A quest of G-continuity in neutrosophic spaces

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1. Introduction

In recent years new technologies are rapidly developing, and for this development many branches of science, including significant contributions of mathematics, work together. Especially, the topics such as logic, data mining, artificial intelligence, quantum physics, machine learning come to the forefront. In this context, the concept of neutrosophic set is a solution to the problems in various fields of the real life.

It is widely known that the concept of continuity and any concept related to continuity have a great importance not only for pure mathematics but also for many other branches of science involving mathematics such as computer science, information theory, biological science and dynamical systems. Many scientists focused on continuity and introduced numerous concepts related to it. These concepts have always been indispensable parts in many studies. Sequential continuity has always been among the significant ones of these concepts. In [1], Connor and Grosse-Erdmann changed the definition of the convergence of sequences on the structure of sequential continuity. Furthermore, Cakalli [2] extended this concept to topological group-valued sequences, gave theorems in this generalized setting some of which were new not only in the setting of topological groups, but also new in the real case (see also [3, 4, 5, 6]). In this paper, our purpose is to extend these ideas to a neutrosophic topological space and make some investigations in this direction.

2. Preliminaries

In this section, we present basic definitions related to neutrosophic set theory and neutrosophic topological spaces. Our notation and terminology are standard and follow mainly the papers [7, 8].

Definition 2.1. ([8]) A *neutrosophic set A* on the universe set X is defined as:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\},$$

where $T, I, F : X \to]^-0, 1^+[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$. T, I and F are called the membership function, indeterminacy function and non-membership function, respectively.

Membership functions T, indeterminacy functions I and non-membership functions F of neutrosophic sets take value from real standard or nonstandard subsets of $]^-0, 1^+[$. However, these subsets are sometimes inconvenient to be used in real life applications such as economic and engineering problems. On account of this fact, we consider the neutrosophic sets, whose membership function, indeterminacy function and non-membership function take values from subsets of $[0, 1]$.

Definition 2.2. ([9]) Let X be a nonempty set. If r, t, s are real standard or non standard subsets of $]^{-}0,1^{+}[$, then the neutrosophic set

$$x_{r,t,s}(x_p) = \begin{cases} (r,t,s), & \text{if } x = x_p \\ (0,0,1), & \text{if } x \neq x_p \end{cases}$$

is called a *neutrosophic point*. $x_p \in X$ is called the *support* of $x_{r,t,s}$, and r denotes the degree of membership value, t the degree of indeterminacy and s the degree of non-membership value of $x_{r,t,s}$.

Definition 2.3. ([7]) Let A be a neutrosophic set over the universe set X. The *complement* of A is denoted by A^c and is defined by:

$$A^c = \left\{\langle x,\ F_{\widetilde{F}(e)}(x), 1\ -\ I_{\widetilde{F}(e)}(x),\ T_{\widetilde{F}(e)}(x)\rangle : x \in X\right\}$$

It is obvious that $[A^c]^c = A$.

Definition 2.4. ([7]) Let A and B be two neutrosophic sets over the universe set X. A is said to be a *neutrosophic subset* of B if

$$T_{A(x)} \leq T_{B(x)}, I_{A(x)} \leq I_{B(x)}, F_{A(x)} \geq F_{B(x)}, \forall x \in X.$$

It is denoted by $A \subseteq B$. A is said to be neutrosophic equal to B if $A \subseteq B$ and $B \subseteq A$. It is denoted by $A = B$.

Definition 2.5. ([7]) Let A_1 and A_2 be two neutrosophic soft sets over the universe set X. Then:

(1) the *union* of A_1 and A_2, denoted by $A_1 \cup A_2 = A_3$, is defined by

$$A_3 = \left\{\left\langle x,\ T_{A_3}(x),\ I_{A_3}(x),\ F_{A_3}(x)\right\rangle : x \in X\right\},$$

where

$$T_{A_3}(x) = \max\{T_{A_1}(x), T_{A_2}(x)\},$$
$$I_{A_3}(x) = \max\{I_{A_1}(x), I_{A_2}(x)\},$$
$$F_{A_3}(x) = \min\{F_{A_1}(x), F_{A_2}(x)\}.$$

(2) the *intersection* of A_1 and A_2, denoted by $A_1 \cap A_2 = A_3$ is defined by

$$A_3 = \left\{\left\langle x,\ T_{A_3}(x),\ I_{A_3}(x),\ F_{A_3}(x)\right\rangle : x \in X\right\},$$

where

$$T_{A_3}(x) = \min\{T_{A_1}(x), T_{A_2}(x)\},$$
$$I_{A_3}(x) = \min\{I_{A_1}(x), I_{A_2}(x)\},$$
$$F_{A_3}(x) = \max\{F_{A_1}(x), F_{A_2}(x)\}.$$

Definition 2.6. ([7]) A neutrosophic set A over the universe set X is said to be

(1) the *null neutrosophic set*, denoted by 0_X, if $T_A(x) = 0,\ I_A(x) = 0,\ F_A(x) = 1,\ \forall x \in X$;

(2) the *absolute neutrosophic set*, denoted by 1_X, if $T_A(x) = 1,\ I_A(x) = 1,\ F_A(x) = 0,\ \forall x \in X$.

Clearly $0_X^c = 1_X$ and $1_X^c = 0_X$.

Definition 2.7. ([7]) The collection τ of the neutrosophic sets over the universe set X is said to be a *neutrosophic topology* on X, if

1.0_X and 1_X belong to τ;
2. The union of any subcollection of τ belongs to τ;
3. The intersection of a finite number of sets in τ belongs to τ.

Then (X,τ) is said to be a *neutrosophic topological space* over X. Each member of τ is said to be a *neutrosophic open set*, and its complement is said to be a *neutrosophic closed set*.

Note. In what follows (X,τ) and (Y,σ) will denote neutrosophic topological spaces. Sometime, when it is clear from the context, we write simply X and Y instead of (X,τ) and (Y,σ).

3. New Definitions

In the following, we give several new definitions that will be required in the next section.

