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SUPPLEMENTARY INFORMATION

The following is a further continuation of the model description

presented in the article *The Mycorrhizal Tragedy of the Commons*.

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Data, parameter estimation, and model testing

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Competition for N among individual plants

Relative difference in N uptake among individual plants in each pot was assumed to be

proportional to the relative differences in their measured ^{15}N values, i.e.

$$\dot{N}_{pi}/\overline{\dot{N}_p} = {}^{15}N_{pi}/ \text{ mean pot } {}^{15}N_{pi} \ . \tag{5}$$

13

C supply to fungi from an individual plant ($\dot{C}_{si}$) was assumed to be proportional to its root

mass (C_{ri})(Rouhier & Read 1998; Neumann & Matzner 2013) and further reduced by

strangling by a factor e_{st}:

17

$$\dot{C}_{si} \propto C_{ri} \cdot (1 - e_{st})$$

$$\tag{6}$$

The strangling effect e_{st} and the discrimination parameters d and z (eq. 4) were estimated by

inserting eq. 6 in eq. 5 and fitting to measured seedling ^{15}N uptake using the NLS function in

the R software (Table 1). z was estimated to 1.038, and did not significantly differ from 1.

Because including z did not increase the $r^2 = 0.58$ for modelled versus measured N

competition effect (lhs versus rhs of eq. 4; Fig. 1), z was removed from the model. We also

tested a model for $\dot{C}_{si}$ including shading effect but this effect was small and insignificant and

was thus excluded from the further analysis.

27

Table 1

Parameter	Estimated value	Std. error	t value	P value

e_{st}	0.551	0.0483	11.41	<2e-16
d	0.945	0.0653	14.47	<2e-16

29

30 **Stand level C-N exchange**

31 We used measured ^{15}N in plant biomass ($^{15}N_p$) to estimate the "measured" plant N uptake

32 rate in whole pots ($\dot{N}_p$) during the experiment based on the assumption that the differences

33 in $^{15}N_p$ relative to the control plots reflect differences in N uptake rate: $\dot{N}_p = ¿\ ^{15}N_p \cdot (N_{up}/$

34 $^{15}N_p)_{control}$, where N uptake in control plants, $N_{up} = (N_{p\ harvest} - N_{p\ planting})$/experiment duration.

35 We assume that $\dot{N}_p$ depends on pot $\dot{C}_s$ according to eq. 3. The effect of strangling on

36 individual plants (e_{st}) was used to estimate the total strangling effect at the pot level based

37 on the fraction of root biomass (C_r) in the pot belonging to strangled plants (f_{stR}). We

38 assumed that there was an additional shading effect (e_{sh}) on $\dot{C}_s$ in shaded pots and an

39 additional baseline C flux, $\dot{C}_{s0}$, in all pots, representing C gained or lost by the fungi due to

40 factors not included in the model, such as plant C export captured by other microbes.

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42 $$\dot{C}_s = C_r \cdot \left(1 - e_{st} \cdot f_{stR}\right)\left(1 - e_{sh}\right) + \dot{C}_{s0} \qquad (7)$$

43

44 $\dot{C}_s$ (eq. 7) was inserted in the equation for $\dot{N}_p$ (eq. 3) and the unknown parameters, e_{sh}, N_a,

45 $\dot{C}_{s0}$, I_f and C_{fh} were estimated by fitting modeled $\dot{N}_p$ to the estimated $\dot{N}_p$ based on ^{15}N in

46 plant biomass (Table 2). The parameter e_{sh} was subsequently excluded from the model

47 because it was small and not significant, and excluding it did not reduce the $r^2 = 0.25$ of

48 modeled versus measured $\dot{N}_p$.

49

50 **Table 2**

Parameter	Estimated value	Std. error	t value	P value
N_a	6.97e-4	1.84e-4	3.786	5.6e-4
C_{fh}	0.052	0.0583	0.891	0.379
I_f	2.74e-4	1.66e-4	1.645	0.109
$\dot{C}_{s0}$	-0.174	0.0359	-4.849	2.38e-5

51

52 While we did not have direct measurements of fungal growth or C supply to mycorrhizal

53 fungi ($\dot{C}_s$) to test eq. 7, we used pot respiration measurement to derive indirect estimates of

54   $\dot{C}_s$ based on a C budget method. This analysis showed a strong relationship between $\dot{C}_s$ (eq.

