New description for the bright, dark periodic solutions to the complex Hirota-dynamical model

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ABSTRACT

In this article, we employ the nonlinear complex Hirota-dynamical model which is one of the famous and important standards to the nonlinear Schrödinger equation in which the third derivative term represent the self-interaction in the high-frequency subsystem. Specially, in plasma this term is isomorphic to the so known self-focusing effect. The bright, dark and periodic optical soliton solutions to this equation will realized successfully for the first time in the framework of the solitary wave ansatz method. Furthermore, in this connection at the same time and parallel the extended simple equation method has been applied successfully to achieve new impressive solitary wave solutions to this model. A comparison between the obtained results and that satisfied in previous work has been established.

Keywords: The nonlinear complex Hirota-dynamical model; the solitary wave ansatz method; the extended simple equation method; optical soliton solutions, Graphical representation

1-Introduction

The main idea of this article focused on how we can realizing the bright, dark and periodic soliton solution to the nonlinear complex (1+1)-dimensional Hirota equation (CHE) arising in various fields of applied sciences. Two important impressive techniques are invited for this purpose, the first one which recently appearing and listed with name the wave ansatz method(SWAM) [1-3] that give surprise results for several problems. In this connection and parallel the other one is introduced with name the extended simple equation method (ESEM) [4] which has been perfectly used to achieve new optical solutions to this model. The wave propagation modeling and wave motion plays a vital roles in coastal, ocean and beaches, floating structures and maritime engineering. This modes can be used to identify various nonlinear phenomena's in different branches of physics and engineering sciences. Specially, optic, plasma physics, electric communication and so on. In addition one of the famous and important physical applications to Hirota equation is the propagation of optical pulses and femtosecond pulse propagation in optical fibers [5]. Hirota dynamical model represents a special case of the well known nonlinear Schrödinger equation arising in mathematical physics which related to the complex modified KdV equation and nonlinear Schrodinger equation which shows interaction of the lower-hybrid large-amplitude waves with finite-frequency density perturbations in plasma physics [6].

It is important to find a mathematical model which represents the nonlinear complex Hirota equation which have many applications in nonlinear physics. According to Seadawy [7] this model can be introduced as,

$$iu_t + u_{xx} + 2\left|u^2\right|u + i\alpha u_{xxx} + 6i\alpha \left|u\right|^2 u_x = 0 \tag{1}$$

Where $u = u(x,t)$ represents wave propagation and α is real constant.

Briefly, when $\alpha = 0$ equation (1) revere to the well known standard Schrödinger equation which widely propagated in mathematical physics. Furthermore, the third derivatives term which in plasma denotes self-focusing effect and in other branches indicates the nonlinear self-interaction in the high-frequency subsystem.

Many methods are established to solve nonlinear problems arising in different branches of science and can be briefly proposed through references [8-26]. Specially, several tries were admitted by small sequences of them through good established efforts to solve Hirota equation by diverse methods [27–29].

2. Description of the solitary wave ansatz method

According to [1, 3] the solitary wave ansatz method solutions can be proposed as follows,

Consider the wave transformation,

$$u(x,t)=\psi(x,t)e^{iR(x,t)} \tag{2}$$

Where $\psi(x,t)$ is the amplitude portion, while $R(x,t)$ is the phase portion of soliton.

Consequently it is easy to get these relations,

$$\begin{aligned} u_t &= (\psi_t + i\psi R_t)e^{iR} \\ u_x &= (\psi_x + i\psi R_x)e^{iR} \\ u_{xx} &= (\psi_{xx} + 2i\psi_x R_x + i\psi R_{xx} - \psi R_x^2)e^{iR} \\ u_{xxx} &= (\psi_{xxx} + 3i\psi_{xx} R_x - i\psi R_x^3 - 3\psi_x R_x^2 + i\psi R_{xxx} + 3i\psi_x R_{xx} - 3\psi R_x R_{xx})e^{iR} \end{aligned} \tag{3}$$

The bright and dark soliton solutions admits as follows,

(I) The bright soliton solutions

$$\psi(x,t)=A_1\operatorname{sech}^R(t_1),\ where\, t_1=B(x-w_1t)\, and\ R(x,t)=kx-\Omega t$$

$$\psi_t = A_1 B w_1 R\operatorname{sech}^R(t_1)\tanh(t_1)$$

$$\psi_x = -A_1 BR\operatorname{sech}^R(t_1)\tanh(t_1)$$

$$\psi_{xx} = -A_1 B^2 R(1+R)\operatorname{sech}^{R+2}(t_1) + A_1 B^2 R^2\operatorname{sech}^R(t_1)$$

$$\psi_{xxx} = A_1 B^3 R(R+1)(R+2)\operatorname{sech}^{R+2}(t_1)\tanh(t_1) - A_1 B^3 R^3\operatorname{sech}^R(t_1)\tanh(t_1)$$

(II) The dark soliton solutions

$$\psi(x,t)=A_2\tanh^R(t_1),\ where\, t_1=B(x-w_2t)\, and\ R(x,t)=kx-\Omega t$$

$$\psi_t = -A_2 BRw_2[\tanh^{R-1}(t_1)-\tanh^{R+1}(t_1)]$$

$$\psi_x = A_2 BR[\tanh^{R-1}(t_1)-\tanh^{R+1}(t_1)]$$

$$\psi_{xx} = A_2 R(R-1)B^2\tanh^{R-2}(t_1) - 2A_2R^2B^2\tanh^R(t_1) + 2A_2R(R+1)B^2\tanh^{R+2}(t_1)$$

$$\begin{aligned}\psi_{xxx} = A_2RB^3[&(R-1)(R-2)\tanh^{R-3}(t_1) - ((R-1)(R-2)+2R^2)\tanh^{R-1}(t_1) \\ &+((R+1)(R+2)+2R^2)\tanh^{R+1}(t_1) - (R+1)(R+2)\tanh^{R+3}(t_1)]\end{aligned} \tag{4}$$

3. Application:

The bright and dark soliton solutions to the nonlinear complex Hirota-dynamical model

Substituting about $u_t, u_x, u_{xx} and\, u_{xxx}$ mentioned in the equation (3) at the proposed equation (1), we get,

$$\begin{aligned}&i(\psi_t + i\psi R_t)e^{iR} + [\psi_{xx} + 2i\psi_x R_x + i\psi R_{xx} - \psi R_x^2]e^{iR} \\ &+ 2\psi^5\psi_x e^{iR} + i\alpha[\psi_{xxx} + 3i\psi_{xx}R_x - 3\psi_x R_x^2 - i\psi R_x^3 \\ &+ i\psi R_{xxx} + 3i\psi_x R_{xx} - 3\psi R_x R_{xx} + 6i\alpha[\psi_x\psi^4 + iR_x\psi^5] = 0\end{aligned} \tag{6}$$

