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RESEARCH ARTICLE

Optimal input filters for iterative learning control systems with additive noises, random delays and data dropouts in both channels†

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Summary

In wireless networked iterative learning control systems, the controller is separated from the plant, and additive noises, random delays and data dropouts arise in both sensor-to-controller and controller-to-actuator channels. In order to guarantee the convergence performance of such systems with the effect of these uncertainties, an input filter is designed based on a proportional iterative learning controller, so that updated inputs can be filtered at the actuator side. Specifically, two data transmission processes are first developed to describe the mix of those uncertainties in both channels by Bernoulli and Gaussian distributed variables with known distributions. Based on state augmentation, the two data transmission processes are further combined with the iterative learning process of controllers to build a unified filtering model. According to this unified model, an optimal filter is designed via the projection theory and implemented at the actuator side to filter the updated inputs in iteration domain. Moreover, the convergence performance of the filtering error covariance matrix is proved theoretically. Finally, some numerical results are given to illustrate the effectiveness of the proposed method.

KEYWORDS:
iterative learning control, convergence performance, optimal filtering, additive noise, delay, data dropout

1 | INTRODUCTION

1.1 | Backgrounds

During last few years, networked control systems (NCSs) have attracted more and more attention. Compared with conventional systems with point-to-point transmission, measurements and inputs in NCSs all need to be transmitted over networks. In particular, the introduction of wireless networks separates the controller from the plant. As a result, such systems not only have such advantages as easy setup and reduced maintenance, but also can be used in a variety of complex environments[1,2,3].

In NCSs, the controller design is a challenging work. Fortunately, iterative learning control (ILC) is an effective strategy for the control object operating repeatedly over a fixed time interval, which was first proposed by Arimoto for the robotic manipulator to track a desired trajectory accurately[4]. The basic mechanism of this strategy is to update inputs for the next operation by adjusting current inputs with tracking errors and proper learning gains. Compared with other control strategies, ILC not only shows better tracking performance, but also requires less system information. As surveyed in[5,6], the efficacy of ILC strategies

†This is an example for title footnote.
[0]**Abbreviations:** ANA, anti-nuclear antibodies; APC, antigen-presenting cells; IRF, interferon regulatory factor

has been researched in a number of existing works, and the concerned topics include initial errors, stochastic noises, parameter optimization and so on.

However, the unreliability of wireless networks also brings some new challenges in guaranteeing the convergence performance of wireless networked ILC systems. Since measured output errors and updated inputs would be distorted during the transmission by unexpected network uncertainties such as additive noises, random delays and data dropouts, outputs fail to track the desired trajectory accurately if the wireless networked ILC systems operate without considering the effect of these uncertainties.

1.2 | Related works

The robustness of ILC systems has been researched for several years [7,8,9], and some results on analyzing and processing of ILC systems with measurement noise and process noise have been reported [10,11,12]. In [13], however, authors highlighted that the additive noise is different from the process noise and the measurement noise because the former is an external disturbance and added on measurements and inputs during the transmission, while the latter two appear inside the ILC system. According to the discovery that additive noises in both sensor-to-controller (SC) and controller-to-actuator (CA) channels are all constrained by the learning gain, authors proposed a learning gain selection method to suppress the effect of additive noises on the convergence performance of wireless networked ILC systems. In [14], authors further pointed out that the additive noise in SC channel is accumulated only in iteration domain, while the additive noise in CA channel is accumulated in both iteration and time domain simultaneously.

For ILC systems with delays, there are three different types involved such as fixed delays, random delays and varying delays. The first is assumed that the delay is a constant, the second is modeled by a set of Bernoulli distributed sequences, and the last one is generally given the lower and upper bounds of delays. Furthermore, the time delay would be suffered by different data. Different from the state delay that is inherent in systems such as bath processes and man-machine systems [15,16], the delay suffered by measurements and inputs occurs in network environment. In [17], the input delay is assumed to be known and compensated in the previous cycle, and the constant measurement delay is compensated in the learning process of controllers. In [18,19], authors discussed the convergence performance of networked ILC systems with random delays. In particular, the delayed measurement or input data was replaced by the synchronous one received in previous iteration.

With regard to data dropouts, the randomness of which can be described by Bernoulli distributed sequences or Markov chains. As to the former, the value of variables means corresponding data is dropped or not [20,21]. As to the latter, inspired by the work reported in [22], the update process of inputs with random dropouts is modeled as a Markov chain [23,24,25]. In addition, a variety of approaches were proposed for guaranteeing the convergence performance of networked ILC systems with random data dropouts, and can be split into two categories: Kalman-type filtering methods and compensation methods. In [26,27,28], authors filtered the effect of measured data dropped dependently or not through selecting the learning gain adaptively. In [17], authors replaced the dropped data in time domain with the one received at last time instant in the same iteration. Motivated by the method proposed in [17] and the nature that inputs of ILC systems converge in iteration domain, authors compensated the dropped data in this domain with the one received at the same time instant in last iteration [29,30,31,32].

It is worthy of our attention that most of aforementioned works were confined to a special case that only one uncertainty in single or both channels is considered. In fact, different uncertainties may be concurrent and jointly cause the performance degradation of ILC systems. And what's more, most of works processed the uncertainty at the controller side, which fails to guarantee the convergence performance of inputs received at the actuator side.

1.3 | Our contribution

Motivated by these two discoveries, it is important to address ILC systems under general communication conditions, and give methods to guarantee the convergence performance of inputs received at the actuator side. In this paper, we study the convergence performance of wireless networked ILC systems with additive noises, random delays and data dropouts in both SC and CA channels, and design an optimal filter so that updated inputs can be filtered at the actuator side with the effect of these uncertainties. To the best of our knowledge, this is the first time to address the convergence performance of ILC systems with those uncertainties simultaneously from the perspective of input filtering. Specifically, main contribution is summarized as follows:

- two data transmission processes are first developed to describe the mix of additive noises, random delays and data dropouts in both SC and CA channels by Bernoulli and Gaussian distributed variables with known distributions;

- Based on state augmentation, a unified filtering model is built only by using the iterative learning process of controllers as well as the two developed data transmission processes. Consequently, three uncertainties in both channels are all embodied in the filtering model;
- According to the unified model, an optimal filter is designed in the linear minimum variance (LMV) sense via projection theory, so updated inputs can be filtered at the actuator side with the effect of mixed uncertainties in both channels.

The rest of this paper is organized as follows: in section 2, the researched problem is formulated by considering a proportional-type iterative learning controller and developing two transmission processes of both measured output errors and updated inputs. In section 3, a unified filtering model is built, and the optimal filer is designed. Convergence performance of the filtering error covariance is analyzed in section 4. Numerical examples are given in section 5. The last section warps up this paper in the conclusion.

Notations: The superscript '-1', 'T' and '$-T$' denote inverse, transpose and combination of inverse and transpose actions, respectively. The symbol 'I' and '0' represent identity and zero matrices with appropriate dimensions. $(\cdot)$ denotes the same contents as that in the previous parenthesis; $Prob\{\cdot\}$ represents the occurrence probability of event '$\cdot$'; '$\perp$' denotes orthogonality.

