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RESEARCH ARTICLE

New intregral transform with Generalized Bessel-Maitland function kernel and its applications

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Summary

In this paper, authors introduce the generalized Bessel-Maitland transform whose kernel is the generalized Bessel-Maitland function. New identities are obtained for special cases of the generalized Bessel-Maitland function. Using these relations, several identities are obtained for generalized Bessel-Maitland integral transform. It is shown that some special cases of them are related with the Laplace transform and the Hankel transform. Also, some examples are given as representations of the outcomes presented here.

KEYWORDS:
Bessel function, Bessel-Maitland function, generalized Bessel-Maitland function, Laplace transform, Hankel transform.

AMS CLASSIFICATION 33E50, 35A22, 44A10, 44A20.

1 | INTRODUCTION, DEFINITIONS AND PRELIMINARIES

Special functions and integral transfoms are frequently used in many mathematics, physics and engineering applications. Generally, integral transforms, which are one of the solution methods and special functions that are the solution of certain types of differential equations, are studied in many disciplines today. Bernoulli gave series solutions of Bessel differential equations in 1703. This solutions are called Bessel functions. Bessel functions were seen in many studies like astronomy, mechanics and physics in the eighteenth century [1,2,3]. In the literature, integral transforms whose kernel involving the Bessel functions are called the Hankel transform, the Bessel transform, the Y and I transforms. There are also integral transforms such as Laplace and Mellin transforms, which are frequently seen in the literature. In 1933, E. M. Wright introduced Wright function and investigated its asymptotic behaviour [1] and a generalization of the Wright function, known as the Bessel-Maitland function was defined in [4,5]. In 2011, Singh [6,7] defined a generalized Bessel-Maitland function as follows:

$$
\begin{aligned}
\mathcal{J}_{\nu,p}^{\mu,\gamma}(z) &= \sum_{n=0}^{\infty} \frac{(\gamma)_{pn}(-z)^n}{n!\Gamma(\mu n+\nu+1)} \\
&= \frac{1}{\Gamma(\gamma)}\, {}_1\Psi_1\left[\left.\begin{array}{c}(\gamma,p)\\(\nu+1,\mu)\end{array}\right|-z\right] \\
&= \frac{1}{\Gamma(\gamma)} H_{1,2}^{1,1}\left[\left.\begin{array}{c}(1-\gamma,p)\\(0,1),(-\nu,\mu)\end{array}\right|z\right],
\end{aligned}
\tag{1}
$$

where $H_{1,2}^{1,1}$ is the Fox $H-$function defined in[8], $\mu, \nu, \gamma \in \mathbb{C}$; $\mathrm{Re}(\mu) \geq 0$, $\mathrm{Re}(\nu) \geq -1$, $\mathrm{Re}(\gamma) \geq 0$, $p \in (0,1) \cup \mathbb{N}$ and $(\gamma)_0 = 1$, $(\gamma)_{pn} = \dfrac{\Gamma(\gamma + pn)}{\Gamma(\gamma)}$. Also, Singh[6, Eq(5.2.9), p.130] proved that the generalized Bessel-Maitland function (1) has following integral representation:

$$\mathcal{J}_{\nu,p}^{\mu,\gamma}(z) = \frac{1}{\Gamma(\gamma)} \frac{1}{2\pi i} \int_L \frac{\Gamma(s)\Gamma(\gamma - ps)}{\Gamma(1 + \upsilon - \mu s)} z^{-s} ds, \qquad (|\arg z| < \pi), \tag{2}$$

where $\mu, \nu, \gamma \in \mathbb{C}$; $\mathrm{Re}(\mu) \geq 0$, $\mathrm{Re}(\nu) \geq -1$, $\mathrm{Re}(\gamma) \geq 0$ and $p \in (0,1) \cup \mathbb{N}$. The contour of integration L, lies from $s = -i\infty$ to $s = +i\infty$, separates poles of integrand at $s = -n \left(n \in \mathbb{N}_0\right)$ to the left and at $s = \dfrac{\gamma + p}{n} \left(n \in \mathbb{N}_0\right)$ to the right. He examined certain properties of this function which got a number of results including differentiation and integration formulas[6,7].

The aim of this paper is to first find some new features of the generalized Bessel Maitland function and then to examine the properties of the integral transform, whose kernel involving the Generalized Bessel-Maitland function, so-called the Generalized Bessel Maitland integral transform.

Now, let us start with basic definitions and properties of special functions and integral transforms to be used in the rest of the article. The Gamma function is defined by

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt, \qquad \mathrm{Re}(z) > 0. \tag{3}$$

The basic properties of the Gamma function were given in[9, p.3].

Pochammer symbol is defined by the following relation,

$$(\alpha)_n = \begin{cases} \alpha(\alpha+1)\ldots(\alpha+n-1), & n = 1,2,3\ldots \\ 1, & n = 0 \end{cases}. \tag{4}$$

where $\alpha \in \mathbb{R}$,

The relationship between the Pochammer symbol and the Gamma function is given by

$$(\alpha)_n = \frac{\Gamma(\alpha + n)}{\Gamma(\alpha)}, \qquad \alpha \neq 0, 1, 2, \ldots . \tag{5}$$

The Beta function is defined by the following formula[9],

$$B(z, \nu) = \int_0^\infty \frac{t^{z-1}}{(1+t)^{\nu+z}} dt = \int_0^1 t^{z-1} (1-t)^{\nu-1} dt, \tag{6}$$

where for $\mathrm{Re}(z) > 0, \mathrm{Re}(\nu) > 0$,.

The relationship between the Beta function and the Gamma function is given by

$$B(z, \nu) = \frac{\Gamma(z)\Gamma(\nu)}{\Gamma(z+\nu)}. \tag{7}$$

The generalized hypergeometric series is defined by[9]

$${}_rF_s\left[\begin{array}{c} \alpha_1, \alpha_2, \ldots, \alpha_r \\ \beta_1, \beta_2, \ldots, \beta_s \end{array} \middle| z\right] = \sum_{n=0}^\infty \frac{(\alpha_1)_n (\alpha_2)_n \cdots (\alpha_r)_n}{(\beta_1)_n (\beta_2)_n \cdots (\beta_s)_n} \frac{z^n}{n!}, \tag{8}$$

where $r, s \in \mathbb{Z}^+ \cup \{0\}$ and α_i , $\beta_j \neq 0, -1, -2, \ldots$ $(1 \leq i \leq r, 1 \leq j \leq s)$.

The Fox-Wright function is a generalization of the hypergeometric function and defined as[10]:

$${}_p\Psi_q\left[\begin{array}{c} (a_1, \alpha_1), (a_2, \alpha_2), \ldots, (a_p, \alpha_p) \\ (b_1, \beta_1), (b_2, \beta_2), \ldots, (b_q, \beta_q) \end{array} \middle| z\right] = \sum_{n=0}^\infty \frac{\Gamma(a_1 + \alpha_1 n)\Gamma(a_2 + \alpha_2 n)\ldots\Gamma(a_p + \alpha_p n)}{\Gamma(b_1 + \beta_1 n)\Gamma(b_2 + \beta_2 n)\ldots\Gamma(b_q + \beta_q n)} \frac{z^n}{n!}, \tag{9}$$

where $p, q \in \mathbb{Z}^+ \cup \{0\}$, $a_i, b_j \in \mathbb{C}$ and $\alpha_i, \beta_j \in \mathbb{R}$ $(1 \leq i \leq p, 1 \leq j \leq q)$.

The Bessel-Maitland function, which is also known as Wright function, is defined as the following series form[4,5]:

$$\mathcal{J}_\nu^\mu(z) = \sum_{n=0}^\infty \frac{(-z)^n}{n!\Gamma(\mu n + \nu + 1)} = {}_0\Psi_1\left[\begin{array}{c} - \\ (\nu+1, \mu) \end{array} \middle| -z\right], \tag{10}$$

where ${}_0\Psi_1$ is the Fox-Wright function. If we set $\mu = 1$ and $z = \frac{z^2}{2}$ in the Bessel-Maitland function and multiply with $\left(\frac{z}{2}\right)^{2\nu}$, we obtain the first kind of Bessel functions as follow:

$$\mathcal{J}_\nu(z) = \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(n+\nu+1)\, n!} \left(\frac{z}{2}\right)^{n+2\nu}. \tag{11}$$

The Hankel transform whose kernel involving Bessel functions defined as follow[11, p.xi]:

$$\mathcal{H}_\nu\{f(t); y\} = \int_0^\infty (yt)^{1/2} \mathcal{J}_\nu(yt) f(t)\, dt, \tag{12}$$

where $\operatorname{Re} y > 0$.

