On a nonlinear fractional order model of novel coronavirus (nCoV-2019) under AB-fractional derivative

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ABSTRACT. Utilizing the model of novel coronavirus given by Chen *et al.* [A mathematical model for simulating the phase-based transmissibility of a novel coronavirus, Infectious Diseases of Poverty, (2020) 9:24], we intend to generalize the model to fractional order derivative in Atangana-Baleanu sense and to show the existence of solution for the fractional model using Schaefer's fixed point theorem and for the uniqueness of solution we make use of Banach fixed point theorem. By using Shehu transform and Picard successive iterative procedure, we explore the iterative solutions and its stability for the considered fractional model.

1. Introduction and Preliminaries

The theory of fractional calculus, especially differential equations of fractional order derivatives have a significance importance in the modeling of many real world problems arising in science and engineering (see e.g. [**1**, **2**, **3**, **4**, **5**, **6**, **7**] and references therein). Fractional derivative due to Riemann-Liouville and Caputo were used widely in the early literature. But due to the presence of singularities in their kernels, some new fractional derivatives were introduced which settled the arisen problem, for details, we refer [**8**, **9**, **10**, **11**, **12**, **13**, **14**, **15**, **16**]. More precisely, to study the complex biological systems and diseases, fractional calculus played an important role as it provides better results than the integer order models (see e.g. [**17**, **18**, **19**, **20**, **21**, **22**].

Very recently, Chen *et al.* [**23**] developed a mathematical model of novel coronavirus. In this paper, we generalized the model of novel coronavirus (nVoC-2019) to the ABC-fractional model proposed by Chen *et al.* [**23**] and explore the existence and uniqueness of its solution using fixed point theory along with a stability result.

We start with some basic notions:

First, we recall the definition of Caputo fractional derivative which can be found in many books (see, e.g., [**2**]).

DEFINITION 1.1. *For a differentiable function h, the Caputo derivative of order $\delta \in (0,1)$ is defined by*

$$ {}^{C}\mathfrak{D}^{\delta} h(t) = \frac{1}{\Gamma(n-\delta)} \int_0^t h^{'}(s) \frac{1}{(t-s)^{\delta}} ds. \tag{1}$$

DEFINITION 1.2. [**9**] *Let $h \in \mathcal{H}^1(0,1)$ and $\delta \in [0,1]$ then the Atangana-Baleanu-Caputo (ABC) fractional derivative is defined by*

$$ {}^{ABC}\mathfrak{D}^{\delta} h(t) = \frac{M(\delta)}{(1-\delta)} \int_0^t h^{'}(\omega) E_{\delta}[-\frac{\delta}{1-\delta}(t-\omega)^{\delta}] d\omega. \tag{2}$$

Key words and phrases. Fractional Atangana-Baleanu derivative, novel coronavirus (nCoV-2019), Shehu transform.

Definition 1.3. [9] *The integral operator associated with ABC-fractional derivative is defined by*

$$^{ABC}\mathfrak{I}^{\delta}h(t) = \frac{(1-\delta)}{M(\delta)}h(t) + \frac{\delta}{M(\delta)\Gamma(\delta)}\int_0^t h(\omega)(t-\omega)^{\delta-1}d\omega, \tag{3}$$

where $M(\delta)$ is the normalization function.

Definition 1.4. [24] *For a function $\xi(t)$ in*

$$A = \{\xi(t) : \text{there exist } \chi, t_1, t_2 > 0, |\xi(t)| < \chi exp\left(\frac{|t|}{t_i}\right), \text{ if } t \in (-1)^j \times [0,\infty)\},$$

the Shehu transform of $\xi(t) \in A$ is given by

$$S_h(\xi(t)) = \int\limits_0^{\infty} exp\left(-\frac{st}{u}\right)\xi(t)dt \;\; u \in (-t_1, t_2). \tag{4}$$

Lemma 1.1. [25] *Assume $h \in H^1(a,b)$, $b > a$, $\gamma \in (0,1)$ and $h(t) \in A$, the Shehu transform (S_h) of Atangana-Baleanu fractional derivative in Caputo sense is*

$$S_h(^{ABC}\mathfrak{D}^{\delta}h(t)) = \frac{M(\gamma)}{1-\gamma+\gamma\left(\frac{u}{s}\right)^{\gamma}}(S_h(h(t)) - \frac{u}{s}h(0)). \tag{5}$$

2. Fractional Model in Atangana-Baleanu Sense

Very recently, Chen *et al.* [23] proposed a mathematical model of a novel coronavirus (nCoV-19) as follows:

$$\begin{aligned}
\frac{d\mathfrak{S}_p}{dt} &= \coprod_p - \omega_p\mathfrak{S}_p - \zeta_p\mathfrak{S}_p(\mathfrak{I}_p + \Psi\mathfrak{A}_p) - \omega_w\mathfrak{S}_p\mathfrak{M},\\
\frac{d\mathfrak{E}_p}{dt} &= \zeta_p\mathfrak{S}_p(\mathfrak{I}_p + \Psi\mathfrak{A}_p) + \omega_w\mathfrak{S}_p\mathfrak{M} - (1-\Phi_p)\eta_p\mathfrak{E}_p - \Phi_p\varrho_p\mathfrak{E}_p - \omega_p\mathfrak{E}_p,\\
\frac{d\mathfrak{I}_p}{dt} &= (1-\Phi_p)\eta_p\mathfrak{E}_p - (\tau_p + \omega_p)\mathfrak{I}_p,\\
\frac{d\mathfrak{A}_p}{dt} &= \Phi_p\varrho_P\mathfrak{E}_p - (\tau_{ap} + \omega_p)\mathfrak{A}_p,\\
\frac{d\mathfrak{R}_p}{dt} &= \tau_p\mathfrak{I}_p + \tau_{ap}\mathfrak{A}_p - \omega_p\mathfrak{R}_p,\\
\frac{d\mathfrak{M}}{dt} &= \phi_p\mathfrak{I}_p + \varpi_p\mathfrak{A}_p - \varphi\mathfrak{M},
\end{aligned} \tag{6}$$

with the initial conditions

$$\begin{gathered}
\mathfrak{S}_p(0) = \mathfrak{S}_p(0) \geq 0, \mathfrak{E}_p(0) = \mathfrak{E}_p(0) \geq 0, \mathfrak{I}_p(0) = \mathfrak{I}_p(0) \geq 0,\\
\mathfrak{A}_p(0) = \mathfrak{A}_p(0) \geq 0, \mathfrak{R}_p(0) = \mathfrak{R}_p(0) \geq 0, \mathfrak{M}(0) = \mathfrak{M}(0) \geq 0.
\end{gathered}$$

We now generalize the model (6) to a fractional order model using Atangana-Baleanu derivative in Caputo sense as follows:

$$
\begin{aligned}
{}^{ABC}\mathfrak{D}^{\delta}\mathfrak{S}_p &= \coprod_p -\omega_p\mathfrak{S}_p - \zeta_p\mathfrak{S}_p(\mathfrak{I}_p + \Psi\mathfrak{A}_p) - \omega_w\mathfrak{S}_p\mathfrak{M}, \\
{}^{ABC}\mathfrak{D}^{\delta}\mathfrak{E}_p &= \zeta_p\mathfrak{S}_p(\mathfrak{I}_p + \Psi\mathfrak{A}_p) + \omega_w\mathfrak{S}_p\mathfrak{M} - (1-\Phi_p)\eta_p\mathfrak{E}_p - \Phi_p\varrho_p\mathfrak{E}_p - \omega_p\mathfrak{E}_p, \\
{}^{ABC}\mathfrak{D}^{\delta}\mathfrak{I}_p &= (1-\Phi_p)\eta_p\mathfrak{E}_p - (\tau_p + \omega_p)\mathfrak{I}_p, \\
{}^{ABC}\mathfrak{D}^{\delta}\mathfrak{A}_p &= \Phi_p\varrho_P\mathfrak{E}_p - (\tau_{ap} + \omega_p)\mathfrak{A}_p, \\
{}^{ABC}\mathfrak{D}^{\delta}\mathfrak{R}_p &= \tau_p\mathfrak{I}_p + \tau_{ap}\mathfrak{A}_p - \omega_p\mathfrak{R}_p, \\
{}^{ABC}\mathfrak{D}^{\delta}\mathfrak{M} &= \phi_p\mathfrak{I}_p + \varpi_p\mathfrak{A}_p - \varphi\mathfrak{M},
\end{aligned}
\tag{7}
$$

where δ denotes the the fractional order parameter and the model variables in (7) are nonnegative and the initial conditions are given by

$$
\begin{aligned}
&\mathfrak{S}_p(0) = \mathfrak{S}_p(0) \geq 0, \mathfrak{E}_p(0) = \mathfrak{E}_p(0) \geq 0, \mathfrak{I}_p(0) = \mathfrak{I}_p(0) \geq 0, \\
&\mathfrak{A}_p(0) = \mathfrak{A}_p(0) \geq 0, \mathfrak{R}_p(0) = \mathfrak{R}_p(0) \geq 0, \mathfrak{M}(0) = \mathfrak{M}(0) \geq 0.
\end{aligned}
$$

Using the initial conditions and fractional integral operator, we convert model (7) into integral equations