2 | PROBLEM FORMULATION

In this paper, we consider the plant controlled over wireless networks with the following proportional iterative learning controller:

$$u_{k+1}^t(t) = u_k^t(t) + \Gamma(t)e_k^r(t+1) \tag{1}$$

where $e_k^r(t)$ stands for the received output error, $u_{k+1}^t(t)$ is the updated input and needs to be transmitted to the actuator, $\Gamma(t)$ is the selected learning gain, $t \in [0, 1, \cdots, T-1]$ denotes the operation time, and k indicates the iteration number.

Due to the unreliability of wireless channels, various uncertainties would arise and distort the received output error and input data, which are denoted as $e_k^r(t)$ and $u_k^r(t)$ respectively in this situation. Taking additive noises, random one-step delays and data dropouts into consideration, two data transmission processes can be developed to describe the mix of these uncertainties as

$$u_k^r(t) = \xi_{1,k}(t)u_k^t(t) + \left(1-\xi_{1,k}(t-1)\right)\xi_{2,k}(t)u_k^t(t-1) + \left(1-\xi_{1,k}(t)\right)u_{k-1}^r(t) + m_k(t) \tag{2a}$$

$$e_k^r(t) = \eta_{1,k}(t)e_k^t(t) + \left(1-\eta_{1,k}(t-1)\right)\eta_{2,k}(t)e_k^t(t-1) + n_k(t) \tag{2b}$$

where $\xi_{1,k}(t)$, $\xi_{2,k}(t)$, $\eta_{1,k}(t)$ and $\eta_{2,k}(t)$ are Bernoulli distributed variables taking value 0 or 1 with probabilities Prob $\left\{\xi_{i,k}(t)=1\right\} = \bar{\xi}_i$ and Prob $\left\{\eta_{i,k}(t)=1\right\} = \bar{\eta}_i$, $i=1,2$, in which $0 < \bar{\xi}_i \leq 1$ and $0 < \bar{\eta}_i \leq 1$. $m_k(t)$ and $n_k(t)$ are Gaussian distributed variables with zero means and variance Q_m and Q_n. Additionally, $\xi_{1,k}(t)$, $\xi_{1,k}(t)$, $\eta_{1,k}(t)$, $\eta_{2,k}(t)$, $m_k(t)$ and $n_k(t)$ are independent of each other for all k, t and i indices.

In the transmission process of updated input data, $\xi_{1,k}(t)$ denotes that the input data is received on time or one-step delayed, and $\xi_{2,k}(t)$ represents the one-step delayed input data is received at current moment or not. Similarly, $\eta_{1,k}(t)$ and $\eta_{2,k}(t)$ are used to describe the effect of the same uncertainties on the transmission process of output error data. Clearly, possible one-step delays and data dropouts in the data transmission processes can be described in four different cases as following:

Case 1 No data is received at current moment, then $\xi_{1,k}(t)$=$\eta_{1,k}(t)$=0, and $\xi_{1,k}(t-1)$=$\eta_{1,k}(t-1)$=1 or $\xi_{2,k}(t)$=$\eta_{2,k}(t)$=0. In this case, it is obvious that $u_k^r(t) = u_{k-1}^r(t) + m_k(t)$ and $e_k^r(t) = n_k(t)$;

Case 2 Only current data is received on time, then $\xi_{1,k}(t)$=$\eta_{1,k}(t)$=1, and $\xi_{1,k}(t-1)$=$\eta_{1,k}(t-1)$=1 or $\xi_{2,k}(t)$=$\eta_{2,k}(t)$=0. In this case, we find that $u_k^r(t) = u_k^t(t) + m_k(t)$ and $e_k^r(t) = e_k^t(t) + n_k(t)$;

Case 3 Only one-step delayed data is received at present moment, then $\xi_{1,k}(t)$=$\eta_{1,k}(t)$=0, $\xi_{1,k}(t-1)$=$\eta_{1,k}(t-1)$=0 and $\xi_{2,k}(t)$=$\eta_{2,k}(t)$=0. In this case, it can be easily seen that $u_k^r(t) = u_k^t(t-1) + u_{k-1}^r(t) + m_k(t)$ and $e_k^r(t) = e_k^t(t-1) + n_k(t)$;

Case 4 The current data and the one-step delayed data are all received at present moment simultaneously, then $\xi_{1,k}(t)$=$\eta_{1,k}(t)$=1, $\xi_{1,k}(t-1)$=$\eta_{1,k}(t-1)$=0, and $\xi_{2,k}(t)$=$\eta_{2,k}(t)$=1. In this case, $u_k^r(t) = u_k^t(t)+u_k^t(t-1)+m_k(t)$ and $e_k^r(t) = e_k^t(t)+e_k^t(t-1)+n_k(t)$.

Remark 1. In the developed data transmission processes, if transmitted input data $u_k^t(t)$ is not received on time, $u_{k-1}^r(t)$ would be used as a standby to drive the actuator whether one-step delayed data $u_k^t(t-1)$ is received or not at current moment. The

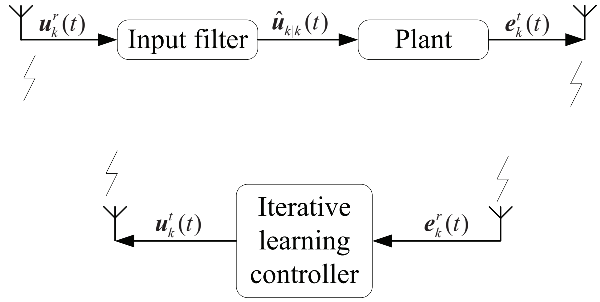

FIGURE 1 Diagram of ILC systems over wireless networks with the input filter.

reason we compensate the input data $u_k^t(t)$ not received on time with $u_{k-1}^r(t)$ is that the actuator needs to be driven by inputs all the time and the inputs converge in iteration domain. As to the controller, if transmitted output error $e_k^t(t)$ is not received on time, no data is used as a standby to replace it, and then the learning process is suspended at this moment. The essence of this compensation strategy is to guarantee the convergence performance of inputs by trading their convergence speed.

Obviously, uncertainties not only distort the updating of input data at the controller side but also contaminate the input data received at the actuator side, so the convergence performance of ILC systems cannot be guaranteed. According to this discovery, the idea behind this paper is to design an input filter, so the updated input $u_k^t(t)$ can be filtered at the actuator side with effect of these uncertainties in both channels. After that, the filtered input $\hat{u}_{k|k}(t)$ is used to drive the actuator. For a better understanding, the idea is further illustrated in Figure 1. In next section, we first build a unified filtering model, and then design an optimal filter based on this model.

