Supporting Information
High-precision calculation of relativistic corrections for hydrogen-like atoms in screened Coulomb potentials

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[1] A. Poszwa. *J. Phys. A: Math. Theor.*, 45:185302, 2012.

[2] A. Poszwa and M. K. Bahar. *Phys. Plasmas*, 22:012104, 2015.

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TABLE I. Non-relativistic energy $E^{(0)}$ and scaled relativistic corrections $E^{(1)}c^2$ for some s-wave states of H atom with SCP as the functions of screening parameter λ. Predictions of the scaled relativistic corrections $E^{(1)}c^2$ by Poszwa Ref. [1] are included for comparison. Numbers in parentheses represent the power of ten.

nl	λ	$E^{(0)}$	$E^{(1)}c^2,\ j=1/2$	Ref. [1]
$1s$	0	-0.50000000000000000000000000	-0.12500000000000000000000000	
	0.1000	-0.40705803061340315675450 70	-0.12275428491853096631564 95	-0.1227542849
	0.2000	-0.32680851136919338488249 54	-0.11682864851391074038792 68	-0.1168286485
	0.3000	-0.25763858630305414887896 40	-0.10815629412864802897056 62	
	0.4000	-0.19837608336185021660841 38	-0.97454087163780040505867 40(-1)	-0.9745408717(-1)
	0.5000	-0.14811702188993261671175 82	-0.85319404617643452531115 39(-1)	
	0.6000	-0.10613590750581419300073 86	-0.72274641041163870844301 18(-1)	-0.7227464104(-1)
	0.7000	-0.71833555904512213040262 12	-0.58791082981019212736712 62(-1)	
	0.8000	-0.44704304497359663200348 76	-0.45303243700354288296577 45(-1)	-0.4530324370(-1)
	0.9000	-0.24314193827502054887384 15	-0.32218283171923406533999 07(-1)	-0.3221828317(-1)
	1.0000	-0.10285789990017696804774 21	-0.19922687487090128548723 00(-1)	-0.1992268748(-1)
	1.1000	-0.22872442340534854634760 81(-2)	-0.87873218440674181649583 66(-2)	
	1.1500	-0.45588902135595778568292 74(-3)	-0.37673570882679565438619 90(-2)	
	1.1900	-0.10303196149897202023678 15(-6)	-0.54617454586004287587606 51(-4)	
	1.1906	-0.42378847720426058453958 65(-10)	-0.11070630795996180881913 79(-5)	
$2s$	0	-0.12500000000000000000000000	-0.39062500000000000000000000(-1)	
	0.0100	-0.11529328516799425622204 55	-0.38957600058254879294278 74(-1)	-0.3895760006(-1)
	0.0200	-0.10614832024469550325070 83	-0.38659184851712463983968 32(-1)	-0.3865818485(-1)
	0.0300	-0.97531786134660862770039 06(-1)	-0.38188714173141909678422 01(-1)	
	0.0400	-0.89414634185159188415714 62(-1)	-0.37564574189539245740717 93(-1)	-0.3756457419(-1)
	0.0500	-0.81771195795253124173489 41(-1)	-0.36802855395830417606855 39(-1)	
	0.0600	-0.74578534412709709694885 17(-1)	-0.35917877514510633828483 61(-1)	-0.3591787752(-1)
	0.0800	-0.61464656212300385911466 25(-1)	-0.33828674762592824862263 49(-1)	-0.3382867476(-1)
	0.1000	-0.49928271331918889234996 68(-1)	-0.31387794417123892209071 92(-1)	-0.3138779442(-1)
	0.1500	-0.27222190725688518250187 26(-1)	-0.24235630384345076664647 27(-1)	
	0.2000	-0.12107865195440464385855 37(-1)	-0.16345360972066594920298 46(-1)	-0.1634536097(-1)
	0.2500	-0.33959062832393077964422 64(-2)	-0.84972831803553688259041 74(-2)	-0.8497283180(-2)
	0.3000	-0.91602443891898903833923 47(-4)	-0.13299294377902153171031 29(-2)	
	0.3100	-0.37992565723646161449664 43(-7)	-0.26723967776381134834231 22(-4)	
	0.3102	-0.74725289159365127376618 84(-10)	-0.11848487473989388602554 39(-5)	
$3s$	0	-0.55555555555555555555555556(-1)	-0.13888888888888888888888889(-1)	
	0.0100	-0.46198857799033191519298 25(-1)	-0.13719014846071542305919 17(-1)	
	0.0250	-0.34329509911543775438062 33(-1)	-0.12947721231637740993145 03(-1)	-0.1294772123(-1)
	0.0500	-0.19352554814752342295397 99(-1)	-0.10733068085219594128768 77(-1)	-0.1073306809(-1)
	0.0800	-0.77758770389577100926393 79(-2)	-0.72255242581123683200173 60(-2)	-0.7225524258(-2)
	0.1000	-0.32080467446902587182135 16(-2)	-0.46958491353271987582318 21(-2)	-0.4695849135(-2)
	0.1200	-0.72747319105291140477234 77(-3)	-0.22121857566040363696126 23(-2)	-0.2212185756(-2)
	0.1300	-0.16543177793915109288361 73(-3)	-0.10391004739795519279360 43(-2)	
	0.1350	-0.35974301727093089280416 59(-4)	-0.47950450178983966202597 50(-3)	
	0.1394	-0.45133134010808170246720 24(-8)	-0.53116833665639313071930 51(-5)	
$4s$	0	-0.31250000000000000000000000(-1)	-0.63476562500000000000000000(-2)	
	0.0100	-0.22356120521237134526537 48(-1)	-0.61237392422249133542245 61(-2)	
	0.0250	-0.12503238318007071505739 69(-1)	-0.51940669415344966764721 94(-2)	-0.5194066942(-2)
	0.0500	-0.30916599001613214846589 50(-2)	-0.28513619202516432671541 42(-2)	-0.2851361920(-2)
	0.0600	-0.12367190970350327739174 45(-2)	-0.18202209338003159546307 39(-2)	-0.1820220934(-2)
	0.0700	-0.25320650045595307911726 43(-3)	-0.81527008967708576952098 36(-3)	
	0.0750	-0.45751958982534574482246 71(-4)	-0.34143955065280933216561 32(-3)	
	0.0788	-0.23867738923201382414714 03(-8)	-0.24233579695580554582379 82(-5)	

TABLE II. Same as Table I but for non-s-wave states of H atom with SCP.

