Optical soliton solutions for the generalized Kudryashov's equation of propagation pulse in optical fiber with power nonlinearities by three integration algorithms

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Abstract

In this paper, we employ three integration algorithms namely, the well known Kudryashove method, the new Kudryashov method and the unified Riccati equation expansion method to extract optical soliton solutions for the generalized Kudryashov's equation with power nonlinearities. Straddled soliton, bright solitons, dark solitons and singular solitons have been found.

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Key words: Generalized Kudryashov's equation; The well known Kudryashov method; The new Kudryashov method; Unified Riccati equation expansion method; Soliton Solutions.

1 Introduction

Nonlinear partial differential equations (NLPDEs) play an important role in various sections of mathematical physical sciences as physics, biology, chemistry, fluid mechanics, plasma, optical fibers and other areas of engineering. The analytical solutions of such equations are essential significance since a lot of mathematical physical model are described by NLPSDEs. In the last times, there were some mathematical models describing the propagation of pulses in optic fibers [1-15]. All these models are the generalization of the nonlinear Schrödinger equation (NLSE) which are used for description of the wave packet envelope. Recently, Kudryashov [16] has proposed the following new equation of an arbitrary power of nonlinearity:

$$\begin{aligned} &iq_t + i\beta_1 q_x + \alpha_1 q_{xx} + i\beta_2 q_{xxx} + \alpha_2 q_{xxxx} + \gamma q \\ &+ \left(\mu_1 |q|^n + \mu_2 |q|^{2n} + \mu_3 |q|^{3n} + \mu_4 |q|^{4n} + \nu_1 |q|^{-n} + \nu_2 |q|^{-2n} + \nu_3 |q|^{-3n} + \nu_4 |q|^{-4n}\right) q = 0, \end{aligned} \tag{1}$$

where γ and $\alpha_l, \beta_l, (l = 1, 2)$ and $\mu_m, \nu_m, (m = 1, 2, 3, 4)$ are parameters, while $i = \sqrt{-1}$. The dependent variable $q = q(x, t)$ is the complex valued describing the pulse profile, while the independent variables x and t represent spatial and temporal variables, respectively. If $\beta_1 = \beta_2 = \alpha_2 = \gamma = \mu_3 = \mu_4 = \nu_3 = \nu_4 = 0$, we have the well-known Kudryashov's equation [17]:

$$iq_t + \alpha_1 q_{xx} + \left(\mu_1 |q|^n + \mu_2 |q|^{2n} + \nu_1 |q|^{-n} + \nu_2 |q|^{-2n}\right) q = 0, \tag{2}$$

which has been studied by many authors, see for example [18,19]. Thus, Eq. (1) is the generalization of Eq. (2). The two equations (1) and (2) describe the propagation of pulses in optic fibers with power nonlinearities. In the present article, we demonstrate that Eq. (1) like Eq. (2) has a solution in the form of

solitary waves, which can be considered as optical solitons.
This article is organized as follows: In section 2, mathematical analysis is discussed. In sections 3-5 we solve Eq. (1) using the well known Kudryashov method, the new Kudryashov method and unified Riccati equation expansion method. In section 6, the numerical simulations are introduced. In section 7, conclusions are obtained. .

2 Mathematical analysis

To this aim, we assume that Eq.(1) has the formal solution:

$$q(x,t)=\phi(\xi)\exp[i\psi(x,t)], \tag{3}$$

where $\phi(\xi)$ and $\psi(x,t)$ are real functions, such that

$$\xi=x-vt,\quad \psi(x,t)=-\kappa x+\omega t+\theta_0, \tag{4}$$

and v,k,ω and θ_0 are real constants. Here $\phi(\xi)$ represents the pulse shape, v is the velocity of the soliton, κ is the soliton frequency, ω is the soliton wave number and θ_0 is a phase constant. Substituting (3) along with (4) into Eq. (1) and separating the real and imaginary parts, one gets the real part in the form:

$$\begin{aligned}&\alpha_2\phi''''+\left(\alpha_1+3\beta_2\kappa-6\alpha_2\kappa^2\right)\phi''+\left(\gamma-\omega+\beta_1\kappa-\alpha_1\kappa^2-\beta_2\kappa^3+\alpha_2\kappa^4\right)\phi\\&+\mu_1\phi^{1+n}+\mu_2\phi^{1+2n}+\mu_3\phi^{1+3n}+\mu_4\phi^{1+4n}+\nu_1\phi^{1-n}+\nu_2\phi^{1-2n}+\nu_3\phi^{1-3n}+\nu_4\phi^{1-4n}=0,\end{aligned} \tag{5}$$

and the imaginary part in the form:

$$\left(4\alpha_2\kappa-\beta_2\right)\phi'''+\left(v-\beta_1+2\alpha_1\kappa-4\alpha_2\kappa^3+3\beta_2\kappa^2\right)\phi'=0. \tag{6}$$

The linearly independent principle is applied on (6) to get

$$4\alpha_2\kappa-\beta_2=0, \tag{7}$$

and

$$v-\beta_1+2\alpha_1\kappa-4\alpha_2\kappa^3+3\beta_2\kappa^2=0. \tag{8}$$

Consequently, the velocity of the soliton is reduced to

$$v=\beta_1-2\alpha_1\kappa-8\alpha_2\kappa^3. \tag{9}$$

Inserting (7) into Eq. (5), one gets

$$\begin{aligned}&\alpha_2\phi''''+\left(\alpha_1+6\alpha_2\kappa^2\right)\phi''+\left(\gamma-\omega+\beta_1\kappa-\alpha_1\kappa^2-3\alpha_2\kappa^4\right)\phi\\&+\mu_1\phi^{1+n}+\mu_2\phi^{1+2n}+\mu_3\phi^{1+3n}+\mu_4\phi^{1+4n}+\nu_1\phi^{1-n}+\nu_2\phi^{1-2n}+\nu_3\phi^{1-3n}+\nu_4\phi^{1-4n}=0.\end{aligned} \tag{10}$$

Balancing ϕ'''' and ϕ^{1+4n} in Eq. (10), yields the balance number $N=\frac{1}{n}$, $n>1$. Since the balance number is not integer, then we take into consideration the transformation

$$\phi(\xi)=\left[U(\xi)\right]^{1/n}, \tag{11}$$

where $U(\xi)$ is a new function of ξ, such that $U(\xi)>0$. Substituting (11) into (10), we have the new equation

$$\begin{aligned}&n^3\alpha_2U^3U''''-4n^2\left(n-1\right)\alpha_2U^2U'U'''+\left[6n\left(n-1\right)\left(2n-1\right)\alpha_2U'^2+n^3\left(\alpha_1+6\alpha_2\kappa^2\right)U^2\right]UU''\\&-3n^2\left(n-1\right)\alpha_2U^2U''^2-\left(3n-1\right)\left(2n-1\right)(n-1)\alpha_2U'^4-n^2(n-1)\left(\alpha_1+6\alpha_2\kappa^2\right)U^2U'^2\\&+n^4\left[\nu_4+\nu_3U+\nu_2U^2+\nu_1U^3+\left(\gamma-\omega+\beta_1\kappa-\alpha_1\kappa^2-3\alpha_2\kappa^4\right)U^4+\mu_1U^5+\mu_2U^6+\mu_3U^7+\mu_4U^8\right]=0.\end{aligned} \tag{12}$$

Balancing U^3U'''' and U^8 in Eq. (12), gives the balance number $N=1$. Now, we will solve Eq. (12) using the following two methods:

3 The well known Kudryashov method

According to this method [20,21], Eq. (12) has the formal solution

$$U(\xi)=\sigma_0+\sigma_1 Q(\xi), \tag{13}$$

where σ_0 and σ_1 are real constants to be determined such that $\sigma_1 \neq 0$. Here $Q(\xi)$ is the solution of the ODE

$$Q'(\xi)=Q^2(\xi)-Q(\xi). \tag{14}$$

It is well known that Eq. (14) has the solution:

$$Q(\xi)=\frac{1}{1+\varepsilon e^{\xi}}, \tag{15}$$

where $\varepsilon=\pm 1$. Substituting (13) along with (14) into Eq. (12), collecting all the coefficients of $Q^r(\xi)$, $(r=0,1,2,...8)$ and setting them to zero, we have a set of algebraic equations, which can be solved by Maple to get the results:

$$\sigma_1=-\frac{4n^4\nu_1}{\Delta_1}, \sigma_0=-\frac{2n^4\nu_1}{\Delta_1}, \tag{16}$$

and

$$\left.\begin{aligned}
&\omega=\frac{8n^4\left(\gamma+\beta_1\kappa-\alpha_1\kappa^2-3\alpha_2\kappa^4\right)+\left[44n^2\left(\alpha_1+6\alpha_2\kappa^2\right)+\left(285n^2+443\right)\alpha_2\right]}{8n^4},\\
&\mu_1=-\frac{(n+2)\left[2n^2\left(10\alpha_2+\alpha_1+6\alpha_2\kappa^2\right)+25\,(n+1)\,\alpha_2\right]\Delta_1}{4n^8\nu_1},\\
&\mu_2=-\frac{(n+1)\left[n^2\left(34\alpha_2+\alpha_1+6\alpha_2\kappa^2\right)+27\,(2n+1)\,\alpha_2\right]\Delta_1^2}{16n^{12}\nu_1^2},\\
&\mu_3=-\frac{(n+1)\,(3n+2)\,(2n+1)\,\alpha_2\Delta_1^3}{16n^{16}\nu_1^3}, \mu_4=-\frac{(n+1)\,(3n+1)\,(2n+1)\,\alpha_2\Delta_1^4}{256n^{20}\nu_1^4},\\
&\nu_1=\nu_1, \nu_2=\frac{9n^4\,(n-1)\,\nu_1^2\left[n^2\left(34\alpha_2+\alpha_1+6\alpha_2\kappa^2\right)-27\alpha_2\,(2n-1)\right]}{\Delta_1^2},\\
&\nu_3=\frac{108n^8\left(6n^3-13n^2+9n-2\right)\alpha_2\nu_1^3}{\Delta_1^3}, \nu_4=\frac{81n^{12}\left(6n^3-11n^2+6n-1\right)\alpha_2\nu_1^4}{\Delta_1^4},
\end{aligned}\right\} \tag{17}$$

where

$$\Delta_1=3\,(n-2)\left[2n^2\left(10\alpha_2+\alpha_1+6\alpha_2\kappa^2\right)-25\,(n-1)\,\alpha_2\right]. \tag{18}$$

Substituting (16) along with (15) into Eq. (13), we have the soliton solutions of Eq. (1) in the form:

$$q(x,t)=\left\{-\frac{2n^4\upsilon_1}{\Delta_1}\left[\frac{3+\varepsilon e^{(x-vt)}}{1+\varepsilon e^{(x-vt)}}\right]\right\}^{1/n} e^{i(-\kappa x+\omega t+\theta_0)}, \tag{19}$$

provided

$$\upsilon_1\Delta_1<0 \quad \text{and} \quad \varepsilon=\pm 1. \tag{20}$$

If $\varepsilon = 1$, then Eq. (1) reveals the dark soliton in the form:

$$q(x,t) = \left\{ -\frac{2n^4 \upsilon_1}{\Delta_1} \left[2 - \tanh\left(\frac{x - vt}{2} \right) \right] \right\}^{1/n} e^{i(-\kappa x + \omega t + \theta_0)}, \tag{21}$$

while, if $\varepsilon = -1$, then Eq. (1) has the singular soliton in the form:

$$q(x,t) = \left\{ -\frac{2n^4 \upsilon_1}{\Delta_1} \left[2 - \coth\left(\frac{x - vt}{2} \right) \right] \right\}^{1/n} e^{i(-\kappa x + \omega t + \theta_0)}. \tag{22}$$

The solutions (19), (21) and (22) exist under the conditions (17).

