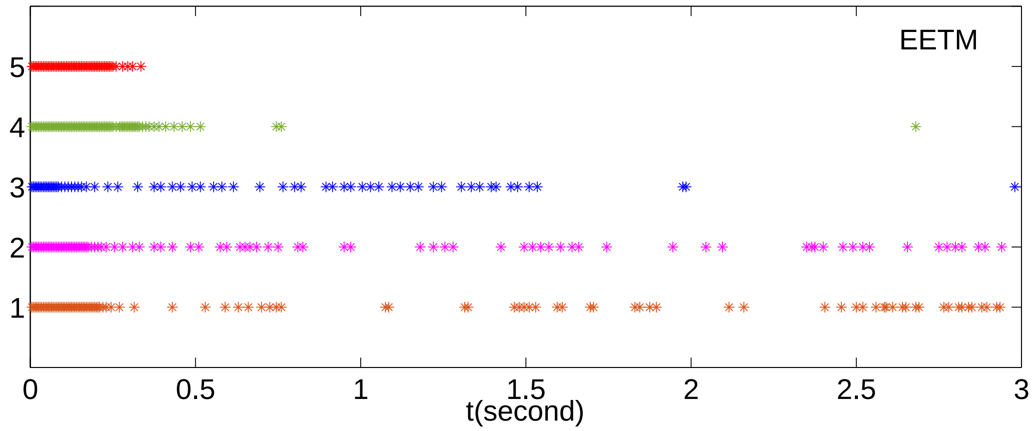

Figure 8: Event-triggered instants based on distributed consensus protocol with separate NETM.

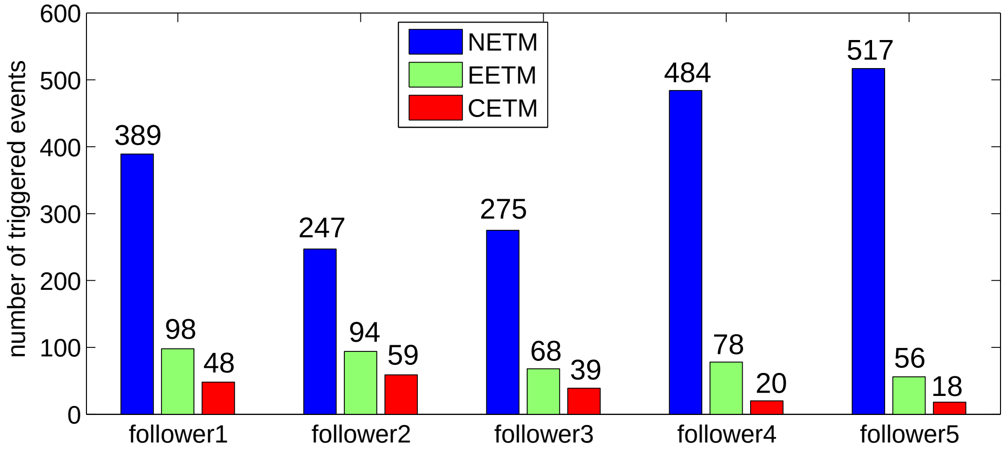

Figure 9: Comparison of the event-triggered numbers for distributed consensus protocol based on three separate event-triggered mechanisms.