Which splits into two real and imaginary parts are shown respectively as:

$$-\psi R_t + \psi_{xx} - \psi R_x^2 + 2\psi^5 - 3\alpha\psi_{xx}R_x + \alpha\psi R_x^3 - \alpha\psi R_{xxx} - 3\alpha\psi_x R_{xx} - 6\alpha\psi^5 R_x = 0, \tag{7}$$

$$\psi_t + 2\psi_x R_x + \psi R_{xx} - 3\alpha\psi_x R_x^2 + \alpha\psi_{xxx} - 3\alpha\psi R_x R_{xx} + 6\alpha\psi_x\psi^4 = 0 \tag{8}$$

Now; use the constructed relations (4) at the real part equation (7) and at the imaginary part equation (8) respectively, we obtain:

$$\begin{aligned}&(\Omega + B^2R - k^2 - 3\alpha k B^2 R^2 + \alpha k^3)A_1 \operatorname{sech}^R \\ &+ [3\alpha k B^2 R(R+1) - B^2 R(R+1)]A_1 \operatorname{sech}^{R+2} + [2 - 6\alpha k]A_1^5 \operatorname{sech}^{5R} = 0\end{aligned} \tag{9}$$

$$\begin{aligned}&[A_1 B R w_1 - 2kA_1 BR - \alpha A_1 B^3 R^3 + 2\alpha k^2 A_1 BR]\operatorname{sech}^R \tanh \\ &+ \alpha A_1 B^3 R(R+1)(R+2)\operatorname{sech}^{R+2} \tanh - 6\alpha A_1^5 R \operatorname{sech}^{5R} \tanh = 0\end{aligned} \tag{10}$$

From equations (9), (10), we can easily obtain:

$$R = \frac{1}{2}, B = \frac{3}{5}, \alpha = \frac{2}{3}, kx - \Omega t = \frac{1}{2}, \Omega = \frac{w_1}{2} - 0.63, k = 1 \tag{11}$$

Thus, according to the constructed method the solution will be,

$$u(x,t) = A_1 e^{i(kx - \Omega t)} \times \operatorname{sech}^R B(x - w_1 t)$$

$$u(x,t) = e^{i(x + 0.13t)} \times \operatorname{sech}^{0.5}(x - t)$$

$$u(x,t) = [\operatorname{Cos}(x + 0.13t) + i\operatorname{Sin}(x + 0.13t)]\operatorname{sech}^{\frac{1}{2}}(x - t) \tag{12}$$

Hence, the real part is

$$\operatorname{Re} u(x,t) = [\operatorname{Cos}(x + 0.13t)]\operatorname{sech}^{\frac{1}{2}}(x - t) \tag{13}$$

And the imaginary part is,

$$\operatorname{Im} u(x,t) = [\operatorname{Sin}(x + 0.13t)]\operatorname{sech}^{\frac{1}{2}}(x - t) \tag{14}$$

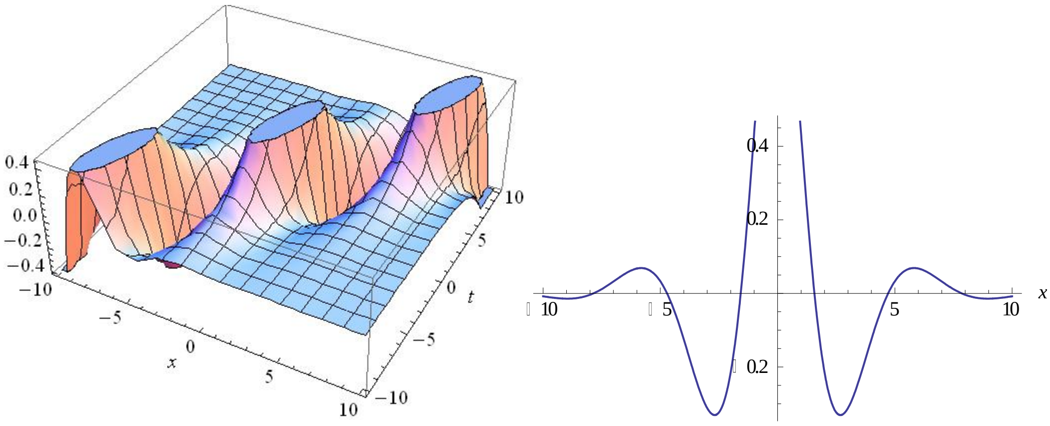

Figure 1. The bright soliton solution of real part Eq.(13) in **2D** and **3D** with values:

$$A_1 = 1,\quad \Omega = -0.13, w_1 = 1, k = 1, B = \frac{3}{5}, \alpha = \frac{2}{3}, R = 0.5$$

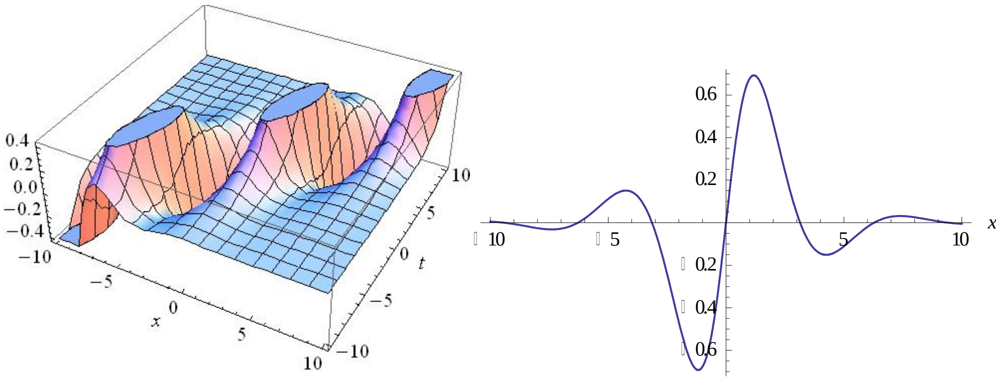

Figure 2. The bright soliton solution imaginary part Eq.(14) in **2D** and **3D** with values:

$$A_1 = 1,\quad \Omega = -0.13, w_1 = 1, k = 1, B = \frac{3}{5}, \alpha = \frac{2}{3}, R = 0.5$$