The Laplace transform defined by[12, p.127]

$$\mathcal{L}\{f(t); s\} = \int_0^\infty e^{-st} f(t)\, dt, \qquad \operatorname{Re} s > 0. \tag{13}$$

2 | IDENTITIES FOR THE SPECIAL CASES OF BESSEL-MAITLAND FUNCTION

In this section, we will consider some new conclusions and their special cases for the generalized Bessel-Maitland function (1). The special cases of the generalized Bessel-Maitland function (1) are given by

$$\mathcal{J}_{0,1}^{0,1}(z) = \frac{1}{1+z}, \tag{14}$$

$$\mathcal{J}_{0,p}^{p,1}(z) = e^{-z}, \tag{15}$$

$$z\mathcal{J}_{1,1}^{2,1}\left(z^2\right) = \sin z, \tag{16}$$

$$z\mathcal{J}_{1,1}^{2,1}\left(-z^2\right) = \sinh z, \tag{17}$$

$$\mathcal{J}_{0,1}^{2,1}\left(z^2\right) = \cos z, \tag{18}$$

$$\mathcal{J}_{0,1}^{2,1}\left(-z^2\right) = \cosh z, \tag{19}$$

$$\left(\frac{z}{2}\right)^\nu \mathcal{J}_{\nu,0}^{1,\gamma}\left(\frac{z^2}{4}\right) = \mathcal{J}_\nu(z), \tag{20}$$

$$\frac{1}{\sin(\pi\nu)}\left[\left(\frac{z}{2}\right)^\nu \mathcal{J}_{\nu,0}^{1,\gamma}\left(\frac{z^2}{4}\right)\cos(\pi\nu) - \left(\frac{z}{2}\right)^{-\nu} \mathcal{J}_{-\nu,0}^{1,\gamma}\left(\frac{z^2}{4}\right)\right] = \mathcal{Y}_\nu(z), \tag{21}$$

where $\mathcal{J}_\nu$ is the first kind[13, Eq(53.6.1), p.537] and $\mathcal{Y}_\nu$ is the second kind of Bessel function[13, Eq(54.3.1), p.568].

Lemma 1. The following identity holds true

$$\mathcal{J}_{\nu,p}^{\mu,\gamma}(z) = \frac{1}{\nu}\mathcal{J}_{\nu-1,p}^{\mu,\gamma}(z) + \frac{(\gamma)_p\, \mu z}{\nu}\mathcal{J}_{\nu+\mu,p}^{\mu,\gamma+p}(z), \tag{22}$$

for $\mu, \nu, \gamma \in \mathbb{C}; \operatorname{Re}(\mu) \geq 0, \operatorname{Re}(\nu) \geq -1, \operatorname{Re}(\gamma) \geq 0$ and $p \in (0,1) \cup \mathbb{N}$, $m \in \mathbb{N}$.

Proof. We have the following identities[6, Eq(5.2.1.1-2), p.127],

$$\frac{d^m}{dz^m}\mathcal{J}_{\nu,p}^{\mu,\gamma}(z) = (-1)^m (\gamma)_{pm}\, \mathcal{J}_{\nu+\mu m,p}^{\mu,\gamma+pm}(z), \tag{23}$$

$$\mathcal{J}_{\nu,p}^{\mu,\gamma}(z) = (\nu+1)\,\mathcal{J}_{\nu+1,p}^{\mu,\gamma}(z) + \mu z \frac{d}{dz}\mathcal{J}_{\nu+1,p}^{\mu,\gamma}(z). \tag{24}$$

If we put $m = 1$ in (23) and make a simple change of variables from ν to $\nu - 1$, where $\nu = \nu - 1$ in (24), we arrive at the relation (22). □

Remark 1. Substituting $\mu = 1$, $p = 0$, $z = z^2/4$ in (22), multiplying by $\left(\frac{z}{2}\right)^{\nu-1}$ and using the special case (20), we obtain

$$\frac{2\nu}{z}\mathcal{J}_\nu(z) = \mathcal{J}_{\nu-1}(z) + \mathcal{J}_{\nu+1}(z), \tag{25}$$

where was obtained earlier in [14, Eq(56), p.12].

Theorem 1. The following identity holds true

$$\frac{d^m}{dz^m}\left(z^{\nu}\mathcal{J}_{\nu,p}^{\mu,\gamma}\left(z^{\mu}\right)\right)=z^{\nu-m}\mathcal{J}_{\nu-m,p}^{\mu,\gamma}\left(z^{\mu}\right)\ , \tag{26}$$

for $\mu,\nu,\gamma\in\mathbb{C}$; $\mathrm{Re}\left(\mu\right)\geq 0$, $\mathrm{Re}\left(\nu\right)\geq -1$, $\mathrm{Re}\left(\gamma\right)\geq 0$, $p\in(0,1)\cup\mathbb{N}$ and $m\in\mathbb{N}$.

Proof. Using the series representation (1), we find

$$\begin{aligned}\frac{d^m}{dz^m}\left(z^{\nu}\mathcal{J}_{\nu,p}^{\mu,\gamma}\left(z^{\mu}\right)\right)&=\sum_{n=0}^{\infty}\frac{(-1)^n\left(\gamma\right)_{pn}}{n!\Gamma\left(\mu n+\nu+1\right)}\frac{d^m}{dz^m}\left(z^{\mu n+\nu}\right)\\&=\sum_{n=0}^{\infty}\frac{(-1)^n\left(\gamma\right)_{pn}}{n!\Gamma\left(\mu n+\nu-m+1\right)}z^{\mu n+\nu-m}\\&=z^{\nu-m}\mathcal{J}_{\nu-m,p}^{\mu,\gamma}\left(z^{\mu}\right).\end{aligned}$$

□

Theorem 2. The function $\mathcal{J}_{\nu,p}^{\mu,\gamma}\left(z\right)$ has the following integral representations,

$$\mathcal{J}_{\nu,p}^{\mu,\gamma}\left(z\right)=\frac{1}{\Gamma\left(\frac{1}{2}+\upsilon\right)}\int_0^1 t^{-1/2}\left(1-t\right)^{\upsilon-1/2}\mathcal{J}_{-1/2,p}^{\mu,\gamma}\left(zt^{\mu}\right)dt\ , \tag{27}$$

$$\mathcal{J}_{\nu,p}^{\mu,\gamma}\left(z\right)=\frac{2}{\Gamma\left(\frac{1}{2}+\upsilon\right)}\int_0^{\pi/2}\sin^{2\upsilon}\theta\,\mathcal{J}_{-1/2,p}^{\mu,\gamma}\left(z\cos^{2\mu}\theta\right)d\theta\ , \tag{28}$$

where $\mu,\nu,\gamma\in\mathbb{C}$; $\mathrm{Re}\left(\mu\right)\geq 0$, $\mathrm{Re}\left(\nu\right)\geq -1/2$, $\mathrm{Re}\left(\gamma\right)\geq 0$ and $p\in(0,1)\cup\mathbb{N}$.

Proof. Using the series representation (1), the relation (7) and the definition of Beta function (6), we get

$$\begin{aligned}\mathcal{J}_{\nu,p}^{\mu,\gamma}\left(z\right)&=\frac{1}{\Gamma\left(\frac{1}{2}+\nu\right)}\sum_{n=0}^{\infty}\frac{(-z)^n\left(\gamma\right)_{pn}}{n!\Gamma\left(\frac{1}{2}+\mu n\right)}B\left(\frac{1}{2}+\nu,\frac{1}{2}+\mu n\right)\\&=\frac{1}{\Gamma\left(\frac{1}{2}+\nu\right)}\int_0^1\frac{(1-t)^{\nu-1/2}}{t^{1/2}}\left[\sum_{n=0}^{\infty}\frac{(-t^{\mu}z)^n\left(\gamma\right)_{pn}}{n!\Gamma\left(\frac{1}{2}+\mu n\right)}\right]dt.\end{aligned} \tag{29}$$

Now, using the series representation (1), we obtain integral representation (27). Making the change of variable $1-t=\sin^2\theta$ in (27), we arrive at (28). □

Remark 2. Substituting $\mu=1$, $p=0$, $z=z^2/4$ into (27), multiplying by $\left(\frac{z}{2}\right)^{\nu}$ and making the change of variable $t=u^2$, we get

$$\left(\frac{z}{2}\right)^{\nu}\mathcal{J}_{\nu,0}^{1,\gamma}\left(\frac{z^2}{4}\right)=\left(\frac{z}{2}\right)^{\nu}\frac{2}{\Gamma\left(\frac{1}{2}+\nu\right)}\int_0^1\left(1-u^2\right)^{\nu-1/2}\mathcal{J}_{-1/2,0}^{1,\gamma}\left(\frac{u^2z^2}{4}\right)du.$$