$$
\begin{aligned}
\mathfrak{S}_p(t) - \mathfrak{S}_p(0) &= {}^{ABC}\mathfrak{J}^{\delta}\left[\coprod_p -\omega_p\mathfrak{S}_p - \zeta_p\mathfrak{S}_p(\mathfrak{I}_p + \Psi\mathfrak{A}_p) - \omega_w\mathfrak{S}_p\mathfrak{M}\right], \\
\mathfrak{E}_p(t) - \mathfrak{E}_p(0) &= {}^{ABC}\mathfrak{J}^{\delta}\Bigg[\zeta_p\mathfrak{S}_p(\mathfrak{I}_p + \Psi\mathfrak{A}_p) + \omega_w\mathfrak{S}_p\mathfrak{M} - (1-\Phi_p)\eta_p\mathfrak{E}_p \\
&\quad -\Phi_p\varrho_p\mathfrak{E}_p - \omega_p\mathfrak{E}_p\Bigg], \\
\mathfrak{I}_p(t) - \mathfrak{I}_p(0) &= {}^{ABC}\mathfrak{J}^{\delta}[(1-\Phi_p)\eta_p\mathfrak{E}_p - (\tau_p + \omega_p)\mathfrak{I}_p], \\
\mathfrak{A}_p(t) - \mathfrak{A}_p(0) &= {}^{ABC}\mathfrak{J}^{\delta}[\Phi_p\varrho_P\mathfrak{E}_p - (\tau_{ap} + \omega_p)\mathfrak{A}_p], \\
\mathfrak{R}_p(t) - \mathfrak{R}_p(0) &= {}^{ABC}\mathfrak{J}^{\delta}[\tau_p\mathfrak{I}_p + \tau_{ap}\mathfrak{A}_p - \omega_p\mathfrak{R}_p], \\
\mathfrak{M}(t) - \mathfrak{M}(0) &= {}^{ABC}\mathfrak{J}^{\delta}[\phi_p\mathfrak{I}_p + \varpi_p\mathfrak{A}_p - \varphi\mathfrak{M}].
\end{aligned}
\tag{8}
$$

For simplicity, we write the kernels

$$
\begin{aligned}
\mathfrak{F}_1(t, \mathfrak{S}_p(t)) &= \coprod_p -\omega_p\mathfrak{S}_p(t) - \zeta_p\mathfrak{S}_p(t)(\mathfrak{I}_p(t) + \Psi\mathfrak{A}_p(t)) - \omega_w\mathfrak{S}_p(t)\mathfrak{M}(t), \\
\mathfrak{F}_2(t, \mathfrak{E}_p(t)) &= \zeta_p\mathfrak{S}_p(t)(\mathfrak{I}_p(t) + \Psi\mathfrak{A}_p(t)) + \omega_w\mathfrak{S}_p(t)\mathfrak{M}(t) - (1-\Phi_p)\eta_p\mathfrak{E}_p(t) \\
&\quad -\Phi_p\varrho_p\mathfrak{E}_p(t) - \omega_p\mathfrak{E}_p(t), \\
\mathfrak{F}_3(t, \mathfrak{I}_p(t)) &= (1-\Phi_p)\eta_p\mathfrak{E}_p(t) - (\tau_p + \omega_p)\mathfrak{I}_p(t), \\
\mathfrak{F}_4(t, \mathfrak{A}_p(t)) &= \Phi_p\varrho_P\mathfrak{E}_p(t) - (\tau_{ap} + \omega_p)\mathfrak{A}_p(t), \\
\mathfrak{F}_5(t, \mathfrak{R}_p(t)) &= \tau_p\mathfrak{I}_p(t) + \tau_{ap}\mathfrak{A}_p(t) - \omega_p\mathfrak{R}_p(t), \\
\mathfrak{F}_6(t, \mathfrak{M}(t)) &= \phi_p\mathfrak{I}_p(t) + \varpi_p\mathfrak{A}_p(t) - \varphi\mathfrak{M}(t)
\end{aligned}
\tag{9}
$$

and the functions

$$
\Upsilon(\delta) = \frac{1-\delta}{M(\delta)}, \ \Lambda(\delta) = \frac{\delta}{\Gamma(\delta)M(\delta)}. \tag{10}
$$

Applying (3), (9) and (10) in (8) and writing state variables in terms of kernels, we obtain

$$
\begin{aligned}
\mathfrak{S}_p(t) &= \mathfrak{S}_p(0)+\Upsilon(\delta)\mathfrak{F}_1(t,\mathfrak{S}_p(t))+\Lambda(\delta)\int_0^t \mathfrak{F}_1(x,\mathfrak{S}_p(x))(t-x)^{\delta-1}dx,\\
\mathfrak{E}_p(t) &= \mathfrak{E}_p(0)+\Upsilon(\delta)\mathfrak{F}_2(t,\mathfrak{E}_p(t))+\Lambda(\delta)\int_0^t \mathfrak{F}_2(x,\mathfrak{E}_p(x))(t-x)^{\delta-1}dx,\\
\mathfrak{I}_p(t) &= \mathfrak{I}_p(0)+\Upsilon(\delta)\mathfrak{F}_3(t,\mathfrak{I}_p(t))+\Lambda(\delta)\int_0^t \mathfrak{F}_3(x,\mathfrak{I}_p(x))(t-x)^{\delta-1}dx,\\
\mathfrak{A}_p(t) &= \mathfrak{A}_p(0)+\Upsilon(\delta)\mathfrak{F}_4(t,\mathfrak{A}_p(t))+\Lambda(\delta)\int_0^t \mathfrak{F}_4(x,\mathfrak{A}_p(x))(t-x)^{\delta-1}dx,\\
\mathfrak{R}_p(t) &= \mathfrak{R}_p(0)+\Upsilon(\delta)\mathfrak{F}_5(t,\mathfrak{R}_p(t))+\Lambda(\delta)\int_0^t \mathfrak{F}_5(x,\mathfrak{R}_p(x))(t-x)^{\delta-1}dx,\\
\mathfrak{M}(t) &= \mathfrak{M}(0)+\Upsilon(\delta)\mathfrak{F}_6(t,\mathfrak{M}(t))+\Lambda(\delta)\int_0^t \mathfrak{F}_6(x,\mathfrak{M}(x))(t-x)^{\delta-1}dx.
\end{aligned}
\tag{11}
$$