Similarly, use the constructed relations (5) at the real part equation (7) and at the imaginary part equation (8) respectively, we obtain:

$$\begin{aligned}&(A_2\Omega - 2A_2B^2R^2 - A_2k^2 + \alpha A_2 + 6\alpha k A_2B^2R^2)\tanh^R + R(R-1)(A_2B^2 - 3\alpha k A_2B^2)\tanh^{R-2} + \\ &\qquad R(R+1)(A_2B^2 - 3\alpha k A_2B^2)\tanh^{R+2} + (2-6\alpha k)A_2^5\tanh^{5R} = 0\end{aligned} \tag{15}$$

$$\begin{aligned}&[-A_2w_2RB + 2kA_2RB - 3\alpha A_2RBk^2 + \alpha RB^3((R-1)(R-2)+2R^2)]\tanh^{R-1} + \\ &[A_2w_2RB - 2kA_2RB + 3\alpha A_2RBk^2 + \alpha RB^3((R+1)(R+2)+2R^2)]\tanh^{R+1} + \\ &\alpha RB^3(R-1)(R-2)\tanh^{R-3} - \alpha RB^3(R+1)(R+2)\tanh^{R+3} + \\ &6\alpha RBA_2^5\tanh^{5R-1} - 6\alpha RBA_2^5\tanh^{5R+1} = 0\end{aligned} \tag{16}$$

From equations (15), (16) we can easily obtain:

$$w_2 = k^2 + \frac{1}{k}, kx - \Omega t = \frac{1}{2}, \Omega = \frac{51}{A_2} + \frac{2}{3} A_2 = \frac{17B^2}{24(w_2 - \frac{2}{3})} \tag{17}$$

The solution according to the proposed method is,

$$u(x,t) = A_2 e^{i(kx-\Omega t)} \times \tanh^R (x-t)$$

$$u(x,t) = \frac{17B^2}{24(w_2 - \frac{2}{3})} e^{i\left(x-(\frac{51}{A_2}+\frac{2}{3})t\right)} \times \tanh^{0.5} B(x - w_2 t)$$

$$u(x,t) = 1.063 \times e^{i(x-48.6t)} \times \tanh^{0.5}(x-1.3t)$$

$$u(x,t) = 1.063 \times [\mathrm{Cos}(x-48.7t) + i\,\mathrm{Sin}(x-48.7t)] \times \tanh^{\frac{1}{2}}(x-1.3t)$$

Hence, the real part is

$$\mathrm{Re}\,u(x,t) = 1.063 \times [\mathrm{Cos}(x-48.7t)] \times \tanh^{\frac{1}{2}}(x-1.3t) \tag{18}$$

And the imaginary part is,

$$\mathrm{Im}\,u(x,t) = 1.063 \times [\mathrm{Sin}(x-48.7t)] \times \tanh^{\frac{1}{2}}(x-1.3t) \tag{19}$$

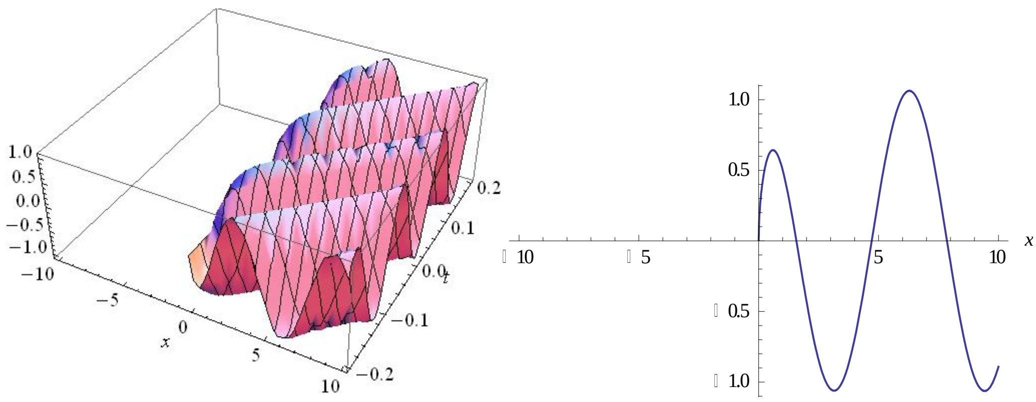

Figure 3. The dark soliton solution of Real part Eq.(18) in **2D** and **3D** with values:

$$A_1 = 1.063, \quad \Omega = 48.7, w_2 = 1.3, k = 1, B = 1, \alpha = \frac{1}{3}, R = 0.5$$

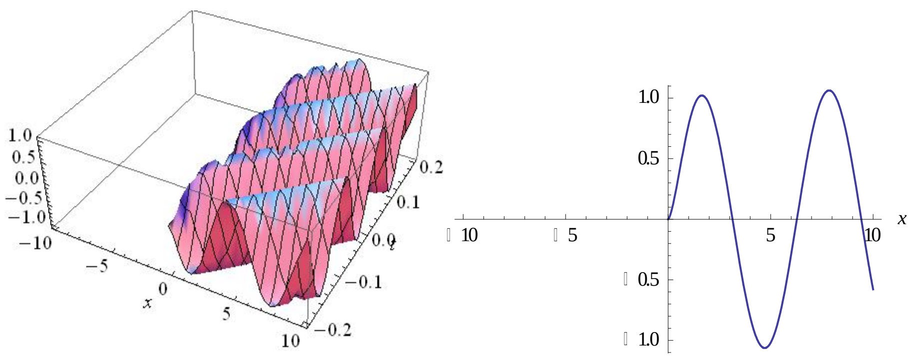

Figure 4 . The dark soliton solution imaginary part Eq.(19) in **2D** and **3D** with values:

$$A_1 = 1.063, \quad \Omega = 48.7, w_2 = 1.3, k = 1, B = 1, \alpha = \frac{1}{3}, R = 0.5$$

4. The extended simple equation method

To propose the description algorithm for the extended simple equation method ESEM [4], let us firstly propose in this section the general form of the nonlinear evolution equation by introducing the function R as a function of h(x, t) and its partial derivatives as,

$$R(h, h_x, h_t, h_{xx}, h_{tt}..........) = 0 \tag{20}$$

That involves the highest order derivatives and nonlinear terms. With the aid of the transformation $h(x,t) = h(\zeta), \zeta = wx + kt$ equation (20) can be reduced to the following ODE:

$$S(h, h', h''..........) = 0 \tag{21}$$

Where S is a function in $h(\zeta)$ and its total derivatives, while $' = \frac{d}{d\zeta}$

The constructed solution according to this method is:

$$V(\zeta) = \sum_{i=-m}^{m} A_i \psi^i(\zeta). \tag{22}$$

Where the positive integer m in *E*q. (22) can be located by balancing the highest order derivative term and the nonlinear term, while the arbitrary constants A_i could be calculated later, the function $\phi(\zeta)$ satisfies the following new ansatz equation

$$\psi'(\zeta) = a_0 + a_1\psi + a_2\psi^2 \tag{23}$$

Where a_0, a_1 *and* a_2 other arbitrary constants which admit these two cases;