Definition 3.1. (1) A neutrosophic point $x_{r,t,s}$ is said to be *neutrosophic quasi-coincident* (neutrosophic q-coincident, for short) with a neutrosophic set A, denoted by $x_{r,t,s}qA$ if $x_{r,t,s} \nsubseteq A^c$. If $x_{r,t,s}$ is not neutrosophic quasi-coincident with A, we write $x_{r,t,s}\tilde{q}A$.

(2) A neutrosophic set A is said to be *neutrosophic quasi-coincident* (neutrosophic q-coincident, for short) with B, denoted by AqB if $A \nsubseteq B^c$. If A is not neutrosophic quasi-coincident with B, we denote it by $A\tilde{q}B$.

Definition 3.2. Let (X,τ) be a neutrosophic topological space. Then:

(1) A neutrosophic set A in (X,τ) is said to be a *neutrosophic q-neighborhood* of a neutrosophic point $x_{r,t,s}$ if there exists a neutrosophic open set B such that $x_{r,t,s}qB \subset A$.

(2) A neutrosophic point $x_{r,t,s}$ in (X,τ) is said to be a *neurosophic cluster point* of a neutrosophic set A if every neutrosophic open q-neighborhood B of $x_{r,t,s}$ is q-coincident with A. The union of all neutrosophic cluster points of A is called the *neutrosophic closure* of A and is denoted by $\overline{A}$.

(3) A neutrosophic point $x_{r,t,s}$ is said to be a *neurosophic boundary point* of a neutrosophic set F if every neutrosophic open q-neighborhood G of $x_{r,t,s}$ is q-coincident with F and F^c. The union of all neutrosophic boundary points of F is called the *neutrosophic boundary* of F and denoted by F^b.

Definition 3.3. Let (X,τ) be a neutrosophic topological space.

(1) A *neutrosophic sequence* in (X,τ) is a function $S:\mathbb{N}\to(X,\tau)$ from the set $\mathbb{N}$ of natural numbers to (X,τ). We write $\mathbf{x}=\{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ to denote a sequence in (X,τ).

(2) A *neutrosophic subsequence* of a neutrosophic sequence $S:\mathbb{N}\to(X,\tau,E)$ is a composition $S\circ P$, where $P:\mathbb{N}\to\mathbb{N}$ is an increasing cofinal function. That is,

(a) $P(n_1)\le(n_2)$, whenever $n_1\le n_2$ (P is increasing),

(b) For each $n_1\in\mathbb{N}$, there exists a naturel number $n_2\in\mathbb{N}$ such that $n_1\le P(n_2)$ (P is cofinal in N). For $k\in\mathbb{N}$, the neutrosophic point $(S\circ P)(k)$ will often be written $\left\{x_{n_{r_n,t_n,s_n}}\right\}$.

(3) A neutrosophic sequence $\mathbf{x}=\{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ in (X,τ) *converges to a neutrosophic point* $x_{r,t,s}$ in (X,τ) (written $x_{n_{r_n,t_n,s_n}}\to x_{r,t,s}$) if for each neutrosophic q-neighbourhood U of $x_{r,t,s}$ there exists $n_0\in\mathbb{N}$ such that $x_{n_{r_n,t_n,s_n}}qU$ for all $n\ge n_0$.

(4) We will use boldface letters $\mathbf{x}$, $\mathbf{y}$, … for neutrosophic sequences. By $\mathsf{s}(\mathsf{X})$ and $\mathsf{c}(\mathsf{X})$ we denote the set of all neutrosophic sequences in (X,τ) and the set of all convergent neutrosophic sequences in (X,τ), respectively.

Definition 3.4. Let (X,τ) be a neutrosophic topological space.

(1) (X,Δ) is a *neutrosophic group* on X if Δ is a binary operation defined on X such that the following conditions hold:

(a) Associativity: For all neutrosophic points $x1_{r_1,t_1,s_1}$, $x2_{r_2,t_2,s_2}$, $x3_{r_3,t_3,s_3}$ in (X,τ), we have

$$x1_{r_1,t_1,s_1}\,\Delta\left(x2_{r_2,t_2,s_2}\,\Delta\, x3_{r_3,t_3,s_3}\right)=\left(x1_{r_1,t_1,s_1}\,\Delta x2_{r_2,t_2,s_2}\right)\Delta\, x3_{r_3,t_3,s_3}.$$

(b) Identity: There exists an identity neutrosophic point $e_{\alpha,\beta,\gamma}$ in (X,τ) such that

$$x1_{r_1,t_1,s_1}\,\Delta\, e_{\alpha,\beta,\gamma}=e_{\alpha,\beta,\gamma}\,\Delta\, x1_{r_1,t_1,s_1}=x1_{r_1,t_1,s_1}$$

for any neutrosophic point $x1_{r_1,t_1,s_1}$ in (X,τ).

(c) Inverse: For any neutrosophic point $x1_{r_1,t_1,s_1}$ in (X,τ), there exists an inverse neutrosophic point $\left(x1_{r_1,t_1,s_1}\right)^{-1}$ in (X,τ) such that

$$x_{1_{r_1,t_1,s_1}} \Delta \left(x_{1_{r_1,t_1,s_1}}\right)^{-1} = e_{\alpha,\beta,\gamma} \text{ and } \left(x_{1_{r_1,t_1,s_1}}\right)^{-1} \Delta x_{1_{r_1,t_1,s_1}} = e_{\alpha,\beta,\gamma}.$$

(2) Let $(\mathsf{s}(\mathsf{X}),*)$ be the group of neutrosophic sequences in X, and (X,Δ) be a neutrosophic group. A *neutrosophic method* is a function G defined on a subgroup $(\mathsf{c}_\mathsf{G}(\mathsf{X}),*)$ of $(\mathsf{s}(\mathsf{X}),*)$ such that $G(\mathbf{x}*\mathbf{y}) = G(\mathbf{x})\Delta G(\mathbf{y})$ for all neutrosophic convergent sequences $\mathbf{x}$, $\mathbf{y}$ in (X,τ).