nl	λ	$E^{(0)}$	$E^{(1)}c^2,\ j=l+1/2$	$E^{(1)}c^2,\ j=l-1/2$	Ref. [1]
$2p$	0.0000	-0.12500000000000000000000000	-0.78125000000000000000000000(-2)	-0.39062500000000000000000000(-1)	
	0.0100	-0.11524522409056418589478 32	-0.77883762901156372172010 80(-2)	-0.38956888227034322101290 84(-1)	-0.3895688823(-1)
	0.0200	-0.10596339817993990475573 10	-0.77192302497573892578648 82(-2)	-0.38653753999955100831405 25(-1)	-0.3865375400(-1)
	0.0300	-0.97131366795691310671783 88(-1)	-0.76093839878772560138136 55(-2)	-0.38171164920130544695148 87(-1)	
	0.0400	-0.88729373582879526287939 14(-1)	-0.74626238233703446685848 42(-2)	-0.37524612576556488844745 20(-1)	-0.3752461258(-1)
	0.0500	-0.80740387037784609712102 74(-1)	-0.72823337446711582656586 49(-2)	-0.36727658607210950231258 62(-1)	
	0.0600	-0.73149619385860625023805 41(-1)	-0.70715849236953503228661 92(-2)	-0.35792365301126266354747 32(-1)	-0.3579236530(-1)
	0.0800	-0.59112804787031234635692 33(-1)	-0.65697842542298646538423 81(-2)	-0.33549186380906083776141 05(-1)	-0.3354918638(-1)
	0.1000	-0.46534390486724608386600 84(-1)	-0.59774647853703338842212 10(-2)	-0.30870664051379791069347 80(-1)	-0.3087066405(-1)
	0.1500	-0.21104888927736242916943 38(-1)	-0.42107055061198396203922 31(-2)	-0.22649082355760053083501 30(-1)	
	0.2000	-0.41016465307840903884466 10(-2)	-0.21631349515400166686899 17(-2)	-0.12477539147106200472626 74(-1)	-0.1247753914(-1)
	0.2100	-0.18087600662817953669872 53(-2)	-0.16948757773010784830831 97(-2)	-0.99860650875027721381109 17(-2)	-0.9986065087(-2)
	0.2200	-0.28697244985229882829234 84(-4)	-0.10295388889344154818867 22(-2)	-0.62345793604042436048655 01(-2)	
	0.2202	-0.21469619036334373695172 36(-5)	-0.97425815672625692536303 75(-3)	-0.59039693023830700627956 15(-2)	
$3p$	0.0000	-0.55555555555555555555555 56(-1)	-0.46296296296296296296296 30(-2)	-0.13888888888888888888888 89(-1)	
	0.0100	-0.46153104829162287315273 87(-1)	-0.45657493378478399470790 96(-2)	-0.13717888126316443767879 77(-1)	
	0.0250	-0.34078910428938130548463 16(-1)	-0.42730635768393370155331 19(-2)	-0.12932147361120508493382 17(-1)	-0.1293214736(-1)
	0.0500	-0.18557751883405996604893 99(-1)	-0.34284858351427352289040 48(-2)	-0.10625597741553993375715 31(-1)	-0.1062559774(-1)
	0.0800	-0.63299954392681132693381 78(-2)	-0.20996926939645336052909 83(-2)	-0.68255885987947131110158 24(-2)	-0.6825588599(-2)
	0.1000	-0.15890015258675602675586 34(-2)	-0.11410804440498917927475 79(-2)	-0.38844136252661234725876 73(-2)	-0.3884413625(-2)
	0.1100	-0.22634084060161183368084 63(-3)	-0.61566123768568226101675 92(-3)	-0.21579800002840990602325 02(-2)	
	0.1127	-0.63489628257450986556715 55(-6)	-0.37296674062671114511741 42(-3)	-0.13192288042205898589657 55(-3)	
$4p$	0.0000	-0.31250000000000000000000 00(-1)	-0.24414062500000000000000 00(-2)	-0.63476562500000000000000 00(-2)	
	0.0100	-0.22313420741967189431061 75(-1)	-0.23471020447384597713757 93(-2)	-0.61222591118362768067153 34(-2)	
	0.0250	-0.12294320436313933670168 89(-1)	-0.19541344366797683499378 33(-2)	-0.51745436025484475881651 11(-2)	-0.5174543603(-2)
	0.0500	-0.25980588525718539652611 91(-2)	-0.97652755341436690234392 51(-3)	-0.27143708898585217219647 74(-2)	-0.2714370890(-2)