(1) If $a_1 = a_3 = 0$ it will transform the Riccati equation [30], [31], which has the following solutions;

$$\psi(\zeta) = \frac{\sqrt{a_0 a_2}}{a_2} \tan(\sqrt{a_0 a_2}(\zeta + \zeta_0), a_0 a_2 \succ 0 \tag{24}$$

$$\psi(\zeta) = \frac{\sqrt{-a_0 a_2}}{a_2} \tanh(\sqrt{-a_0 a_2}\zeta - \frac{\rho \ln \zeta_0}{2}), a_0 a_2 \prec 0, \zeta \succ 0, \rho = \pm 1 \tag{25}$$

(2) If $a_0 = a_3 = 0,$ it will transform the Bernoulli equation [32], which has the following solutions;

$$\psi(\zeta) = \frac{a_1\, Exp[a_1(\zeta + \zeta_0)]}{1 - a_2 Exp[a_1(\zeta + \zeta_0)]}, a_1 \succ 0 \tag{26}$$

$$\psi(\zeta) = \frac{-a_1\, Exp[a_1(\zeta + \zeta_0)]}{1 + a_2 Exp[a_1(\zeta + \zeta_0)]}, a_1 \prec 0 \tag{27}$$

And the general solution to ansatz equation (23) is as follows:

$$\psi(\zeta) = -\frac{1}{a_2}\left(a_1 - \sqrt{4a_1a_2 - a_1^2}\, \tan\left(\frac{\sqrt{4a_1a_2 - a_1^2}}{2}(\zeta + \zeta_0) \right) \right), 4a_1a_2 \succ a_1^2, a_2 \succ 0, \tag{28}$$

$$\psi(\zeta)=\frac{1}{a_2}\left(a_1+\sqrt{4a_1a_2-a_1^2}\tanh\left(\frac{\sqrt{4a_1a_2-a_1^2}}{2}(\zeta+\zeta_0)\right)\right), 4a_1a_2 \succ a_1^2, a_2 \prec 0, \qquad (29)$$

Where the integer ζ_0 is constancy of integration

Finally, substituting for equation (22) at equation (23) and equating the coefficients of different powers of ψ^i to zero, we can easily obtain a system of algebraic equations, when one solves it, he can get the values of the unknown parameters mentioned in these methods. Furthermore, substituting about these obtained parameters at equation (23) then the required solution has been realized.

5. The exact solutions according to the extended simple equation method

In this section, we will apply the proposed method to the suggested equation (1) mentioned above,

$$iu_t+u_{xx}+2\left|u^2\right|u+i\alpha u_{xxx}+6i\alpha\left|u\right|^2u_x=0$$

According to this method, the solution is:

$$u(x,t)=\phi(\zeta)\,e^{i\mu(x,t)}, \zeta=kx+wt, \mu=qx+\delta t \qquad (30)$$

$$u_t=i\delta\phi\,e^{i\mu}+w\phi'\,e^{i\mu}, \qquad (31)$$

$$u_x=iq\phi\,e^{i\mu}+k\phi'\,e^{i\mu}, \qquad (32)$$

$$u_{xx}=-q^2\phi\,e^{i\mu}+2ikq\phi'\,e^{i\mu}+k^2\phi''e^{i\mu}, \qquad (33)$$

$$u_{xxx}=-iq^3\phi\,e^{i\mu}-3kq^2\phi'\,e^{i\mu}+3iqk^2\phi''e^{i\mu}+k^3\phi'''e^{i\mu}, \qquad (34)$$

$$\left|u^2\right|=\phi^2, \left(\left|u^2\right|\right)_x=2k\phi\phi'. \qquad (35)$$

Substituting about the above relation at the nonlinear complex Hirota- dynamical model we get:

$$\begin{aligned}&-\delta\phi\,e^{i\mu}+iw\phi'\,e^{i\mu}+k^2\phi''e^{i\mu}+2ikq\phi'\,e^{i\mu}-q^2\phi\,e^{i\mu}+2\varphi^3\,e^{i\mu}\\&+\alpha q^3\phi\,e^{i\mu}-3i\alpha kq^2\phi'\,e^{i\mu}-3\alpha qk^2\phi''e^{i\mu}+i\alpha k^3\phi'''\,e^{i\mu}+12i\alpha k\phi^2\phi'\,e^{i\mu}=0,\end{aligned} \qquad (36)$$

From which we can separate the following real and imaginary parts respectively

$$\mathrm{Re.}\quad -\delta\phi-q^2\phi+k^2\phi''+2\varphi^3+\alpha q^3\phi-3\alpha qk^2\phi''=0, \qquad (37)$$

$$\mathrm{Im}\quad (\alpha q^3-q^2-\delta)\phi+k^2(1-3\alpha qk^2)\phi''+2\varphi^3=0, \qquad (38)$$

Firstly we will study the real part,

$$\mathrm{Re.}\quad -\delta\phi-q^2\phi+k^2\phi''+2\varphi^3+\alpha q^3\phi-3\alpha qk^2\phi''=0,$$

Balancing the nonlinear term and the higher order derivatives term at equation (37) implies $3m=m+2$ from which we get $m=1$, hence the solution is,

$$\varphi(\zeta)=\frac{A_{-1}}{v}+A_0+A_1v \qquad (39)$$

Where $v'=a_0+a_1v+a_2v^2$

Case 5-1: When $a_1=0 \Rightarrow v'=a_0+a_2v^2$**,** consequently

$$\varphi'=-\frac{a_0A_{-1}}{v^2}+A_1a_0+A_1a_2v^2-a_2A_{-1} \qquad (40)$$

$$\varphi'' = \frac{2a_0^2 A_{-1}}{v^3} + \frac{2a_0 a_2 A_{-1}}{v} + 2A_1 a_0 a_2 v + 2A_1 a_2^2 v^3 \tag{41}$$

$$\varphi^3 = A_1^3 v^3 + 3A_0 A_1^2 v^2 + 3(A_{-1}A_1^2 + A_1 A_0^2)v + 6A_0 A_1 A_{-1}$$
$$A_0^3 + \frac{A_{-1}^3}{v^3} + \frac{3A_0 A_{-1}^2}{v^2} + 3(A_1 A_{-1}^2 + A_{-1}A_0^2)\frac{1}{v} \tag{42}$$

Substituting for equations (39-42) at equation (37) and collecting and equating the coefficients of different powers of ψ^i to zero, we can easily obtain this system of algebraic equations

$$A_1^2 + k^2 a_2^2 (1 - 3\alpha q) = 0$$
$$6A_1^2 A_0 = 0$$
$$2a_0 a_2 k^2 (1 - 3\alpha q) + 6(A_{-1}A_1 + A_0^2) + (\alpha q^3 - q^2 - \delta) = 0$$
$$(\alpha q^3 - q^2 - \delta) + 12A_1 A_{-1} + 2A_0^2 = 0$$
$$2a_0 a_2 k^2 (1 - 3\alpha q) + 6(A_{-1}A_1 + A_0^2) + (\alpha q^3 - q^2 - \delta) = 0$$
$$6A_{-1}^2 A_0 = 0$$
$$A_{-1}^2 + k^2 a_0^2 (1 - 3\alpha q) = 0 \tag{43}$$

