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RESEARCH PAPER

Mixed problem with dynamical transmission condition for a one-dimensional hyperbolic equation with strong dissipation †

Akbar B. Aliev *[1,2] | Gulshan Kh. Shafieva[1,3]

[1]Deparment of Differential Equations, Institute of Mathematics and Mechanics of NAS of Azerbaijan, Baku, Azerbaijan
[2]Deparment of Mathematics, Azerbaijan Technical University, Baku, Azerbaijan
[3]Deparment of Computational Mathematics, Baku State University, Baku, Azerbaijan

Correspondence
*Akbar B. Aliev, Deparment of Differential Equations,Institute of Mathematics and Mechanics of NAS of Azerbaijan, Baku, Azerbaijan. Email: alievakbar@gmail.com

Summary

In this paper, we study a mixed problem for the nonlinear hyperbolic equation with a strong dissipation and a dynamical transmission condition. The existence and uniqueness theorems of local and global solutions are proved.

KEYWORDS:
strongly damped wave equation, mixed problem, dynamical transmission condition, global solution

1 | INTRODUCTION

Hyperbolic equations with strong dissipation arise in the study of various problems of mechanics and physics with viscosity [1,2,3]. A mixed problem with dynamic boundary conditions or with dynamic transmission conditions also arises in the mathematical modelling of various problems of mechanics [4,5,6,7,8,9,10,11,12].
The mixed problem for wave equations with strong dissipation was studied in [13,14,15]. A mixed problem with dynamic boundary conditions for one - dimensional wave equations with strong dissipation was studied in [16]. The asymptotic of the solutions of the mixed problem for wave equations with strong dissipation was studied in [14,15,17,18].
Recently, numerous studies of nonlocal problems for various evolution equations have been carried out. Among these problems, mixed problems with integral boundary conditions are of particular interest (see [19], as well as the literature cited in these works).
In this paper, we study a mixed problem for one - dimensional wave equations with strong dissipation and a dynamic transmission condition. We investigate the correctness of the considered problem in L_p – type spaces.

2 | STATEMENT OF THE PROBLEM AND MAIN RESULT

In the domain $Q_T = [0, T] \times [0, 2]$ we consider the mixed problem:

$$u_{tt} - (\mu_1(x)u_x)_{xt} - \eta_1(x)u_{xx} = f_1(u) + \xi_1(t, x), \quad 0 \le t \le T, \quad 0 \le x \le 1, \tag{1}$$

$$\upsilon_{tt} - (\mu_2(x)\upsilon_x)_{xt} - \eta_2(x)\upsilon_{xx} = f_2(\upsilon) + \xi_2(t, x), \quad 0 \le t \le T, \quad 1 \le x \le 2 \tag{2}$$

with the boundary conditions

$$u(t, 0) = 0, \quad \upsilon(t, 2) = 0, \tag{3}$$

†This is an example for title footnote.

the transmission conditions

$$u(t,1) = v(t,1) = \phi(t), \tag{4}$$

$$\phi_{tt}(t) + \gamma_1 u_{xt}(t,1) - \gamma_2 v_{xt}(t,1) + \beta_1 u_x(t,1) - \beta_2 v_x(t,1) = h(\phi) + g(t) \tag{5}$$

and the initial conditions

$$u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), \tag{6}$$

$$\phi'(0) = \phi_1. \tag{7}$$

Assume that the following conditions are satisfied:

(i) $\mu_1(\cdot) \in C^1[0,1], \quad \mu_2(\cdot) \in C^1[1,2]$;

(ii) $\mu_1(x) > 0, \quad 0 \le x \le 1, \quad \mu_2(x) > 0, \quad 1 \le x \le 2$;

(iii) $\eta_1(\cdot) \in L_\infty(0,1), \quad \eta_2(\cdot) \in L_\infty(1,2)$;

(iv) $\gamma_1 > 0, \quad \gamma_2 > 0$;

(v) $\beta_1, \beta_2 \in R$;

(vi) $\xi_1(\cdot) \in C^1([0,T] \times L_p(0,1))$;

(vii) $\xi_2(\cdot) \in C^1([0,T] \times L_p(1,2))$;

(viii) $g(\cdot) \in C^1[0,T]$;

(ix) $\left|f_k(\tau_2) - f_k(\tau_1)\right| \le c_1(\tau_2,\tau_1) \cdot \left|\tau_2 - \tau_1\right|, \quad k = 1,2, \quad c_1(\cdot) \in C(R \times R)$;

(x) $\left|h_k(\tau_2) - h_k(\tau_1)\right| \le c_2(\tau_2,\tau_1) \cdot \left|\tau_2 - \tau_1\right|, \quad k = 1,2, \quad c_2(\cdot) \in R \times R$.

We denote by $\|\cdot\|_{p,1}$ the norm in the space $L_p(0,1)$, and by $\|\cdot\|_{p,2}$ the norm in the space $L_p(1,2)$, respectively, i.e.

$$\|u\|_{p,1} = \left[\int_0^1 |u(x)|^p\,dx\right]^{\frac{1}{p}}, \|v\|_{p,2} = \left[\int_1^2 |v(x)|^p\,dx\right]^{\frac{1}{p}}.$$

By $W_p^1((a,b);c)$, where $c \in [a,b]$ denote the following subspace of the Sobolev space $W_p^1(a,b)$, i.e.