Using the relation (20) and the known formula $\mathcal{J}_{-1/2}\left(z\right)=\frac{2^{1/2}}{\pi^{1/2}z^{1/2}}\cos\left(z\right)$ [14, Eq(56), p.12], we find the known formula of Bessel function [13, Eq(53.3.2), p.555] as follows:

$$\mathcal{J}_{\nu}\left(z\right)=\left(\frac{z}{2}\right)^{\upsilon}\frac{2}{\sqrt{\pi}\Gamma\left(\frac{1}{2}+\upsilon\right)}\int_0^1\left(1-u^2\right)^{\upsilon-1/2}\cos\left(uz\right)du\ , \tag{30}$$

where $\operatorname{Re}(\nu) \geq -1/2$. Applying similar calculations in (28), we find the known formula of Bessel function[13, Eq(53.3.3), p.555] as follows:

$$\mathcal{J}_{\nu}(z) = \left(\frac{z}{2}\right)^{\nu} \frac{2}{\sqrt{\pi}\Gamma\left(\nu+\frac{1}{2}\right)} \int_{0}^{\pi/2} \sin^{2\nu}\theta \cos(z\cos\theta)\, d\theta, \tag{31}$$

where $\operatorname{Re}(\nu) \geq -1/2$.

3 | GENERALIZED BESSEL-MAITLAND INTEGRAL TRANSFORM

In this section, generalized Bessel-Maitland Integral transform will be introduced and some basic properties of it will be given.

Definition 1. The ${}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}$−transform is defined as follows:

$${}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\{f(t);s\} = \int_{0}^{\infty} (st)^{\alpha} \mathcal{J}_{\nu,p}^{\mu,\gamma}(st) f(t)\, dt, \tag{32}$$

where $\alpha, \mu, \nu, \gamma, s \in \mathbb{C}$; $\operatorname{Re}(\alpha) \geq 0$, $\operatorname{Re}(\mu) \geq 0$, $\operatorname{Re}(\gamma) \geq 0$, $\operatorname{Re}(\nu) \geq -1$, $\operatorname{Re}(s) \geq 0$, and $p \in (0,1) \cup \mathbb{N}$.

Lemma 2. The following identities,

$${}_{\alpha}\mathcal{H}_{0,p}^{p,1}\{f(t);s\} = s^{\alpha}\mathcal{L}\{t^{\alpha}f(t);s\}\ , \tag{33}$$

$${}_{\frac{\nu}{2}-\frac{1}{4}}\mathcal{H}_{\nu,0}^{1,\gamma}\{f(t);s^{2}\} = \frac{1}{s\sqrt{2}}\mathcal{H}_{\nu}\left\{f\left(\frac{t^{2}}{4}\right);s\right\}\ , \tag{34}$$

hold true, provided that the integrals involved converge absolutely, where $\mathcal{L}$ is the Laplace transform[12, p.127] and $\mathcal{H}_{\nu}$ is the Hankel transform[11, p.xi].

Proof. Setting $\nu = 0, \gamma = 1$ into the definition of the generalized Bessel-Maitland function (1) and using the definition of Laplace transform and (32), we obtain (33). Secondly, making the change of variable $t = u^{2}/4$ on the right hand side of (32), we have

$${}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\{f(t);s\} = \int_{0}^{\infty} \left(s\frac{u^{2}}{4}\right)^{\alpha} \mathcal{J}_{\nu,p}^{\mu,\gamma}\left(s\frac{u^{2}}{4}\right) f\left(\frac{u^{2}}{4}\right) \frac{u}{2} du. \tag{35}$$

Making the change of variables $s = s^{2}$, $\alpha = \dfrac{\nu}{2} - \dfrac{1}{4}$ into (35) and using the special case of the generalized Bessel-Maitland function (20), we obtain (34). ☐

Lemma 3. The following identities,

$${}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\{f(t)+g(t);s\} = {}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\{f(t);s\} + {}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\{g(t);s\}\ , \tag{36}$$

$${}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\{f(at);s\} = \frac{1}{a}\, {}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{f(t);\frac{s}{a}\right\}\ , \tag{37}$$

hold true, provided that the integrals involved converge absolutely.

Proof. Using the Definition1 of the generalized Bessel-Maitland integral transform and the linearity of the integral, we obtain (36). Making the change of variable $at = u$ in relation (32), we arrive at (37). ☐

Lemma 4. The following equations hold true,

$${}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{t^{\beta}f(t);s\right\} = s^{-\beta}\, {}_{\alpha+\beta}\mathcal{H}_{\nu,p}^{\mu,\gamma}\{f(t);s\}, \tag{38}$$

$${}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{t^{\beta}f(t);s\right\} = \frac{s^{-\beta}}{\nu}\, {}_{\alpha+\beta}\mathcal{H}_{\nu-1,p}^{\mu,\gamma}\{f(t);s\} + \frac{\mu s^{-\beta}(\gamma)_{p}}{\nu}\, {}_{\alpha+\beta+1}\mathcal{H}_{\nu+\mu,p}^{\mu,\gamma+p}\{f(t);s\}, \tag{39}$$

where $\alpha, \beta, \mu, \nu, \gamma, s \in \mathbb{C}$; $\operatorname{Re}(\alpha+\beta) \geq -1$, $\operatorname{Re}(\mu) \geq 0$, $\operatorname{Re}(\gamma) \geq 0$, $\operatorname{Re}(\nu) \geq -1$, $\operatorname{Re}(s) \geq 0$ and $p \in (0,1) \cup \mathbb{N}$, provided that the integrals involved converge absolutely.

Proof. We obtain (38) easily using the Definition1. Similarly, using the relation (22) of Lemma1 and the linearity of the integral, we arrive at (39). □

Remark 3. Setting $\beta = -\frac{1}{2}, f(t) = f\left(2\sqrt{t}\right)$ in (39), and using the relation (34), we obtain the known following relation

$$\mathcal{H}_\nu\left\{\frac{2\nu f(t)}{t}; s\right\} = s\mathcal{H}_{\nu-1}\{f(t); s\} + s\mathcal{H}_{\nu+1}\{f(t); s\}\ , \tag{40}$$

where $\mathcal{H}_\nu$ is the Hankel transform[11, Eq(5), p.5].

Theorem 3. The following identities for the generalized Bessel-Maitland integral transform,

$$ {}_\alpha\mathcal{H}^{\mu,\gamma}_{\nu,p}\{f'(t); s\} = s\left(\frac{\nu}{\mu} - \alpha\right){}_{\alpha-1}\mathcal{H}^{\mu,\gamma}_{\nu,p}\{f(t); s\} - \frac{s}{\mu}{}_{\alpha-1}\mathcal{H}^{\mu,\gamma}_{\nu-1,p}\{f(t); s\}\ , \tag{41}$$

$$ {}_\alpha\mathcal{H}^{\mu,\gamma}_{\nu,p}\{f'(t); s\} = s\left((\gamma)_p - \frac{(\gamma)_p\mu\alpha}{\nu}\right){}_\alpha\mathcal{H}^{\mu,\gamma+p}_{\nu+\mu,p}\{f(t); s\} - \left(\frac{s\alpha}{\nu}\right){}_{\alpha-1}\mathcal{H}^{\mu,\gamma}_{\nu-1,p}\{f(t); s\}\ , \tag{42}$$

hold true, provided that the integrals involved converge absolutely, where $\alpha, \beta, \mu, \nu, \gamma, s \in \mathbb{C}$; $\mathrm{Re}(\mu) \geq 0$, $\mathrm{Re}(\gamma) \geq 0$, $\mathrm{Re}(\nu) \geq -1$, $\mathrm{Re}(s) \geq 0$ and $p \in (0, 1) \cup \mathbb{N}$.