4 New Kudryashov method

According to this new method [22], Eq. (12) has the solution:

$$U(\xi) = \delta_0 + \delta_1 R(\xi), \tag{23}$$

where δ_0 and δ_1 are real constants to be determined such that $\delta_1 \neq 0$. Here $R(\xi)$ is the solution of the ODE

$$R'^2(\xi) = R^2(\xi) \left[1 - \chi R^2(\xi) \right], \tag{24}$$

where χ is a constant. It is well known that Eq. (24) has the solution:

$$R(\xi) = \frac{4a}{4a^2 e^{\xi} + \chi e^{-\xi}}, \tag{25}$$

where a is a nonzero constant. Substituting (23) along with (24) into Eq. (12), collecting all the coefficients of $R^r(\xi) R'^s(\xi)$, $(r = 0, 1, 2, ...8; s = 0, 1)$ and setting them to zero, we have a set of algebraic equations, which can be solved by Maple to get the results:

$$\delta_1 = -\frac{4n^4 \nu_1}{\Delta_2}, \delta_0 = -\frac{2n^4 \nu_1}{\Delta_2}, \tag{26}$$

and

$$\left.\begin{aligned}
&\omega = \frac{8n^4\left(\gamma+\beta_1\kappa-\alpha_1\kappa^2-3\alpha_2\kappa^4\right)-\left[60\left(n^2+1\right)\chi-5\left(5n^2+7\right)\chi^2-8\right]\alpha_2-4n^2(3\chi-2)\left(\alpha_1+6\alpha_2\kappa^2\right)}{8n^4}, \\
&\mu_1 = -\frac{(n+2)\chi\Delta_2\left\{2n^2\left[\alpha_2(3\chi-4)-\left(\alpha_1+6\alpha_2\kappa^2\right)\right]+(n+1)(7\chi-6)\alpha_2\right\}}{8n^4\nu_1}, \\
&\mu_2 = -\frac{(n+1)\chi\Delta_2^2\left\{n^2\left[\alpha_2(9\chi-4)-\left(\alpha_1+6\alpha_2\kappa^2\right)\right]+(2n+1)(7\chi-62)\alpha_2\right\}}{16n^{12}\nu_1^2}, \\
&\mu_3 = -\frac{(n+1)(3n+2)(2n+1)\alpha_2\chi^2\Delta_2^3}{32n^{16}\nu_1^3}, \mu_4 = -\frac{(n+1)(3n+1)(2n+1)\alpha_2\chi^2\Delta_2^4}{256n^{20}\nu_1^4}, \nu_1 = \nu_1, \\
&\nu_2 = \frac{n^4(n-1)\nu_1^2\left\{n^2\left[\alpha_2\left(9\chi^2-40\chi+28\right)-(\chi-4)\left(\alpha_1+6\alpha_2\kappa^2\right)\right]-(2n-1)\left(7\chi^2-30\chi+24\right)\alpha_2\right\}}{\Delta_2^2}, \\
&\nu_3 = \frac{2n^8(n-1)(3n-2)(2n-1)(\chi-2)(\chi-4)\alpha_2\nu_1^3}{\Delta_2^3}, \nu_4 = \frac{n^{12}(n-1)(3n-1)(2n-1)(\chi-4)^2\alpha_2\nu_1^4}{\Delta_2^4},
\end{aligned}\right\} \tag{27}$$

where

$$\Delta_2 = \frac{(n-2)}{2} \left[2n^2 \alpha_2 \left(3\chi^2 - 10\chi + 2 \right) - 2n^2 \left(\chi - 2 \right) \left(\alpha_1 + 6\alpha_2 \kappa^2 \right) - (n-1) \left(7\chi^2 - 20\chi + 8 \right) \alpha_2 \right]. \tag{28}$$

Substituting (26) along with (25) into Eq. (23), we have the soliton solutions of Eq. (1) in the form:

$$q(x,t)=\left\{-\frac{2n^4\upsilon_1}{\Delta_2}\left[1+\frac{8a}{4a^2e^{(x-vt)}+\chi e^{-(x-vt)}}\right]\right\}^{1/n}e^{i(-\kappa x+\omega t+\theta_0)}, \tag{29}$$

provided

$$\upsilon_1\Delta_2<0. \tag{30}$$

In particular if $\chi=4a^2$, then Eq. (1) reveals the bright soliton in the form:

$$q(x,t)=\left\{-\frac{2n^4\upsilon_1}{a\Delta_2}\left[a+\operatorname{sech}(x-vt)\right]\right\}^{1/n}e^{i(-\kappa x+\omega t+\theta_0)}, \tag{31}$$

while, if $\chi=-4a^2$, then Eq. (1) has the singular soliton in the form:

$$q(x,t)=\left\{-\frac{2n^4\upsilon_1}{a\Delta_2}\left[a+\operatorname{csch}(x-vt)\right]\right\}^{1/n}e^{i(-\kappa x+\omega t+\theta_0)}. \tag{32}$$

The solutions (29), (31) and (32) exist under the conditions (27).