$$W_p^1((a,b);c) = \left\{w : w \in W_p^1(a,b),\ w(c) = 0\right\},$$

We will also use the following spaces:

$$X_p = \left\{w : w = (u,v,\alpha),\ u \in L_p(0,1),\ v \in L_p(1,2),\ \alpha \in C\right\}$$

with norm

$$\|w\|_{X_p} = \left[\int_0^1 \|u(x)\|_p^p\,dx\right]^{\frac{1}{p}} + \left[\int_1^2 \|v(x)\|_p^p\,dx\right]^{\frac{1}{p}} + |\alpha|,$$

and

$$Y_p = \Big\{w : w = (u,v,\phi),\ u \in W_p^2(0,1) \bigcap W_p^1((0,1);0),$$
$$v \in W_p^2(1,2) \bigcap W_p^1((1,2);2),\ u(1) = v(1) = \phi\Big\}$$

with norm

$$\|w\|_{Y_p} = \left\|u_{xx}\right\|_{p,1} + \left\|u_x\right\|_{p,1} + \left\|v_{xx}\right\|_{p,2} + \left\|v_x\right\|_{p,2}.$$

Theorem 1. Let the conditions (i) - (x) be satisfied, then for any $u_0(\cdot) \in W_p^2(0,1) \bigcap W_p^1((0,1);0)$, $u_1(\cdot) \in L_p(0,1)$, $\upsilon_0(\cdot) \in W_p^2(1,2) \bigcap W_p^1((1,2);2)$, $\upsilon_1(\cdot) \in L_p(1,2)$, $\phi_1 \in R$ there is such T_0, that problem (1) - (5) has a unique solution (u, υ, ϕ) such that

$$\begin{aligned}
&u(\cdot) \in C([0,T_0] \times W_p^1((0,1);0)) \bigcap C^1((0,T_0) \times L_p[0,1]) \bigcap \\
&\bigcap C^1((0,T_0] \times W_p^2(0,1) \bigcap W_p^1((0,1);0)) \bigcap C^2((0,T_0) \times L_p[0,1]),\\
&\upsilon(\cdot) \in C([0,T_0] \times W_p^1((1,2);2)) \bigcap C^1((0,T_0) \times L_p[1,2]) \bigcap \\
&\bigcap C^1((0,T_0] \times W_p^2(1,2) \bigcap W_p^1((1,2);2)) \bigcap C^2((0,T_0) \times L_p[1,2]),\\
&\phi(\cdot) \in C[0,T_0] \bigcap C^1(0,T_0] \bigcap C^2(0,T_0), u_x(\cdot,1),\ \upsilon_x(\cdot,1) \in C(0,T_0),\\
&u_{tt}(\cdot,1),\ \upsilon_{tt}(\cdot,1),\ u_{xt}(\cdot,1),\ \upsilon_{xt}(\cdot,1) \in C(0,T_0).
\end{aligned}$$

If T' the length of the maximum interval for the existence of a global solution, then one of the following statements is true:

a) $T' = +\infty$;

b) $\lim_{t\to T'-0} \sup\{\|u_{xx}\|_{p,1} + \|u_t\|_{p,1} + \|\upsilon_{xx}\|_{p,2} + \|\upsilon_t\|_{p,2}\} = +\infty$.

Proof. In the space X_p we define a linear operator A, where

$$D(A) = Y_p = \Big\{ w : w = (u,\upsilon,\phi),\ u \in W_p^2(0,1) \bigcap W_p^1((0,1);0),$$
$$\upsilon \in W_p^2(1,2) \bigcap W_p^1((1,2);2),\ u(1) = \upsilon(1) = \phi \Big\},$$
$$Aw = (-(\mu_1(x)u_x)_x(x),\ -(\mu_2(x)\upsilon_x)_x(x),\ \gamma_1 u_x(1) - \gamma_2 \upsilon_x(1)), \quad w = (u,\upsilon,\phi),$$

similarly linear operator B, where

$$D(B) = Y_p = \Big\{ w : w = (u,\upsilon,\phi),\ u \in W_p^2(0,1) \bigcap W_p^1((0,1);0),$$
$$\upsilon \in W_p^2(1,2) \bigcap W_p^1((1,2);2),\ u(1) = \upsilon(1) = \phi \Big\},$$
$$Bw = (-\eta_1(x)u_{xx}(x),\ -\eta_2(x)\upsilon_{xx}(x),\ \beta_1 u_x(1) - \beta_2 \upsilon_x(1)),$$
$$w = (u,\upsilon,\phi).$$

The mixed problem (1) - (5) can be written as the following Cauchy problem in the space X_p:

$$w'' + Aw' + Bw = F(w) + G(t), \tag{8}$$

$$w(0) = w_0, \quad w'(0) = w_1, \tag{9}$$

where $w_0 = (u_0, \upsilon_0, \phi_0)$, $\phi_0 = u_0(1) = \upsilon_0(1)$, $w_1 = (u_1, \upsilon_1, \phi_1)$, $F(w) = (f_1(u),\ f_2(\upsilon),\ h(\phi))$, $G(t) = (\xi_1(t,\cdot),\ \xi_2(t,\cdot),\ g(t))$. □

Lemma 1. Let the conditions (i), (ii), (iv) be satisfied and $p \geq 1$, then A - is a sectorial operator in X_p.

Note that the definition of a sectorial operator can be found in[20,21], as well as in the literature cited there. In these papers, the main properties of a sectorial operator are also is given.

Lemma 2. Let the conditions (iii) - (v) be satisfied and $p \geq 1$, then B - linear bounded operator, acting from Y_p in X_p.

The following lemmas are obtained from conditions (viii)-(x), by virtue of the embedding theorems.

Lemma 3. Let the conditions (ix) - (xii) be satisfied and $p \geq 1$, then nonlinear operator F acting from Y_p in X_p satisfies the local Lipshits condition, i.e. for all w_1, w_2 inequality is true

$$\|F(w_1) - F(w_2)\|_{X_p} \leq c(\|w_1\|_{Y_p}, \|w_2\|_{Y_p}) \|w_1 - w_2\|_{Y_p}.$$

Lemma 4. Let the conditions (vi) - (viii) be satisfied, then $G(t) \in C^1([0,T]; X_p)$.