When one solves this system, he can obtain,

The second and the six part of equation (43) implies $A_0 = 0$**,** hence substitute at the other parts of equation about $A_0 = 0$ and solve them analytically we get,

$$a_0 = \pm\frac{2}{3}, a_2 = 0.5, k = 1, A_1 = \pm 1, A_{-1} = \pm\frac{2}{3}, A_0 = 0, \delta = \sqrt{0.7}, \tag{44}$$

From which we can get 7-differents solutions namely, we will plot only one case which is (1)

(5.1.a) $a_0 = \frac{2}{3}, a_2 = 0.5, k = 1, A_1 = 1, A_{-1} = \frac{2}{3}, A_0 = 0, \delta = \sqrt{0.7},$

$$v(\zeta) = \frac{\sqrt{a_0 a_2}}{a_2}\tan(\sqrt{a_0 a_2}(\zeta + \zeta_0)), \ \therefore v(\zeta) = 1.15\tan[0.57(x + t + 1)],$$
$$\varphi_{11}(\zeta) = 0.57\cot[0.58(x + t + 1)] + 1.15\tan[0.58(x + t + 1)] \tag{45}$$

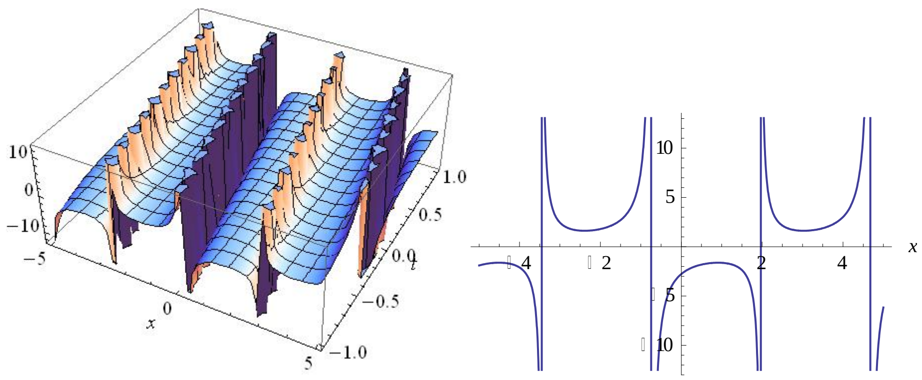

Figure 5.1.a The periodic soliton solution real part Eq.(45) in **2D** and **3D** with values:

$$a_0 = \frac{2}{3}, a_2 = 0.5, k = 1, A_1 = 1, A_{-1} = \frac{2}{3}, A_0 = 0, \delta = \sqrt{0.7}, \zeta_0 = 2$$

(5.1-b) $a_0 = -\frac{2}{3}, a_2 = 0.5, k = 1, A_1 = 1, A_{-1} = \frac{2}{3}, A_0 = 0, \delta = \sqrt{0.7},$

$$\psi(\zeta)=\frac{\sqrt{-a_0a_2}}{a_2}\tanh(\sqrt{-a_0a_2}\,\zeta-\frac{\rho\ln\zeta_0}{2}),\ \ \therefore v(\zeta)=1.15\tanh[0.57(x+t+0.34)],$$

$$\varphi_{12}(\zeta)=0.57\coth[0.58(x+t+0.34)]+1.15\tanh[0.58(x+t+0.34)] \tag{45}$$

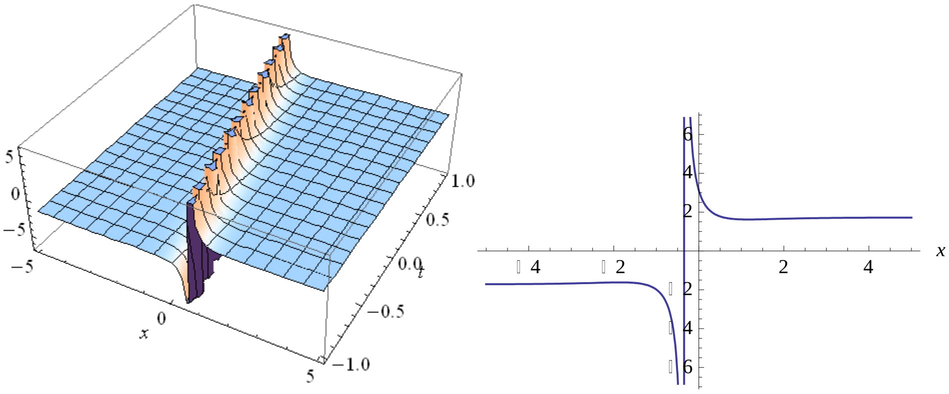

Figure 5.1.b The periodic soliton solution real part Eq.(45) in **2D** and **3D** with values:

$$a_0=-\frac{2}{3},a_2=0.5,k=1,A_1=1,A_{-1}=\frac{2}{3},A_0=0,\delta=\sqrt{0.7},\rho=-1,\zeta_0=2$$

By the same manner we can easily drown the other 6-cases.

Case 5- 2: Now we will study the second case of the constructed method for the real part: When $a_0=0\Rightarrow v'=a_1v+a_2v^2$**,** consequently from the above equation,

$$\varphi^2=\frac{A_{-1}^2}{v^2}+A_0^2+A_1^2v^2+\frac{2A_0A_{-1}}{v^2}+2A_1A_{-1}+2A_0A_1v \tag{46}$$

$$\varphi'=A_1a_2v^2+A_1a_1v-\frac{a_1A_{-1}}{v^2}-a_2A_{-1} \tag{47}$$

$$\varphi''=2A_1a_2^2v^3+3A_1a_1a_2v^2+A_1a_1^2v+\frac{A_{-1}a_1a_2}{v}+\frac{a_1^2A_{-1}}{v^2} \tag{48}$$

$$\begin{aligned}\varphi^3&=A_1^3v^3+3A_0A_1^2v^2+3(A_{-1}A_1^2+A_1A_0^2)v+6A_0A_1A_{-1}\\&A_0^3+\frac{A_{-1}^3}{v^3}+\frac{3A_0A_{-1}^2}{v^2}+3(A_1A_{-1}^2+A_{-1}A_0^2)\frac{1}{v}\end{aligned} \tag{49}$$

Substituting for equations (46-49) at equation (37) and collecting and equating the coefficients of different powers of ψ^i to zero, we can easily obtain this system of algebraic equations

$$\begin{aligned}&k^2a_2^2(1-3\alpha q)+A_1^2=0\\&k^2a_1a_2(1-3\alpha q)+2A_0A_1=0\\&(\alpha q^3-q^2-\delta)+k^2a_1^2(1-3\alpha q)+6(A_{-1}A_1+A_0^2)=0\\&(\alpha q^3-q^2-\delta)++2A_0^2+12A_{-1}A_1=0\\&(\alpha q^3-q^2-\delta)+k^2a_1a_2(1-3\alpha q)+6(A_{-1}A_1+A_0^2)=0\\&k^2a_1^2(1-3\alpha q)+6A_0A_{-1}^2=0\\&2A_{-1}^3=0\end{aligned} \tag{50}$$

The last part of equation (50) implies $A_{-1}=0$**,** hence when we substitute at the six part give one of probability values a_1 which is $a_1 = 0$ and in this case there are no solutions.