5 Unified Riccati equation expansion method

According to the unified Riccati equation expansion method [23], Eq. (12) has the formal solution:

$$U(\xi)=E_0+E_1F(\xi), \tag{33}$$

where E_0 and E_1 are constants to be determined, such that $E_1\neq 0$ and $F(\xi)$ satisfies the Riccati equation:

$$F'(\xi)=h_0+h_1F(\xi)+h_2F^2(\xi). \tag{34}$$

Here h_0, h_1 and h_2 are constants to be determined such that $h_2\neq 0$. It is well known that the Riccati equation (34) has the following fractional solutions:

$$F(\xi)=\begin{cases}-\dfrac{h_1}{2h_2}-\dfrac{\sqrt{\Delta}\left[r_1\tanh\left(\frac{\sqrt{\Delta}}{2}\xi\right)+r_2\right]}{2h_2\left[r_1+r_2\tanh\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right]}, & \text{if } \Delta>0 \text{ and } r_1^2+r_2^2\neq 0,\\[2ex] -\dfrac{h_1}{2h_2}+\dfrac{\sqrt{-\Delta}\left[r_3\tan\left(\frac{\sqrt{-\Delta}}{2}\xi\right)-r_4\right]}{2h_2\left[r_3+r_4\tan\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right]}, & \text{if } \Delta<0 \text{ and } r_3^2+r_4^2\neq 0,\\[2ex] -\dfrac{h_1}{2h_2}+\dfrac{1}{h_2\xi+r_5}, & \text{if } \Delta=0,\end{cases} \tag{35}$$

where $\Delta=h_1^2-4h_0h_2$ and $r_i(i=1,2,...,5)$ are arbitrary constants. Substituting (33) along with (34) into Eq. (12), collecting all the coefficients of $F^j(\xi)$ $(j=0,1,...,8)$ and setting them to zero, we have a set of algebraic equations, which can be solved by Maple to get the results:

$$E_1=\frac{2}{n^2\Delta_2}, E_0=\frac{1}{n^2\Delta_2}, h_0=-\sqrt{\frac{2\alpha_2}{n\left(\alpha_1+6\alpha_2\kappa^2\right)}}, h_1=0, h_2=\sqrt{\frac{2\alpha_2}{n\left(\alpha_1+6\alpha_2\kappa^2\right)}}, \tag{36}$$

and

$$\left.\begin{aligned}
&\omega=\gamma+\beta_1\kappa-\alpha_1\kappa^2-3\alpha_2\kappa^4-\frac{\alpha_2}{n^3}-\frac{\left(15n^2+37\right)\alpha_2^3}{2n^6\left(\alpha_1+6\alpha_2\kappa^2\right)^2},\\
&\mu_1=\frac{(n+2)\,\alpha_2\Delta_3\left[n^3\left(\alpha_1+6\alpha_2\kappa^2\right)^2-5\left(2n^2+n+1\right)\alpha_2^2\right]}{n^4\left(\alpha_1+6\alpha_2\kappa^2\right)^2},\\
&\mu_2=-\frac{(n+1)\,\alpha_2\Delta_3^2\left[n^3\left(\alpha_1+6\alpha_2\kappa^2\right)^2+2\left(n^2+6n+3\right)\alpha_2^2\right]}{2n^2\left(\alpha_1+6\alpha_2\kappa^2\right)^2},\\
&\mu_3=\frac{(n+1)\,(3n+2)\,(2n+1)\,\alpha_2^3\Delta_3^3}{\left(\alpha_1+6\alpha_2\kappa^2\right)^2},\mu_4=-\frac{n^2\,(n+1)\,(3n+1)\,(2n+1)\,\alpha_2^3\Delta_3^4}{4\left(\alpha_1+6\alpha_2\kappa^2\right)^2},\\
&\nu_1=\nu_1,\nu_2=\frac{9\alpha_2\,(n-1)\left[n^3\left(\alpha_1+6\alpha_2\kappa^2\right)^2+2\left(n^2-6n+3\right)\alpha_2^2\right]}{2n^{10}\Delta_3^2\left(\alpha_1+6\alpha_2\kappa^2\right)^2},\\
&\nu_3=\frac{27\alpha_2^3\left(6n^3-13n^2+9n-2\right)}{n^{12}\Delta_3^3\left(\alpha_1+6\alpha_2\kappa^2\right)^2},\nu_4=\frac{81\alpha_2^3\left(6n^3-11n^2+6n-1\right)}{4n^{14}\Delta_3^4\left(\alpha_1+6\alpha_2\kappa^2\right)^2},
\end{aligned}\right\}\tag{37}$$