By virtue of the Lemma 1 linear operator A generates the analytic semigroup $U(t) = e^{-tA}$ in the space X_p. It is known that

$$\left\| tAe^{-tA} \right\|_{X_p \to X_p} \leq C, \quad 0 \leq t \leq T (see^{20,21?}), \tag{10}$$

where $C > 0$ is a constant.
The problem (8), (9) can be reduced to the problem

$$\theta' = S\theta + \Phi(\theta) + \Psi(t), \tag{11}$$

$$\theta(0) = \theta_0 \tag{12}$$

in the Banach space $\mathrm{E} = Y_p \times X_p$, where

$$\theta = \theta(t) = \begin{pmatrix} v_1(t) \\ v_2(t) \end{pmatrix} = \begin{pmatrix} w(t) \\ w_t(t) \end{pmatrix}, \theta_0 = \begin{pmatrix} w_0 \\ w_1 \end{pmatrix}, \Phi(\theta) = \begin{pmatrix} 0 \\ F(w) \end{pmatrix}, \Psi(t) = \begin{pmatrix} 0 \\ G(t) \end{pmatrix},$$

$$S = S_0 + S_1, S_0 = \begin{pmatrix} 0 & I \\ 0 & -A \end{pmatrix}, S_1 = \begin{pmatrix} 0 & 0 \\ -B & 0 \end{pmatrix}.$$

In the space $\mathrm{E} = Y_p \times X_p$ the linear operator S_0 generates the strongly continuous semigroup

$$e^{tS_0}\theta_0 = (w_0 + \int_0^t e^{-\tau A} w_1 d\tau, e^{-tA} w_1).$$

Taking (10) into account, we see that

$$\left\| tS_0 e^{tS_0} \right\|_{E\to E} \le C, \quad 0 \le t \le T.$$

It follows that S_0 generates an analytic semigroup.
On the other hand, the linear operator S_1 is bounded on the space $\mathrm{E} = Y_p \times X_p$, hence, the operator S also generates an analytic semigroup[20,21,22]
Taking into account the Lemma 3 and Lemma 4

$$\left\|\Phi(\theta_1) - \Phi(\theta_2)\right\|_E \le c(\left\|\theta_1\right\|_E, \left\|\theta_2\right\|_E) \left\|\theta_1 - \theta_2\right\|_E,$$
$$\Psi(t) \in C^1([0,T]; E).$$

Thus, all conditions for the existence and uniqueness of a local solution for nonlinear equations in a Banach space are satisfied[20,22?].

Theorem 2. Let $p = 2$ and the conditions (i) - (x) be satisfied. Suppose, that the following conditions are additionally satisfied:

(xi) $\left|F_k(t)\right| \le c(1 + |t|^{v_k}), \quad 0 \le v_k < 2$, where $F_k(t) = \int_0^t f_k(s)ds, \quad k = 1, 2$;

(xii) $|H(t)| \le c(1 + |t|^{v}), \quad 0 \le v < 2$, where $H(t) = \int_0^t h(s)ds$;

(xiii) $\begin{vmatrix} \mu_1(1) & \eta_1(1) \\ \gamma_1 & \beta_1 \end{vmatrix} = 0, \quad \begin{vmatrix} \mu_2(1) & \eta_2(1) \\ \gamma_2 & \beta_2 \end{vmatrix} = 0.$

Then for any $u_0(\cdot) \in W_2^2(0,1) \bigcap W_2^1((0,1);0)$, $u_1(\cdot) \in L_2(0,1)$, $v_0(\cdot) \in W_2^2(1,2) \bigcap W_2^1((1,2);2)$, $v_1(\cdot) \in L_2(1,2)$, $\phi_1 \in R$ problem (1) - (5) has a unique solution (u, v, ϕ) such that

$$\begin{aligned}
&u(\cdot) \in C([0,T] \times W_2^1((0,1);0)) \bigcap C^1((0,T) \times L_2[0,1]) \bigcap \\
&\bigcap C^1((0,T] \times W_2^2(0,1) \bigcap W_2^1((0,1);0)) \bigcap C^2((0,T) \times L_2[0,1]), \\
&v(\cdot) \in C([0,T] \times W_2^1((1,2);2)) \bigcap C^1((0,T) \times L_2[1,2]) \bigcap \\
&\bigcap C^1((0,T] \times W_2^2(1,2) \bigcap W_2^1((1,2);2) \bigcap C^2((0,T) \times L_2[1,2]), \\
&\phi(\cdot) \in C[0,T] \bigcap C^1(0,T] \bigcap C^2(0,T), u_x(\cdot,1), \ v_x(\cdot,1) \in C(0,T), \\
&u_{tt}(\cdot,1), \ v_{tt}(\cdot,1), \ u_{xt}(\cdot,1), \ v_{xt}(\cdot,1) \in C(0,T).
\end{aligned}$$

Proof. If for a local solution the following a priori estimate is true

$$\int_0^1 \left|u_t(t,x)\right|^2 dx + \int_1^2 \left|v_t(t,x)\right|^2 dx + \int_0^1 \left|u_{xx}(t,x)\right|^2 dx + \int_1^2 \left|v_{xx}(t,x)\right|^2 dx \le C, \quad 0 \le t \le T', \tag{13}$$

then, by virtue of Theorem 1, this solution can be globally continued on the entire interval $[0,T]$.
In order to get an a priori estimate (13), first, we multiply both sides of (1) by $\frac{\gamma_1}{\mu_1(1)} u_t(t,x)$ and integrate in $[0,t] \times [0,1]$. Then,

we multiply both sides of (2) by $\frac{\gamma_2}{\mu_2(1)} v_t(t,x)$ and integrate in $[0,t]\times[1,2]$. Lastly we multiply both sides of (5) by $\phi_t(t)$ and integrate in $[0,t]$. Next, by applying the integration by parts and using the conditions (3), (4), (6) and (7) we can get