But for the other parts of equations (50) if $A_{-1} = 0$ the other parts then we can easily analytically obtain,

$$a_1 = \pm\sqrt{\frac{\alpha q^3 - q^2 - \delta}{k^2(3\alpha q - 1)}}, a_2 = 1, k = 1, A_1^2 = (3\alpha q - 1)k^2 a_2^2, A_{-1} = 0, A_0 = 0,$$

$$a_1 = \pm\sqrt{\frac{\alpha q^3 - q^2 - \delta}{k^2(3\alpha q - 1)}}, a_2 = 1, k = 1, A_1 = \pm\sqrt{(3\alpha q - 1)k^2 a_2^2}, A_{-1} = 0, A_0 = 0,$$

$$a_1 = \pm 2, a_2 = \pm 1, k = 1, A_1 = \pm 2, k = 1, A_{-1} = 0, A_0 = 0, \delta = 0.7, \alpha = \frac{1}{3}, q = 5 \tag{51}$$

These obtained results admits 7-solutions, we will choose only two from of them,

Case 5-2.a:

$$a_1 = 2, a_2 = 1, k = 1, A_1 = 2, k = 1, A_{-1} = 0, A_0 = 0, \delta = 0.7, \alpha = \frac{1}{3}, q = 5, \zeta_0 = 1$$

$$v(\zeta) = -\frac{1}{a_2}\left(a_1 - \sqrt{4a_1a_2 - a_1^2}\tan\left(\frac{\sqrt{4a_1a_2 - a_1^2}}{2}(\zeta + \zeta_0)\right)\right), 4a_1a_2 \succ a_1^2, a_2 \succ 0,$$

$$v(\zeta) = -2 + 2\tan(x + t + 1),$$

$$\varphi_{21}(\zeta) = -4 + 4\tan(x + t + 1), \tag{52}$$

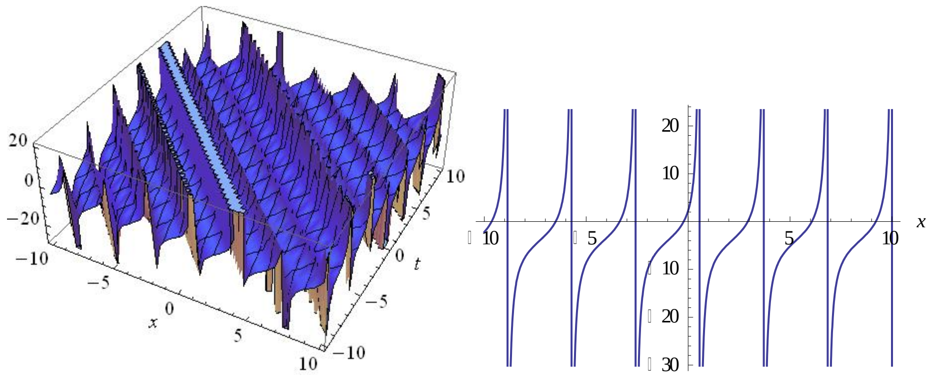

Figure 5.2.a The periodic soliton solution real part Eq.(52) in **2D** and **3D** with values:

$$a_1 = 2, a_2 = 1, k = 1, A_1 = 2, k = 1, A_{-1} = 0, A_0 = 0, \delta = 0.7, \alpha = \frac{1}{3}, q = 5, \zeta_0 = 1$$

Case 5-2.b:

$$a_1 = -2, a_2 = -1, k = 1, A_1 = 2, k = 1, A_{-1} = 0, A_0 = 0, \delta = 0.7, \alpha = \frac{1}{3}, q = 5 \tag{53}$$

$$v(\zeta) = \frac{1}{a_2}\left(a_1 + \sqrt{4a_1a_2 - a_1^2}\tanh\left(\frac{\sqrt{4a_1a_2 - a_1^2}}{2}(\zeta + \zeta_0)\right)\right), 4a_1a_2 \succ a_1^2, a_2 \prec 0,$$

$$v(\zeta) = 2 - 2\tanh(x + t + 1),$$

$$\varphi_{22}(\zeta) = 4 - 4\tanh(x+t+1), \tag{54}$$

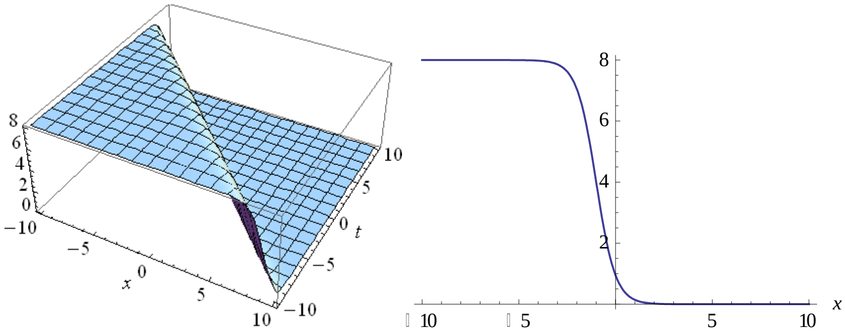

Figure 5.2.b The periodic soliton solution real part Eq.(54) in **2D** and **3D** with values:

$$a_1 = -2, a_2 = -1, k = 1, A_1 = 2, k = 1, A_{-1} = 0, A_0 = 0, \delta = 0.7, \alpha = \frac{1}{3}, q = 5$$

By the same manner we can easily drown the other 6-cases.