55   7) and estimated mycorrhizal respiration, explaining 39% of its variation among pots

56   (Supplementary information).

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58

59   **Testing modeled C export to mycorrhiza against respiration measurements**

60   While we did not have direct measurements of mycorrhizal fungi C to test against our model

61   results, as an indirect test we compared measured respiration with modeled respiration,

62   based on a C budget approach.

63

64   To measure dark respiration, pots were enclosed in a sealed black chamber (chamber

65   volume = 22 liters) containing axial fans and an infrared gas analyzer, IRGA (Vaisala

66   CARBOCAP® , Vaisala Oyj, Helsinki, Finland), for 15 minutes. Throughout incubation, internal

67   chamber CO_2 concentration was logged every minute. The respiration measurements were

68   repeated on three occasions (pre-treatment, 1 week after treatment, and 3 weeks after

69   treatment, fig. S1, S2).

70

71   We assumed that three sources contributed to the total respiration measured in each pot

72   (R_{tot}): autotropic respiration by the plants (R_a), heterotrophic respiration from the added pot

73   soil independent of the plants and mycorrhiza (R_s), and respiration by mycorrhiza fungi

74   fueled by the plants (R_f). $R_{tot} = R_a + R_s + R_f$. R_a was assumed proportional to measured plant

75   growth rate (G) based on plant biomasses at the start and end of the experiment and a

76   factor 0.7 based on a reasonable value of C use efficiency for plant seedlings (Manzoni *et al*.

77   2018), i.e. $R_a = 0.7 * G$. R_s was assumed equal for all pots and to decline exponentially with

78   time (t) as substrate is consumed $R_s = R_{s0} \cdot e^{r_s t}$. Respiration by mycorrhizal fungi was

79   assumed to be proportional to our previous estimates of C export to mycorrhiza for each pot

80   ($\dot{C}_s$, eq. s8), i.e. $R_f = r_f \dot{C}_s$. The full model is:

81

82   $$R_{tot} = R_{s0} \cdot e^{r_s t} + 0.7 \cdot G + r_f \dot{C}_s \cdot \qquad\qquad\qquad (s1)$$

83

84   The parameters of the model (eq. s1) were determined by fitting the model to measured

85   values of pot respiration (R_{tot}) measured twice, once before and once during the treatments,

86   using the NLS function in the R Software (Table S1, Fig. S4). Based on the results we

87   calculated the contributions of each source to total respiration, which were R_f = 31%, R_a=

88   36%, R_s = 33% during the treatment period. Compared to a null-model with a common mean

89   R_f for all pots, our model improved r^2 for modeled versus measured R_{tot} from 0.48 to 0.60, i.e.

90   by 12%. Given that R_f contributed 31% of R_{tot} this suggest that our model of C export to

91   mycorrhizal fungi ($\dot{C}_s$, eq. 7) explains 12/31 = 39% of the variation in R_f among pots.

92

93   **Table S1**

Parameter	Estimated value	Std. error	t value	P value
R_{s0}	0.32	0.17	1.797	0.076
r_s	0.029	0.076	3.84	2.5e-4
r_f	0.0074	0.0021	3.53	7.1e-4

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98   Manzoni, S., Čapek, P., Porada, P., Thurner, M., Winterdahl, M., Beer, C., *et al.* (2018).
99   Reviews and syntheses: Carbon use efficiency from organisms to ecosystems –
100   definitions, theories, and empirical evidence. *Biogeosciences*, 15, 5929–5949.
101 Neumann, J. & Matzner, E. (2013). Biomass of extramatrical ectomycorrhizal mycelium and
102   fine roots in a young Norway spruce stand — a study using ingrowth bags with
103   different substrates. *Plant and Soil*, 371, 435–446.
104 Rouhier, H. & Read, D.J. (1998). Plant and fungal responses to elevated atmospheric carbon
105   dioxide in mycorrhizal seedlings of Pinus syl6estris, 10.
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