	0.0600	-0.72917276663620057296857 43(-3)	-0.54969340167972104261841 52(-3)	-0.15713222162200872294225 84(-2)	-0.1571322216(-2)
	0.0678	-0.34041515122519116188232 16(-5)	-0.16182415701772775377361 83(-3)	-0.47486058745865934829799 16(-3)	
$3d$	0.0000	-0.55555555555555555555555 56(-1)	-0.15432098765432098765432 10(-2)	-0.46296296296296296296296 30(-2)	
	0.0100	-0.46061454160659627138130 27(-1)	-0.15199324834545820427396 39(-2)	-0.45648640036638927946244 10(-2)	
	0.0250	-0.33573122086666003102965 09(-1)	-0.14112839987967492973867 36(-2)	-0.42611307181164430558356 29(-2)	-0.4261130718(-2)
	0.0500	-0.16915570569815842886114 75(-1)	-0.10893165295247654064258 61(-2)	-0.33490518704843552941648 04(-2)	-0.3349051870(-2)
	0.0800	-0.32483604287519935724675 25(-2)	-0.56055845666155271040187 94(-3)	-0.18022209854753649578945 58(-2)	-0.1802220986(-2)
	0.0900	-0.31291350263492163421638 55(-3)	-0.34723097402741306347049 49(-3)	-0.11489162721460847536209 11(-2)	
	0.0913	-0.10033762275160703324711 22(-4)	-0.31080923478303822151622 60(-3)	-0.10338388706453805985760 98(-2)	
$4d$	0.0000	-0.31250000000000000000000 00(-1)	-0.11393229166666666666666 67(-2)	-0.24414062500000000000000 00(-2)	
	0.0100	-0.22227792489804398453786 85(-1)	-0.10908976831914854256000 09(-2)	-0.23458533535402291262070 33(-2)	-0.2345853354(-2)
	0.0250	-0.11870448936190599940851 14(-1)	-0.88638322461233643984048 60(-3)	-0.19380427776748494179667 98(-2)	-0.1938042778(-2)
	0.0500	-0.15808716268710049528839 48(-2)	-0.37133712959293370539984 04(-3)	-0.86291463342401806263123 30(-3)	-0.8629146334(-3)
	0.0550	-0.49188376726527085040836 61(-3)	-0.25393787217525365680427 88(-3)	-0.60197537293924056759309 37(-3)	
	0.0581	-0.65226702119533922336452 84(-6)	-0.16712121218584727219139 34(-3)	-0.40212908402146088824142 42(-3)	
$4f$	0.0000	-0.31250000000000000000000 00(-1)	-0.48828125000000000000000 00(-3)	-0.11393229166666666666666 67(-2)	
	0.0100	-0.22098770463802567552796 83(-1)	-0.46605782730121838552978 15(-3)	-0.10898907049991486066902 82(-2)	-0.1089890705(-2)
	0.0200	-0.14491978017490743686655 39(-1)	-0.40840594663388739816340 37(-3)	-0.96095838598990551842408 36(-3)	-0.9609583860(-3)
	0.0300	-0.82821524667796329592811 24(-1)	-0.32596407134260060894962 90(-3)	-0.77493343525005012164273 02(-3)	-0.7749334352(-3)
	0.0400	-0.34095449122776511766303 32(-2)	-0.22661243617522600430084 22(-3)	-0.54768796261617731532085 08(-3)	
	0.0450	-0.14873597433347200485517 30(-2)	-0.17173797549275196494270 39(-3)	-0.42024272107631336769207 75(-3)	
	0.0498	-0.81823981405461653988930 61(-5)	-0.11241945358959627824379 38(-3)	-0.28005255781836149189261 90(-3)	

TABLE III. Non-relativistic energy $E^{(0)}$ and scaled relativistic corrections $E^{(1)}c^2$ for some s-wave states of H atom with ECSCP as the functions of screening parameter λ. Predictions of Poszwa and Bahar Ref. [2] included for comparison is obtained by subtracting the non-relativisitc energy from the fully relativistic calculations of the Dirac equation and scaled by c^2, where only four digits are significant. Numbers in parentheses represent the power of ten.