3 | MAIN RESULTS

3.1 | Building of the unified filtering model

In order to design the input filter, the filtering model needs to be built first. Fortunately, the iterative learning process is known when the controller is established, and can be used to describe the the state equation in filtering model. Additionally, due to the random one-step delay occurs in time domain but the dropped input data is compensated in iteration domain, two iterative learning processes at the neighboring time instants in the same iteration are used simultaneously. Furthermore, in order to embrace the three uncertainties in both channels in the state equation, these two iterative learning processes are combined with developed data transmission processes based on state augmentation. After that, the measurement equation in filtering model is represented by the transmission process of updated inputs. With these two equations, the unified filtering model can be built. For the sake of concise expression, we let $\tilde{\xi}_{2,k}(t) = \left(1 - \xi_{1,k}(t-1)\right)\xi_{2,k}(t)$, $\tilde{\eta}_{2,k}(t) = \left(1 - \eta_{1,k}(t-1)\right)\eta_{2,k}(t)$. Since ILC systems converge in iteration domain, the input filtering is also designed in this domain, and the time index t is omitted. Consequently, the filtering model can be written in an augmented form as

$$X_{k+1} = A_k X_k + B_{1,k} U_k + B_{2,k} W_k \tag{3}$$

$$Y_k = C_k X_k + V_k \tag{4}$$

where $X_{k+1}=\begin{bmatrix} u_{k+1}^{t}(t)\\ u_{k+1}^{t}(t-1)\\ u_{k}^{r}(t)\\ e_{k}^{r}(t+1)\end{bmatrix}$, $U_k=\begin{bmatrix} e_{k}^{t}(t+1)\\ e_{k}^{t}(t)\\ e_{k}^{t}(t-1)\end{bmatrix}$, $Y_k=u_k^r(t)$, $W_k=\begin{bmatrix} m_k(t)\\ n_k(t)\\ n_k(t+1)\end{bmatrix}$, $V_k=m_k(t)$,

$A_k=\begin{bmatrix} I & 0 & 0 & 0\\ 0 & I & 0 & 0\\ \xi_{1,k}(t)I & \tilde{\xi}_{2,k}(t))I & \left(1-\xi_{1,k}(t)\right)I & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$, $B_{1,k}=\begin{bmatrix} \eta_{1,k}(t+1)\Gamma(t) & \tilde{\eta}_{2,k}(t+1)\Gamma(t) & 0\\ 0 & \eta_{1,k}(t)\Gamma(t-1) & \tilde{\eta}_{2,k}(t)\Gamma(t-1)\\ 0 & 0 & 0\\ \eta_{1,k}(t+1)I & \tilde{\eta}_{2,k}(t+1)I & 0\end{bmatrix}$,

$B_{2,k}=\begin{bmatrix} 0 & 0 & \Gamma(t)\\ 0 & \Gamma(t-1) & 0\\ I & 0 & 0\\ 0 & 0 & I\end{bmatrix}$, $C_k=\begin{bmatrix}\xi_{1,k}(t)I & \tilde{\xi}_{2,k}(t)I & \left(1-\xi_{1,k}(t)\right)I & 0\end{bmatrix}$.

In view of the previously mentioned assumptions about network uncertainties, it can be easily seen that $E\left\{W_kW_k^T\right\}=Q_W=\begin{bmatrix} Q_m & 0 & 0\\ 0 & Q_n & 0\\ 0 & 0 & Q_n\end{bmatrix}$, $E\left\{V_kV_k^T\right\}=Q_V=Q_m$, $E\left\{W_kV_k^T\right\}=Q_S=\begin{bmatrix} Q_m & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$, $E\left\{\xi_{1,k}(t)\right\}=\bar{\xi}_1=\alpha_1$, $E\left\{\tilde{\xi}_{2,k}(t)\right\}=\left(1-\bar{\xi}_1\right)\bar{\xi}_2=\alpha_2$, $E\left\{\eta_{1,k}(t)\right\}=\bar{\eta}_1=\beta_1$, $E\left\{\tilde{\eta}_{2,k}(t)\right\}=\left(1-\bar{\eta}_1\right)\bar{\eta}_2=\beta_2$.

3.2 | Design of the optimal input filter

Based on the filtering model built in last subsection, an optimal input filter is designed in the LMV sense by using the projection theory. Before presenting the optimal input filter for augmented model (3) and (4), we first define $\bar{A}_k$, $\bar{B}_{1,k}$, $\bar{B}_{2,k}$ and $\bar{C}_k$ are mathematical expectations of A_k, $B_{1,k}$, $B_{2,k}$ and C_k respectively, which can be computed by replacing the stochastic parameters with their mathematical expectations. After that, the following lemmas are introduced.

Lemma 1. For unified filtering model (3) and (4), we define $\Delta A_k \triangleq A_k-\bar{A}_k$, $\Delta B_{1,k}\triangleq B_{1,k}-\bar{B}_{1,k}$, $\Delta B_{2,k}\triangleq B_{2,k}-\bar{B}_{2,k}$, $\Delta C_k\triangleq C_k-\bar{C}_k$, and the following results can be derived:

$$\begin{aligned}
&\Delta A_k=\left(\xi_{1,k}(t)-\alpha_1\right)\Psi_1+\left(\tilde{\xi}_{2,k}(t)-\alpha_2\right)\Psi_2\\
&\Delta B_{1,k}=\left(\eta_{1,k}(t+1)-\beta_1\right)\Psi_3+\left(\tilde{\eta}_{2,k}(t+1)-\beta_2\right)\Psi_4+\left(\eta_{1,k}(t)-\beta_1\right)\Psi_5+\left(\tilde{\eta}_{2,k}(t)-\beta_2\right)\Psi_6\\
&\Delta B_{2,k}=0\\
&\Delta C_k=\left(\xi_{1,k}(t)-\alpha_1\right)\Psi_7+\left(\tilde{\xi}_{2,k}(t)-\alpha_2\right)\Psi_8
\end{aligned}\tag{5}$$

with

$\Psi_1=\begin{bmatrix} 0&0&0&0\\ 0&0&0&0\\ I&0&-I&0\\ 0&0&0&0\end{bmatrix}$, $\Psi_2=\begin{bmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&I&0&0\\ 0&0&0&0\end{bmatrix}$, $\Psi_3=\begin{bmatrix} \Gamma(t)&0&0\\ 0&0&0\\ 0&0&0\\ I&0&0\end{bmatrix}$, $\Psi_4=\begin{bmatrix} 0&\Gamma(t)&0\\ 0&0&0\\ 0&0&0\\ 0&I&0\end{bmatrix}$, $\Psi_5=\begin{bmatrix} 0&0&0\\ 0&\Gamma(t-1)&0\\ 0&0&0\\ 0&0&0\end{bmatrix}$, $\Psi_6=\begin{bmatrix} 0&0&0\\ 0&0&\Gamma(t-1)\\ 0&0&0\\ 0&0&0\end{bmatrix}$,
$\Psi_7=\begin{bmatrix} I & 0 & -I & 0\end{bmatrix}$, $\Psi_8=\begin{bmatrix} 0 & -I & 0 & 0\end{bmatrix}$.