where

$$\Delta_3=\frac{3\alpha_2\,(n-2)\left[n^3\left(\alpha_1+6\alpha_2\kappa^2\right)^2-5\alpha_2^2\left(2n^2-n+1\right)\right]}{n^8\nu_1\left(\alpha_1+6\alpha_2\kappa^2\right)^2},\tag{38}$$

provided

$$\alpha_2\left(\alpha_1+6\alpha_2\kappa^2\right)>0.\tag{39}$$

By the aid of solutions (35), we find the following solutions for Eq. (1):

Since $\Delta>0$, then substituting (36) along with (35) into Eq. (33), we have straddled soliton of Eq. (1) in the form:

$$q(x,t)=\left\{\frac{1}{n^2\Delta_3}\left[1-\frac{2\left[r_1\tanh\left(\sqrt{\frac{2\alpha_2}{n\left(\alpha_1+6\alpha_2\kappa^2\right)}}\,(x-vt)\right)+r_2\right]}{r_1+r_2\tanh\left(\sqrt{\frac{2\alpha_2}{n\left(\alpha_1+6\alpha_2\kappa^2\right)}}\,(x-vt)\right)}\right]\right\}^{1/n}e^{i(-\kappa x+\omega t+\theta_0)}.\tag{40}$$

In particular, if $r_1\neq0$ and $r_2=0$ in (40), then Eq. (1) reveals the dark soliton in the form:

$$q(x,t)=\left\{\frac{1}{n^2\Delta_3}\left[1-2\tanh\left(\sqrt{\frac{2\alpha_2}{n\left(\alpha_1+6\alpha_2\kappa^2\right)}}\,(x-vt)\right)\right]\right\}^{1/n}e^{i(-\kappa x+\omega t+\theta_0)},\tag{41}$$

while if $r_1=0$ and $r_2\neq0$, Eq. (1) gives the singular soliton:

$$q(x,t)=\left\{\frac{1}{n^2\Delta_3}\left[1-2\coth\left(\sqrt{\frac{2\alpha_2}{n\left(\alpha_1+6\alpha_2\kappa^2\right)}}\,(x-vt)\right)\right]\right\}^{1/n}e^{i(-\kappa x+\omega t+\theta_0)}.\tag{42}$$

The solutions (40) – (42) exist under the conditions (37).

6 Numerical simulations

In this section, we present the graphs of some solutions for Eq.(1). Let us now examine Figures, 1-6. as it illustrates some of our solutions obtained in this paper. To this aim, we select some special values of the obtained parameters.

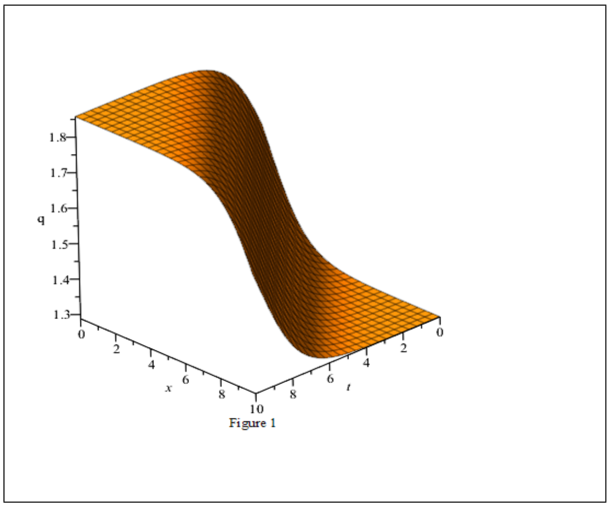

Figure 1: The numerical simulations of the dark soliton solution (21) with the parameter values $\kappa = 1, \alpha_1 = -0.5, \alpha_2 = 0.25, \beta_1 = 2, \nu_1 = -2, n = 3, \gamma = 2, \omega = 1.521604939, v = 1$.