The Picard iterations are given by

$$
\begin{aligned}
\mathfrak{S}_p^{j+1}(t) &= \Upsilon(\delta)\mathfrak{F}_1(t,\mathfrak{S}_p^{j}(t))+\Lambda(\delta_1)\int_0^t \mathfrak{F}_1(x,\mathfrak{S}_p^{j}(x))(t-x)^{\delta-1}dx,\\
\mathfrak{E}_p^{j+1}(t) &= \Upsilon(\delta)\mathfrak{F}_2(t,\mathfrak{E}_p^{j}(t))+\Lambda(\delta_2)\int_0^t \mathfrak{F}_2(x,\mathfrak{E}_p^{j}(x))(t-x)^{\delta-1}dx,\\
\mathfrak{I}_p^{j+1}(t) &= \Upsilon(\delta)\mathfrak{F}_3(t,\mathfrak{I}_p^{j}(t))+\Lambda(\delta_3)\int_0^t \mathfrak{F}_3(x,\mathfrak{I}_p^{j}(x))(t-x)^{\delta-1}dx,\\
\mathfrak{A}_p^{j+1}(t) &= \Upsilon(\delta)\mathfrak{F}_4(t,\mathfrak{A}_p^{j}(t))+\Lambda(\delta_4)\int_0^t \mathfrak{F}_4(x,\mathfrak{A}_p^{j}(x))(t-x)^{\delta-1}dx,\\
\mathfrak{R}_p^{j+1}(t) &= \Upsilon(\delta)\mathfrak{F}_5(t,\mathfrak{R}_p^{j}(t))+\Lambda(\delta_5)\int_0^t \mathfrak{F}_5(x,\mathfrak{R}_p^{j}(x))(t-x)^{\delta-1}dx,\\
\mathfrak{M}^{j+1}(t) &= \Upsilon(\delta)\mathfrak{F}_6(t,\mathfrak{M}^{j}(t))+\Lambda(\delta_6)\int_0^t \mathfrak{F}_6(x,\mathfrak{M}^{j}(x))(t-x)^{\delta-1}dx.
\end{aligned}
\tag{12}
$$

In order to show the existence and uniqueness of solution of the model (7), we make use of fixed point theory. First, we re-write the model (7) in the following way:

$$
\begin{cases}
{}^{ABC}\mathfrak{D}^{\delta}\zeta(t)=\mathfrak{F}(t,\zeta(t)),\\
\zeta(0)=\zeta_0,\ 0<t<T<\infty.
\end{cases}
\tag{13}
$$

The vector $\zeta(t)=(\mathfrak{S}_p,\mathfrak{E}_p,\mathfrak{I}_p,\mathfrak{A}_p,\mathfrak{R}_p,\mathfrak{M})$ and $\mathfrak{F}$ in (13) represent the state variables and a continuous vector function respectively defined as follows:

$$
\mathfrak{F}=\begin{pmatrix}\mathfrak{F}_1\\ \mathfrak{F}_2\\ \mathfrak{F}_3\\ \mathfrak{F}_4\\ \mathfrak{F}_6\\ \mathfrak{F}_6\end{pmatrix}=\begin{pmatrix}
\coprod_p-\omega_p\mathfrak{S}_p(t)-\zeta_p\mathfrak{S}_p(t)(\mathfrak{I}_p(t)+\Psi\mathfrak{A}_p(t))-\omega_w\mathfrak{S}_p(t)\mathfrak{M}(t)\\
\zeta_p\mathfrak{S}_p(t)(\mathfrak{I}_p(t)+\Psi\mathfrak{A}_p(t))+\omega_w\mathfrak{S}_p(t)\mathfrak{M}(t)-(1-\Phi_p)\eta_p\mathfrak{E}_p(t)\\
-\Phi_p\varrho_p\mathfrak{E}_p(t)-\omega_p\mathfrak{E}_p(t)\\
(1-\Phi_p)\eta_p\mathfrak{E}_p(t)-(\tau_p+\omega_p)\mathfrak{I}_p(t)\\
\Phi_p\varrho_P\mathfrak{E}_p(t)-(\tau_{ap}+\omega_p)\mathfrak{A}_p(t)\\
\tau_p\mathfrak{I}_p(t)+\tau_{ap}\mathfrak{A}_p(t)-\omega_p\mathfrak{R}_p(t)\\
\phi_p\mathfrak{I}_p(t)+\varpi_p\mathfrak{A}_p(t)-\varphi\mathfrak{M}(t)
\end{pmatrix}
\tag{14}
$$

with initial conditions $\zeta_0(t) = (\mathfrak{S}_p(0), \mathfrak{E}_p(0), \mathfrak{I}_p(0), \mathfrak{A}_p(0), \mathfrak{R}_p(0), \mathfrak{M}(0))$. Corresponding to (13), the integral equation is give by

$$\zeta(t) = \zeta_0 + \Upsilon(\delta)\mathfrak{F}(t,\zeta(t)) + \Lambda(\delta)\int_0^t \mathfrak{F}(x,\zeta(x))(t-x)^{\delta-1}dx. \tag{15}$$