(3) A neutrosophic sequence $\mathbf{x} = \left\{x_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ is said to be *G-convergent* to $x_{r,t,s}$, if $\mathbf{x} \in \mathsf{c}_\mathsf{G}(\mathsf{X})$ and $G(\mathbf{x}) = x_{r,t,s}$.

(4) A neutrosophic method G is called *neutrosophic regular* if every convergent neutrosophic sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ is G-convergent with $G(\mathbf{x}) = x_{r,t,s}$, where $\mathbf{x}$ converges to $x_{r,t,s}$.

If (X,Δ) is a neutrosophic group on a neutrosophic topological space (X,τ), and A, B neutrosophioc sets in (X,τ), then

$$A\Delta B = \{x_{1_{r_1,t_1,s_1}} : x_{1_{r_1,t_1,s_1}} = x_{2_{r_2,t_2,s_2}} \Delta\, x_{3_{r_3,t_3,s_3}},\ x_{2_{r_2,t_2,s_2}} \in A,\ x_{3_{r_3,t_3,s_3}} \in B\}$$

and

$$A^{-1} = \{(x_{r,t,s})^{-1} : x_{r,t,s} \in A\}.$$

Definition 3.5. A neutrosophic point $x_{r,t,s}$ is said to be a *neutrosophic sequential cluster point* of a neutrosophic set A if there exists a neutrosophic sequence of neutrosophic points in A that converges to $x_{r,t,s}$. The union of all neutrosophic sequential cluster points of A is called the *neutrosophic sequential closure* of A and denoted by $\overline{F}^{seq}$.

Definition 3.6. Let f be a function from (X,τ) to (Y,σ). Then for a neutrosophic sets A in X and B in Y we have:

(1) the *image* of A under f, written as $f(A)$, is a neutrosophic subset of Y whose membership function, indeterminacy function and non-membership function are defined as

$$T_{f(A)}(y) = \begin{cases} \sup_{z\in f^{\leftarrow}(y)}\{T_A(z)\}, & \text{if } f^{\leftarrow}(y) \text{ is not empty,} \\ 0, & \text{if } f^{\leftarrow}(y) \text{ is empty,} \end{cases}$$

$$I_{f(A)}(y) = \begin{cases} \sup_{z\in f^{\leftarrow}(y)}\{I_A(z)\}, & \text{if } f^{\leftarrow}(y) \text{ is not empty,} \\ 0, & \text{if } f^{\leftarrow}(y) \text{ is empty,} \end{cases}$$

$$F_{f(A)}(y) = \begin{cases} \inf_{z\in f^{\leftarrow}(y)}\{F_A(z)\}, & \text{if } f^{\leftarrow}(y) \text{ is not empty,} \\ 1, & \text{if } f^{\leftarrow}(y) \text{ is empty} \end{cases}$$

for all y in Y (where $f^{\leftarrow}(y) = \{x : f(x) = y\}$).

(2) the *inverse image* (or *preimage*) of B under f, written as $f^{\leftarrow}(B)$, is a neutrosophic subset of X defined as

$$\begin{array}{c} T_{f^{\leftarrow}(B)}(x) = T_B(f(x)), \\ I_{f^{\leftarrow}(B)}(x) = I_B(f(x)), \\ F_{f^{\leftarrow}(B)}(x) = F_B(f(x)) \end{array}$$

for all x in X.

4. Sequental definitions of continuity in neutrosophic topological spaces

In this section, we introduce some new concepts and investigate their properties. Also, we give several counterexamples.

Definition 4.1. Let A be a neutrosophic set and $x_{r,t,s}$ be a neutrosophic point in (X,τ). Then, $x_{r,t,s}$ is in the *neutrosophic G- sequential closure of* A (or *neutrosophic G-hull of* A), if there is a neutrosophic sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ of neutrosophic points in A such that $G(\mathbf{x}) = x_{r,t,s}$. We denote neutrosophic G-sequential closure of a neutrosophic set A by $\overline{A}^G$.

We say that a neutrosophic set A is *G-sequentially neutrosophic closed* if it contains all the neutrosophic points in its neutrosophic G-closure, that is if $\overline{A}^G \subseteq A$.

It is clear that $\overline{\emptyset}^G = 0_X$ and $\overline{X}^G = 1_X$. If G is a neutrosophic regular method, then $A \subset \overline{A} \subset \overline{A}^G$. Even for neutrosophic regular methods, it is not always true that $\overline{(\overline{A}^G)}^G = \overline{A}^G$ as the following example shows.

Example 4.2. Let $X = [0,1]$ and let τ be a neiutrosophic topology on X. Consider the neutrosophic method G defined by

$$G(\mathbf{x}) = u_{0.5,0.5,0.5}, \text{ where } u = \lim_{n\to\infty} \frac{x_n + x_{n+1}}{2}$$

for a neutrosophic sequence $\mathbf{x} = \left\{x_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$ in X. Further, consider a neutrosophic set $A = 0_{0.5,0.5,0.5} \cup 1_{0.5,0.5,0.5}$. Then

$$\overline{A}^G = 0_{0.5,0.5,0.5} \cup 0.5_{0.5,0.5,0.5} \cup 1_{0.5,0.5,0.5}$$

and

$$\overline{\left(\overline{A}^G\right)}^G = 0_{0.5,0.5,0.5} \cup 0.25_{0.5,0.5,0.5} \cup 0.5_{0.5,0.5,0.5} \cup 0.75_{0.5,0.5,0.5} \cup 1_{0.5,0.5,0.5}.$$

So, $\overline{\left(\overline{A}^G\right)}^G \neq \overline{A}^G$.

Definition 4.3. A subset A of X is called *neutrosophic G-sequentially compact* if for any neutrosophic sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ in A there is a subsequence $\mathbf{y} = \{x_{n_{r_{n_k},t_{n_k},s_{n_k}}}\}_{k\in\mathbb{N}}$ of $\mathbf{x}$ with $G(\mathbf{y}) \in A$.