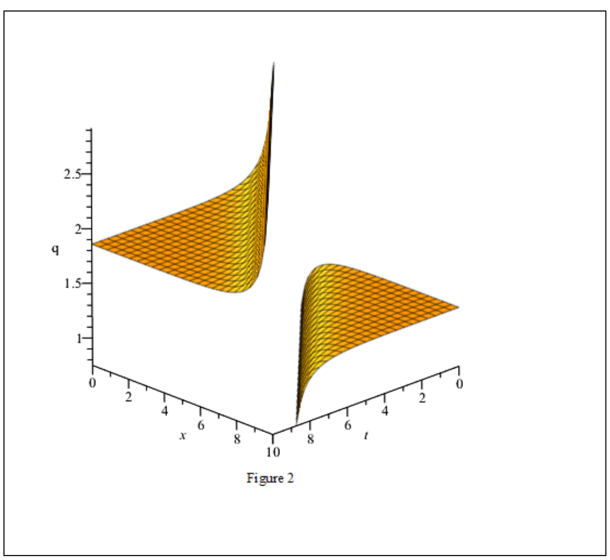

Figure 2: The numerical simulations of the singular soliton solution (22) with the parameter values $\kappa = 1, \alpha_1 = -0.5, \alpha_2 = 0.25, \beta_1 = 2, \nu_1 = -2, n = 3, \gamma = 2, \omega = 1.521604939, v = 1$.

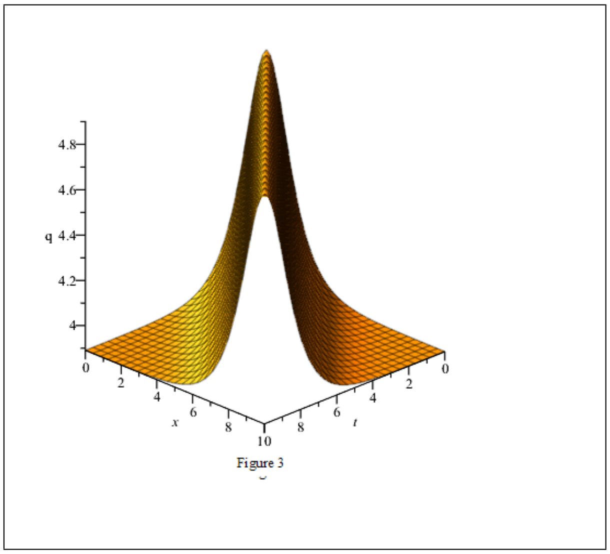

Figure 3: The numerical simulations of the bright soliton solution (31) with the parameter values $\kappa = 1, \alpha_1 = -0.5, \alpha_2 = 0.25, \beta_1 = 2, \nu_1 = 2, n = 3, \gamma = 2, \chi = 4, a = 1, \omega = 1.521604939, v = 1$.

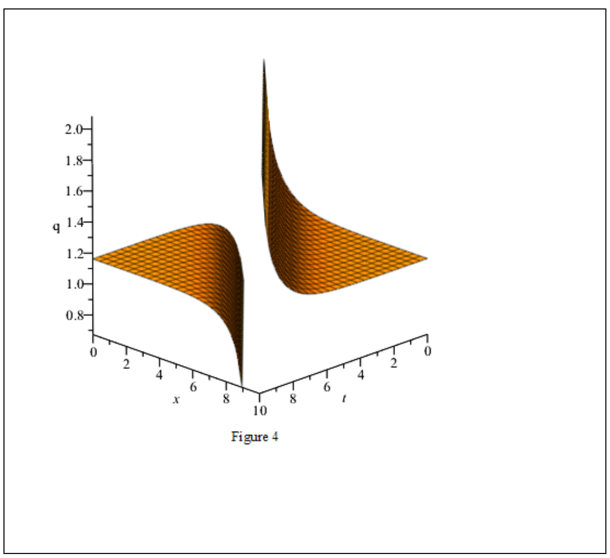

Figure 4: The numerical simulations of the singular soliton solution (32) with the parameter values $\kappa = 1, \alpha_1 = -0.5, \alpha_2 = 0.25, \beta_1 = 2, \nu_1 = -2, n = 3, \gamma = 2, \chi = -4, a = 1, \omega = 1.521604939, v = 1$.