$$\begin{gathered}
\frac{\gamma_1}{2\mu_1(1)}\int_0^1 \left|u_t(t,x)\right|^2 dx+\frac{\gamma_1}{2\mu_1(1)}\int_0^1 \eta_1(x)\left|u_x(t,x)\right|^2 dx+ \\
+\frac{\gamma_2}{2\mu_2(1)}\int_1^2 \left|v_t(t,x)\right|^2 dx+\frac{\gamma_2}{2\mu_2(1)}\int_1^2 \eta_2(x)\left|v_x(t,x)\right|^2 dx+ \\
+\frac{\gamma_1}{\mu_1(1)}\int_0^t\int_0^1 \mu_1(x)\left|u_{xt}(s,x)\right|^2 dxds+\frac{\gamma_2}{\mu_2(1)}\int_0^t\int_1^2 \mu_2(x)\left|v_{xt}(s,x)\right|^2 dxds+ \\
+\frac{\gamma_1}{\mu_1(1)}\int_0^t\int_0^1 \eta_{1x}(x)u_x(s,x)u_t(s,x)dxds+\frac{\gamma_2}{\mu_2(1)}\int_0^t\int_1^2 \eta_{2x}(x)v_x(s,x)v_t(s,x)dxds+ \\
+\frac{1}{2}\left|\phi_t(t)\right|^2=\int_0^1 F_1(u(t,x))dx+\int_1^2 F_2(v(t,x))dx+H(\phi(t))+ \\
+\frac{\gamma_1}{\mu_1(1)}\int_0^t\int_0^1 \xi_1(s,x)u_t(s,x)dxds+\frac{\gamma_2}{\mu_2(1)}\int_0^t\int_1^2 \xi_2(s,x)v_t(s,x)dxds+ \\
+\int_0^t g(s)\phi_t(s)ds+\frac{\gamma_1}{2\mu_1(1)}\int_0^1 \left|u_1(x)\right|^2 dx+\frac{\gamma_1}{2\mu_1(1)}\int_0^1 \eta_1(x)\left|u_{0x}(x)\right|^2 dx+ \\
+\frac{\gamma_2}{2\mu_2(1)}\int_1^2 \left|v_1(x)\right|^2 dx+\frac{\gamma_2}{2\mu_2(1)}\int_1^2 \eta_2(x)\left|v_{0x}(x)\right|^2 dx+\frac{1}{2}\left|\phi_1(t)\right|^2- \\
-\int_0^1 F_1(u_0(x))dx-\int_1^2 F_2(v_0(x))dx-H(\phi(0)). \qquad (14)
\end{gathered}$$

Taking into account conditions (xi), (xii) and applying the Holder and Young inequalities, we obtain that

$$\int_0^1 F_1(u(t,x))dx\le c\int_0^1 (1+|u(t,x)|^{v_1})dx\le c\left(1+\left(\frac{1}{\varepsilon}\right)^{\frac{2}{2-v_1}}\right)+c\varepsilon\int_0^1 \left|u_x(t,x)\right|^2 dx; \qquad (15)$$

$$\int_1^2 F_2(v(t,x))dx\le c\left(1+\left(\frac{1}{\varepsilon}\right)^{\frac{2}{2-v_2}}\right)+c\varepsilon\int_1^2 \left|v_x(t,x)\right|^2 dx; \qquad (16)$$

$$|H(\phi(t))|\le c_1\left(\frac{1}{\varepsilon}+\varepsilon\,|\phi(t)|^2\right), \qquad (17)$$

also following inequality

$$\left|\int_0^t\int_0^1 \xi_1(s,x)u_t(s,x)dxds\right|\le\frac{1}{\varepsilon}\int_0^t\int_0^1 \left|\xi_1(s,x)\right|^2 dxds+\varepsilon\int_0^t\int_0^1 \left|u_t(s,x)\right|^2 dxds; \qquad (18)$$

$$\left|\int_0^t\int_1^2 \xi_2(s,x)v_t(s,x)dxds\right|\le\frac{1}{\varepsilon}\int_0^t\int_1^2 \left|\xi_2(s,x)\right|^2 dxds+\varepsilon\int_0^t\int_1^2 \left|v_t(s,x)\right|^2 dxds; \qquad (19)$$

$$\left|\int_0^t\int_0^1 \eta_{1x}(x)u_x(s,x)u_t(s,x)dxds\right|\le c\left[\int_0^t\int_0^1 \left|u_t(s,x)\right|^2 dxds+\int_0^t\int_0^1 \left|u_x(s,x)\right|^2 dxds\right]; \qquad (20)$$

$$\left|\int_0^t\int_1^2 \eta_{2x}(x)v_x(s,x)v_t(s,x)dxds\right| \le c\left[\int_0^t\int_1^2 \left|v_t(s,x)\right|^2 dxds + \int_0^t\int_1^2 \left|v_x(s,x)\right|^2 dxds\right]. \tag{21}$$

Taking into account inequalities (15) - (21) in (14) and applying the Gronwall's lemma, we obtain the following a priori estimate

$$\int_0^1 \left|u_t(t,x)\right|^2 dx + \int_0^1 \left|u_x(t,x)\right|^2 dx + \int_1^2 \left|v_t(t,x)\right|^2 dx + \int_1^2 \left|v_x(t,x)\right|^2 dx +$$

$$+\int_0^t\int_0^1 \left|u_{xt}(s,x)\right|^2 dxds + \int_0^t\int_1^2 \left|v_{xt}(s,x)\right|^2 dxds + \left|\phi_t(t)\right|^2 \le C. \tag{22}$$