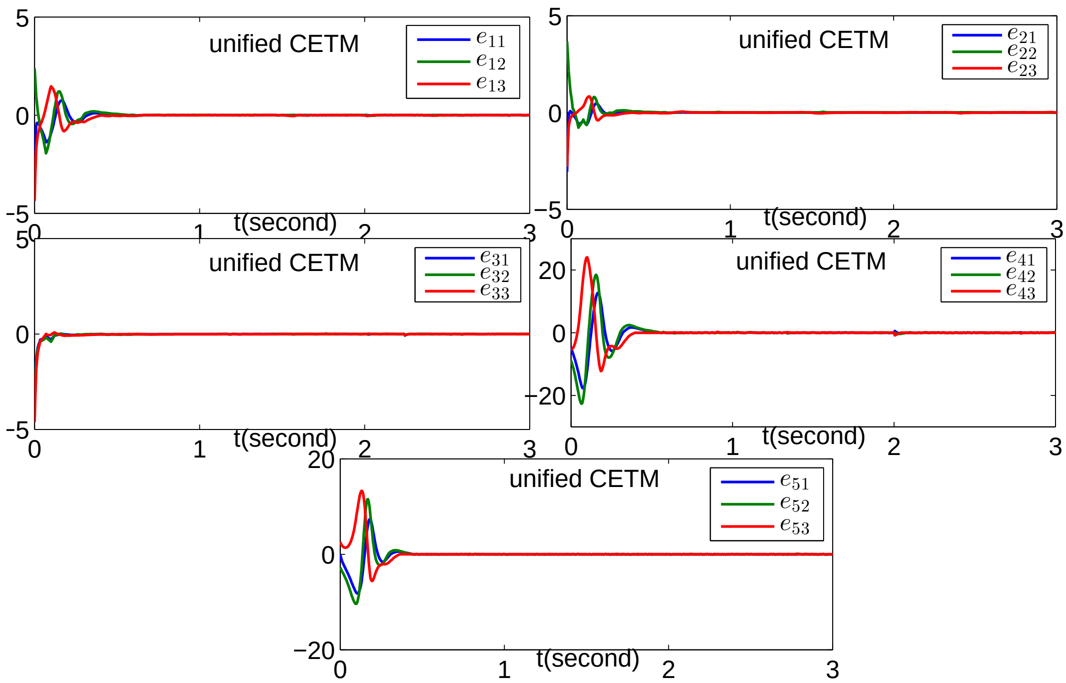

Figure 10: Consensus error of five follower agents based on distributed consensus protocol with unified CETM.

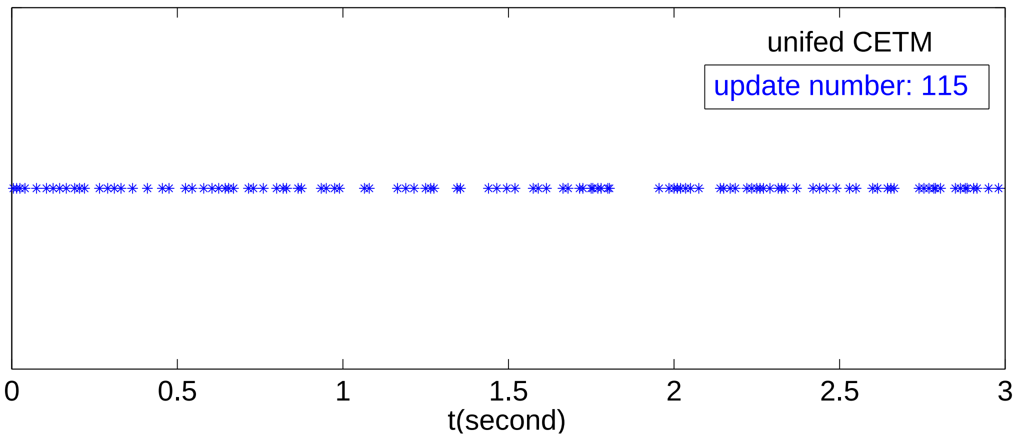

Figure 11: Event-triggered instants based on distributed consensus protocol with unified CETM.

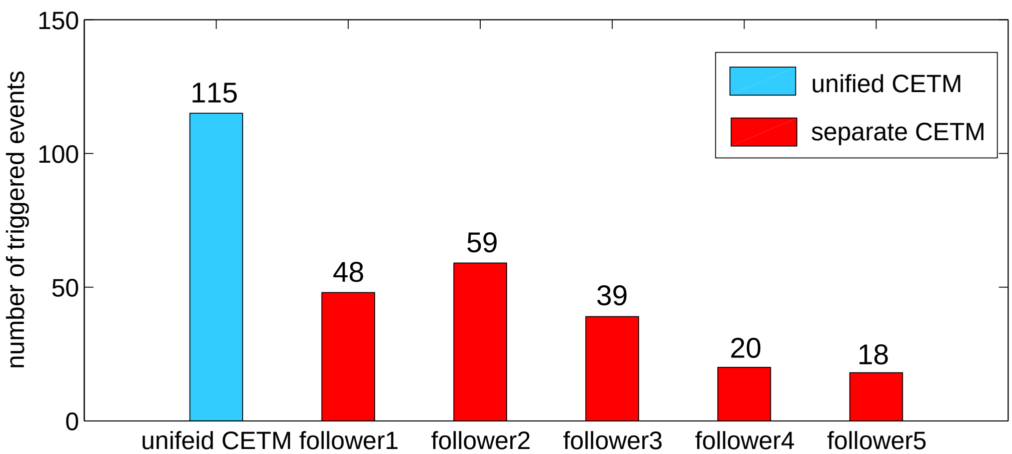

Figure 12: Comparison of the event-triggered numbers for distributed consensus protocols based on separate CETM and unified CETM.

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