3. Existence Results

Consider $A = [0,T]$, $\mathcal{E} = \mathcal{C}(A,\mathbb{R}^6)$ and the Picard operator $\mathcal{P} : \mathcal{E} \to \mathcal{E}$ be given by

$$\mathcal{P}[\zeta(t)] = \zeta_0 + \Upsilon(\delta)\mathfrak{F}(t,\zeta(t)) + \Lambda(\delta)\int_0^t \mathfrak{F}(x,\zeta(x))(t-x)^{\delta-1}dx. \tag{16}$$

Together with the supremum norm $\|\cdot\|_{\mathcal{C}}$, on ζ is defined by

$$\|\zeta(t)\|_{\mathcal{C}} = \sup_{t\in A}\|\zeta(t)\|,\ \zeta(t) \in \mathcal{E}, \tag{17}$$

$\mathcal{E}$ defines a Banach space. Assume the following

[$\mathcal{A}_1$**:**] Let $\mathfrak{F} : A \times \mathbb{R}^6 \to \mathbb{R}^6$ is continuous.

[$\mathcal{A}_2$**:**] There exists $C_{\mathfrak{F}} > 0$ such that

$$|\mathfrak{F}(t,\zeta) - \mathfrak{F}(x,\zeta')| \leq C_{\mathfrak{F}}|\zeta - \zeta'|$$

for all $\zeta, \zeta' \in \mathbb{R}^6, t \in A$.

[$\mathcal{A}_3$**:**] There exist a constant $L > 0$ such that $|\mathfrak{F}(x,\zeta)| \leq L(1+|\zeta|)$ for each $x \in A$ and all $\zeta \in \mathbb{R}^6$.

We prove the existence of solution of (13) by Schaefer's fixed point theorem.

Theorem 3.1. *Assuming [$\mathcal{A}_1$]-[$\mathcal{A}_3$] together with* $1-\Upsilon(\delta)L > 0$, *(13) has at least one solution.*

Proof. We first show that the operator $\mathcal{P}$ given in (16) is continuous. Consider a sequence (ζ_j) such that $\zeta_j \to \zeta$ in $\mathcal{E}$. Now

$$\begin{aligned}
|\mathcal{P}\zeta_j(t) - \mathcal{P}\zeta(t)| &= \bigg|\Upsilon(\delta)\mathfrak{F}(t,\zeta_j(t)) + \Lambda(\delta)\int_0^t \mathfrak{F}(x,\zeta_j(x))(t-x)^{\delta-1}dx \\
&\quad -\Upsilon(\delta)\mathfrak{F}(t,\zeta(t)) - \Lambda(\delta)\int_0^t \mathfrak{F}(x,\zeta(x))(t-x)^{\delta-1}dx\bigg| \\
&\leq \Upsilon(\delta)\Big|(\mathfrak{F}(t,\zeta_j(t)) - \mathfrak{F}(t,\zeta(t)))\Big| \\
&\quad +\Lambda(\delta)\bigg|\int_0^t \mathfrak{F}(x,\zeta_j(x))(t-x)^{\delta-1}dx - \int_0^t \mathfrak{F}(x,\zeta(x))(t-x)^{\delta-1}dx\bigg| \\
&\leq \Upsilon(\delta)\Big|(\mathfrak{F}(t,\zeta_j(t)) - \mathfrak{F}(t,\zeta(t)))\Big| + \Lambda(\delta)\int_0^t |\mathfrak{F}(x,\zeta_j(x)) - \mathfrak{F}(x,\zeta(x))|(t-x)^{\delta-1}dx \\
&\leq \Upsilon(\delta)C_{\mathfrak{F}}\|\mathfrak{F}(x,\zeta_j(x)) - \mathfrak{F}(x,\zeta(x))\|_{\mathcal{C}} + \Lambda(\delta)C_{\mathfrak{F}}\|\mathfrak{F}(x,\zeta_j(x)) - \mathfrak{F}(x,\zeta(x))\|_{\mathcal{C}}\frac{t^\delta}{\delta} \\
&\leq \left(\Upsilon(\delta) + \frac{\Lambda(\delta)T^\delta}{\delta}\right)C_{\mathfrak{F}}\|\mathfrak{F}(x,\zeta_j(x)) - \mathfrak{F}(x,\zeta(x))\|_{\mathcal{C}}.
\end{aligned}$$

Continuity of $\mathfrak{F}$ implies the continuity of $\mathcal{P}$.