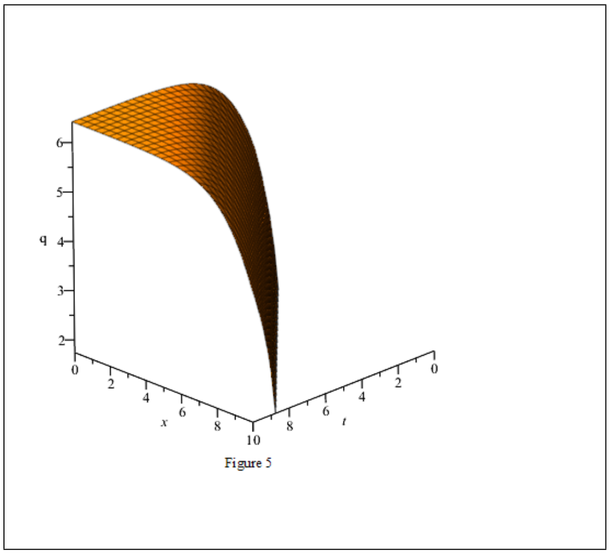

Figure 5: The numerical simulations of the dark soliton solution (41) with the parameter values $\kappa = 1, \alpha_1 = -0.5, \alpha_2 = 0.25, \beta_1 = 2, \nu_1 = 2, n = 3, \gamma = 2, \omega = 1.521604939, v = 1$.

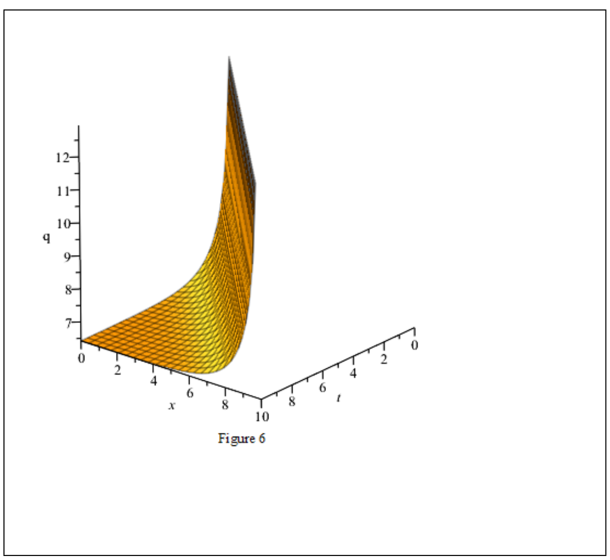

Figure 6: The numerical simulations of the singular soliton solution (42) with the parameter values $\kappa = 1, \alpha_1 = -0.5, \alpha_2 = 0.25, \beta_1 = 2, \nu_1 = 2, n = 3, \gamma = 2, \omega = 1.521604939, v = 1$.

From the above Figures, one can see that the obtained solutions possess the dark soliton solutions, the bright soliton solutions and the singular soliton solutions of Eq. (1). Also, these figures express the behavior of these solutions which give some perspective reader show the behavior solutions are produced.

7 Conclusions

This paper introduced the optical soliton solutions to the generalized Kudryashov's equation with power nonlinearities (1). The well known Kudryashov method, the new Kudryashov method and the unified Riccati equation expansion method have been applied in this paper to find straddled soliton, bright solitons, dark solitons and singular solitons to Eq. (1). We demonstrate that Eq. (1) has a solution in the form of solitary waves, which can be considered as optical solitons. Therefore, integrability for Eq. (1) was possible with phase-matching condition. These very valuable outcomes sequentially are going to be presented.

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