Secondly, we will study the imaginary term:

$$\text{Im.} \qquad w\phi' + 2kq\phi' - 3\alpha kq^2\phi' + \alpha k^3\phi''' + 12\alpha k\varphi^2\phi' = 0, \tag{55}$$

Equation (55) implies that

$$(w + 2kq - 3\alpha kq^2)\phi' + 12\alpha k\varphi^2\phi' + \alpha k^3\phi''' = 0,$$

Integrating once, we get

$$(w + 2kq - 3\alpha kq^2)\varphi + 4\alpha k\varphi^3 + \alpha k^3\phi'' = 0, \tag{56}$$

Balancing the nonlinear term and the highest order derivative term at equation (56) implies that $3m = m+2$ hence $m = 1$ and the solution is,

$$\varphi(\zeta) = \frac{A_{-1}}{v} + A_0 + A_1 v \tag{57}$$

Where $v' = a_0 + a_1 v + a_2 v^2$

Case 6.1: When $a_1 = 0 \Rightarrow v' = a_0 + a_2 v^2$ **,** consequently

$$\varphi'' = \frac{2a_0^2 A_{-1}}{v^3} + \frac{2a_0 a_2 A_{-1}}{v} + 2A_1 a_0 a_2 v + 2A_1 a_2^2 v^3 \tag{58}$$

$$\begin{aligned} \varphi^3 &= A_1^3 v^3 + 3A_0 A_1^2 v^2 + 3(A_{-1}A_1^2 + A_1 A_0^2)v + 6A_0 A_1 A_{-1} \\ &\quad A_0^3 + \frac{A_{-1}^3}{v^3} + \frac{3A_0 A_{-1}^2}{v^2} + 3(A_1 A_{-1}^2 + A_{-1}A_0^2)\frac{1}{v} \end{aligned} \tag{59}$$

Substituting for equations (57-59) at equation (56) and collecting and equating the coefficients of different powers of ψ^i to zero, we can easily obtain this system of algebraic equations

$$2A_1^2 + k^2 a_2^2 = 0$$

$$4\alpha k(3A_1^2 A_0) = 0$$

$$2\alpha a_0 a_2 k^3 + 12\alpha k(A_{-1}A_1 + A_0^2) + (w_2 + 2qk - 3\alpha kq^2) = 0$$

$$(w_2 + 2qk - 3\alpha kq^2) + 24\alpha k A_1 A_{-1} + 4\alpha k A_0^2 = 0$$

$$2A_{-1}^2 + k^2 a_0^2 = 0$$

$$4\alpha k(3A_{-1}^2 A_0)=0$$

$$(w_2+2qk-3\alpha kq^2)+2\alpha a_0 a_2 k^3+12\alpha k(A_{-1}A_1+A_0^2)=0 \qquad (60)$$

The second and the six part of equation (60) implies $A_0=0$, hence substitute at the other parts of equation about $A_0=0$ and solve them analytically we get,

$$a_0=\pm 13\sqrt{2}\,i, a_2=\pm\frac{1}{4\sqrt{2}\,i}, k=1, A_1=\frac{1}{8}, A_{-1}=13, A_0=0, w_2=2, q=5, \alpha=\frac{1}{3} \qquad (61)$$

From which we can get 4-differents solutions namely, we will plot only one case which is (1)

Case (6-1.a)

$$a_0=13\sqrt{2}\,i, a_2=\frac{1}{4\sqrt{2}\,i}, k=1, A_1=\frac{1}{8}, A_{-1}=13, A_0=0, w_2=2, q=5, \alpha=\frac{1}{3}$$

$$\psi(\zeta)=\frac{\sqrt{a_0a_2}}{a_2}\tan(\sqrt{a_0a_2}(\zeta+\zeta_0)),$$

$$\psi(\zeta)=10.2\tan(1.8\zeta+2),$$

$$\varphi_{21}(\zeta)=\frac{13}{10.2}\cot[1.8x+1.8t+2]+\frac{10.2}{8}\tan[1.8x+1.8t+2]$$

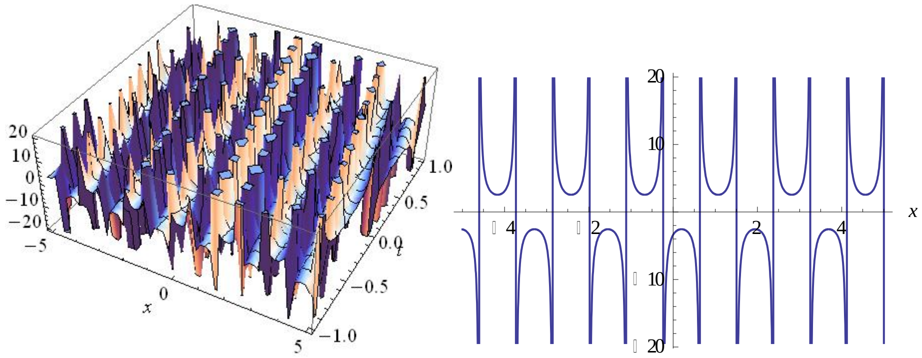

Figure 6.1.a. The periodic soliton solution imaginary part Eq.(62) in **2D** and **3D** with values:

$$a_0=13\sqrt{2}\;i, a_2=\frac{1}{4\sqrt{2}\;i}, k=1, A_1=\frac{1}{8}, A_{-1}=13, A_0=0, w_2=2, \zeta_0=2$$

Case (6.1.b)

$$a_0=13i\sqrt{2}, a_2=\frac{-1}{4i\sqrt{2}}, k=1, A_1=\frac{1}{8}, A_{-1}=13, A_0=0, w_2=2, q=5, \alpha=\frac{1}{3}, \rho=-1$$

$$\psi(\zeta)=\frac{\sqrt{-a_0a_2}}{a_2}\tanh(\sqrt{-a_0a_2}\,\zeta-\frac{\rho\ln\zeta_0}{2}),$$

$$\psi(\zeta)=10.2\tanh(1.8\zeta+0.34),$$

$$\varphi_{21}(\zeta)=\frac{-13}{10.2}\coth[1.8x+1.8t+0.34]+\frac{10.2}{8}\tanh[1.8x+1.8t+0.34]$$

$$\varphi_{21}(\zeta)=-1.27\coth[1.8x+1.8t+0.34]+1.28\tanh[1.8x+1.8t+0.34] \qquad (62)$$

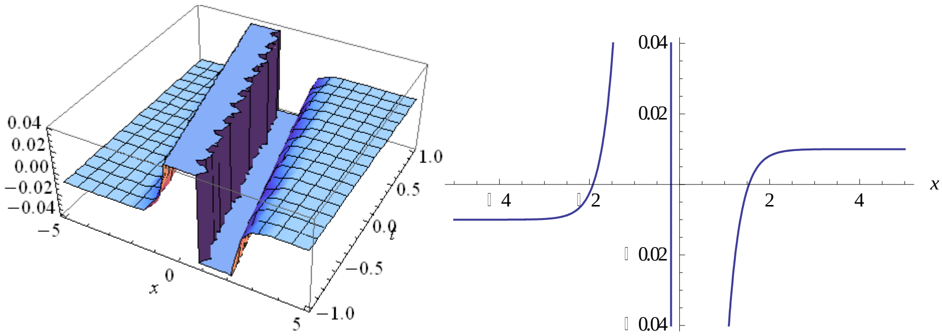

Figure 6. 1.b. The periodic soliton solution imaginary part Eq.(62) in **2D** and **3D** with values:

$$a_0 = 13\sqrt{2}\ i, a_2 = \frac{1}{4\sqrt{2}\ i}, k = 1, A_1 = \frac{1}{8}, A_{-1} = 13, A_0 = 0, w_2 = 2, \zeta_0 = 2$$

In the same manner, we can plot the other 3-cases.