Definition 4.4. A function $f : X \to Y$ is *neutrosophic G-sequentially continuous at a neutrosophic point $u_{r,t,s}$*, if for any sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ in X , $G(\mathbf{x}) = u_{r,t,s}$ implies $G(f(\mathbf{x})) = f(u_{r,t,s})$. We say that f is G-sequentially continuous on a neutrosophic subset D of X, if it is neutrosophic G-sequentially continuous at every $u_{r,t,s} \in D$ and is neutrosophic G-sequentially continuous if it is neutrosophic G-sequentially continuous on X.

Theorem 4.5. *The image of any neutrosophic G-sequentially compact subset of X under a neutrosophic G-sequentially continuous function is neutrosophic G-sequentially compact.*

Proof. Let f be any neutrosophic G-sequentially continuous function on X and A be any neutrosophic G-sequentially compact subset of X. Take any neutrosophic sequence $\mathbf{y} = \{y_{n_{r_{n_k},t_{n_k},s_{n_k}}}\}_{n\in\mathbb{N}} = \{f(x_{n_{r_{n_k},t_{n_k},s_{n_k}}})\}_{n\in\mathbb{N}}$ of neutrosophic points in $f(A)$. Since A is neutrosophic G-sequentially compact, there exists a subsequence $\mathbf{z} = \{x_{n_{r_{n_k},t_{n_k},s_{n_k}}}\}_{k\in\mathbb{N}}$ of $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ with $G(\mathbf{z}) \in A$. Then the sequence $f(\mathbf{z}) = \{f(x_{n_{r_{n_k},t_{n_k},s_{n_k}}})\}_{k\in\mathbb{N}}$ is a subsequence of the sequence $\mathbf{y}$. Since f is neutrosophic G-sequentially continuous, $G(f(\mathbf{z})) \in f(A)$. Thus, $f(A)$ is neutrosophic G-sequentially compact. $\square$

Definition 4.6. A neutrosophic method G is called *neutrosophic subsequential* if for any neutrosophic sequence $\mathbf{x}$ such that $G(\mathbf{x}) = x_{r,t,s}$, there exists a subsequence $\{x_{n_{r_{n_k},t_{n_k},s_{n_k}}}\}_{k\in\mathbb{N}}$ of $\mathbf{x}$ that converges to $x_{r,t,s}$.

Definition 4.7. Let $u_{r,t,s}$ be a neutrosophic point in (X,τ). Then:

(1) $u_{r,t,s}$ is in the *neutrosophic sequential derived set of A* (called also *sequential accumulation point of A*) if there exists a neutrosophic sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ of neutrosophic points in A such that where $x_{n_{r_n,t_n,s_n}} q(u_{r,t,s})^c$ for all $n \in \mathbb{N}$ and $x_{n_{r_n,t_n,s_n}} \to u_{r,t,s}$. We denote neutrosophic sequential derived set of a neutrosophic set A by A'.

(2) $u_{r,t,s}$ is in the *neutrosophic G-sequential derived set of A* (also called a *neutrosophic G-sequential accumulation point of A*) if there exists a neutrosophic sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ in A such that $x_{n_{r_n,t_n,s_n}} q(u_{r,t,s})^c$ for all $n \in \mathbb{N}$ and $G(\mathbf{x}) =$

$u_{r,t,s}$. We denote neutrosophic G-sequential derived set of a neutrosophic set A by $(A')^G$.

(3) $u_{r,t,s}$ is a *neutrosophic G-sequential boundary point* of a neutrosophic set A if $u_{r,t,s}$ lies in both the neutrosophic G-sequential closure of A and neutrosophic G-sequential closure of the complement of A. We denote neutrosophic G-sequential boundary set of A by $((A)^b)^G$.

The following three theorems give characterizations of a neutrosophic subsequential method.

Theorem 4.8. *Let G be a neutrosophic regular method and A be any neutrosophic subset of X. Then* $\overline{A}^G = \overline{A}$ *if and only if G is a neutrosophic subsequential method, where* $\overline{A}$ *denotes the usual closure of A.*

Proof. Suppose that G is a neutrosophic subsequential method and that $a_{r,t,s} \in \overline{A}^G$. Then, there is a neutrosophic sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ in A such that $G(\mathbf{x}) = a_{r,t,s}$. As G is a neutrosophic subsequential method, there is a subsequence $\{x_{n_{r_{n_k},t_{n_k},s_{n_k}}}\}_{k\in\mathbb{N}}$ of $\mathbf{x}$ such that $x_{n_{r_{n_k},t_{n_k},s_{n_k}}} \to a_{r,t,s}$, and hence $a_{r,t,s} \in \overline{A}$. As G is neutrosophic regular, it follows that $\overline{A}^G = \overline{A}$.

Conversely, let $\overline{A}^G = \overline{A}$ for every neutrosophic subset A of X. Let $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ be a neutrosophic G-convergent sequence with $G(\mathbf{x}) = a_{r,t,s}$. Since G is a neutrosophic regular method, $a_{r,t,s} \in \overline{\bigcup_{n\geq m} x_{n_{r_n,t_n,s_n}}}^G$ for any $m \in \mathbb{N}$. As $\overline{\bigcup_{n\geq m} x_{n_{r_n,t_n,s_n}}}^G = \overline{\bigcup_{n\geq m} x_{n_{r_n,t_n,s_n}}}$, it follows that $a_{r,t,s} \in \bigcap_m \overline{\bigcup_{n\geq m} x_{n_{r_n,t_n,s_n}}}$. Hence, there is a neutrosophic subsequence $\{x_{n_{r_{n_k},t_{n_k},s_{n_k}}}\}_{k\in\mathbb{N}}$ of $\mathbf{x}$ such that $x_{n_{r_{n_k},t_{n_k},s_{n_k}}} \to a_{r,t,s}$. □

Theorem 4.9. *Let G be a neutrosophic regular method. Then, G is a neutrosophic subsequential method if and only if* $A' = (A')^G$ *for every neutrosophic subset A of X.*

Proof. First, suppose that $A' = (A')^G$ for every neutrosophic subset A of X. Then $\overline{A} = A \cup A' = A \cup (A')^G = \overline{A}^G$. It follows from Theorem 4.8 that G is a neutrosophic subsequential method.