nl	λ	$E^{(0)}$	$E^{(1)}c^2,\ j=1/2$	Ref. [2]
$1s$	0.0100	-0.490000987578423397430519 4	-0.124999426383286822907135 6	-0.12495
	0.0200	-0.480007802609296926896814 3	-0.124995487489837372917422 5	-0.12494
	0.0250	-0.475015144823300165896914 0	-0.124991260091148199988569 8	-0.12493
	0.0300	-0.470026008002314877491553 6	-0.124985023328128923630421 9	
	0.0400	-0.460060889023123495760437 1	-0.124965087159462120186770 5	
	0.0500	-0.450117466420687668045276 3	-0.124932932726102734454196 5	-0.12486
	0.0600	-0.440200510290238543398389 0	-0.124886000922749194810454 7	
	0.0700	-0.430314554280137516733675 3	-0.124821906363611305055959 0	
	0.0800	-0.420463909843642977551398 8	-0.124738424169269093148490 8	
	0.1000	-0.400884774639478191787512 3	-0.124505123844778259011743 1	-0.12445
	0.2000	-0.306334488457804462064564 6	-0.121583022589373848325580 5	
	0.2500	-0.261768085698874238242720 8	-0.118749823228369307005115 1	
	0.3000	-0.219415692006939877341358 4	-0.114830185345752355151145 5	
	0.4000	-0.142439176030952888526827 6	-0.103268570945519901511343 2	
	0.5000	-0.776836846501795522165209 9(-1)	-0.857647702337793156421633 8(-1)	
	0.6000	-0.283049644279871858774740 2(-1)	-0.599032754864690712833222 8(-1)	
	0.7000	-0.115044272276665461899497 0(-2)	-0.156386411159662757566266 3(-1)	
	0.7200	-0.835882746246450105476901 8(-6)	-0.463828140472546821910564 3(-3)	
	0.7205	-0.176956375112083893377332 2(-8)	-0.214405198358803496087918 8(-4)	
$2s$	0.0100	-0.115013461037210709331307 5	-0.390535943025970503050028 5(-1)	
	0.0200	-0.105103587692221196555205 8	-0.389950807763156535334053 7(-1)	
	0.0250	-0.100198490523700251361485 9	-0.389343441777073901149675 8(-1)	
	0.0300	-0.953365864822536798816199 7(-1)	-0.388468791838297258884403 6(-1)	
	0.0400	-0.857689988968008900332571 0(-1)	-0.385772504223751380280464 4(-1)	
	0.0500	-0.764495962502477443840668 8(-1)	-0.381606965751154836249737 2(-1)	
	0.0600	-0.674211051489114228793455 6(-1)	-0.375761659117705937340082 2(-1)	
	0.0700	-0.587217363987317201971002 3(-1)	-0.368055234037483623880724 5(-1)	
	0.0800	-0.503865619256789400397938 5(-1)	-0.358321543642888375961390 4(-1)	
	0.1000	-0.349413119472280783940172 7(-1)	-0.332092297214516475684325 1(-1)	
	0.1600	-0.149810877666326875956290 8(-2)	-0.156334059754591738634043 5(-1)	
	0.1666	-0.405227103515924045720134 1(-7)	-0.230592185252628306075631 5(-3)	
$3s$	0.0100	-0.456190795446594271087018 9(-1)	-0.138567554131381269742621 7(-1)	
	0.0200	-0.360251051130600359410645 5(-1)	-0.136595988979863278177178 1(-1)	
	0.0250	-0.314389612564796070080308 8(-1)	-0.134643197356596452094493 5(-1)	
	0.0300	-0.270284214121325145265754 7(-1)	-0.131912656713921725025304 7(-1)	
	0.0400	-0.188230633341783738052412 7(-1)	-0.123787063025851636239212 2(-1)	
	0.0500	-0.115755642070607958475556 6(-1)	-0.111519618141529860277995 1(-1)	
	0.0600	-0.546208748981547244802997 5(-2)	-0.939351553722017420809858 5(-2)	
	0.0700	-0.774004464291481829604496 4(-3)	-0.655981385596111821559008 3(-2)	
	0.0724	-0.284557795298944738579482 3(-5)	-0.224957452184563159438740 0(-2)	
$4s$	0.0100	-0.214374651555558898664066 33(-1)	-0.627369035795843241101436 0(-2)	
	0.0200	-0.125717773152090172594412 8(-1)	-0.585301814109949638308957 6(-2)	
	0.0250	-0.869315296529837407794181 1(-2)	-0.545013816524002487628070 0(-2)	
	0.0300	-0.527040168968764625673228 8(-2)	-0.488645924020579785005032 3(-2)	
	0.0400	-0.140269548278051835024715 6(-3)	-0.286650909920366951145533 5(-2)	
	0.0404	-0.621015053476511743706647 2(-5)	-0.225611490522877137119106 3(-2)	

TABLE IV. Same as Table III but for non-s-wave states of H atom with ECSCP.