Now suppose that $W=\{\zeta\in\mathcal{E}:\|\zeta\|\leq c>0\}$. We now show that $\mathcal{P}[W]$ is bounded, i.e. there exists $d>0$ such that for every $\zeta\in W$, $\|\mathcal{P}\zeta\|\leq d$. For any $t\in A$, we have

$$\begin{aligned}
|\mathcal{P}\zeta(t)| &= \left|\zeta_0+\Upsilon(\delta)\mathfrak{F}(t,\zeta_j(t))+\Lambda(\delta)\int_0^t\mathfrak{F}(x,\zeta_j(x))(t-x)^{\delta-1}dx\right|\\
&\leq |\zeta_0|+\Upsilon(\delta)|\mathfrak{F}(t,\zeta(t))|+\Lambda(\delta)\left|\int_0^t\mathfrak{F}(x,\zeta(x))(t-x)^{\delta-1}dx\right|\\
&\leq |\zeta_0|+\Upsilon(\delta)|\mathfrak{F}(t,\zeta(t))|+\Lambda(\delta)\int_0^t|\mathfrak{F}(x,\zeta(x))|(t-x)^{\delta-1}dx\\
&\leq |\zeta_0|+\Upsilon(\delta)L(1+|\zeta|)+\Lambda(\delta)L\int_0^t(1+|\zeta(x)|)(t-x)^{\delta-1}dx\\
&\leq |\zeta_0|+\Upsilon(\delta)L(1+\|\zeta\|)+\Lambda(\delta)L(1+\|\zeta\|)\frac{T^\delta}{\delta}\\
&\leq |\zeta_0|+\Upsilon(\delta)L(1+c)+\Lambda(\delta)L(1+c)\frac{T^\delta}{\delta}\\
&= |\zeta_0|+\left(\Upsilon(\delta)+\Lambda(\delta)\frac{T^\delta}{\delta}\right)L(1+c)=d,
\end{aligned}$$

which implies

$$|\mathcal{P}\zeta(t)|\leq d.$$

For the equicontinuity of $\mathcal{P}$, let $t_1,t_2\in A$ with $0\leq t_1,t_2\leq T$ and $\zeta\in W$. Utilizing $[\mathcal{A}_\ni]$, we have

$$\begin{aligned}
|\mathcal{P}\zeta(t_1)-\mathcal{P}\zeta(t_2)| = &\bigg|\Upsilon(\delta)\mathfrak{F}(t_1,\zeta(t_1))+\Lambda(\delta)\int_0^{t_1}\mathfrak{F}(x,\zeta(x))(t_1-x)^{\delta-1}dx\\
&-\Upsilon(\delta)\mathfrak{F}(t_2,\zeta(t_2))-\Lambda(\delta)\int_0^{t_2}\mathfrak{F}(x,\zeta(x))(t_2-x)^{\delta-1}dx\bigg|\\
\leq\ &\Upsilon(\delta)\Big|(\mathfrak{F}(t_1,\zeta(t_1))-\mathfrak{F}(t_2,\zeta(t_2)))\Big|\\
&+\Lambda(\delta)\left|\int_0^{t_1}\mathfrak{F}(x,\zeta(x))(t_1-x)^{\delta-1}dx-\int_0^{t_2}\mathfrak{F}(x,\zeta(x))(t_2-x)^{\delta-1}dx\right|\\
\leq\ &\Upsilon(\delta)\Big|(\mathfrak{F}(t_1,\zeta(t_1))-\mathfrak{F}(t_2,\zeta(t_2)))\Big|+\Lambda(\delta)\bigg|\int_0^{t_1}\mathfrak{F}(x,\zeta(x))(t_1-x)^{\delta-1}dx\\
&-\int_0^{t_1}\mathfrak{F}(x,\zeta(x))(t_2-x)^{\delta-1}dx-\int_{t_1}^{t_2}\mathfrak{F}(x,\zeta(x))(t_2-x)^{\delta-1}dx\bigg|\\
\leq\ &\Upsilon(\delta)\Big|(\mathfrak{F}(t_1,\zeta(t_1))-\mathfrak{F}(t_2,\zeta(t_2)))\Big|+\Lambda(\delta)\left|\int_0^{t_1}\mathfrak{F}(x,\zeta(x))[(t_1-x)^{\delta-1}-(t_2-x)^{\delta-1}]dx\right|\\
&+\Lambda(\delta)\left|\int_{t_1}^{t_2}\mathfrak{F}(x,\zeta(x))(t_2-x)^{\delta-1}dx\right|
\end{aligned}$$

$$\leq \Upsilon(\delta)\Big|(\mathfrak{F}(t_1,\zeta(t_1)) - \mathfrak{F}(t_2,\zeta(t_2)))\Big| + \Lambda(\delta)\Big|\int_0^{t_1}\mathfrak{F}(x,\zeta(x))[(t_1-x)^{\delta-1} - (t_2-x)^{\delta-1}]dx\Big|$$
$$+ \Lambda(\delta)L(1+|\zeta|)\int_{t_1}^{t_2}|(t_2-x)^{\delta-1}|dx$$
$$\leq \Upsilon(\delta)\Big|(\mathfrak{F}(t_1,\zeta(t_1)) - \mathfrak{F}(t_2,\zeta(t_2)))\Big| + \Lambda(\delta)\Big|\int_0^{t_1}\mathfrak{F}(x,\zeta(x))[(t_1-x)^{\delta-1} - (t_2-x)^{\delta-1}]dx\Big|$$
$$+ \Lambda(\delta)L(1+d)\frac{(t_2-t_1)^{\delta}}{\delta}.$$