Proof. Equation (5) follows directly from (3). □

Lemma 2. For unified filtering model (3) and (4), we define the mean of X_k and U_k as $\bar{X}_k=E\left\{X_k\right\}$ and $\bar{U}_k=E\left\{U_k\right\}$, and the second-order moment matrix of X_k as $q_{k+1}=E\left\{X_{k+1}X_{k+1}^T\right\}$, which satisfies the following recursion:

$$\begin{aligned}
q_{k+1}=&\bar{A}_kq_k\bar{A}_k^T+\bar{A}_k\bar{X}_k\bar{U}_k^T\bar{B}_{1,k}^T+\bar{B}_{1,k}\bar{U}_k\bar{X}_k^T\bar{A}_k^T+\bar{B}_{1,k}\bar{U}_k\bar{U}_k^T\bar{B}_{1,k}^T+\bar{B}_{2,k}Q_W\bar{B}_{2,k}^T\\
&+\left(\alpha_1-\alpha_1^2\right)\Psi_1q_k\Psi_1^T+\left(\alpha_2-\alpha_2^2\right)\Psi_2q_k\Psi_2^T+\left(\beta_1-\beta_1^2\right)\Psi_3\bar{U}_k\bar{U}_k^T\Psi_3^T\\
&+\left(\beta_2-\beta_2^2\right)\Psi_4\bar{U}_k\bar{U}_k^T\Psi_4^T+\left(\beta_1-\beta_1^2\right)\Psi_5\bar{U}_k\bar{U}_k^T\Psi_5^T+\left(\beta_2-\beta_2^2\right)\Psi_6\bar{U}_k\bar{U}_k^T\Psi_6^T
\end{aligned}\tag{6}$$

Proof. Equation (3) can be rewritten as

$$X_{k+1}=\left(\bar{A}_k+\Delta A_k\right)X_k+\left(\bar{B}_{1,k}+\Delta B_{1,k}\right)U_k+B_{2,k}W_k\tag{7}$$

Substituting (7) into $q_{k+1} = E\left\{X_{k+1}X_{k+1}^T\right\}$, and noting that $X_k \perp W_k$ and $U_k \perp W_k$, then q_{k+1} can be computed by

$$\begin{aligned} q_{k+1} =& \bar{A}_k q_k \bar{A}_k^T + \bar{A}_k \bar{X}_k \bar{U}_k^T \bar{B}_{1,k}^T + \bar{B}_{1,k}\bar{U}_k \bar{X}_k^T \bar{A}_k^T + \bar{B}_{1,k}\bar{U}_k\bar{U}_k^T \bar{B}_{1,k}^T \\ &+ E\left\{\Delta A_k q_k \Delta A_k^T\right\} + E\left\{\left(\Delta B_{1,k}\bar{U}_k\right)(\cdot)^T\right\} + E\left\{B_{2,k}W_kW_k^T B_{2,k}^T\right\} \end{aligned} \tag{8}$$

$$E\left\{\Delta A_k q_k \Delta A_k^T\right\} = \left(\alpha_1 - \alpha_1^2\right)\Psi_1 q_k \Psi_1^T + \left(\alpha_2 - \alpha_2^2\right)\Psi_2 q_k \Psi_2^T \tag{9}$$

$$\begin{aligned} E\left\{\left(\Delta B_{1,k}U_k\right)(\cdot)^T\right\} =& \left(\beta_1 - \beta_1^2\right)\Psi_3\bar{U}_k\bar{U}_k^T\Psi_3^T + \left(\beta_2 - \beta_2^2\right)\Psi_4\bar{U}_k\bar{U}_k^T\Psi_4^T \\ &+ \left(\beta_1 - \beta_1^2\right)\Psi_5\bar{U}_k\bar{U}_k^T\Psi_5^T + \left(\beta_2 - \beta_2^2\right)\Psi_6\bar{U}_k\bar{U}_k^T\Psi_6^T \end{aligned} \tag{10}$$

$$E\left\{B_{2,k}W_kW_k^T B_{2,k}^T\right\} = \bar{B}_{2,k}Q_W\bar{B}_{2,k}^T \tag{11}$$

Substituting (9)-(11) into (8), (6) is derived. □

In the following, we design the optimal filter according to the projection theory[33]. For the sake of easy manipulation, we first define ε_k as the innovation sequence with covariance $\theta_k = E\left\{\varepsilon_k\varepsilon_k^T\right\}$, $K_k = E\left\{X_k\varepsilon_k^T\right\}\theta_k^{-1}$ and $L_k = E\left\{X_{k+1}\varepsilon_k^T\right\}\theta_k^{-1}$ as the filtering and prediction gain matrices, $P_{k+1|k}$ and $P_{k|k}$ are one-step prediction and filtering error covariance matrices, $\hat{X}_{k|k-1} = \min\limits_{\theta_{1,k}} E\left\{\left(\theta_{1,k} - X_k\right)(\cdot)^T\right\}$, $\hat{X}_{k+1|k-1} = \min\limits_{\theta_{2,k}} E\left\{\left(\theta_{2,k} - X_{k+1}\right)(\cdot)^T\right\}$, $\hat{Y}_{k|k-1} = \min\limits_{\theta_{3,k}} E\left\{\left(\theta_{3,k} - Y_k\right)(\cdot)^T\right\}$, where $\theta_{1,k}$, $\theta_{2,k}$ and $\theta_{3,k}$ are linear functions of Y_1, Y_2, …, and Y_{k-1}.

Theorem 1. For the unified model (3) and (4), the recursive optimal filter is given by

$$\hat{X}_{k|k} = \hat{X}_{k|k-1} + K_k\varepsilon_k \tag{12}$$

$$\hat{X}_{k+1|k} = \hat{X}_{k+1|k-1} + L_k\varepsilon_k \tag{13}$$

$$\varepsilon_k = Y_k - \hat{Y}_{k|k-1} \tag{14}$$

$$\hat{X}_{k+1|k-1} = \overline{A}_k X_{k|k-1} + \overline{B}_{1,k}U_k \tag{15}$$

$$\hat{Y}_{k|k-1} = \overline{C}_k\hat{X}_{k|k-1} \tag{16}$$

$$\theta_k = \overline{C}_k P_{k|k-1}\overline{C}_k^T + J_k \tag{17}$$

$$K_k = P_{k|k-1}\overline{C}_k^T\theta_k^{-1} \tag{18}$$

$$L_k = \left(\overline{A}_k P_{k|k-1}\overline{C}_k^T + H_k\right)\theta_k^{-1} \tag{19}$$

$$\begin{aligned} P_{k+1|k} =& \left(\overline{A}_k - L_k\overline{C}_k\right)P_{k|k-1}\left(\overline{A}_k - L_k\overline{C}_k\right)^T \\ &+ L_kQ_VL_k^T + \Omega_{1,k} + \Omega_{2,k} + \Omega_{3,k} - \Omega_{4,k} - \Omega_{4,k}^T \end{aligned} \tag{20}$$