Case 6.2: Now we will study the second case of the constructed method for the imaginary part:

Case 6.2.a: When $a_0 = 0 \Rightarrow v' = a_1 v + a_2 v^2$**,** consequently from the above equation,

$$\varphi^2 = \frac{A_{-1}^2}{v^2} + A_0^2 + A_1^2 v^2 + \frac{2A_0 A_{-1}}{v^2} + 2A_1 A_{-1} + 2A_0 A_1 v \tag{63}$$

$$\varphi' = A_1 a_2 v^2 + A_1 a_1 v - \frac{a_1 A_{-1}}{v^2} - a_2 A_{-1} \tag{64}$$

$$\varphi'' = 2A_1 a_2^2 v^3 + 3A_1 a_1 a_2 v^2 + A_1 a_1^2 v + \frac{A_{-1} a_1 a_2}{v} + \frac{a_1^2 A_{-1}}{v^2} \tag{65}$$

$$\begin{aligned} \varphi^3 &= A_1^3 v^3 + 3A_0 A_1^2 v^2 + 3(A_{-1} A_1^2 + A_1 A_0^2) v + 6A_0 A_1 A_{-1} \\ & A_0^3 + \frac{A_{-1}^3}{v^3} + \frac{3A_0 A_{-1}^2}{v^2} + 3(A_1 A_{-1}^2 + A_{-1} A_0^2)\frac{1}{v} \end{aligned} \tag{66}$$

Substituting for equations (63-66) at equation (38) and collecting and equating the coefficients of different powers of ψ^i to zero, we can easily obtain this system of algebraic equations

$$\begin{aligned} & k^2 a_2^2 + 2A_1^2 = 0 \\ & k^2 a_1 a_2 + 4A_0 A_1 = 0 \\ & (w_2 + 2qk - 3\alpha k q^3) + \alpha k^3 a_1^2 + 12\alpha k (A_{-1} A_1 + A_0^2) = 0 \\ & (w_2 + 2qk - 3\alpha k q^3) + \alpha k^3 A_0^2 + 6\alpha k^3 A_{-1} A_1 = 0 \\ & (w_2 + 2qk - 3\alpha k q^3) + \alpha k^3 a_1 a_2 + 12\alpha k (A_{-1} A_1 + A_0^2) = 0 \\ & k^2 a_1^2 + 12 A_0 A_{-1} = 0 \\ & 4\alpha k A_{-1}^3 = 0 \end{aligned} \tag{67}$$

The last part of equation (67) implies $A_{-1} = 0$**,** hence when we substitute at the six part give one of probability values a_1 which is $a_1 = 0$ but for the other parts of equations (67) if $A_{-1} = 0, A_0 \neq 0$ the other parts then we can easily analytically obtain,

$$a_1 = 2\ i, a_2 = 2, k = 1, A_1 = \pm \frac{i}{\sqrt{39}}, A_{-1} = 0, A_0 = \pm\sqrt{39}\ i, w_2 = 2, \zeta_0 = 2, k = 1, q = 5$$

$$v(\zeta)=-\frac{1}{a_2}\left(a_1-\sqrt{4a_1a_2-a_1^2}\,\tan\left(\frac{\sqrt{4a_1a_2-a_1^2}}{2}(\zeta+\zeta_0)\right)\right),$$

$$v(\zeta)=-1+1.8\tan(1.7\zeta+2),$$

$$\varphi_{22}(\zeta)=\sqrt{39}+\frac{1}{\sqrt{39}}[-1+1.8\tan(1.7\zeta+2)],$$

$$\varphi_{22}(\zeta)=6+0.4\tan(1.7\zeta+2)],$$

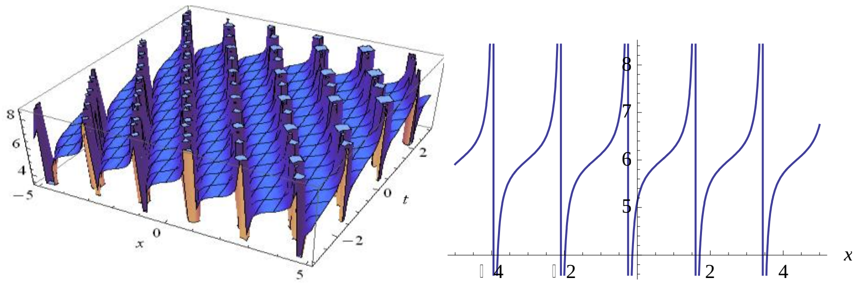

Figure 6.2.a. The periodic soliton solution imaginary part Eq.(62) in **2D** and **3D** with values:

$$a_1=2\,i,a_2=2,k=1,A_1=\frac{i}{\sqrt{39}},A_{-1}=0,A_0=\sqrt{39}\,i,w_2=2,\zeta_0=2,k=1,q=5$$

7-Results and Discussion

We briefly summarize the results in this paper which splits into two distinct parts as follows:

- In the first part, the bright and dark soliton solutions of the nonlinear complex Hirota-dynamical model which play a significant role in different branches of physics have been established for the first time in the framework of the solitary wave ansatz method.

  When we compare our obtained solutions with these previously realized through different authors [27-29] using various methods, we find that the proposed method gives a new accurate solitary solutions.
- While the second part concerned with demonstrating the new impressive abundant exact and hence solitary wave solutions according to the ESEM which have never been achieved before.

8-Conclusion

In this article, we use the (SWAM) as a new basic technique successfully for the first time to obtain the bright and dark soliton solutions as new solitary solutions to the nonlinear complex Hirota- dynamical model. Our obtained results are new and more accurate compared with these realized by other authors [27- 29] whose used different techniques to discussing this model. In a similar vein and in parallel direction, new impressive solitary wave solutions have been demonstrated for the first time in the framework of the ESEM. Furthermore, on a related subject, the new solitary solutions realized with the aid of these two different techniques will add future studies to all phenomena concerned with this model experimentally. The performance to each one of these two methods is reliable and effective and can be applied to many other nonlinear complex evolution equations as well as it can be considered as benchmark against the numerical solutions.

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