Now suppose that G is a neutrosophic subsequential method and take any neutrosophic subset A of X. Let $u_{r,t,s}$ be any neutrosophic point in A'. Then there is a neutrosophic sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ of neutrosophic points in A such that $x_{n_{r_n,t_n,s_n}} q (u_{r,t,s})^c$ for all $n \in \mathbb{N}$ and $x_{n_{r_n,t_n,s_n}} \to x_{r,t,s}$. As G is regular, $G(\mathbf{x}) = x_{r,t,s}$. Hence, $x_{r,t,s} \in (A')^G$. To prove that $(A')^G \subset A'$, take any neutrosophic point $u_{r,t,s}$ in $(A')^G$. Then there is a neutrosophic sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ of neutrosophic points in A whose supports are different from $u_{r,t,s}$ such that $G(\mathbf{x}) = u_{r,t,s}$. As G is a neutrosophic subsequential method, there is a neutrosophic subsequence (x_{n_k}) of $\mathbf{x}$ with $x_{n_{r_{n_k},t_{n_k},s_{n_k}}} \to u_{r,t,s}$. Hence $u_{r,t,s} \in A'$. This completes the proof. □

Theorem 4.10. *Let G be a neutrosophic regular method. Then G is a neutrosophic subsequential method if and only if* $A^b = (A^b)^G$ *for every neutrosophic subset A of X.*

Proof. Firstly, suppose that $A^b = (A^b)^G$ for every neutrosophic subset A of X. Then $\overline{A} = A \cup A^b = A \cup (A^b)^G = \overline{A}^G$. It follows from Theorem 4.8 that G is a neutrosophic subsequential method.

Suppose now that G is a neutrosophic subsequential method and take any neutrosophic subset A of X. Let $u_{r,t,s}$ be any neutrosophic point in A^b. Then $u_{r,t,s}$ is in both $\overline{A}$ and $\overline{A^c}$. Hence, there exist a neutrosophic sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ of neutrosophic points in A and a neutrosophic sequence $\mathbf{y} = \{y_{n_{p_n,v_n,m_n}}\}_{n\in\mathbb{N}}$ of neutrosophic points in A^c such that

$$x_{n_{r_n,t_n,s_n}} \to u_{r,t,s} \text{ and } y_{n_{p_n,v_n,m_n}} \to u_{r,t,s}.$$

As G is neutrosophic regular, $G(\mathbf{x}) = u_{r,t,s}$ and $G(\mathbf{y}) = u_{r,t,s}$, hence $u_{r,t,s} \in (A^b)^G$.

To prove that $(A^b)^G \subset A^b$, take any neutrosophic point $u_{r,t,s}$ of $(A^b)^G$. Then $u_{r,t,s}$ is in both $\overline{A}$ and $\overline{A^c}$. Hence there exist a neutrosophic sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ in A and a neutrosophic sequence $\mathbf{y} = \{y_{n_{p_n,v_n,m_n}}\}_{n\in\mathbb{N}}$ in A^c such that $G(\mathbf{x}) = u_{r,t,s}$ and $G(\mathbf{y}) = u_{r,t,s}$. As G is a neutrosophic subsequential method, there are neutrosophic subsequences $\{x_{n_{k_{r_{n_k},t_{n_k},s_{n_k}}}}\}_{k\in\mathbb{N}}$ of $\mathbf{x}$ and $\{y_{n_{k_{p_{n_k},v_{n_k},m_{n_k}}}}\}_{k\in\mathbb{N}}$ of $\mathbf{y}$ such that

$$x_{n_{k_{r_{n_k},t_{n_k},s_{n_k}}}} \to u_{r,t,s} \text{ and } y_{n_{k_{p_{n_k},v_{n_k},m_{n_k}}}} \to u_{r,t,s}.$$

Hence, $u_{r,t,s} \in A^b$. This completes the proof. □

Theorem 4.11. *Let (X,τ) be a neutrosophic topological space and (X,Δ) be a neutrosophic group whose identity neutrosophic point is $e_{\alpha,\beta,\gamma}$. If G is a neutrosophic regular method, then for any neutrosophic sets A, B in (X,τ) the following are satisfied:*

(1) *If $A \subset B$, then $\overline{A}^G \subset \overline{B}^G$;*

(2) $\overline{A}^G \cup \overline{B}^G \subset \overline{A\cup B}^G$;

(3) $\overline{A\cap B}^G \subset \overline{A}^G \cap \overline{B}^G$;

(4) $\overline{A}^G \Delta \overline{B}^G \subset \overline{A\Delta B}^G$;

(5) $\overline{A^{-1}}^G = (\overline{A}^G)^{-1}$;

(6) *Neutrosophic G-sequential closure of a neutrosophic subgroup of (X,Δ) is also a neutrosophic subgroup of (X,Δ);*

(7) *$A\Delta\overline{e_{\alpha,\beta,\gamma}} = A$, if A is neutrosophic G-closed.*

Proof. We will prove only (2) and (4) because the proofs of other properties are similar and easy.

(2) Take any element $u_{r,t,s}$ of $\overline{A}^G \cap \overline{B}^G$. Then $u_{r,t,s}$ is either in $\overline{A}^G$ or in $\overline{B}^G$. Suppose $u_{r,t,s} \in \overline{A}^G$. There is a neutrosophic sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ in A, hence in $A\cup B$, with $G(\mathbf{x}) = u_{r,t,s}$. Hence, $u_{r,t,s}\overline{A\cap B}^G$.