Proof. Using the definition (32), integration by parts and the relation (23) for $m = 1$[6, Eq(5.2.1), p.127], we arrive at (41). Substituting (21) into (41), we obtain (42). □

Remark 4. Changing the variable from t to $2\sqrt{t}$ and setting $p = 0$, $\mu = 1$ and $s = s^2$ in (42), we find

$$ {}_\alpha\mathcal{H}^{1,\gamma}_{\nu,0}\left\{\frac{f'\left(2\sqrt{t}\right)}{\sqrt{t}}; s^2\right\} = s^2\left(1 - \frac{\alpha}{\nu}\right){}_\alpha\mathcal{H}^{1,\gamma}_{\nu+1,0}\left\{f\left(2\sqrt{t}\right); s^2\right\} - \left(\frac{s^2\alpha}{\nu}\right){}_{\alpha-1}\mathcal{H}^{1,\gamma}_{\nu-1,0}\left\{f\left(2\sqrt{t}\right); s^2\right\}. \tag{43}$$

On the other hand, setting $f(t) = f\left(2\sqrt{t}\right)$, $\beta = -1/2$, $p = 0$, $\mu = 1$ and $s = s^2$ in (38), we get

$$ {}_\alpha\mathcal{H}^{1,\gamma}_{\nu,0}\left\{\frac{f'\left(2\sqrt{t}\right)}{\sqrt{t}}; s^2\right\} = s\,{}_{\alpha-1/2}\mathcal{H}^{1,\gamma}_{\nu,0}\left\{f'\left(2\sqrt{t}\right); s^2\right\}. \tag{44}$$

Using (43) and (44) together, we have

$$ {}_{\alpha-1/2}\mathcal{H}^{1,\gamma}_{\nu,0}\left\{\frac{f'\left(2\sqrt{t}\right)}{\sqrt{t}}; s^2\right\} = s\left(1 - \frac{\alpha}{\nu}\right){}_\alpha\mathcal{H}^{1,\gamma}_{\nu+1,0}\left\{f\left(2\sqrt{t}\right); s^2\right\} - \left(\frac{s\alpha}{\nu}\right){}_{\alpha-1}\mathcal{H}^{1,\gamma}_{\nu-1,0}\left\{f\left(2\sqrt{t}\right); s^2\right\}. \tag{45}$$

Setting $\alpha = \frac{\nu}{2} + \frac{1}{4}$ in (45) and using the relation (34), we obtain the known following relation,

$$\mathcal{H}_\nu\left\{2\nu f'(t); s\right\} = s\left(\nu - \frac{1}{2}\right)\mathcal{H}_{\nu+1}\{f(t); s\} - s\left(\nu + \frac{1}{2}\right)\mathcal{H}_{\nu-1}\{f(t); s\}\ , \tag{46}$$

where $\mathcal{H}_\nu$ is the Hankel transform[11, Eq(10), p.6].

Theorem 4. The following identities for the ${}_\alpha\mathcal{H}^{\mu,\gamma}_{\nu,p}$–transform,

$$\frac{d}{ds}\left({}_\alpha\mathcal{H}^{\mu,\gamma}_{\nu,p}\{f(t); s\}\right) = \frac{\alpha}{s}{}_\alpha\mathcal{H}^{\mu,\gamma}_{\nu,p}\{f(t); s\} - \frac{(\gamma)_p}{s}{}_{\alpha+1}\mathcal{H}^{\mu,\gamma+p}_{\nu+\mu,p}\{f(t); s\}, \tag{47}$$

$$\frac{d}{ds}\left({}_\alpha\mathcal{H}^{\mu,\gamma}_{\nu,p}\{f(t); s\}\right) = \alpha\,{}_{\alpha-1}\mathcal{H}^{\mu,\gamma}_{\nu,p}\{tf(t); s\} - (\gamma)_p\,{}_\alpha\mathcal{H}^{\mu,\gamma+p}_{\nu+\mu,p}\{tf(t); s\}, \tag{48}$$

hold true, provided that the integrals involved converge absolutely, where $\alpha, \beta, \mu, \nu, \gamma, s \in \mathbb{C}$; $\mathrm{Re}(\mu) \geq 0$, $\mathrm{Re}(\gamma) \geq 0$, $\mathrm{Re}(\nu) \geq -1$, $\mathrm{Re}(s) \geq 0$ and $p \in (0, 1) \cup \mathbb{N}$.

Proof. Using the definition of (32), changing the order of integration and derivation and making use of the relation (22), we obtain (47). Setting $\beta = 1$ in (38) and using the relation (47), we arrive at (48). □

Corollary 1. The following identity holds true for the ${}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}-$transform,

$$\frac{d^m}{ds^m}\left({}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{f\left(t\right);s\right\}\right)=s^{-m}\sum_{k=0}^{m}\binom{m}{k}\frac{\left(-1\right)^k\left(\gamma\right)_{pk}\Gamma\left(\alpha+1\right)}{\Gamma\left(\alpha-\left(m-k-1\right)\right)}\,{}_{\alpha+k}\mathcal{H}_{\nu+\mu k,p}^{\mu,\gamma+pk}\left\{f\left(t\right);s\right\}\;. \tag{49}$$

Proof. Using the definition of (32) and the Leibniz derivative formula[13, Eq(0:10:8),p.8], we obtain (49). □

4 | ILLUSTRATIVE EXAMPLES

In this section, we will give some illustrative examples of the generalized Bessel-Maitland integral transform. The results of the examples in this section are obtained using the integral representation of the generalized Bessel-Maitland function (2) and different advanced methods such as residue calculus.

Example 5. We show that

$$\begin{aligned}{}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{\frac{1}{a^2+t^2};s\right\}=&\frac{\pi a^{\alpha-1}s^{\alpha}}{2\cos\left(\frac{\pi\alpha}{2}\right)\Gamma\left(\gamma\right)}\,{}_2\Psi_2\left[\begin{matrix}\left(\gamma,2p\right),\left(1,1\right)\\\left(1+\upsilon,2\mu\right),\left(1,2\right)\end{matrix}\middle|-a^2s^2\right]\\&+\frac{\pi a^{\alpha}s^{\alpha+1}}{2\sin\left(\frac{\pi\alpha}{2}\right)\Gamma\left(\gamma\right)}\,{}_2\Psi_2\left[\begin{matrix}\left(\gamma+p,2p\right),\ \left(1,1\right)\\\left(1+\upsilon+\mu,2\mu\right),\ \left(2,2\right)\end{matrix}\middle|-a^2s^2\right]\\&-\frac{\pi s}{\sin\left(\pi\alpha\right)\Gamma\left(\gamma\right)}\,{}_2\Psi_2\left[\begin{matrix}\left(\gamma-p\alpha+p,2p\right),\ \left(1,1\right)\\\left(1+\nu-\mu\alpha+\mu,2\mu\right),\ \left(2-\alpha,2\right)\end{matrix}\middle|-a^2s^2\right],\end{aligned} \tag{50}$$

where $\mathrm{Re}a>0$ and ${}_2\Psi_2$ is the Fox-Wright function (9).

Proof. Setting $f\left(t\right)=\dfrac{1}{a^2+t^2}$ $(\mathrm{Re}a>0)$ in (32) and using the definition of generalized Bessel-Maitland transform (32) and integral representation of generalized Bessel-Maitland function (2), we obtain

$${}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{f(t);s\right\}=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\frac{\Gamma\left(u\right)\Gamma\left(\gamma-pu\right)s^{\alpha-u}}{\Gamma\left(\gamma\right)\Gamma\left(1+\nu-\mu u\right)}\left(\int_{0}^{\infty}\frac{t^{\alpha-u}}{a^2+t^2}dt\right)du. \tag{51}$$

Using the formula of Mellin transform[12, Eq(11), p.309], and the known property of Gamma function[9, Eq(6), p.3], we have

$${}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{\frac{1}{a^2+t^2};s\right\}=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}g\left(u\right)du=\sum_{i}Res\left(g(u),u_i\right), \tag{52}$$

where

$$g\left(u\right)=\frac{\Gamma(u)\Gamma\left(\gamma-pu\right)s^{\alpha-u}a^{\alpha-u-1}}{\Gamma\left(\gamma\right)\Gamma\left(1+\nu-\mu u\right)}\frac{\pi}{2}\sec\left(\frac{\pi}{2}\left(\alpha-u\right)\right),$$

which has poles at $u_1=-n$ and $u_2=\alpha-1-2n\,(n\in\mathbb{N})$. The residues of $g(u)$ at $u_1=-n\,(n\in\mathbb{N})$ are

$$\mathrm{Res}\{g\left(u\right),u_1\}=\frac{\left(-1\right)^n\Gamma\left(\gamma+pn\right)a^{\alpha+n-1}s^{\alpha+n}}{\Gamma\left(\gamma\right)\Gamma\left(1+\nu+\mu n\right)n!}\frac{\pi}{2}\sec\left(\frac{\pi}{2}\left(\alpha+n\right)\right), \tag{53}$$

where we have for $k\in\mathbb{N}$,

$$\sec\left(\frac{\pi}{2}\left(\alpha+n\right)\right)=\begin{cases}\left(-1\right)^k\sec\left(\frac{\pi\alpha}{2}\right),\ n=2k\\\left(-1\right)^{k+1}\csc\left(\frac{\pi\alpha}{2}\right),\ n=2k+1\end{cases}.$$