$$P_{k|k} = P_{k|k-1} - K_k\theta_kK_k^T \tag{21}$$

where

$$H_k = \left(\alpha_1 - \alpha_1^2\right)\Psi_1 q_k\Psi_7^T + \left(\alpha_2 - \alpha_2^2\right)\Psi_2 q_k\Psi_8^T + \overline{B}_{2,k}Q_S$$

$$J_k = \left(\alpha_1 - \alpha_1^2\right)\Psi_7 q_k\Psi_7^T + \left(\alpha_2 - \alpha_2^2\right)\Psi_8 q_k\Psi_8^T + Q_V$$

$$\Omega_{1,k} = \left(\alpha_1 - \alpha_1^2\right)\left(\Psi_1 - L_k\Psi_7\right)q_k\left(\Psi_1 - L_k\Psi_7\right)^T + \left(\alpha_2 - \alpha_2^2\right)\left(\Psi_2 - L_k\Psi_8\right)q_k\left(\Psi_2 - L_k\Psi_8\right)^T$$

$$\Omega_{2,k} = \left(\beta_1 - \beta_1^2\right)\Psi_3U_kU_k^T\Psi_3^T + \left(\beta_1 - \beta_1^2\right)\Psi_4U_kU_k^T\Psi_4^T + \left(\beta_2 - \beta_2^2\right)\Psi_5U_kU_k^T\Psi_5^T + \left(\beta_2 - \beta_2^2\right)\Psi_6U_kU_k^T\Psi_6^T$$

$$\Omega_{3,k} = \bar{B}_{2,k}Q_W\bar{B}_{2,k}^T$$

$$\Omega_{4,k} = \bar{B}_{2,k} Q_S L_k^T$$

Proof. By projection, we have (12)-(14) naturally. Taking projection of both sides of (3) onto the linear space spanned by $\left(Y_1, Y_2, \dots, Y_{k-1}\right)$ and noting $E\left\{W_k\right\} = 0$, (15) can be given by

$$\hat{X}_{k+1|k-1} = \overline{A}_k X_{k|k-1} + \overline{B}_{1,k} U_k$$

Taking projection of both sides of (4) onto the linear space spanned by $\left(Y_1, Y_2, \dots, Y_{k-1}\right)$ and noting $E\left\{V_k\right\} = 0$, (16) can be shown as

$$\hat{Y}_{k|k-1} = \overline{C}_k \hat{X}_{k|k-1}$$

Substituting (4) and (16) into (14), the innovation ε_k can be represented as

$$\varepsilon_k = \Delta C_k X_k + \overline{C}_k \tilde{X}_{k|k-1} + V_k \tag{22}$$

where $\tilde{X}_{k|k-1} = X_k - \hat{X}_{k|k-1}$. Because $\tilde{X}_{k|k-1} \perp V_k$, $X_k \perp V_k$, and ΔC_k, $\tilde{X}_{k|k-1}$ and V_k are zero-mean, thus the innovation covariance matrix in (17) can be obtained from (22) as

$$\begin{aligned}\theta_k &= E\left\{\varepsilon_k \varepsilon_k^T\right\} \\ &= E\left\{\Delta C_k q_k \Delta C_k^T\right\} + \bar{C}_k P_{k|k-1} \bar{C}_k^T + Q_V \\ &= \left(\alpha_1 - \alpha_1^2\right) \Psi_7 q_k \Psi_7^T + \left(\alpha_2 - \alpha_2^2\right) \Psi_8 q_k \Psi_8^T + \bar{C}_k P_{k|k-1} \bar{C}_k^T + Q_V\end{aligned}$$

Using $\tilde{X}_{k|k-1} \perp \hat{X}_{k|k-1}$ and $E\left\{V_k\right\} = 0$, we have

$$\begin{aligned}E\left\{X_k \varepsilon_k^T\right\} &= E\left\{X_k \tilde{X}_{k|k-1}^T\right\} \bar{C}_k^T \\ &= E\left\{\tilde{X}_{k|k-1} \tilde{X}_{k|k-1}^T\right\} \bar{C}_k^T \\ &= P_{k|k-1} \bar{C}_k^T\end{aligned} \tag{23}$$

Substituting (23) and the covariance matrix of innovation into the definition of K_k, we have (18). The prediction gain L_k can be computed by

$$\begin{aligned}L_k &= E\left\{X_{k+1} \varepsilon_k^T\right\} \theta_k^{-1} \\ &= E\left\{A_k X_k \varepsilon_k^T + B_{1,k} U_k \varepsilon_k^T + B_{2,k} W_k \varepsilon_k^T\right\} \theta_k^{-1}\end{aligned} \tag{24}$$

Substituting (22) into $E\left\{A_k X_k \varepsilon_k^T\right\}$, we have

$$\begin{aligned}E\left\{A_k X_k \varepsilon_k^T\right\} &= E\left\{A_k q_k \Delta C_k^T\right\} \\ &\quad + E\left\{A_k X_k \tilde{X}_{k|k-1}^T C_k^T\right\}\end{aligned} \tag{25}$$

with

$$\begin{aligned}E\left\{A_k q_k \Delta C_k^T\right\} &= E\left\{\Delta A_k q_k \Delta C_k^T\right\} \\ &= \left(\alpha_1 - \alpha_1^2\right) \Psi_1 q_k \Psi_7^T + \left(\alpha_2 - \alpha_2^2\right) \Psi_2 q_k \Psi_8^T\end{aligned}$$

$$\begin{aligned}E\left\{A_k X_k \tilde{X}_{k|k-1}^T C_k^T\right\} &= \overline{A}_k E\left\{\tilde{X}_{k|k-1} \tilde{X}_{k|k-1}^T\right\} \overline{C}_k^T \\ &= \overline{A}_k P_{k|k-1} \overline{C}_k^T\end{aligned}$$

Substituting (22) into $E\left\{B_{1,k} U_k \varepsilon_k^T\right\}$ and $E\left\{B_{2,k} W_k \varepsilon_k^T\right\}$, we have

$$\begin{aligned}E\left\{B_{1,k} U_k \varepsilon_k^T\right\} &= E\left\{B_{1,k} U_k X_k^T \Delta C_k^T\right\} + E\left\{B_{1,k} U_k \tilde{X}_{k|k-1}^T \bar{C}_k^T\right\} + E\left\{B_{1,k} U_k V_k^T\right\} \\ &= 0\end{aligned} \tag{26}$$

$$\begin{aligned}E\left\{B_{2,k} W_k \varepsilon_k^T\right\} &= E\left\{B_{2,k} W_k X_k^T \Delta C_k^T\right\} + E\left\{B_{2,k} W_k \tilde{X}_{k|k-1}^T \bar{C}_k^T\right\} + E\left\{B_{2,k} W_k V_k^T\right\} \\ &= \overline{B}_{2,k} Q_S\end{aligned} \tag{27}$$