Now multiply both sides of (1) by $\gamma_1 u_{xx}(t,x)$ and integrate in $[0,t]\times[0,1]$. Then, we multiply both sides of (2) by $\gamma_2 v_{xx}(t,x)$ and integrate in $[0,t]\times[1,2]$. Lastly we multiply both sides of (5) by $\phi_t(t)$ and integrate in $[0,t]$.
Next, by applying the integration by parts and using the conditions (3), (4), (6) and (7) we can get

$$-\frac{\gamma_1}{2}\int_0^1 \mu_1(x)\left|u_{xx}(t,x)\right|^2 dx - \frac{\gamma_2}{2}\int_1^2 \mu_2(x)\left|v_{xx}(t,x)\right|^2 dx -$$

$$-\gamma_1\int_0^t\int_0^1 \mu_{1x}(x)u_{xx}(s,x)u_{xt}(s,x)dxds - \gamma_2\int_0^t\int_1^2 \mu_{2x}(x)v_{xx}(s,x)v_{xt}(s,x)dxds +$$

$$+\gamma_1\int_0^1 u_{xx}(t,x)u_t(t,x)dx + \gamma_1\int_0^t\int_0^1 \left|u_{xt}(s,x)\right|^2 dxds + \gamma_2\int_1^2 v_{xx}(t,x)v_t(t,x)dx +$$

$$+\gamma_2\int_0^t\int_1^2 \left|v_{xt}(s,x)\right|^2 dxds - \gamma_1\int_0^t\int_0^1 \eta_1(x)\left|u_{xx}(s,x)\right|^2 dxds - \gamma_2\int_0^t\int_1^2 \eta_2(x)\left|v_{xx}(s,x)\right|^2 dxds +$$

$$+\frac{1}{2}\left|\phi_t(t)\right|^2 = \gamma_1\int_0^1 u_{0xx}(x)u_1(x)dx + \gamma_2\int_1^2 v_{0xx}(x)v_1(x)dx + \frac{1}{2}\left|\phi_1\right|^2 +$$

$$+\int_0^t\int_0^1 [f_1(u(s,x)) + \xi_1(u(s,x))]u_{xx}(s,x)dxds + \int_0^t\int_1^2 [f_2(v(s,x)) + \xi_2(v(s,x))]v_{xx}(s,x)dxds +$$

$$+\int_0^t [h(\phi(s)) + g(s)]\phi_t(s)ds. \tag{23}$$

Using Holder inequality and taking into account a priori estimate (22) from (23), we have

$$\int_0^1 \left|u_{xx}(t,x)\right|^2 dx + \int_1^2 \left|v_{xx}(t,x)\right|^2 dx \le C_1 + C_2\int_0^t\left[\int_0^1 \left|u_{xx}(t,x)\right|^2 dx + \int_1^2 \left|v_{xx}(t,x)\right|^2 dx\right]dt, \tag{24}$$

where $C_1 > 0$ and $C_2 > 0$ constants independent of $t,\ x,\ u$ and v.
From (24), in view of Gronwall lemma, it follows

$$\int_0^1 \left|u_{xx}(t,x)\right|^2 dx + \int_1^2 \left|v_{xx}(t,x)\right|^2 dx \le C, \quad 0 \le t \le T. \tag{25}$$

It follows from (22), (24) that the local solution presented in Theorem 1 satisfies a priori estimate (13).

$\square$

3 | PROOF OF THE LEMMA 1

According to the definition, for showing that the operator A is a sectorial, we must evalute its resolvent.
To estimate the resolvent of the operator A, we consider the equation

$$\lambda\tilde{u} + A\tilde{u} = G, \tag{26}$$

where $G = (\xi_1(\cdot), \xi_2(\cdot), \alpha) \in X_p$.
Equation (26) is equivalent to the boundary value problem:

$$\lambda u(x) - (\mu_1(x)u'(x))' = \xi_1(x), \quad 0 \le x \le 1, \tag{27}$$

$$\lambda \upsilon(x) - (\mu_2(x)\upsilon'(x))' = \xi_2(x), \quad 1 \le x \le 2, \tag{28}$$

$$u(1) = \upsilon(1) = \phi, \tag{29}$$

$$u(0) = 0, \tag{30}$$

$$\upsilon(2) = 0, \tag{31}$$

$$\lambda\phi + \gamma_1 u'(1) - \gamma_2 \upsilon'(1) = \alpha, \tag{32}$$

where $\lambda \in C$, $\xi_1(x) \in L_p(0,1)$, $\xi_2(x) \in L_p(1,2)$, $\alpha \in R$.
Let $u(x)$ and $\upsilon(x)$ smooth functions satisfy (27) - (32). Following [23], we multiply both sides of (27) by the function $\bar{u}(x)\,|u(x)|^{p-2}$.
Further, integrating by parts, we obtain

$$\lambda \int_0^1 |u(x)|^p\,dx + \int_0^1 \mu_1(x)u'(x)(\bar{u}(x)\,|u(x)|^{p-2})_x dx - \mu_1(1)u'(1)(\bar{u}(1)\,|u(1)|^{p-2}) =$$

$$= \int_0^1 \xi_1(x)\bar{u}(x)\,|u(x)|^{p-2}\,dx. \tag{33}$$

Multiplying both sides of (28) by $\bar{\upsilon}(x)\,|\upsilon(x)|^{p-2}$ and integrating by parts, we see that

$$\lambda \int_1^2 |\upsilon(x)|^p\,dx + \int_1^2 \mu_2(x)\upsilon'(x)(\bar{\upsilon}(x)\,|\upsilon(x)|^{p-2})_x dx + \mu_2(1)\upsilon'(1)(\bar{\upsilon}(1)\,|\upsilon(1)|^{p-2}) =$$

$$= \int_1^2 \xi_2(x)\bar{\upsilon}(x)\,|\upsilon(x)|^{p-2}\,dx. \tag{34}$$

Next, we multiply both sides of (32) by $\bar{\phi}\,|\phi|^{p-2}$, we get

$$\lambda\,|\phi|^p + \left[\gamma_1 u'(1) - \gamma_2\upsilon'(1)\right]\bar{\phi}\,|\phi|^{p-2} = \alpha\bar{\phi}\,|\phi|^{p-2}. \tag{35}$$