nl	λ	$E^{(0)}$	$E^{(1)}c^2,\ j=l+1/2$	$E^{(1)}c^2,\ j=l-1/2$
$2p$	0.0100	-0.11500965664057866258800 63	-0.781075373621313593208688 8(-2)	-0.390549586798182924585353 6(-1)
			-0.7812(-2)	-0.3906(-1)
	0.0200	-0.10507463830618907109938 98	-0.779918546812412442483177 3(-2)	-0.390049885967138036692465 7(-1)
			-0.7793(-2)	-0.3899(-1)
	0.0250	-0.10014333757742097411049 71	-0.778710107955340951584879 0(-2)	-0.389527711220590387403031 8(-1)
			-0.7784(-2)	-0.3895(-1)
	0.0300	-0.95243609673210075420379 42(-1)	-0.776961680847342248583633 9(-2)	-0.388771906590060408668933 2(-1)
	0.0400	-0.85559137218710782356015 32(-1)	-0.771532658849118162099377 3(-2)	-0.386422950019481059745508 6(-1)
	0.0500	-0.76059012441731047354936 57(-1)	-0.763069487220927653308827 6(-2)	-0.382755083553551432370528 9(-1)
			-0.7643(-2)	-0.3821(-1)
	0.0600	-0.66777520524790545498837 81(-1)	-0.751088373025643262339060 5(-2)	-0.377550069786035641273162 2(-1)
	0.0700	-0.57746643072308465526923 94(-1)	-0.735154658631919406602007 5(-2)	-0.370605300231350341931282 1(-1)
	0.0800	-0.48997247808301078012886 67(-1)	-0.714853670075577654820360 1(-2)	-0.361719209029594020116045 1(-1)
	0.0900	-0.40560373723686039158750 17(-1)	-0.689756213355369125515096 7(-2)	-0.350673398848030923274798 3(-1)
	0.1000	-0.32468805180794407604985 77(-1)	-0.659368622315849260661712 0(-2)	-0.337206094149382369633841 0(-1)
			-0.6582(-2)	-0.3332(-1)
	0.1200	-0.17476455652277472821484 96(-1)	-0.579791457663668536894120 7(-2)	-0.301401272074282190662919 8(-1)
	0.1400	-0.44698757311627568435945 41(-2)	-0.461814634138128888256975 2(-2)	-0.246458286657405293147927 8(-1)
	0.1482	-0.23429758086515061009565 03(-5)	-0.355325942923030388397364 6(-2)	-0.192944154423434468683970 4(-1)
$3p$	0.0100	-0.45611041324099095230270 22(-1)	-0.461866988555972459910601 7(-2)	-0.138587550177205632373270 8(-1)
	0.0200	-0.35967603433584811049721 17(-1)	-0.455075942690746855970764 6(-2)	-0.136722561861836043600703 1(-1)
	0.0250	-0.31332673584770417536043 36(-1)	-0.448304097177961921897461 5(-2)	-0.134862017854114311734837 8(-1)
	0.0300	-0.26854522009110744817267 66(-1)	-0.438795666075569965802791 8(-2)	-0.132245577333903542087900 4(-1)
	0.0400	-0.18453352989114131843507 37(-1)	-0.410358895036233078493142 9(-2)	-0.124381391426834288279551 7(-1)
	0.0500	-0.10929329822556161248564 73(-1)	-0.367244725350093659545364 1(-2)	-0.112318432414472915632029 2(-1)
	0.0600	-0.44725751127770886052097 72(-2)	-0.305119174198044599503801 3(-2)	-0.945728095680970334324852 0(-2)
	0.0680	-0.30452300679172342517232 93(-3)	-0.226293878375566712337614 5(-2)	-0.712298641183159007212604 8(-2)
	0.0687	-0.48755998691544197804679 91(-5)	-0.205895974374110358508836 3(-2)	-0.649208958108509142843807 9(-2)
$4p$	0.0100	-0.21424374599520029460896 52(-1)	-0.241216122122743120478004 4(-2)	-0.627601129716022764556205 3(-2)
	0.0200	-0.12485752309217609658915 90(-1)	-0.224452598716184542019897 5(-2)	-0.586469606915498192373221 3(-2)
	0.0250	-0.85409020733089673315348 97(-2)	-0.208354321666652618665931 8(-2)	-0.546723773686049689394171 2(-2)
	0.0300	-0.50328473296714377602726 85(-2)	-0.185837353640263737328564 7(-2)	-0.490616555870345712416067 0(-2)
	0.0390	-0.10681464743429254179200 03(-3)	-0.117825418497910973726742 9(-2)	-0.316504776280529927651034 3(-2)
	0.0392	-0.25158469869864392962674 38(-4)	-0.113955985496554175138130 0(-2)	-0.306272625608391438296809 6(-2)
$3d$	0.0100	-0.45594839384336572165137 36(-1)	-0.153979970720126944249282 2(-2)	-0.462029955061649561718275 8(-2)
	0.0200	-0.35850662310886846217811 63(-1)	-0.151827587762249020682951 9(-2)	-0.456130871426632553792191 2(-2)
	0.0250	-0.31115387910124599330834 68(-1)	-0.149651029903580785457497 9(-2)	-0.450153298914720027519918 6(-2)