where the fact ΔC_k, $\tilde{X}_{k|k-1}$ and V_k are zero-mean has been applied. Putting (25)-(27) into (24), (19) is obtained. Subtracting $\hat{X}_{k+1|k}$ in (13) from X_{k+1} in (3), we have the one-step prediction error equation

$$\tilde{X}_{k+1|k} = \left(\bar{A}_k - L_k\bar{C}_k\right)\tilde{X}_{k|k-1} + \left(\Delta A_k - L_k\Delta C_k\right)X_k + \Delta B_{1,k}U_k + B_{2,k}W_k - L_kV_k$$

$$\begin{aligned} P_{k+1|k} = {} & \left(\bar{A}_k - L_k\bar{C}_k\right)P_{k|k-1}\left(\bar{A}_k - L_k\bar{C}_k\right)^T + E\left\{\left(\Delta A_k - L_k\Delta C_k\right)q_k(\cdot)^T\right\} + E\left\{\Delta B_{1,k}U_k^T\Delta B_{1,k}^T\right\} \\ & + E\left\{B_{2,k}W_kW_k^TB_{2,k}^T\right\} + L_kQ_VL_k^T + E\left\{\left(\Delta A_k - L_k\Delta C_k\right)X_kU_k^T\Delta B_{1,k}^T\right\} \\ & + E\left\{\left(\Delta A_k - L_k\Delta C_k\right)X_kU_k^T\Delta B_{1,k}^T\right\}^T - E\left\{B_{2,k}W_kV_k^TL_k^T\right\} - E\left\{B_{2,k}W_kV_k^TL_k^T\right\}^T \end{aligned} \tag{28}$$

Using Lemma 1 and statistic characteristics of uncertainty, we further have

$$\begin{aligned} E\left\{\left(\Delta A_k - L_k\Delta C_k\right)q_k\left(\Delta A_k - L_k\Delta C_k\right)^T\right\} = {} & \left(\alpha_1 - \alpha_1^2\right)\left(\Psi_1 - L_k\Psi_7\right)q_k\left(\Psi_1 - L_k\Psi_7\right)^T \\ & + \left(\alpha_2 - \alpha_2^2\right)\left(\Psi_2 - L_k\Psi_8\right)q_k\left(\Psi_2 - L_k\Psi_8\right)^T \end{aligned} \tag{29}$$

$$\begin{aligned} E\left\{\Delta B_{1,k}U_kU_k^T\Delta B_{1,k}^T\right\} = {} & \left(\beta_1 - \beta_1^2\right)\Psi_3U_kU_k^T\Psi_3^T + \left(\beta_2 - \beta_2^2\right)\Psi_4U_kU_k^T\Psi_4^T \\ & + \left(\beta_1 - \beta_1^2\right)\Psi_5U_kU_k^T\Psi_5^T + \left(\beta_2 - \beta_2^2\right)\Psi_6U_kU_k^T\Psi_6^T \end{aligned} \tag{30}$$

$$E\left\{B_{2,k}W_kW_k^TB_{2,k}^T\right\} = \bar{B}_{2,k}Q_W\bar{B}_{2,k}^T \tag{31}$$

$$E\left\{\left(\Delta A_k - L_k\Delta C_k\right)X_kU_k^T\Delta B_{1,k}^T\right\} = 0 \tag{32}$$

$$E\left\{B_{2,k}W_kV_k^TL_k^T\right\} = \bar{B}_{2,k}Q_SL_k^T \tag{33}$$

Putting (29)-(33) into (28), we obtain (20). According to (12), the filtering error can be written as

$$\tilde{X}_{k|k} = \tilde{X}_{k|k-1} - K_k\varepsilon_k$$

So the filtering error covariance matrix can be represented as

$$P_{k|k} = P_{k|k-1} + K_k\theta_kK_k^T - K_kE\left\{\varepsilon_k\tilde{X}_{k|k-1}^T\right\} - E\left\{\tilde{X}_{k|k-1}\varepsilon_k^T\right\}K_k^T \tag{34}$$

From the definition of K_k, we have

$$\begin{aligned} E\left\{\tilde{X}_{k|k-1}\varepsilon_k^T\right\} &= E\left\{X_k\varepsilon_k^T\right\} \\ &= K_k\theta_k \end{aligned} \tag{35}$$

Substituting (35) into (34), we have (21). □

The optimal filter for the unified model given in (3) and (4) has been designed. In next section, we would analyze the convergence performance of the filtering error covariance matrix.

4 | CONVERGENCE ANALYSIS

In this section, the convergence performance of the filtering error covariance matrix is analyzed and described in the following theorem.

Theorem 1. With the given filtering matrix K_k, the one-step filtering error covariance matrix satisfies $\lim_{k\to\infty}\left\|P_{k|k}\right\| = 0, \forall k$.

Proof. Substituting (18) into (21), the relation between $P_{k|k}$ and $P_{k|k-1}$ can be represented as

$$\begin{aligned} P_{k|k} &= P_{k|k-1} - K_k\theta_k\left(P_{k|k-1}\overline{C}_k^T\theta_k^{-1}\right)^T \\ &= P_{k|k-1} - K_k\theta_k\theta_k^{-1}\overline{C}_kP_{k|k-1} \\ &= \left(I - K_k\overline{C}_k\right)P_{k|k-1} \end{aligned} \tag{36}$$

Meanwhile, the one-step prediction error $\tilde{X}_{k|k-1}$ can be expressed as

$$\begin{aligned}\tilde{X}_{k|k-1} &= A_k - \hat{X}_{k|k-1} \\ &= A_{k-1}X_{k-1} + B_{1,k-1}U_{k-1} + B_{2,k-1}W_{k-1} \\ &- A_{k-1}\hat{X}_{k-1} - B_{1,k-1}U_{k-1} \\ &= A_{k-1}\tilde{X}_{k-1} + B_{2,k-1}W_{k-1}\end{aligned} \tag{37}$$

According to (37), the relation between $P_{k|k-1}$ and P_{k-1} is obtained as

$$\begin{aligned}P_{k|k-1} &= E\left\{\tilde{X}_{k|k-1}\tilde{X}^T_{k|k-1}\right\} \\ &= \left\{A_{k-1}\tilde{X}_{k-1} + B_{2,k-1}W_{k-1}\right\}\{\cdot\}^T \\ &= A_{k-1}P_{k-1}A^T_{k-1} + B_{2,k-1}Q_W B^T_{2,k-1}\end{aligned} \tag{38}$$

The recursive relation of one-step filtering error covariance matrices in iteration domain is derived. It can be readily found that the convergence of $\left\|P_{k|k}\right\|$ is determined by $I - K_k\overline{C}_k$. Next, we prove the convergence of $\left\|P_{k|k}\right\|$ through analyzing eigenvalues of $I - K_k\overline{C}_k$. Substituting (17) into (18), the filtering gain matrix K_k can be rewritten as

$$K_k = P_{k|k-1}\overline{C}^T_k\left(\overline{C}_k P_{k|k-1}\overline{C}^T_k + J_k\right)^{-1} \tag{39}$$