As t_1 tends to t_2, continuity of $\mathfrak{F}$ tends R.H.S of above inequality to zero. Hence $\mathcal{P}$ is equicontinuous. Therefore, we conclude by Arzela-Ascoli Theorem that $\mathcal{P}$ is completely continuous.
Finally, we show that the set $Q(\mathcal{P}) = \{\zeta \in \mathcal{E} : \zeta = \vartheta\mathcal{P}\zeta \text{ for some } \vartheta \in (0,1)\}$ is bounded. For each $t \in A$, we have

$$\begin{aligned}
|\mathcal{P}\zeta(t)| &= \Big|\zeta_0 + \Upsilon(\delta)\mathfrak{F}(t,\zeta_j(t)) + \Lambda(\delta)\int_0^t \mathfrak{F}(x,\zeta_j(x))(t-x)^{\delta-1}dx\Big| \\
&\leq |\zeta_0| + \Upsilon(\delta)|\mathfrak{F}(t,\zeta(t))| + \Lambda(\delta)\Big|\int_0^t \mathfrak{F}(x,\zeta(x))(t-x)^{\delta-1}dx\Big| \\
&\leq |\zeta_0| + \Upsilon(\delta)|\mathfrak{F}(t,\zeta(t))| + \Lambda(\delta)\int_0^t |\mathfrak{F}(x,\zeta(x))|(t-x)^{\delta-1}dx \\
&\leq |\zeta_0| + \Upsilon(\delta)L(1+|\zeta(t)|) + \Lambda(\delta)L\int_0^t (1+|\zeta(x)|)(t-x)^{\delta-1}dx \\
&\leq |\zeta_0| + \Upsilon(\delta)L(1+|\zeta(t)|) + \Lambda(\delta)L\frac{T^{\delta}}{\delta} + \Lambda(\delta)L\int_0^t |\zeta(x)|(t-x)^{\delta-1}dx \\
&= |\zeta_0| + \Upsilon(\delta)L + \Upsilon(\delta)L|\zeta(t)| + \Lambda(\delta)L\frac{T^{\delta}}{\delta} + \Lambda(\delta)L\int_0^t |\zeta(x)|(t-x)^{\delta-1}dx.
\end{aligned}$$

Writing $S = |\zeta_0| + \Upsilon(\delta)L + \Lambda(\delta)L\frac{T^{\delta}}{\delta}$ and since $1 - \Upsilon(\delta)L > 0$, we can have

$$|\mathcal{P}\zeta(t)| \leq \frac{S}{1-\Upsilon(\delta)L} + \frac{\Lambda(\delta)L}{1-\Upsilon(\delta)L}\int_0^t |\zeta(x)|(t-x)^{\delta-1}dx,$$

utilizing Gronwall's inequality, we obtain

$$|\mathcal{P}\zeta(t)| \leq \frac{S}{1-\Upsilon(\delta)L} exp\Big(\frac{\Lambda(\delta)LT^{\delta}}{(1-\Upsilon(\delta)L)\delta}\Big).$$

Therefore $Q(\mathcal{P})$ is bounded. Consequently, by Schaefer's theorem $\mathcal{P}$ has a fixed point which is infact a solution of (13). $\square$

We now show by using Banach contraction principle that solution of (13) is unique.

THEOREM 3.2. *Assuming [$\mathcal{A}_1$]-[$\mathcal{A}_2$] together with $\Big(\Upsilon(\delta) + \frac{\Lambda(\delta)T^{\delta}}{\delta}\Big) C_{\mathfrak{F}} < 1$, there exists a unique solution of (13).*

PROOF. Considering (3) together with (13), we have

$$\zeta(t) = \mathcal{P}[\zeta(t)]. \tag{18}$$

The operator $\mathcal{P}$ given in (16), is well defined by $[\mathcal{A}_1]$. Now for all $\zeta, \zeta' \in \mathcal{E}$, we have

$$\begin{aligned}
& |\mathcal{P}[\zeta(t)] - \mathcal{P}[\zeta'(t)]| \\
= & \left|\Upsilon(\delta)(\mathfrak{F}(t,\zeta(t)) - \mathfrak{F}(t,\zeta'(t))) + \Lambda(\delta)\int_0^t (\mathfrak{F}(x,\zeta(x)) - \mathfrak{F}(x,\zeta'(x)))(t-x)^{\delta-1}dx\right| \\
\leq & \Upsilon(\delta)|\mathfrak{F}(t,\zeta(t)) - \mathfrak{F}(t,\zeta'(t))| + \Lambda(\delta)\int_0^t |\mathfrak{F}(x,\zeta(x)) - \mathfrak{F}(x,\zeta(x))|(t-x)^{\delta-1}dx \\
\leq & \Upsilon(\delta)C_{\mathfrak{F}}|\zeta(t) - \zeta'(t)| + \Lambda(\delta)C_{\mathfrak{F}}\int_0^t |\zeta(x) - \zeta'(x)|(t-x)^{\delta-1}dx \\
\leq & \Upsilon(\delta)C_{\mathfrak{F}}\|\zeta - \zeta'\|_{\mathcal{C}} + \Lambda(\delta)C_{\mathfrak{F}}\|\zeta - \zeta'\|_{\mathcal{C}}\int_0^t (t-x)^{\delta-1}dx \\
\leq & (\Upsilon(\delta) + \frac{\Lambda(\delta)T^\delta}{\delta})C_{\mathfrak{F}}\|\zeta - \zeta'\|_{\mathcal{C}} \\
= & \mathcal{A}\|\zeta - \zeta'\|_{\mathcal{C}},
\end{aligned}$$

where

$$\mathcal{A} = \left(\Upsilon(\delta) + \frac{\Lambda(\delta)T^\delta}{\delta}\right)C_{\mathfrak{F}}.$$