Now, multiplying both sides of (33) by $\frac{\gamma_1}{\mu_1(1)}$, and multiplying both sides of (34) by $\frac{\gamma_2}{\mu_2(1)}$. If we add up the equalities that got and (35), we obtain

$$\lambda\frac{\gamma_1}{\mu_1(1)}\int_0^1 |u(x)|^p\,dx + \lambda\frac{\gamma_2}{\mu_2(1)}\int_1^2 |\upsilon(x)|^p\,dx + \lambda\,|\phi|^p + \int_0^1 J_1 dx + \int_1^2 J_2 dx =$$

$$= \frac{\gamma_1}{\mu_1(1)}\int_0^1 \xi_1(x)\bar{u}(x)\,|u(x)|^{p-2}\,dx + \frac{\gamma_2}{\mu_2(1)}\int_1^2 \xi_2(x)\bar{\upsilon}(x)\,|\upsilon(x)|^{p-2}\,dx + \alpha\bar{\phi}\,|\phi|^{p-2}, \tag{36}$$

where

$$J_1 = \frac{\gamma_1\mu_1(x)}{\mu_1(1)}[u'(x)\bar{u}'(x)\,|u(x)|^{p-2} + u'(x)\bar{u}(x)(|u(x)|^{p-2})_x], \quad 0 \le x \le 1,$$

$$J_2 = \frac{\gamma_2 \mu_2(x)}{\mu_2(1)} [v'(x)\bar{v}'(x) \left|v(x)\right|^{p-2} + v'(x)\bar{v}(x)(\left|v(x)\right|^{p-2})_x], \qquad 1 \le x \le 2.$$

It can be verified directly that

$$\left|Im\ J_1\right| \le \frac{\gamma_1 \mu_1(x)}{\mu_1(1)} \left|p-2\right| \left|u(x)\right|^{p-2} \left|Im \frac{u'(x)\bar{u}(x)}{u(x)}\right| \left|(\left|u(x)\right|)_x\right|,$$

$$Re\ J_1 = \frac{\gamma_1 \mu_1(x)}{\mu_1(1)} \left[(p-1) \left|u(x)\right|^{p-2} \left|(\left|u(x)\right|)_x\right|^2 + \left|u(x)\right|^{p-2} \left|Im \frac{u'(x)\bar{u}(x)}{u(x)}\right|^2 \right].$$

Hence we have

$$\frac{\left|Im\ J_1\right|}{Re\ J_1} \le \frac{\left|p-2\right| \frac{1}{2\sqrt{p-1}} 2\sqrt{p-1} \left|Im \frac{u'(x)\bar{u}(x)}{u(x)}\right| \left|(\left|u(x)\right|)_x\right|}{(p-1)\left|(\left|u(x)\right|)_x\right|^2 + \left|Im \frac{u'(x)\bar{u}(x)}{u(x)}\right|^2} \le \frac{\left|p-2\right|}{2\sqrt{p-1}},$$

i.e.

$$\left|Im\ J_1\right| \le \frac{\left|p-2\right|}{2\sqrt{p-1}} Re\ J_1, \tag{37}$$

$$\left|Im\ J_2\right| \le \frac{\left|p-2\right|}{2\sqrt{p-1}} Re\ J_2. \tag{38}$$

On the other hand, from (36) we obtain

$$Re\ \lambda(\frac{\gamma_1}{\mu_1(1)} \left\|u\right\|_{p,1}^p + \frac{\gamma_2}{\mu_2(1)} \left\|v\right\|_{p,2}^p + \left|\phi\right|^p) + Re \int_0^1 J_1 dx + Re \int_1^2 J_2 dx =$$

$$= Re\left[\frac{\gamma_1}{\mu_1(1)} \int_0^1 \xi_1(x)\bar{u}(x) \left|u(x)\right|^{p-2} dx + \frac{\gamma_2}{\mu_2(1)} \int_1^2 \xi_2(x)\bar{v}(x) \left|v(x)\right|^{p-2} dx + \alpha\bar{\phi} \left|\phi\right|^{p-2} \right], \tag{39}$$

$$Im\ \lambda(\frac{\gamma_1}{\mu_1(1)} \left\|u\right\|_{p,1}^p + \frac{\gamma_2}{\mu_2(1)} \left\|v\right\|_{p,2}^p + \left|\phi\right|^p) + Im \int_0^1 J_1 dx + Im \int_1^2 J_2 dx =$$

$$= Im\left[\frac{\gamma_1}{\mu_1(1)} \int_0^1 \xi_1(x)\bar{u}(x) \left|u(x)\right|^{p-2} dx + \frac{\gamma_2}{\mu_2(1)} \int_1^2 \xi_2(x)\bar{v}(x) \left|v(x)\right|^{p-2} dx + \alpha\bar{\phi} \left|\phi\right|^{p-2} \right]. \tag{40}$$

Inviewing of (39), (40) we can write

$$Re \int_0^1 J_1 dx + Re \int_1^2 J_2 dx - \eta \left| Im \int_0^1 J_1 dx \right| - \eta \left| Im \int_1^2 J_2 dx \right| \ge 0,$$

where $0 \le \eta \le \frac{2\sqrt{p-1}}{|p-2|}$.
Taking into account this inequality, we see from (39), (40) that

$$(Re\ \lambda + \eta \left|Im\lambda\right|)(\frac{\gamma_1}{\mu_1(1)} \left\|u\right\|_{p,1}^p + \frac{\gamma_2}{\mu_2(1)} \left\|v\right\|_{p,2}^p + \left|\phi\right|^p) \le$$

$$\le \frac{\gamma_1}{\mu_1(1)}(1+\eta) \left(\int_0^1 \left|\xi_1(x)\right|^p dx \right)^{\frac{1}{p}} \left(\int_0^1 \left|u(x)\right|^p dx \right)^{\frac{p-1}{p}} +$$

$$+ \frac{\gamma_2}{\mu_2(1)}(1+\eta) \left(\int_1^2 \left|\xi_2(x)\right|^p dx \right)^{\frac{1}{p}} \left(\int_1^2 \left|v(x)\right|^p dx \right)^{\frac{p-1}{p}} + \alpha \left|\phi\right|^{p-1}.$$