(4) Let $u_{r,t,s} \in \overline{A}^G \Delta \overline{B}^G$. Hence, there exist $u1_{r_1,t_1,s_1} \in \overline{A}^G$ and $u2_{r_2,t_2,s_2} \in \overline{B}^G$ such that $u_{r,t,s} = u1_{r_1,t_1,s_1} \Delta u2_{r_2,t_2,s_2}$. Then, there are neutrosophic sequences $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ in A and $\mathbf{y} = \{y_{n_{p_n,v_n,m_n}}\}_{n\in\mathbb{N}}$ in B such that $G(\mathbf{x}) = u1_{r_1,t_1,s_1}$ and $G(\mathbf{y}) = u2_{r_2,t_2,s_2}$. Now define a sequence $\mathbf{z} = \mathbf{x}\Delta\mathbf{y}$. From the additivity of G, we get $G(\mathbf{z}) = G(\mathbf{x}\Delta\mathbf{y}) = G(\mathbf{x})\Delta G(\mathbf{y}) = u1_{r_1,t_1,s_1}\Delta u2_{r_2,t_2,s_2} \in \overline{A\Delta B}^G$. □

Example 4.12. Take the neutrosophic topological space (X,τ) and the neutrosophic method G as in Example 4.2. Consider neutrosophic sets $A = 0_{0.5,0.5,0.5}$ and $B = 1_{0.5,0.5,0.5}$. Then

$$\overline{A}^G \cup \overline{B}^G = 0_{0.5,0.5,0.5} \cup 1_{0.5,0.5,0.5} \text{ and } \overline{A\cup B}^G = 0_{0.5,0.5,0.5} \cup 0,5_{0.5,0.5,0.5} \cup 1_{0.5,0.5,0.5} .$$

This means that the converse inclusion in (2) of Theorem 4.11 is not always true.

Example 4.13. Take any neutrosophic topological space (X,τ). Consider a neutrosophic method G defined as $G(\mathbf{x}) = x1_{r_1,t_1,s_1}$ for any neutrosophic sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ in X. Then, any neutrosophic subset of X is neutrosophic G-sequentially closed. This shows that the neutrosophic G-sequential closure of a neutrosophic set may be different from the neutrosophic sequential closure of the set.

The proof of the following theorem is omitted because it is similar to the proof of Theorem 4.11.

Theorem 4.14. *Let G be a neutrosophic regular method and $\{A_i : i \in I\}$ be any collection of neutrosophic subsets of X , where I is an index set. Then the followings are satisfied:*

(1) $\bigcup_{i\in I}\overline{A_i}^G \subset \overline{\bigcup_{i\in I} A_i}^G$;

(2) $\overline{\bigcap_{i\in I} A_i}^G \subset \bigcap_{i\in I}\overline{A_i}^G$;

(3) $\overline{A_{i_1}}^G \Delta \overline{A_{i_2}}^G \Delta \overline{A_{i_3}}^G \Delta \ldots \Delta \overline{A_{i_k}}^G \Delta \ldots \subset \overline{A_{i_1} \Delta A_{i_2} \Delta A_{i_3} \Delta \ldots \Delta A_{i_k} \Delta \ldots}^G$,
where $i_k \in I$, $k \in \mathbb{N}$.

Theorem 4.15. *Let G be a neutrosophic regular method and A and B neutrosophic subsets of X. Then the following are satisfied:*
(1) *if $A \subset B$ then $(A')^G \subset (B')^G$;*
(2) $\overline{A}^G = A \cup (A')^G$*;*
(3) $\overline{A}^G = A \cup (A^b)^G$.

Proof. (1) The proof is easy and thus omitted.

(2) Take any neutrosophic point $u_{r,t,s}$ of $\overline{A}^G$. If $u_{r,t,s} \in A$, then it is in $A \cup (A')^G$. If $u_{r,t,s} \notin A$, there exists a neutrosophic sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n \in \mathbb{N}}$ of neutrosophic points in A such that $x_{n_{r_n,t_n,s_n}} q\, (u_{r,t,s})^c$ for all $n \in \mathbb{N}$ and $G(\mathbf{x}) = u_{r,t,s}$. Thus, $u_{r,t,s} \in (A')^G$.

On the other hand, we get $A \cup (A')^G \subset \overline{A}^G$ since $A \subset \overline{A}^G$ and $(A')^G \subset \overline{A}^G$.

(3) Let $u_{r,t,s} \in A \cup (A^b)^G$. If $u_{r,t,s}$ is in A, there is nothing to prove since $A \subset \overline{A}^G$ for a neutrosophic regular method G. If $u_{r,t,s} \in (A^b)^G$, then $u_{r,t,s} \in \overline{A}^G \cap \overline{(A^c)}^G$. Hence, $u_{r,t,s} \in \overline{A}^G$.

Conversely, take any neutrosophic point $u_{r,t,s}$ of $\overline{A}^G$. Thus, $u_{r,t,s} \tilde{q} A^c$ or $u_{r,t,s} q A^c$. If $u_{r,t,s} \tilde{q} A^c$, then $u_{r,t,s} \in A$ and there is nothing to prove. If $u_{r,t,s} q A^c$, $u_{r,t,s} \in \overline{A^c}$. Then, there exists a neutrosophic sequence $\mathbf{y} = \{y_{n_{p_n,v_n,m_n}}\}_{n \in mathbbN}$ in A^c such that $y_{n_{p_n,v_n,m_n}} \to u_{r,t,s}$. As G is neutrosophic regular, $G(\mathbf{y}) = u_{r,t,s}$. Hence $u_{r,t,s} \in (A^b)^G$. □

Remark 4.16. For a neutrosophic regular method G, a neutrosophic subset A of X is neutrosophic closed if and only if $(A')^G \subset A$. But, we note that $\left(((A')^G)'\right)^G$ is not always a neutrosophic subset of $A \cup (A')^G$.

Corollary 4.17. *Let G be a neutrosophic regular method. Then the intersection of any collection of neutrosophic G-sequentially closed subsets of X is again a neutrosophic G-sequentially closed subset of X.*

Notice that for neutrosophic regular methods G, the union of two neutrosophic G-sequentially closed subsets of X need not be a neutrosophic G-sequentially closed. Example 4.12 shows it.