The residues of $g(u)$ at $u_2=\alpha-1-2n\,(n\in\mathbb{N})$ are

$$\mathrm{Res}\{g\left(u\right),u_2\}=\frac{\pi}{\sin\left(\pi\alpha\right)}\frac{\left(-1\right)^{n+1}\Gamma\left(\gamma-p\alpha+p+2pn\right)a^{2n}s^{2n+1}}{\Gamma\left(\gamma\right)\Gamma\left(2-\alpha+2n\right)\Gamma\left(1+\nu-\mu\alpha+\mu+2\mu n\right)}. \tag{54}$$

Substituting (53) and (54) into (52) and using the definition of the Fox-Wright function (9), we obtain (50). □

Example 6. We show that

$$
\begin{aligned}
{}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{\frac{1}{(1+at)^{\beta}};s\right\} = &-\frac{a^{-\alpha-1}(s)^{\alpha+1}\pi}{\Gamma(\beta)\Gamma(\gamma)\sin(\pi(\beta-\alpha))} \\
&\times {}_2\Psi_2\left[\begin{array}{c}(\gamma,p),(\alpha+1,1)\\(1+\nu,\mu),(2-\beta+\alpha,1)\end{array}\middle|\frac{s}{a}\right] \\
&+\frac{a^{-\beta}(s)^{\beta-1}\pi}{\Gamma(\beta)\Gamma(\gamma)\sin(\pi(\beta-\alpha))} \\
&\times {}_2\Psi_2\left[\begin{array}{c}(\gamma-p(-\beta+\alpha+1),p),(\beta,1)\\(1+\nu-\mu(-\beta+\alpha+1),\mu),(\beta-\alpha,1)\end{array}\middle|\frac{s}{a}\right],
\end{aligned}
\tag{55}
$$

where $0<\mathrm{Re}s<\mathrm{Re}\beta$, $|\arg a|<\pi$ and ${}_2\Psi_2$ is the Fox-Wright function (9).

Proof. Setting $f(t)=\dfrac{1}{(1+at)^{\beta}}$ in (32) and using the definitions (2) and (6) and property of beta function (7), we have

$$
{}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{\frac{1}{(1+at)^{\beta}};s\right\} = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} g(u)\,du = \sum_i Res\left(g(u),u_i\right), \tag{56}
$$

where

$$
g(u)=\frac{\Gamma(u)\Gamma(\gamma-pu)\Gamma(\alpha-u+1)\,s^{\alpha-u}a^{1+u-\alpha}}{\Gamma(\beta)\Gamma(\gamma)\Gamma(1+\nu-\mu u)\Gamma(2-\beta+\alpha-u)}\frac{\pi}{\sin(\pi(\beta-\alpha+u-1))},
$$

which has poles at $u_1=-n$ and $u_2=-\beta+\alpha+1-n\,(n\in\mathbb{N})$. The residues of $g(u)$ at $u_1=-n\,(n\in\mathbb{N})$ are

$$
\mathrm{Res}\{g(u),u_1\}=\frac{s^{\alpha+n}\Gamma(\gamma+pn)\Gamma(\alpha+n+1)\,\pi\csc(\pi(\beta-\alpha-1))}{a^{\alpha+n-1}\Gamma(\beta)\Gamma(\gamma)\Gamma(1+\nu+\mu n)\Gamma(2-\beta+\alpha+n)\,n!}. \tag{57}
$$

The residues of $g(u)$ at $u_2=-\beta+\alpha+1-n\,(n\in\mathbb{N})$ are

$$
\mathrm{Res}\{g(u),u_2\}=\frac{s^{\beta-1+n}\Gamma(\gamma-p(-\beta+\alpha+1)+pn)\Gamma(\beta+n)\,\pi\csc(\pi(\beta-\alpha))}{a^{\beta+n}n!\Gamma(\beta)\Gamma(\gamma)\Gamma(\beta-\alpha+n)\Gamma(1+\nu-\mu(-\beta+\alpha+1)+\mu n)}. \tag{58}
$$

Setting (57) and (58) in (56) and using the definition (9), we arrive at (55). □

Example 7. We show that

$$
{}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{e^{-at};s\right\}=\frac{s^{\alpha}}{a^{\alpha+1}\Gamma(\gamma)}\,{}_2\Psi_1\left[\begin{array}{c}(\gamma,p),(\alpha+1,1)\\(1+\nu,\mu)\end{array}\middle|\frac{-s}{a}\right], \tag{59}
$$

where $\mathrm{Re}a>0$ and ${}_2\Psi_1$ is the Fox-Wright function (9).

Proof. Setting $f(t)=e^{-at}$ in (32) and using the definitions (2) and (3) and property of Gamma function[9, Eq(6), p.3], we have

$$
{}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{e^{-at};s\right\}=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty} g(u)\,du=\sum_i Res\left(g(u),u_i\right), \tag{60}
$$

where

$$
g(u)=\frac{\Gamma(u)\Gamma(\gamma-pu)\Gamma(\alpha-u+1)}{\Gamma(\gamma)\Gamma(1+\nu-\mu u)}\frac{s^{\alpha-u}}{a^{\alpha-u+1}},
$$

which has poles at $u=-n\,(n\in\mathbb{N})$. The residues of $g(u)$ at $u=-n\,(n\in\mathbb{N})$ are

$$
\mathrm{Res}\{g(u),u\}=\frac{(-1)^n\Gamma(\gamma+pn)\Gamma(\alpha+n+1)}{\Gamma(\gamma)\Gamma(1+\nu+\mu n)\,n!}\frac{s^{\alpha+n}}{a^{\alpha+n+1}}. \tag{61}
$$

Substituting (61) into (60), we arrive at (59). □

Example 8. We show that

$$
{}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\{\cos(at);s\}=\frac{s^{\alpha}}{a^{\alpha+1}\Gamma(\gamma)}\sum_{n=0}^{\infty}\frac{\Gamma(\gamma+pn)\Gamma(\alpha+n+1)}{n!\Gamma(1+\nu+\mu n)}\left(-\frac{s}{a}\right)^n\cos\left(\frac{\pi}{2}(n+\alpha+1)\right) \tag{62}
$$

and

$$ {}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{\sin(at);s\right\} = \frac{s^{\alpha}}{a^{\alpha+1}\Gamma(\gamma)}\sum_{n=0}^{\infty}\frac{\Gamma(\gamma+pn)\Gamma(\alpha+n+1)}{n!\Gamma(1+\nu+\mu n)}\left(-\frac{s}{a}\right)^{n}\sin\left(\frac{\pi}{2}(n+\alpha+1)\right), \tag{63} $$

where $\mathrm{Re}a > 0$.

Proof. Changing the parameters in (59) from a to ia, where $a \to ia$ and se-parating the relation into real and imaginary parts, we obtain (62) and (63), respectively. □

Example 9. We show that

$$ {}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{\mathcal{J}_{\beta}(at);s\right\} = \frac{s^{\alpha}}{a^{\alpha+1}}\frac{\sin(\pi(\beta/2-\alpha))}{\pi\Gamma(\gamma)}\,{}_{3}\Psi_{1}\left[\begin{array}{c}(\gamma,p),(\alpha+1\mp\beta/2,1)\\(1+\upsilon,\mu)\end{array}\middle|\frac{s}{a}\right], \tag{64} $$

where $a > 0, -\mathrm{Re}(\beta) < \mathrm{Re}(s) < 3/2$ and ${}_{3}\Psi_{1}$ is the Fox-Wright function (9).