Hence we obtain

$$(Re\ \lambda + \eta \left|Im\lambda\right|)(\left\|u\right\|_{p,1}^p + \left\|v\right\|_{p,2}^p + \left|\phi\right|^p) \le$$

$$\leq C\left[(1+\eta)\left(\int_0^1 \left|\xi_1(x)\right|^p dx\right)^{\frac{1}{p}} + (1+\eta)\left(\int_1^2 \left|\xi_2(x)\right|^p dx\right)^{\frac{1}{p}} + \alpha\right]\times$$

$$\times\left[\left(\int_0^1 |u(x)|^p dx\right)^{\frac{1}{p}} + \left(\int_1^2 |v(x)|^p dx\right)^{\frac{1}{p}} + |\phi|\right]^{p-1},$$

where

$$C = \frac{\max\left\{\frac{\gamma_1}{\mu_1(1)}, \frac{\gamma_2}{\mu_2(1)}\right\}}{\min\left\{\frac{\gamma_1}{\mu_1(1)}, \frac{\gamma_2}{\mu_2(1)}\right\}}.$$

From here we get

$$(Re\ \lambda + \eta\,|Im\lambda|)(\|u(\cdot)\|_{p,1}^p + \|v(\cdot)\|_{p,2}^p + |\phi|^p) \leq$$

$$\leq 3C(1+\eta)\left[\left(\int_0^1 \left|\xi_1(x)\right|^p dx\right)^{\frac{1}{p}} + \left(\int_1^2 \left|\xi_2(x)\right|^p dx\right)^{\frac{1}{p}} + \alpha\right]\times$$

$$\times\left[\left(\int_0^1 |u(x)|^p dx\right)^{\frac{1}{p}} + \left(\int_1^2 |v(x)|^p dx\right)^{\frac{1}{p}} + |\phi|\right]^{p-1},$$

i.e.

$$(Re\ \lambda + \eta\,|Im\lambda|)(\|u(\cdot)\|_{p,1} + \|v(\cdot)\|_{p,2} + |\phi|) \leq$$
$$\leq 3C(1+\eta)(\left\|\xi_1(\cdot)\right\|_{p,1} + \left\|\xi_2(\cdot)\right\|_{p,2} + |\alpha|).$$

Hence, taking into account (27), (28) and (32), we have

$$(Re\ \lambda + \eta\,|Im\lambda|)(\|u(\cdot)\|_{p,1} + \|v(\cdot)\|_{p,2} + |\phi|) \leq$$
$$\leq 3C(1+\eta)(\left\|\lambda u(\cdot) - u''(\cdot)\right\|_{p,1} + \left\|\lambda v(\cdot) - v''(\cdot)\right\|_{p,2} +$$

$$+\left|\lambda\phi + u'(1) - v'(1)\right|). \tag{41}$$

We consider problem (27) - (32) at the point $\lambda = 0$. Solving the corresponding problem, we obtain

$$u(x) = \mu_1(1)K_1\int_0^x \frac{dy}{\mu_1(y)} + \int_0^x \frac{1}{\mu_1(y)}\left(\int_y^1 \xi_1(s)ds\right)dy, \quad 0 \leq x \leq 1,$$

$$v(x) = -\mu_2(1)K_2\int_x^2 \frac{dy}{\mu_2(y)} + \int_x^2 \frac{1}{\mu_2(y)}\left(\int_1^y \xi_2(s)ds\right)dy, \quad 1 \leq x \leq 2,$$

$$\phi = \left[\frac{\gamma_1}{\mu_1(1)\int_0^1 \frac{dy}{\mu_1(y)}} + \frac{\gamma_2}{\mu_2(1)\int_1^2 \frac{dy}{\mu_2(y)}}\right]^{-1}\left[\alpha - \frac{\gamma_1}{\mu_1(1)\int_0^1 \frac{dy}{\mu_1(y)}}\int_0^1 \frac{1}{\mu_1(y)}\left(\int_y^1 \xi_1(s)ds\right)dy + \right.$$

$$\left. + \frac{\gamma_2}{\mu_2(1)\int_1^2 \frac{dy}{\mu_2(y)}}\int_1^2 \frac{1}{\mu_2(y)}\left(\int_1^y \xi_2(s)ds\right)dy\right],$$

where

$$K_1 = \left[\gamma_2\mu_1(1)\int_0^1 \frac{dy}{\mu_1(y)} + \gamma_1\mu_2(1)\int_1^2 \frac{dy}{\mu_2(y)}\right]^{-1}\times$$

$$\times\left[\alpha\mu_2(1)\int\limits_1^2\frac{dy}{\mu_2(y)}+\gamma_2\int\limits_1^2\frac{1}{\mu_2(y)}\left(\int\limits_1^y\xi_2(s)ds\right)dy+\gamma_2\int\limits_0^1\frac{1}{\mu_1(y)}\left(\int\limits_y^1\xi_1(s)ds\right)dy\right],$$

$$K_2=\left[\gamma_2\mu_1(1)\int\limits_0^1\frac{dy}{\mu_1(y)}+\gamma_1\mu_2(1)\int\limits_1^2\frac{dy}{\mu_2(y)}\right]^{-1}\times$$

$$\times\left[\alpha\mu_1(1)\int\limits_0^1\frac{dy}{\mu_1(y)}+\gamma_1\int\limits_1^2\frac{1}{\mu_2(y)}\left(\int\limits_1^y\xi_2(s)ds\right)dy-\gamma_1\int\limits_0^1\frac{1}{\mu_1(y)}\left(\int\limits_y^1\xi_1(s)ds\right)dy\right].$$

Taking conditions (i) - (viii), we have

$$\|u(\cdot)\|_{p,1}+\|v(\cdot)\|_{p,2}+|\phi|\le c\left[\|\xi_1(\cdot)\|_{p,1}+\|\xi_2(\cdot)\|_{p,2}+|\alpha|\right],$$

i.e.

$$\|w\|_{X_p}\le c\,\|Aw\|_{X_p},\quad w\in D(A).$$

Thus, $\lambda=0$ belongs to the resolvent set of the operator A. Therefore, by virtue of (24)

$$\Re=\{\lambda:\ Re\,\lambda+\eta\,|Im\,\lambda|>0\}$$

is contained in the resolvent set of the linear operator A and there exists an $M>0$, such that

$$\left\|(\lambda+A)^{-1}\right\|_{X_p\to X_p}\le\frac{M}{|\lambda|},\lambda\in\Re,$$

i.e., A is a sectorial operator in X_p (see[20,21]).