Theorem 4.18. *Let G be a neutrosophic regular method. If a function $f : X \to Y$ is neutrosophic G-sequentially continuous at a neutrosophic point $u_{r,t,s}$, then $u_{r,t,s} \in \overline{A}^G$ implies $f(u_{r,t,s}) \in \overline{f(A)}^G$ for every neutrosophic subset A of X.*

Proof. Suppose that f is neutrosophic G-sequentially continuous at a neutrosophic point $u_{r,t,s}$. Let A be any neutrosophic subset of X and $u_{r,t,s} \in \overline{A}^G$. Then, there is a neutrosophic sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n \in \mathbb{N}}$ of neutrosophic points in A such that $G(\mathbf{x}) = u_{r,t,s}$. Since f is neutrosophic G-sequentially continuous at $u_{r,t,s}$k, $G(f(\mathbf{x})) = f(u_{r,t,s})$. Thus $f(u_{r,t,s}) \in \overline{f(A)}^G$. □

Corollary 4.19. *Let G be a neutrosophic regular method. If a neutrosophic function $f : X \to Y$ is neutrosophic G sequentially continuous, then $f(\overline{A}^G) \subset \overline{f(A)}^G$ for every neutrosophic subset A of X.*

For neutrosophic regular subsequential methods the converse of Theorem 4.18, as well as of Corollary 4.19 is also valid, i.e. a function f is neutrosophic G-sequentially continuous at a neutrosophic point $u_{r,t,s}$ if and only if the $u_{r,t,s} \in \overline{A}^G$ implies $f(u_{r,t,s}) \in \overline{f(A)}^G$, and a function f is neutrosophic G-sequentially continuous on X if and only if $f(\overline{A}^G) \subset \overline{f(A)}^G$ for every neutrosophic subset A of X.

Corollary 4.20. *Let G be a neutrosophic regular method. If a bijection $f : X \to Y$ is neutrosophic G-sequentially continuous on X, then $f((A')^G) \subset (f(A'))^G$ for every neutrosophic subset A of X.*

Proof. Take a neutrosophic point $f(u_{r,t,s})$ in $f((A')^G)$. This means that $u_{r,t,s} \in (A')^G$. So, there exists a neutrosophic sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ inn$A$ such that where $x_{n_{r_n,t_n,s_n}} q(u_{r,t,s})^c$ for all $n \in \mathbb{N}$ and $G(\mathbf{x}) = u_{r,t,s}$. Then, $f(\mathbf{x}) = \{f(x_{n_{r_n,t_n,s_n}}))\}_{n\in\mathbb{N}}$ is a neutrosophic sequence in $f(A)$. Since f is a bijection and neutrosophic G-sequentially continuous on X, $f(x_{n_{r_n,t_n,s_n}}))q(f(u_{r,t,s}))^c$ for all $n \in \mathbb{N}$ in $f(A)$ and $G(f(\mathbf{x})) = f(u_{r,t,s})$. Hence, $f(u_{r,t,s}) \in (f(A'))^G$. □

From this corollary and Theorem 4.18 we have the following.

Corollary 4.21. *Let G be a neutrosophic regular method. If a bijection f is neutrosophic G-sequentially continuous, then $f(((A)^b)^G) \subset ((f(A))^b)^G$ for every neutrosophic subset A of X.*

For neutrosophic regular subsequential methods the converse of Corollaries 4.20 and 4.21 is also true.

Theorem 4.22. *Let G be a neutrosophic regular method. If a function $f : X \to Y$ is neutrosophic G-sequentially continuous on X, then the preimage of any neutrosophic G-sequentially closed subset of Y is neutrosophic G-sequentially closed, i.e. $f^{\leftarrow}(U)$ is neutrosophic G-sequentially closed for every neutrosophic G-sequentially closed subset U of Y.*

Proof. Take a neutrosophic G-sequentially closed subset U of X. Let $V = f^{\leftarrow}(U)$ and suppose that $u_{r,t,s} \in \overline{V}^G$. Then, there exists a neutrosophic sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ of neutrosophic points in V such that $G(\mathbf{x}) = u_{r,t,s}$. Since $G(f(\mathbf{x})) = f(u_{r,t,s})$, $f(\mathbf{x})$ is a neutrosophic sequence of neutrosophic point is in U and since U is neutrosophic G-closed, we obtain that $f(u_{r,t,s}) \in U$. This implies that $u_{r,t,s} \in V$. Hence, $\overline{V}^G \subset V$. □

We omit the proof of the following simple result.

Theorem 4.23. *Let G be a neutrosophic regular subsequential method. Then every neutrosophic G-sequentially continuous function is neutrosophic continuous in the ordinary sense.*

Theorem 4.24. *Let G be a neutrosophic regular method. If every continuous function is neutrosophic G-sequentially continuous, then G is a neutrosophic subsequential method.*

Proof. Suppose that G is not a neutrosophic subsequential method. We are going to find a function which is neutrosophic continuous but not neutrosophic G-sequentially continuous. As G is not neutrosophic subsequential there is a neutrosophic subset A of X whose closure is a proper neutrosophic subset of its neutrosophic G-sequential closure. Take $u1_{r_1,t_1,s_1} \in \overline{A}^G \cap \overline{A^c}$ and $u2_{r_2,t_2,s_2} \in \overline{A}$. Define a function f as $f(x_{r,t,s}) = u2_{r_2,t_2,s_2}$ for all $x_{r,t,s} \in \overline{A}$ and $f(x) = u1_{r_1,t_1,s_1}$ for $x_{r,t,s} \in \overline{A^c}$. It is clear that f is not neutrosophic G-sequentially continuous and f is neutrosophic continuous in the ordinary sense. This completes the proof. □

Now we give the following definition.

Definition 4.25. A function f is called neutrosophic G-sequentially closed if $f(K)$ is neutrosophic G-sequentially closed for every neutrosophic G-sequentially closed neutrosophic subset K of X.