Proof. Setting $f(t) = \mathcal{J}_{\beta}\left(2\sqrt{at}\right)$ in (32) and using the definition (2), we get

$$ {}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{\mathcal{J}_{\beta}\left(2\sqrt{at}\right);s\right\} = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\frac{\Gamma(u)\Gamma(\gamma-pu)s^{\alpha-u}}{\Gamma(\gamma)\Gamma(1+\nu-\mu u)}\left(\int_{0}^{\infty}t^{\alpha-u}\mathcal{J}_{\beta}\left(2\sqrt{at}\right)dt\right)du. \tag{65} $$

Using the relations [12, Eq(1), p.326] and [12, Eq(5), p.207], which are given for the Mellin integral transform, we have

$$ \mathcal{M}\left\{\mathcal{J}_{\beta}\left(2\sqrt{at}\right);s\right\} = \frac{\Gamma\left(s+\frac{\beta}{2}\right)}{a^{s}\Gamma\left(\frac{\beta}{2}-s+1\right)}, \tag{66} $$

where $-\mathrm{Re}(\beta) < \mathrm{Re}(2s) < 3/2$. Using the relation (66) and (65) together, we obtain

$$ {}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{\mathcal{J}_{\beta}\left(2\sqrt{at}\right);s\right\} = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty}g(u)\,du = \sum_{i}Res\left(g(u),u_{i}\right), \tag{67} $$

where $\alpha-\beta/2 \notin \mathbb{N}$ and

$$ g(u) = \frac{\sin(\pi(\beta/2-\alpha-u))}{\sin(\pi u)}\frac{\Gamma(\gamma-pu)\Gamma(\alpha-u+1+\beta/2)\Gamma(1-\beta/2+\alpha-u)}{\Gamma(1-u)\Gamma(\gamma)\Gamma(1+\nu-\mu u)}\frac{s^{\alpha-u}}{a^{\alpha-u+1}}, $$

which has the pole at $u = -n$ $(n \in \mathbb{N})$. The residues of $g(u)$ at $u = -n\,(n \in \mathbb{N})$ are

$$ \mathrm{Res}\{g(u),u\} = \frac{\sin(\pi(\beta/2-\alpha))}{\pi}\frac{\Gamma(\gamma+pn)\Gamma(\alpha+n+1+\beta/2)\Gamma(1-\beta/2+\alpha+n)}{\Gamma(1+n)\Gamma(\gamma)\Gamma(1+\nu-\mu n)}\frac{s^{\alpha+n}}{a^{\alpha+n+1}} \tag{68} $$

Putting (68) into (67), we obtain (64). □

5 | APPENDIX-I

Using the definition of generalized Bessel-Maitland transform (32) and integral representation of generalized Bessel-Maitland function (2) and the residue theorem, we obtain (AI-1) and (AI-3). Using the equation (38) in (AI-1), (59), (62), (63), (64), respectively, we arrive at (AI-2),((AI-4)-(AI-7)).

$$ {}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\{1;s\} = \frac{\Gamma(\alpha+1)\Gamma(\gamma-p(\alpha+1))}{\Gamma(\gamma)\Gamma(1+\nu-\mu(\alpha+1))}\frac{1}{s}. \tag{AI-1} $$

$$ {}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{t^{\beta};s\right\} = \frac{\Gamma(\alpha+\beta+1)\Gamma(\gamma-p(\alpha+\beta+1))}{\Gamma(\gamma)\Gamma(1+\nu-\mu(\alpha+\beta+1))}\frac{1}{s^{\beta+1}}. \tag{AI-2} $$

$$
{}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{\frac{1}{a+t};s\right\}=\frac{\pi\,(as)^{\alpha}}{\sin\left(\pi\left(\alpha+1\right)\right)\Gamma\left(\gamma\right)}\,{}_{1}\Psi_{1}\left[\left.\begin{array}{c}\left(\gamma,p\right)\\ \left(1+\upsilon,\mu\right)\end{array}\right|as\right]
+\frac{\pi}{\sin\left(\pi\alpha\right)\Gamma\left(\gamma\right)}\,{}_{2}\Psi_{2}\left[\left.\begin{array}{c}\left(\gamma-p\alpha,p\right),\ \left(1,1\right)\\ \left(1+\upsilon-\mu,\mu\right),\ \left(1-\alpha,1\right)\end{array}\right|as\right]. \tag{AI-3}
$$

$$
{}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{t^{\beta}e^{-at};s\right\}=\frac{1}{a^{\alpha+\beta+1}}\frac{s^{\alpha}}{\Gamma\left(\gamma\right)}\,{}_{2}\Psi_{1}\left[\left.\begin{array}{c}\left(\gamma,p\right),\left(\alpha+\beta+1,1\right)\\ \left(1+\upsilon,\mu\right)\end{array}\right|-\frac{s}{a}\right]. \tag{AI-4}
$$

$$
{}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{t^{\beta}\cos\left(at\right);s\right\}=\frac{s^{\alpha}}{a^{\alpha+\beta+1}\Gamma\left(\gamma\right)}\sum_{n=0}^{\infty}\frac{\Gamma\left(\gamma+pn\right)\Gamma\left(\alpha+\beta+n+1\right)}{n!\Gamma\left(1+\nu+\mu n\right)}
\times\left(-\frac{s}{a}\right)^{n}\cos\left(\tfrac{\pi}{2}\left(n+\alpha+\beta+1\right)\right). \tag{AI-5}
$$

$$
{}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{t^{\beta}\sin\left(at\right);s\right\}=\frac{s^{\alpha}}{a^{\alpha+\beta+1}\Gamma\left(\gamma\right)}\sum_{n=0}^{\infty}\frac{\Gamma\left(\gamma+pn\right)\Gamma\left(\alpha+\beta+n+1\right)}{n!\Gamma\left(1+\nu+\mu n\right)}
\times\left(-\frac{s}{a}\right)^{n}\sin\left(\tfrac{\pi}{2}\left(n+\alpha+\beta+1\right)\right). \tag{AI-6}
$$

$$
{}_{\alpha}\mathcal{H}_{\nu,p}^{\mu,\gamma}\left\{t^{\delta}\mathcal{J}_{\beta}\left(2\sqrt{at}\right);s\right\}=\frac{s^{\alpha}}{a^{\alpha+\delta+1}}\frac{\sin\left(\pi\left(\beta/2-\alpha-\delta\right)\right)}{\pi\Gamma\left(\gamma\right)}
\times\,{}_{3}\Psi_{1}\left[\left.\begin{array}{c}\left(\gamma,p\right),\left(\alpha+\delta+1\mp\beta/2,1\right)\\ \left(1+\upsilon,\mu\right)\end{array}\right|\frac{s}{a}\right]. \tag{AI-7}
$$

6 | APPENDIX-II

Using the identity (33) for $\alpha = 0$ in (AI-1), we get (AII-1)[12, Eq(1), p.133]. Using the identity (33) $\alpha = 0$ in (AI-2), we have (AII-2)[12, Eq(1), p.137]. Using the identity (34) for $\beta = -1/4$ and $\beta = \beta/4$, respectively, we obtain (AII-3) and (AII-4)[11, Eq(3), p.72 and Eq(7), p.22]. Using the identity (33) in (AI-3), we get (AII-5)[12, Eq(7), p.137]. Using the identity (34) for $a = a^2/4$ in (AI-3), we obtain (AII-6)[11, Eq(21), $\lambda = 3/2, \mu = 0$, p.24]. Using the identity (33) in (50) for $a = 1$, we have (AII-7)[12, Eq(9), p.138]. Taking $a = a^2/4$ in (50) and using the identity (34), we arrive at (AII-8)[3, p.435]. Using the identity (33) in (55) for $a = 1/a, \alpha = \nu - 1$, $\beta = \nu - 1/2$ and $a = 1/a, \alpha = \nu - 1$, $\beta = \nu + 1/2$, respectively, we have (AII-9) and (AII-10)[12, Eq(20)-(21), p.139]. Using the identity (34) in (55) for $a = 4/a^2, \beta = \beta + 1$, we get (AII-11)[11, Eq(21), $\lambda = 3/2$, p.24]. Firstly, using the identity (33) in (59), we get (AII-12)[12, Eq(3), p.144], then taking $\alpha = 0$ and $\alpha = 1$, respectively, we obtain (AII-13) and (AII-14)[12, Eq(1), p.143 and Eq(2) p.144]. Using the identity (34) in (AI-4) for $a = 4a$, $\beta = \mu/2 - 1/4$, we have (AII-15)[11, Eq(30), p.30]. Using the identity (33) for $\alpha = 0$ in (62) and (63), respectively, we obtain (AII-16) and (AII-17)[12, Eq(43), p.154 and Eq(1), p.150]. Using the identity (33) in (62) and (63), respectively, we obtain (AII-18) and (AII-19)[12, Eq(58), p.157 and Eq(15), p.145]. Using the identity (34) in (AI-5) and (AI-6), respectively, for $a = 4a$, $\beta = -1/4$, $\beta = 1/4$ and $\beta = \nu/2 - 1/4$, res-pectively, we obtain (AII-20)-(AII-25)[11, Eq(38)-(40), p,38 and Eq(13)-(15), p.34]. Using the identity (33) in (64) for $\alpha = \mu - 1, \beta = 2\nu$, we get (AII-26), for $\alpha = \mu - 1/2, \beta = 2\nu$, we have (AII-27), for $\alpha = \nu/2 + n, \beta = \nu$, we obtain (AII-28), for $\alpha = \nu/2 - 1$, $\beta = 2\nu$, we arrive at (AII-29), for $\alpha = -\nu/2, \beta = \nu$, we get (AII-30), for $\alpha = \nu/2, \beta = \nu$, we have (AII-31), for $\alpha = -1/2, \beta = \nu$, we obtain (AII-32), for $\alpha = 1/2, \beta = 1$, we get (AII-33), for $\alpha = 0, \beta = \nu$, we get (AII-34)[12, Eq(26)-(35), p.185-186]. Using the identity (34) in (AI-7), we obtain (AII-35).