4 | PROOF OF THE LEMMA 2

By the definition of the operator B and the norm in X_p, we have

$$\|Bw\|_{X_p}=\|\eta_1(x)u_{xx}(x)\|_{p,1}+\|\eta_2(x)v_{xx}(x)\|_{p,2}+\left|\beta_1u_x(1)+\beta_2v_x(1)\right|.$$

Further, taking into account conditions (iii), (v) we obtain that

$$\|Bw\|_{X_p}\le c[\|u_{xx}(x)\|_{p,1}+\|v_{xx}(x)\|_{p,2}+|u_x(1)|+|v_x(1)|].\tag{42}$$

By virtue of embedding theorems

$$|u_x(1)|\le c\,\|u\|_{W_p^2(0,1)}\le c\,\|w\|_{Y_p},\tag{43}$$

$$|v_x(1)|\le c\,\|v\|_{W_p^2(1,2)}\le c\,\|w\|_{Y_p}.\tag{44}$$

It follows from (42) - (44) that the linear operator B is a bounded operator acting from Y_p in X_p.

5 | MIXED PROBLEM WITH CONDITION OF DYNAMICAL FUNCTIONAL TRANSMISSION FOR A ONE-DIMENSIONAL WAVE EQUATION WITH STRONG DISSIPATION

In the domain $Q_T=[0,T]\times[0,2]$ we consider the mixed problem:

$$u_{tt}-(\mu_1(x)u_x)_{xt}-\eta_1(x)u_{xx}=f(t,x),\quad 0\le t\le T,\quad 0\le x\le 1,\tag{45}$$

$$v_{tt}-(\mu_2(x)v_x)_{xt}-\eta_2(x)v_{xx}=g(t,x),\quad 0\le t\le T,\quad 1\le x\le 2\tag{46}$$

with the boundary conditions

$$u(t,0)=0,\quad v(t,2)=0,\tag{47}$$

the transmission conditions

$$u(t,1) = v(t,1) = \phi(t), \tag{48}$$

$$\phi_{tt}(t) + \gamma_1 u_{xt}(t,1) - \gamma_2 v_{xt}(t,1) + K[u(t,\cdot), v(t,\cdot)] = h(t) \tag{49}$$

and the initial conditions

$$u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), \tag{50}$$

$$\phi_t(0) = \phi_1, \tag{51}$$

where $K[\cdot,\cdot]$ linear continuous functional, acting from Y_p in R (see [19]).
The mixed problem (45) - (51) can be written as the following Cauchy problem in the space X_p:

$$w'' + Aw' + B_1 w = F(w) + G(t), \tag{52}$$

$$w(0) = w_0, \quad w'(0) = w_1, \tag{53}$$

where

$$\begin{aligned} D(B_1) = Y_p = \Big\{ & w : w = (u, v, \phi),\ u \in W_p^2(0,1) \bigcap W_p^1((0,1);0), \\ & v \in W_p^2(1,2) \bigcap W_p^1((1,2);2),\ u(1) = v(1) = \phi \Big\}, \\ & B_1 w = (-\eta_1(x)u_{xx}(x),\ -\eta_2(x)v_{xx}(x),\ K[u(\cdot), v(\cdot)]), \\ & w = (u, v, \phi). \end{aligned}$$

Thus, it is B_1 which a linear bounded operator from Y_p in X_p, then for problem (44) - (50) all the statements of Theorem 1 and Theorem 2 are valid.

Note that instead of $K[u(\cdot), v(\cdot)]$ we can take an arbitrary functional. For example, a functional like the following

$$K[u(\cdot), v(\cdot)] = \sum_{k=0}^{1} a_k \int_0^1 D_x^k u(x)dx + \sum_{k=0}^{1} b_j \int_1^2 D_x^j v(x)dx, \quad a_k, b_j \in R, \quad k = 0,1, \quad j = 0,1$$

that satisfying all the requirements of Theorem 1 and Theorem 2.

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AUTHOR BIOGRAPHY

A.B. Aliev Professor Akbar A. Aliyev is the head of the Department of Differential Equations of Azerbaijan National Academy of Sciences. He works in in the field of partial differential equations. His main research is defining global solvability for nonlinear hyperbolic equations and the study of the asymptotes of those solutions. He has achieved a number of results in obtaining Fudcita-type criteria that determine the presence and absence of global solutions for semilinear hyperbolic systems. More recently, he has been conducting research to determine the presence and absence of global solutions to complex problems for semilinear hyperbolic systems in a mixed problem with dynamic transmission conditions.

G.Kh. Shafieva Docent of Baku State University, fac. of Mathematics and Mechanics, Ph.D. More recently, G.Kh. Shafieva carried out research on the discovery of the existence and uniqueness of classical solutions of inverse boundary value problems for pseudoparabolic equations. She also investigated the continuous dependence of the obtained solutions on the initial data. The modern sphere of scientific activity of G.Kh. Shafieva - investigation of a mixed problem for nonlinear hyperbolic equations, proof of the existence and uniqueness of local and global solutions of the problems under consideration, as well as investigation of the asymptotes of these solutions.

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