Theorem 4.26. *Let G be a neutrosophic regular method. A function f is G-sequentially closed if and only if $\overline{(f(A))}^G \subset f(\overline{A}^G)$ for every neutrosophic subset A of X.*

Proof. From the neutrosophic regularity of G, we have $A \subset \overline{A}^G$. Since f is neutrosophic G-sequentially closed, we deduce $\overline{(f(A))}^G \subset f(\overline{A}^G)$.

Now suppose that $\overline{(f(A))}^G \subset f(\overline{A}^G)$. Let C be any neutrosophic G-sequentially closed neutrosophic subset of X. Then $\overline{(f(C))}^G \subset f(\overline{C}^G) = f(C)$. Hence f is neutrosophic G-sequentially closed. This completes the proof. □

Theorem 4.27. *Let (X, Δ) be a neutrosophic group in a neutrosophic topological space (X, τ). Let G_1 and G_2 be two neutrosophic methods of neutrosophic sequential convergence with $c_{G_1}(X) = c_{G_2}(X)$. Then, $\overline{A}^{G_1 \Delta G_2} \subset \overline{A}^{G_1} \Delta \overline{A}^{G_2}$.*

Proof. Let $u_{r,t,s} \in \overline{A}^{G_1 \Delta G_2}$. Then there is a sequence $\mathbf{x} = \{x_{n_{r_n,t_n,s_n}}\}_{n\in\mathbb{N}}$ such that $(G_1 \Delta G_2)(\mathbf{x}) = u_{r,t,s}$. Hence $G_1(\mathbf{x}) \Delta G_2(\mathbf{x}) = u_{r,t,s}$. Write $G_1(\mathbf{x}) = u1_{r_1,t_1,s_1}$ and $G_2(\mathbf{x}) = u2_{r_2,t_2,s_2}$. Therefore $u1_{r_1,t_1,s_1} \in \overline{A}^{G_1}$ and $u2_{r_2,t_2,s_2} \in \overline{A}^{G_2}$. Hence $u_{r,t,s} = u1_{r_1,t_1,s_1} \Delta u2_{r_2,t_2,s_2} \in \overline{A}^{G_1} \Delta \overline{A}^{G_2}$. □

Corollary 4.28. *Let G_1 and G_2 be two subsequential methods of sequential convergence with $c_{G_1}(X) = c_{G_2}(X)$. Then:*

(1) $(A')^{G_1+G_2} \subset (A')^{G_1} + (A')^{G_2}$;
(2) $(A^b)^{G_1+G_2} \subset (A^b)^{G_1} + (A^b)^{G_2}$.

Theorem 4.29. *Let (X, Δ) be a neutrosophic group in a neutrosophic topological space (X, τ). A function f on (X, τ) is additive if and only if for every methods G_1 and G_2 of sequential convergence with $c_{G_1}(X) = c_{G_2}(X)$, G_1-sequential continuity and G_2-sequential continuity of f together imply $(G_1 \Delta G_2)$-sequential continuity of f.*

Proof. Let $(G_1 \Delta G_2)(\mathbf{x}) = u_{r,t,s}$. Write $G_1(\mathbf{x}) = u1_{r_1,t_1,s_1}$ and $G_2(\mathbf{x}) = u2_{r_2,t_2,s_2}$. As f is G_1-continuous and G_2-continuous, $G_1(f(\mathbf{x})) = f(u1_{r_1,t_1,s_1})$ and $G_2(f(\mathbf{x})) = f(u2_{r_2,t_2,s_2})$. Hence

$$(G_1 \Delta G_2)(f(\mathbf{x})) = G_1(f(\mathbf{x}))\Delta G_2(f(\mathbf{x})) = f(u1_{r_1,t_1,s_1})\Delta f(u2_{r_2,t_2,s_2}) = f(u1_{r_1,t_1,s_1} \Delta u2_{r_2,t_2,s_2}) = f(u_{r,t,s}).$$

Now suppose that f is not additive. There are elements $u1_{r_1,t_1,s_1}$ and $u2_{r_2,t_2,s_2}$ with

$$f(u1_{r_1,t_1,s_1} \Delta u2_{r_2,t_2,s_2}) \neq f(u1_{r_1,t_1,s_1})\Delta f(u2_{r_2,t_2,s_2}).$$

Define $G_1(\mathbf{x}) = u1_{r_1,t_1,s_1}$ and $G_2(\mathbf{x}) = u2_{r_2,t_2,s_2}$, where $\mathbf{x} = \left\{x_{n_{r_n,t_n,s_n}}\right\}_{n\in N}$. Then

$$(G_1 \Delta G_2)(\mathbf{x}) = G_1(\mathbf{x})\Delta G_2(\mathbf{x}) = u1_{r_1,t_1,s_1} \Delta u2_{r_2,t_2,s_2}$$

but

$$(G_1 \Delta G_2)(f(\mathbf{x})) = f(u1_{r_1,t_1,s_1})\Delta f(u2_{r_2,t_2,s_2})$$

which is different from $f(u1_{r_1,t_1,s_1} \Delta u2_{r_2,t_2,s_2})$. This completes the proof. □

5. Conclusion

We introduced the concept of neutrosophic G-sequential continuity. The definitions of neutrosophic sequence, neutrosophic quasi-coincidence, neutrosophic q-neighborhood, neutrosophic cluster point, neutrosophic boundary point, neutrosophic sequential closure, neutrosophic group, neutrosophic method are also given. Using these definitions, we define the concepts of neutrosophic G- sequential closure and neutrosophic G-sequential derived set. Additionally, the concept of G-sequentially neutrosophic compactness is introduced. Their properties are analyzed and some implications are given. It is also shown by counterexamples that the converse statements of these implications are not always true.

Since topological structures of neutrosophic sets carry great importance for numerous mathematicians, various concepts related to the other types of topological spaces, which constitute advantageous situations in different fields, have been adapted to neutrosophic topological spaces. Our expectation is that many scientists will take advantage of using these detections to advance their research in mathematics and also in different disciplines that apply mathematical methods. We also hope that these findings may constitute a general framework for their applications in practical life.

Conflict of interests

The authors declare that there is no conflict of interests.

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