$$
\mathcal{L}\left\{1;s\right\}=\frac{1}{s},\qquad \mathrm{Re}s>0. \tag{AII-1}
$$

$$
\mathcal{L}\left\{t^{\beta};s\right\}=\frac{\Gamma\left(\beta+1\right)}{s^{\beta+1}},\qquad \mathrm{Re}s>0. \tag{AII-2}
$$

$$
\mathcal{H}_{\nu}\left\{t^{-1/2};s\right\}=s^{-1/2},\qquad \mathrm{Re}\nu>-1,\ s>0. \tag{AII-3}
$$

$$
\mathcal{H}_{\nu}\left\{t^{\beta};s\right\}=\frac{2^{\beta+1/2}}{s^{\beta+1}}\frac{\Gamma\left(\beta/2+\nu/2+3/4\right)}{\Gamma\left(\nu/2-\beta/2+1/4\right)},
\quad -\mathrm{Re}\nu-3/2<\mathrm{Re}\mu<-1/2, s>0. \tag{AII-4}
$$

$$\mathcal{L}\left\{\frac{t^{\alpha}}{a+t};s\right\}=\Gamma(\alpha+1)\,a^{\alpha}e^{as}\Gamma(-\alpha,as),$$
$$|\arg a|<\pi,\mathrm{Re}\alpha>-1,\mathrm{Re}s>0. \tag{AII-5}$$

$$\mathcal{H}_{\upsilon}\left\{\frac{1}{a^{2}+t^{2}};s\right\}=\frac{a^{\upsilon-\frac{1}{2}}s^{\upsilon+\frac{1}{2}}\Gamma\left(\frac{\upsilon}{2}+\frac{3}{4}\right)\Gamma\left(-\frac{\upsilon}{2}+\frac{1}{4}\right)}{2^{\upsilon+1}\Gamma(1+\upsilon)}\,{}_{0}F_{1}\left(-;1+\upsilon;\frac{a^{2}s^{2}}{4}\right)$$
$$+\frac{\Gamma\left(\frac{\upsilon}{2}-\frac{1}{4}\right)s}{\Gamma\left(\frac{\upsilon}{2}+\frac{5}{4}\right)2^{3/2}}\,{}_{1}F_{2}\left(1;\frac{\upsilon}{2}+\frac{5}{4},-\frac{\upsilon}{2}+\frac{5}{4};\frac{a^{2}s^{2}}{4}\right),$$
$$\mathrm{Re}a>0,-\mathrm{Re}\nu<3/2,s>0. \tag{AII-6}$$

$$\mathcal{L}\left\{\frac{t^{\alpha}}{1+t^{2}}\right\}=\frac{\pi}{\sin(\pi(\alpha+1))}V_{\alpha+1}(2s,0),\ \mathrm{Re}\nu>0,\mathrm{Re}s>0. \tag{AII-7}$$

$$\mathcal{H}_{\upsilon}\left\{\frac{16}{4a^{4}+t^{4}};s\right\}=\frac{\pi a^{\upsilon-\frac{5}{2}}s^{\upsilon+\frac{1}{2}}}{\sin\left(\pi\left(\frac{\upsilon}{2}+\frac{3}{4}\right)\right)2^{\frac{\upsilon}{2}-\frac{7}{4}}}$$
$$\times\sum_{n=0}^{\infty}\frac{1}{\Gamma(1+\upsilon+n)\,n!}\left(\frac{a^{2}s^{2}}{2}\right)^{n}\cos\left(\pi\left(\frac{\upsilon}{4}+\frac{3}{4}+\frac{n}{2}\right)\right)$$
$$+\frac{\pi s^{3}\sqrt{2}}{\sin\left(\pi\left(\frac{\upsilon}{2}+\frac{3}{4}\right)\right)}\sum_{n=0}^{\infty}\frac{(-1)^{n}}{\Gamma\left(\frac{9}{4}+\frac{\nu}{2}+2n\right)\Gamma\left(\frac{9}{4}-\frac{\nu}{2}+2n\right)}\left(\frac{as}{\sqrt{2}}\right)^{4n},$$
$$|\arg a|<\pi/4,\mathrm{Re}\nu>-3/2. \tag{AII-8}$$

$$\mathcal{L}\left\{\frac{t^{\nu-1}}{(a+t)^{\nu-1/2}};s\right\}=2^{\nu-1/2}\Gamma(\nu)\,e^{as/2}s^{-1/2}D_{1-2\nu}\left(\sqrt{2as}\right),$$
$$\mathrm{Re}\nu>0,|\arg a|<\pi,\mathrm{Re}s>0. \tag{AII-9}$$

$$\mathcal{L}\left\{\frac{t^{\nu-1}}{(a+t)^{\nu+1/2}};s\right\}=2^{\nu}\Gamma(\nu)\,a^{-1/2}e^{as/2}D_{-2\nu}\left(\sqrt{2as}\right),$$
$$\mathrm{Re}\nu>0,|\arg a|<\pi,\mathrm{Re}s>0. \tag{AII-10}$$

$$\mathcal{H}_{\nu}\left\{\frac{1}{\left(a^{2}+t^{2}\right)^{\beta+1}};s\right\}=\frac{a^{-2\beta+\nu-1/2}s^{\nu+1/2}\Gamma\left(\frac{3}{4}+\frac{\nu}{2}\right)\Gamma\left(\beta-\frac{\nu}{2}+\frac{1}{4}\right)}{2^{\nu+1}\Gamma(\beta+1)\Gamma(\nu+\upsilon)}$$
$$\times\ {}_{1}F_{2}\left(\frac{3}{4}+\frac{\upsilon}{2};1+\upsilon,\frac{3}{4}+\frac{\upsilon}{2}-\beta;\frac{a^{2}s^{2}}{4}\right)$$
$$+\frac{s^{1+2\beta}2^{-3/2-2\beta}\Gamma\left(-\frac{1}{4}+\frac{\upsilon}{2}-\beta\right)}{\Gamma\left(\frac{5}{4}+\frac{\upsilon}{2}+\beta\right)}$$
$$\times\ {}_{1}F_{2}\left(\beta+1;\frac{5}{4}+\frac{\upsilon}{2}+\beta,\ \frac{5}{4}-\frac{\upsilon}{2}+\beta;\frac{a^{2}s^{2}}{4}\right),$$
$$-\mathrm{Re}\beta<3/2<2\mathrm{Re}\mu+7/2,\mathrm{Re}a>0,s>0. \tag{AII-11}$$

$$\mathcal{L}\left\{t^{\alpha}e^{-at};s\right\}=\frac{\Gamma(\alpha+1)}{(a+s)^{\alpha+1}},\mathrm{Re}\alpha>-1,\mathrm{Re}s>-\mathrm{Re}a. \tag{AII-12}$$

$$\mathcal{L}\left\{e^{-at};s\right\}=\frac{1}{a+s},\qquad \mathrm{Re}s>-\mathrm{Re}a. \tag{AII-13}$$

$$\mathcal{L}\left\{te^{-at};s\right\}=\frac{1}{(a+s)^{2}}\qquad \mathrm{Re}s>-\mathrm{Re}a. \tag{AII-14}$$

$$\mathcal{H}_{\upsilon}\left\{t^{\mu-1/2}e^{-at^{2}};s\right\}=\frac{s^{\upsilon+1/2}\Gamma\left(\frac{\upsilon}{2}+\frac{\mu}{2}+\frac{1}{2}\right)}{2^{\upsilon+1}a^{\upsilon/2+\mu/2+1/2}\Gamma(1+\upsilon)}\,{}_{1}F_{1}\left[\left.\begin{matrix}\frac{\upsilon}{2}+\frac{\mu}{2}+\frac{1}{2}\\ 1+\upsilon\end{matrix}\right|-\frac{s^{2}}{4a}\right],$$
$$\mathrm{Re}a>0,\mathrm{Re}(\nu+\mu)>-1,s>0. \tag{AII-15}$$

$$\mathcal{L}\{\cos(at);s\} = \tfrac{s}{a^2+s^2}, \qquad \mathrm{Re}s > |\mathrm{Im}a|. \tag{AII-16}$$