According to a well-known matrix inversion method $\left(A - UD^{-1}V\right)^{-1} = A^{-1} + A^{-1}U\left(D - VA^{-1}U\right)^{-1}VA^{-1}$ [34] and (39), it can be readily seen that

$$\begin{aligned}& I - K_k\overline{C}_k \\ &= \left(I - P_{k|k-1}\overline{C}^T_k\left(\overline{C}_k P_{k|k-1}\overline{C}^T_k + J_k\right)^{-1}\overline{C}_k\right)^{-1} \\ &= \left(I^{-1} + I^{-1}P_{k|k-1}\overline{C}^T_k\left(\overline{C}_k P_{k|k-1}\overline{C}^T_k + J_k - \overline{C}^T_k I^{-1}P_{k|k-1}\overline{C}^T_k\right)^{-1}\overline{C}^T_k I^{-1}\right)^{-1} \\ &= \left(I + P_{k|k-1}\overline{C}^T_k J^{-1}_k\overline{C}^T_k\right)^{-1}\end{aligned} \tag{40}$$

According to (40), it can be easily derived that $I - K_0\overline{C}_0 = \left(I + P_{0|-1}\overline{C}^T_0 J^{-1}_0\overline{C}_0\right)^{-1}$. Because J_0 is a positive-definite and symmetric matrix, and $\overline{C}_0$ is a full rank matrix, $\overline{C}^T_0 J^{-1}_0\overline{C}_0$ is also a positive-definite and symmetric matrix. If the initial one-step prediction error covariance matrix $P_{0|-1}$ is positive-definite and symmetric, eigenvalues of $P_{0|-1}\overline{C}^T_0 J^{-1}_0\overline{C}_0$ are all greater than zero. As a result, eigenvalues of $I - K_0\overline{C}_0$ are all greater than zero and less than one. Equation (35) and (37) indicate $P_{0|0}$ and $P_{1|0}$ are also positive-definite and symmetric matrices, thus eigenvalues of $I - K_1\overline{C}_1$ are all greater than zero and less than one. Followed by analogy, it can be derived that

$$\left\|I - K_k\overline{C}_k\right\| < 1, \forall k \tag{41}$$

From (36) and (38), it can be easily seen that

$$\lim_{k\to\infty}\left\|P_{k|k}\right\| = 0, \forall k \tag{42}$$

□

So far, we have proved the convergence performance of the norm of filtering error covariance matrix. In other words, when the filtered input $\hat{u}_{k|k}(t)$, which is the first component in the filtered state $\hat{X}_{k|k}$, is used to drive the actuator, the convergence performance of wireless networked ILC systems with additive noises, random delays and data dropouts can be improved significantly.

5 | SIMULATION RESULTS

In this section, the following linear system given in[35] is used, so the desired inputs without the effect of uncertainties can be easily calculated and shown as a reference to illustrate the effectiveness of the proposed filtering method.

$$\begin{cases} x_k(t+1) = \begin{bmatrix} -0.5 & 0 & 0 \\ 1 & 1.24 & -0.87 \\ 0 & 0.87 & 0 \end{bmatrix} x_k(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u_k(t) \\ y_k(t) = \begin{bmatrix} 2 & 2.6 & -2.8 \end{bmatrix} x_k(t) \end{cases}$$

where $x_k(t)$ is the state, $u_k(t)$ represents the received or filtered input, and $y_k(t)$ is the measured output. $t \in [0, 1, \cdots, T-1]$ is the discrete time for periodic trials, and $k = 0, 1, \cdots$ denotes the iteration number of the system.

The desired output is given by

$$y_d(t) = 10 * (1 + \sin(2\pi t/T) - \pi/2)$$

The proportional controller given in (1) is used. We set initial state $x_k(0)$ as $[0,0,0]^T$, initial input $u_0(t)$ as 0, $\Gamma(t) = 0.2$, and $T = 100$. The additive noise $m_k(t)$ and $n_k(t)$ are Gaussian distributed with zero mean and covariance 0.05. The probabilities of data dropouts and one-step delays are given as $\bar{\xi}_i = \bar{\eta}_i = 0.95$, $i = 1, 2$. In the filtering, the initial prediction state $\hat{X}_{0|-1} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T$ and initial prediction error covariance matrix $P_{0|-1} = 1I_4$, where I_4 is the identify matrix.

Figure 2 shows the profiles of inputs received at the actuator side at different iterations as well as the desired inputs. It can be readily seen that the inputs are sensitive to the effect of uncertainties in both channels. As a result, the outputs fail to track the desired trajectory, which are shown in Figure 3. With the proposed method, however, the inputs filtered at the actuator side converge to the desired inputs, so the outputs track the desired trajectory accurately under the effect of uncertainties in both channels, which are shown in Figure 4 and 5 respectively.

The effectiveness of the proposed filtering method is also illustrated from the perspective of averaged error profiles. Figure 6 compares the absolute mean of input errors without or with filtering along iteration axis. As shown in Figure 6, the convergence performance of absolute mean of input errors with filtering is improved significantly. Correspondingly, the convergence performance of absolute mean of output errors with filtering is also improved, which is shown in Figure 7. In the comparison, two absolute means of input errors are defined as $\frac{1}{T}\sum_{t=0}^{T-1}\left|u_d(t) - u_k^r(t)\right|$ and $\frac{1}{T}\sum_{t=0}^{T-1}\left|u_d(t) - \hat{u}_{k|k}(t)\right|$, and two absolute means of output errors are defined as $\frac{1}{T}\sum_{t=0}^{T-1}\left|y_d(t) - y_k(t)\right|$ and $\frac{1}{T}\sum_{t=0}^{T-1}\left|y_d(t) - \hat{y}_{k|k}(t)\right|$ respectively, in which $\hat{y}_{k|k}(t)$ means the output with the filtered input $\hat{u}_{k|k}(t)$.

In addition, in order to demonstrate the effect of random probability of data dropouts and one-step delays on the convergence speed of ILC systems with the proposed method, the absolute mean of both input errors and output errors with filtering are compared for $\bar{\xi}_i = \bar{\eta}_i = 0.95$, 0.8 and 0.7, $i = 1, 2$, which are shown in Figure 8 and 9 respectively. Two facts are observed. First, it can be easily seen that the convergence speed of all absolute means decreases with the increase of random probability. This observation coincides with intuitive judgment about the effect of iteration domain compensation on the convergence speed of inputs, which is pointed out in Remark 1. Second, all the absolute means of input errors and output errors converge to zero, which also show the effectiveness of the proposed input filtering method.

6 | CONCLUSION

In this paper, we addressed the convergence performance of wireless networked ILC systems in presence of additive noises, random delays and data dropouts. In order to deal with the effect of these uncertainties, a proportional-type iterative learning controller was considered, and two data transmission processes were developed to describe the mix of these uncertainties in both SC and CA channels. After that, a filter model was built taking full advantage of the iterative learning process and the two data transmission processes. Based on this model, an optimal input filter was designed based on the projection theory so updated inputs can be filtered at the actuator side with the effect of uncertainties. Moreover, the convergence performance of filtering error covariance was proved. The simulation on a linear system was given to verify the theoretical results.