	0.0300	-0.26496920285694406596522 02(-1)	-0.146563799011771206951563 1(-2)	-0.441656713585314682441134 7(-2)
	0.0400	-0.17682064255727231796173 69(-1)	-0.137184587882238714351659 6(-2)	-0.415723243418933430314837 5(-2)
	0.0500	-0.95548793227378694409214 07(-2)	-0.122654832472796486288528 3(-2)	-0.375189039234082888694582 0(-2)
	0.0600	-0.23109477331785567264694 38(-2)	-0.100993854078484743275845 0(-2)	-0.313821156190217501945635 4(-2)
	0.0630	-0.36191397530356197311137 26(-3)	-0.920059513716656108486814 7(-3)	-0.287905874373805681216298 4(-2)
	0.0635	-0.50368277873339190629023 50(-4)	-0.901915215482741997523213 4(-3)	-0.282612923362397598484824 1(-2)
$4d$	0.0100	-0.21397977632437718000376 11(-1)	-0.112567357427788222932473 5(-2)	-0.241419755886604058824795 2(-2)
	0.0200	-0.12310266476555062752400 12(-1)	-0.104608494229018187811766 5(-2)	-0.225515356581754036974182 2(-2)
	0.0250	-0.82278086243073149793348 91(-2)	-0.968810558861449012453690 3(-3)	-0.209964767417426241408869 4(-2)
	0.0300	-0.45392732329984048286391 51(-2)	-0.859931231775989159292111 0(-3)	-0.187829807708510774045398 8(-2)
	0.0370	-0.21178795771350467492838 15(-3)	-0.628051341788700240676909 8(-3)	-0.139571607363702568449185 2(-2)
	0.0374	-0.26025931988507090358872 18(-5)	-0.608033243982455615143890 1(-3)	-0.135303913805316891296626 5(-2)
$4f$	0.0100	-0.21357840268783741439564 53(-1)	-0.482852057484411446906151 9(-3)	-0.112741943696664728194676 9(-2)
	0.0200	-0.12038187814414136458752 53(-1)	-0.450259778097103828399647 6(-3)	-0.105568546090352544140632 6(-2)
	0.0300	-0.37481460109203518793775 06(-2)	-0.371593907452791549067744 6(-3)	-0.881021689460671731055426 9(-3)
	0.0350	-0.16053787182024716837980 39(-3)	-0.305497705892777008291720 1(-3)	-0.732405971772191375970839 2(-3)
	0.0352	-0.27348140675739395966093 90(-4)	-0.302188266939766478744386 4(-3)	-0.724906036392490087246624 0(-3)

TABLE V. Non-relativistic energy $E^{(0)}$ and scaled relativistic corrections $E^{(1)}c^2$ for some s-wave states of H atom with HP as the functions of screening parameter λ. Predictions of the scaled relativistic corrections $E^{(1)}c^2$ by Poszwa Ref. [1] are included for comparison. The notation $0.\dot{5}$ refers to $0.5555\cdots$. Numbers in parentheses represent the power of ten.

nl	λ	$E^{(0)}$	$E^{(0)}_{exact}$	$E^{(1)}c^2,\ j=1/2$	Ref. [1]
$1s$	0.0100	-0.4950125000000000000000000	-0.4950125	-0.12499583335416685887579 06	-0.1249958334
	0.1000	-0.4512500000000000000000000	-0.45125	-0.12458354185856233770157 16	-0.1235835419
	0.2000	-0.4050000000000000000000000	-0.405	-0.12333667888805133060682 99	-0.1233366789
	0.3000	-0.3612500000000000000000000	-0.3612	-0.12126701310214439932256 17	-0.1212670131
	0.4000	-0.3200000000000000000000000	-0.32	-0.11838743424220790579898 05	-0.1183874342
	0.5000	-0.2812500000000000000000000	-0.28125	-0.11471643006450845119759 153	-0.1147164306
	1.0000	-0.1250000000000000000000000	-0.125	-0.85584634096108627517808 15(-1)	-0.855846341(-1)
	1.5000	-0.3125000000000000000000000(-1)	-0.3125(-1)	-0.43488966706756922065591 66(-1)	-0.4348896671(-1)
	1.9000	-0.1250000000000000000000000(-2)	-0.125(-2)	-0.80243036011054408488287 56(-2)	-0.8024303601(-2)
	1.9900	-0.1250000000000000000000000(-4)	-0.125(-3)	-0.77544950665393523448156 98(-3)	-0.7754495067(-3)
	1.9999	-0.1250000000000000000000000(-8)	-0.125(-8)	-0.77218987505548119266421 40(-5)	
$2s$	0.0100	-0.1200500000000000000000000	-0.12005	-0.39044271854175123888925 18(-1)	-0.3904427185(-1)
	0.0500	-0.1012500000000000000000000	-0.10125	-0.38607408986266474051659 96(-1)	
	0.1000	-0.8000000000000000000000000(-1)	-0.8	-0.37249800112460279141183 05(-1)	-0.3724980011(-1)