$$\mathcal{L}\{\sin(at);s\} = \tfrac{a}{a^2+s^2}, \qquad \mathrm{Re}s > |\mathrm{Im}a|. \tag{AII-17}$$

$$\mathcal{L}\left\{t^{\nu-1}\cos(at);s\right\} = \frac{\Gamma(\nu)}{2}\left[\frac{1}{(s-ia)^{\nu}} + \frac{1}{(s+ia)^{\nu}}\right], \quad \mathrm{Re}\nu > 0, \mathrm{Re}s > |\mathrm{Im}a|. \tag{AII-18}$$

$$\mathcal{L}\left\{t^{\nu-1}\sin(at);s\right\} = \frac{\Gamma(\nu)}{2i}\left[\frac{1}{(s-ia)^{\nu}} - \frac{1}{(s+ia)^{\nu}}\right], \quad \mathrm{Re} > -1, \mathrm{Re}s > |\mathrm{Im}a|. \tag{AII-19}$$

$$\mathcal{H}_{\nu}\left\{t^{-1/2}\cos\left(at^2\right);s\right\} = \frac{1}{2}\sqrt{\frac{\pi s}{a}}\cos\left(\tfrac{\pi}{2}\left(\tfrac{\nu}{2}+\tfrac{1}{2}\right) - \tfrac{s^2}{4a}\right)\mathcal{J}_{\frac{\nu}{2}}\left(\frac{s^2}{8a}\right), \quad a > 0, \mathrm{Re}\nu > -1. \tag{AII-20}$$

$$\mathcal{H}_{\nu}\left\{t^{-1/2}\sin\left(at^2\right);s\right\} = \frac{1}{2}\sqrt{\frac{\pi s}{a}}\sin\left(\tfrac{\pi}{2}\left(\tfrac{\nu}{2}+\tfrac{1}{2}\right) - \tfrac{s^2}{4a}\right)\mathcal{J}_{\frac{\nu}{2}}\left(\frac{s^2}{8a}\right), \quad a > 0, \mathrm{Re}\nu > -3. \tag{AII-21}$$

$$\mathcal{H}_{\nu}\left\{t^{1/2}\cos\left(at^2\right);s\right\} = \frac{\pi^{1/2}s^{3/2}}{2^3a^{3/2}}(\sin\left(\tfrac{s^2}{8a} - \tfrac{\pi\nu}{4}\right)\mathcal{J}_{\frac{\nu}{2}-\frac{1}{2}}\left(\tfrac{s^2}{8a}\right) + \cos\left(\tfrac{s^2}{8a} - \tfrac{\pi\upsilon}{4}\right)\mathcal{J}_{\frac{\nu}{2}+\frac{1}{2}}\left(\tfrac{s^2}{8a}\right)), \quad a > 0, \mathrm{Re}\nu > -2. \tag{AII-22}$$

$$\mathcal{H}_{\nu}\left\{t^{1/2}\sin\left(at^2\right);s\right\} = \frac{\pi^{1/2}s^{3/2}}{2^3a^{3/2}}(\cos\left(\tfrac{s^2}{8a} - \tfrac{\pi\upsilon}{4}\right)\mathcal{J}_{\frac{\upsilon}{2}-\frac{1}{2}}\left(\tfrac{s^2}{8a}\right) - \sin\left(\tfrac{s^2}{8a} - \tfrac{\pi\upsilon}{4}\right)\mathcal{J}_{\frac{\nu}{2}+\frac{1}{2}}\left(\tfrac{s^2}{8a}\right)), \quad a > 0, \mathrm{Re}\nu > -4. \tag{AII-23}$$

$$\mathcal{H}_{\nu}\left\{t^{\nu+1/2}\cos\left(at^2\right);s\right\} = \frac{s^{\nu+1/2}}{2^{\upsilon+1}a^{\upsilon+1}}\sin\left(\tfrac{s^2}{4a} - \tfrac{\pi\upsilon}{2}\right), \quad -1 < \mathrm{Re}\nu < 1/2. \tag{AII-24}$$

$$\mathcal{H}_{\nu}\left\{t^{\nu+1/2}\sin\left(at^2\right);s\right\} = \frac{s^{\nu+1/2}}{2^{\nu+1}a^{\nu+1}}\cos\left(\tfrac{s^2}{4a} - \tfrac{\pi\nu}{2}\right), \quad -2 < \mathrm{Re}\nu < 1/2. \tag{AII-25}$$

$$\mathcal{L}\left\{t^{\mu-1}\mathcal{J}_{2\nu}\left(2\sqrt{at}\right);s\right\} = \frac{a^{\nu}}{s^{\mu+\upsilon}}\frac{\Gamma(\mu+\nu)}{\Gamma(1+2\upsilon)}\,{}_1F_1\left[\begin{array}{c} \upsilon+\mu \\ 1+2\nu \end{array}\middle| -\frac{a}{s}\right], \quad \mathrm{Re}(\nu+\mu) > 0, \mathrm{Re}s > 0. \tag{AII-26}$$

$$\mathcal{L}\left\{t^{\mu-1/2}\mathcal{J}_{2\nu}\left(2\sqrt{at}\right);s\right\} = \frac{\Gamma(\mu+1/2+\nu)\,e^{-a/2s}}{\Gamma(1+2\upsilon)\,a^{1/2}s^{\mu}}M_{\mu,\nu}\left(\frac{a}{s}\right), \quad \mathrm{Re}(\nu+\mu) > -1/2, \mathrm{Re}s > 0. \tag{AII-27}$$

$$\mathcal{L}\left\{t^{\nu/2+n}\mathcal{J}_{\nu}\left(2\sqrt{at}\right);s\right\} = n!\frac{a^{\nu/2}}{s^{\nu+n+1}}\,e^{-a/s}L_n^{\nu}\left(\frac{a}{s}\right), \quad \mathrm{Re}(\nu) + n > 0, \mathrm{Re}s > 0. \tag{AII-28}$$

$$\mathcal{L}\left\{t^{\nu/2-1}\mathcal{J}_{\nu}\left(2\sqrt{at}\right);s\right\} = a^{-\nu/2}\,\gamma\left(\upsilon;\frac{a}{s}\right), \mathrm{Re}\nu > 0, \mathrm{Re}s > 0. \tag{AII-29}$$

$$\mathcal{L}\left\{t^{-\nu/2}\mathcal{J}_{\nu}\left(2\sqrt{at}\right);s\right\} = e^{i\pi\nu}\frac{a^{-\nu/2}}{s^{1-\nu}}\frac{e^{-a/s}}{\Gamma(\nu)}\gamma\left(\nu;-\frac{a}{s}\right), \mathrm{Re}s > 0. \tag{AII-30}$$

$$\mathcal{L}\left\{t^{\nu/2}\mathcal{J}_{\nu}\left(2\sqrt{at}\right);s\right\} = \frac{a^{\nu/2}}{s^{1+\nu}}e^{-a/s}, \ \mathrm{Re}\nu > -1, \mathrm{Re}s > 0. \tag{AII-31}$$

$$\mathcal{L}\left\{t^{-1/2}\mathcal{J}_{\nu}\left(2\sqrt{at}\right);s\right\} = \frac{\sqrt{\pi}}{s^{1/2}}e^{-a/2s}\,\mathcal{I}_{\nu/2}\left(\frac{a}{2s}\right), \ \mathrm{Re}\nu > -1, \mathrm{Re}s > 0. \tag{AII-32}$$

$$\mathcal{L}\left\{t^{1/2}\mathcal{J}_1\left(2\sqrt{at}\right);s\right\} = a^{1/2}s^{-2}e^{-a/s}, \qquad \mathrm{Re}s > 0. \tag{AII-33}$$

$$\mathcal{L}\left\{\mathcal{J}_\nu\left(2\sqrt{at}\right);s\right\} = \frac{1}{s}e^{-a/s}, \qquad \mathrm{Re}s > 0. \tag{AII-34}$$

$$\begin{aligned}\mathcal{H}_\nu\left\{t^{-2\lambda-1/2}\mathcal{J}_\nu(at);s\right\} &= \frac{s^{\nu+1/2}2^{-2\lambda}}{a^{\nu-2\lambda+1}}\frac{\Gamma\left(\nu-\lambda+\frac{1}{2}\right)}{\Gamma\left(\lambda+\frac{1}{2}\right)\Gamma(\nu+1)}\\ &\times {}_2F_1\left(\upsilon-\lambda+\frac{1}{2},-\lambda+\frac{1}{2};1+\nu;\frac{s^2}{a^2}\right),\\ &\mathrm{Re}\nu+1/2 > \mathrm{Re}\lambda > -1/2, a > 0, s > 0.\end{aligned} \tag{AII-35}$$

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