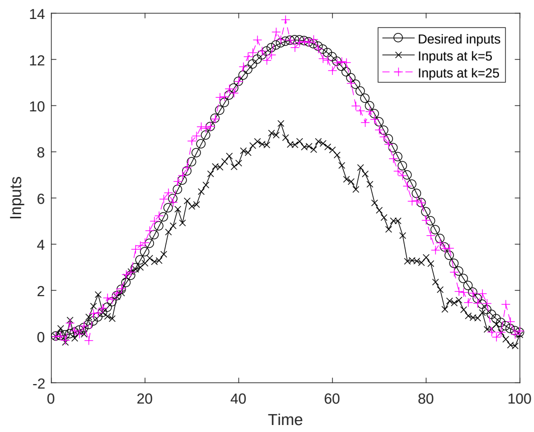

FIGURE 2 Inputs received at the actuator side

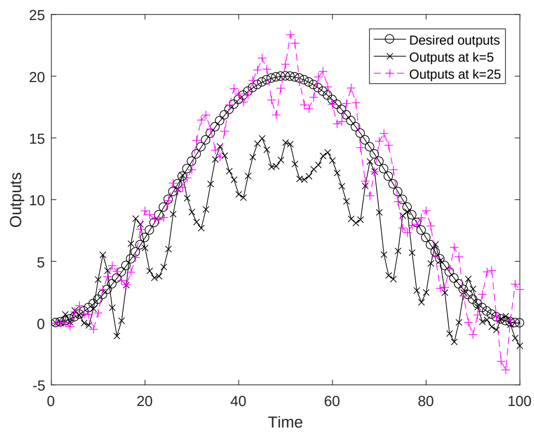

FIGURE 3 Outputs with received inputs

It deserves attention that although the input filtering method was proposed based on a proportional iterative learning controller, similar input filtering methods can be easily derived for other types of controllers. Additionally, since the method design does not use any plant information, the proposed input filtering method can also be applied to other ILC systems.

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Author contributions

None reported.

Financial disclosure

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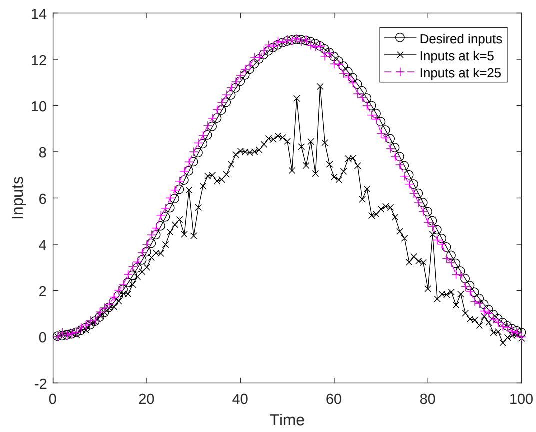

FIGURE 4 Inputs filtered at the actuator side

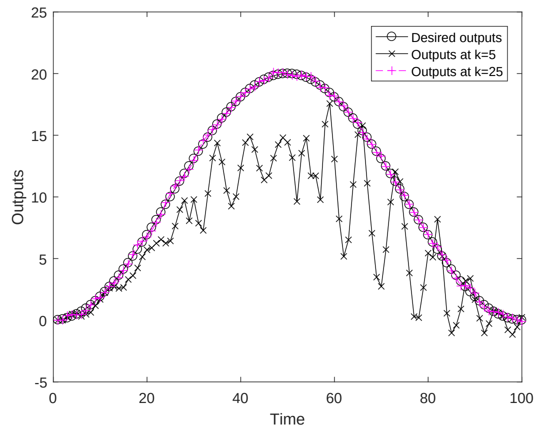

FIGURE 5 Outputs with filtered inputs

Conflict of interest

The authors declare no potential conflict of interests.

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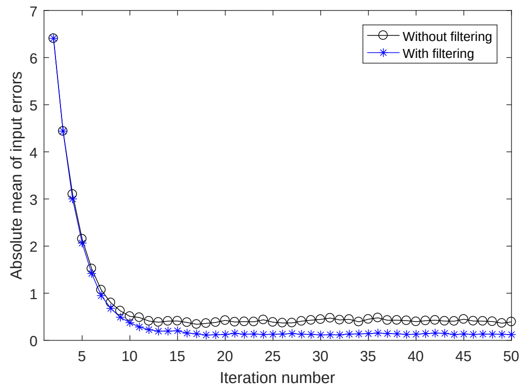

FIGURE 6 Absolute mean of input errors without or with input filtering

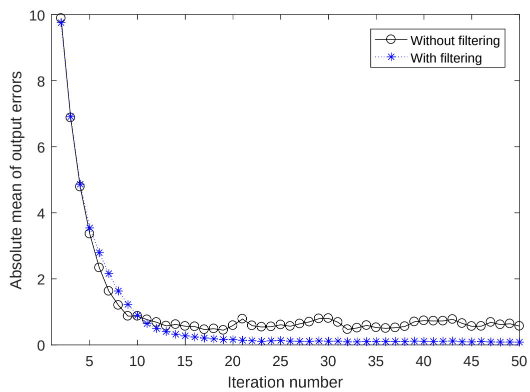

FIGURE 7 Absolute mean of output errors without or with input filtering

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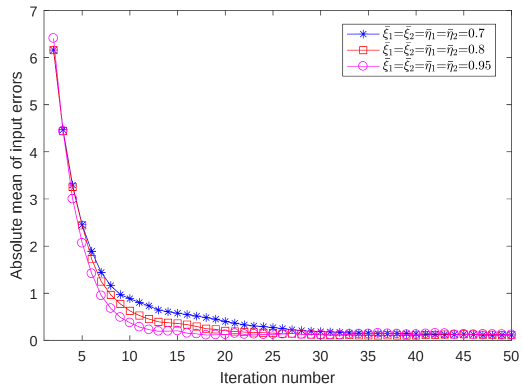

FIGURE 8 Absolute mean of input errors for different probabilities with input filtering

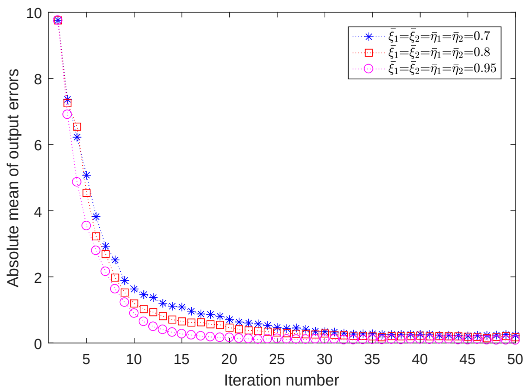

FIGURE 9 Absolute mean of output errors for different probabilities with input filtering

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