	0.2000	-0.4500000000000000000000000(-1)	-0.45(-1)	-0.31934705238560514044993 09(-1)	-0.3193470524(-1)
	0.3000	-0.2000000000000000000000000(-1)	-0.2(-1)	-0.23489231222043102262535 34(-1)	-0.2348923122(-1)
	0.4000	-0.5000000000000000000000000(-2)	-0.5(-2)	-0.12543340613085302324627 36(-1)	-0.1254334061(-1)
	0.4500	-0.1250000000000000000000000(-2)	-0.125(-2)	-0.64036979805864387202923 69(-2)	-0.6403697981(-2)
	0.4900	-0.5000000000000000000000000(-4)	-0.5(-4)	-0.12945079217685105404925 89(-2)	-0.1294507922(-2)
	0.4999	-0.5000000000000000000000000(-8)	-0.5(-8)	-0.12968413890110656376542 93(-4)	
$3s$	0.0100	-0.5066805555555555555555556(-1)	-0.5066805̇(-1)	-0.13857876182958604769793 67(-1)	-0.1385787618(-1)
	0.0250	-0.4375868055555555555555556(-1)	-0.43758680̇5(-1)	-0.13695250220482531074371 45(-1)	
	0.0500	-0.3336805555555555555555556(-1)	-0.333680̇5(-1)	-0.13117060120512008150799 85(-1)	-0.1311706012(-1)
	0.1000	-0.1680555555555555555555556(-1)	-0.1680̇5(-1)	-0.10845251230239902408086 61(-1)	-0.1084525123
	0.1500	-0.5868055555555555555555556(-2)	-0.586805̇(-2)	-0.72050124127794375362196 62(-2)	-0.7205012413(-2)
	0.2000	-0.5555555555555555555555556(-3)	-0.5̇(-3)	-0.24174462524256090676277 98(-2)	-0.2417446252(-2)
	0.2200	-0.5555555555555555555555556(-5)	-0.5̇(-5)	-0.24827913784875569870547 39(-3)	-0.2482791378(-3)
	0.2222	-0.5555555555555555555555556(-9)	-0.5̇(-9)	-0.24894084932262204307554 47(-5)	
$4s$	0.0100	-0.2645000000000000000000000(-1)	-0.2645(-1)	-0.63040555421466629840351 26(-2)	-0.6304055542(-2)
	0.0250	-0.2000000000000000000000000(-1)	-0.2(-1)	-0.60757781122823311178958 15(-2)	-0.6075778112(-2)
	0.0500	-0.1125000000000000000000000(-1)	-0.1125(-1)	-0.52690961339173753070664 10(-2)	-0.5269096134(-2)
	0.0750	-0.5000000000000000000000000(-2)	-0.5(-2)	-0.39545118425703907360110 23(-2)	-0.3954511843(-2)
	0.1000	-0.1250000000000000000000000(-2)	-0.125(-2)	-0.21770192927045541679051 47(-2)	-0.2177019293(-2)
	0.1200	-0.5000000000000000000000000(-4)	-0.5(-4)	-0.46369253275491988865498 08(-3)	-0.4636925328(-3)
	0.1249	-0.2000000000000000000000000(-7)	-0.2(-7)	-0.94012486044456313964542 33(-5)	

TABLE VI. Same as Table V but for non-s-wave states of H atom with HP.

nl	λ	$E^{(0)}$	$E^{(1)}\cdot c^2, j=l+1/2$	$E^{(1)}\cdot c^2, j=l-1/2$	Ref. [1]
$2p$	0.0100	-0.12004166791693063780911153	-0.78083334860842921274687 22(-2)	-0.39044270860886386257315 13(-1)	
	0.0250	-0.11276046555934342561975 21	-0.77864642946461513078401 22(-2)	-0.38948568738392475531816 10(-1)	-0.3894856874(-1)
	0.0500	-0.10104245207235783687933 90	-0.77084283940356465125675 24(-2)	-0.38606784640031648493130 59(-1)	-0.3860678464(-1)
	0.0750	-0.89847752885906144304385 69(-1)	-0.75786034554753021461559 74(-2)	-0.38037156120748871058782 44(-1)	-0.3803715612(-1)
	0.1000	-0.79179439105145937524790 40(-1)	-0.73973325399068675950761 33(-2)	-0.37239624544464820397876 92(-1)	-0.3723962454(-1)
	0.1500	-0.59441517852607600646996 22(-1)	-0.68823802129320403905367 74(-2)	-0.34959456252695345802176 27(-1)	-0.3495945625(-1)
	0.2000	-0.41886049217867004860632 10(-1)	-0.61680141684275247368682 49(-2)	-0.31757232526322042645980 41(-1)	-0.3175723253(-1)
	0.2500	-0.26611051350910183726258 73(-1)	-0.52575770343162209508671 83(-2)	-0.27599461947554543343115 21(-1)	-0.2759946195(-1)
	0.3000	-0.13790034782404699158753 65(-1)	-0.41447097438666919786080 81(-2)	-0.22371007745091540628255 12(-1)	-0.2237100895(-1)
	0.3500	-0.37930981470217339586072 27(-2)	-0.27616873515320352090985 91(-2)	-0.15552431414493297884272 20(-1)	-0.1555243142(-1)
	0.3769	-0.35049497823430676418874 04(-5)	-0.14886350422289857591808 96(-2)	-0.86984043338629428929340 96(-2)	
$3p$	0.0100	-0.50659729727901087106427 13(-1)	-0.46180583378081904118520 06(-2)	-0.13857871956493366456893 43(-1)	
	0.0250	-0.43706891586462308141294 03(-1)	-0.45574001952758523087894 80(-2)	-0.13695084243097759966201 54(-1)	-0.1369508424(-1)
	0.0500	-0.33164501183867202344100 10(-1)	-0.43420048522838051828550 96(-2)	-0.13114351597848140079805 30(-1)	-0.1311435160(-1)
	0.0750	-0.23939747261830537729569 80(-1)	-0.39872393240429918119345 85(-2)	-0.12148295327338202122082 83(-1)	-0.1214829533(-1)
	0.1000	-0.16053726638917253120820 51(-1)	-0.34990486411202824132798 47(-2)	-0.10797720081242885650107 58(-1)	-0.1079772008(-1)
	0.1500	-0.44663087853523108131347 70(-2)	-0.21465926539011021251244 66(-2)	-0.68994854758486599214879 08(-2)	-0.5899485271(-2)
	0.1864	-0.44361338462547528602895 53(-5)	-0.60307464686288029392947 75(-3)	-0.20370486805199118696638 93(-2)	
$4p$	0.0100	-0.26441691717959478711947 42(-1)	-0.24231889989555604798227 153(-2)	-0.63040445575495985346874 21(-2)	
	0.0250	-0.19948905991591497285687 05(-1)	-0.23279376944096708230689 76(-2)	-0.60753419638123848723639 96(-2)	-0.6075341964(-2)
	0.0500	-0.11058170219449860057316 74(-1)	-0.19929777819272627749023 97(-2)	-0.52616484968634364949926 91(-2)	-0.5261648497(-2)
	0.0750	-0.46219264532259712221812 44(-2)	-0.14513853193936892124886 09(-2)	-0.39110494199743986058110 45(-2)	-0.3911049420(-2)
	0.1000	-0.75499338535057242181443 01(-3)	-0.71099435342766984478273 01(-3)	-0.19778855277704128565856 59(-2)	-0.1977885528(-2)
	0.1104	-0.31395439279287656351977 86(-5)	-0.24600958654970911395348 52(-3)	-0.69683333641825095899572 77(-3)	
$3d$	0.0100	-0.50643071323050417732671 31(-1)	-0.15390437714306565318378 24(-2)	-0.46180559946514540358474 21(-2)	
	0.0250	-0.43603050099100057801958 12(-1)	-0.15171899478673471699040 43(-2)	-0.45573080649619883006825 78(-2)	-0.4557308065(-2)
	0.0500	-0.32753184225127904255118 09(-1)	-0.14393788357060238607395 00(-2)	-0.43404939881663469056366 65(-2)	-0.4340493988(-2)
	0.0750	-0.23030704076838084236822 01(-1)	-0.13104126594666913199139 41(-2)	-0.39792286685541656079875 31(-2)	-0.3979228668(-2)
	0.1000	-0.14484226905249705674298 01(-1)	-0.11308214561010955790033 20(-2)	-0.34716481037174617765641 18(-2)	-0.3471648104(-2)
	0.1500	-0.13965924657365547375226 33(-2)	-0.59924042331093274741855 79(-3)	-0.19248119078171220968192 09(-2)	-0.1924811908(-2)
	0.1576	-0.10205913293984966980963 67(-4)	-0.47078846398941040622062 03(-3)	-0.15334984360442321277534 10(-2)	
$4d$	0.0100	-0.26425063173532301232275 47(-1)	-0.11300840101006734258825 28(-2)	-0.24231811527555366704578 589(-2)	
	0.0250	-0.19846254274766466787699 76(-1)	-0.10817705310681558517507 64(-2)	-0.23276254621148870657973 01(-2)	-0.2327625462(-2)
	0.0500	-0.10667404172560599405741 45(-1)	-0.91167762865128131478524 87(-3)	-0.19875852920015328422554 22(-2)	-0.1987585292(-2)
	0.0750	-0.38345330769212164964121 43(-2)	-0.63441909527081664215465 58(-3)	-0.14185581588000408265589 49(-2)	-0.1418558159(-2)
	0.0975	-0.66224378589064785379357 89(-5)	-0.26026074929992251691393 82(-3)	-0.60662578213042659971456 17(-3)	
$4f$	0.0100	-0.26400090317113823941570 55(-1)	-0.48411607324511276436334 75(-3)	-0.11300796392874648760486 06(-2)	
	0.0250	-0.19691095298151444342346 80(-1)	-0.46229558445222779700706 78(-3)	-0.10815961730582220538117 43(-2)	-0.1081596173(-2)
	0.0500	-0.10061964550932802432661 35(-1)	-0.38484789414927020242018 71(-3)	-0.90861985344977122096766 77(-3)	-0.9086198534(-3)
	0.0750	-0.25562969780724235357506 29(-2)	-0.25482876578404808114899 68(-3)	-0.61416970948808956651242 47(-3)	-0.6141697095(-3)
	0.0864	-0.97614928214770450918264 56(-6)	-0.16922081568997625889134 40(-3)	-0.41587717521071733009854 34(-3)	