This implies

$$\|\mathcal{P}[\zeta(t)] - \mathcal{P}[\zeta'(t)]\|_{\mathcal{C}} \leq \mathcal{A}\|\zeta - \zeta'\|_{\mathcal{C}}, \tag{19}$$

Thus the defined operator $\mathcal{P}$ is a contraction, and hence Banach contraction principle guarantees that $\mathcal{P}$ has a unique fixed point which is the solution model (13). □

4. Special Solution by Iterative Approach

We obtain iterative solution of the model (7). Apply Shehu transforms (S_h) on both sides of (7), we get

$$\begin{aligned}
S_h[{}^{ABC}\mathfrak{D}^\delta\mathfrak{S}_p] &= S_h\left[\coprod_p -\omega_p\mathfrak{S}_p - \zeta_p\mathfrak{S}_p(\mathfrak{I}_p + \Psi\mathfrak{A}_p) - \omega_w\mathfrak{S}_p\mathfrak{M}\right], \\
S_h[{}^{ABC}\mathfrak{D}^\delta\mathfrak{E}_p] &= S_h\left[\zeta_p\mathfrak{S}_p(\mathfrak{I}_p + \Psi\mathfrak{A}_p) + \omega_w\mathfrak{S}_p\mathfrak{M} - (1-\Phi_p)\eta_p\mathfrak{E}_p - \Phi_p\varrho_p\mathfrak{E}_p - \omega_p\mathfrak{E}_p\right], \\
S_h[{}^{ABC}\mathfrak{D}^\delta\mathfrak{I}_p] &= S_h[(1-\Phi_p)\eta_p\mathfrak{E}_p - (\tau_p + \omega_p)\mathfrak{I}_p], \\
S_h[{}^{ABC}\mathfrak{D}^\delta\mathfrak{A}_p] &= S_h[\Phi_p\varrho_P\mathfrak{E}_p - (\tau_{ap} + \omega_p)\mathfrak{A}_p], \\
S_h[{}^{ABC}\mathfrak{D}^\delta\mathfrak{R}_p] &= S_h[\tau_p\mathfrak{I}_p + \tau_{ap}\mathfrak{A}_p - \omega_p\mathfrak{R}_p], \\
S_h[{}^{ABC}\mathfrak{D}^\delta\mathfrak{M}] &= S_h[\phi_p\mathfrak{I}_p + \varpi_p\mathfrak{A}_p - \varphi\mathfrak{M}].
\end{aligned} \tag{20}$$

Using definition of Shehu transforms of ABC-derivative, we get

$$
\begin{aligned}
\frac{M(\delta)}{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}\left[S_h(\mathfrak{S}_p)-\left(\frac{u}{s}\right)\mathfrak{S}_p(0)\right] &= S_h\left[\coprod_p -\omega_p\mathfrak{S}_p-\zeta_p\mathfrak{S}_p(\mathfrak{I}_p+\Psi\mathfrak{A}_p)-\omega_w\mathfrak{S}_p\mathfrak{M}\right],\\
\frac{M(\delta)}{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}\left[S_h(\mathfrak{E}_p)-\left(\frac{u}{s}\right)\mathfrak{E}_p(0)\right] &= S_h\Big[\zeta_p\mathfrak{S}_p(\mathfrak{I}_p+\Psi\mathfrak{A}_p)+\omega_w\mathfrak{S}_p\mathfrak{M}-(1-\Phi_p)\eta_p\mathfrak{E}_p\\
&\quad -\Phi_p\varrho_p\mathfrak{E}_p-\omega_p\mathfrak{E}_p\Big],\\
\frac{M(\delta)}{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}\left[S_h(\mathfrak{I}_p)-\left(\frac{u}{s}\right)\mathfrak{I}_p(0)\right] &= S_h[(1-\Phi_p)\eta_p\mathfrak{E}_p-(\tau_p+\omega_p)\mathfrak{I}_p],\\
\frac{M(\delta)}{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}\left[S_h(\mathfrak{A}_p)-\left(\frac{u}{s}\right)\mathfrak{A}_p(0)\right] &= S_h[\Phi_p\varrho_P\mathfrak{E}_p-(\tau_{ap}+\omega_p)\mathfrak{A}_p],\\
\frac{M(\delta)}{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}\left[S_h(\mathfrak{R}_p)-\left(\frac{u}{s}\right)\mathfrak{R}_p(0)\right] &= S_h[\tau_p\mathfrak{I}_p+\tau_{ap}\mathfrak{A}_p-\omega_p\mathfrak{R}_p],\\
\frac{M(\delta)}{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}\left[S_h(\mathfrak{M}_p)-\left(\frac{u}{s}\right)\mathfrak{M}_p(0)\right] &= S_h[\phi_p\mathfrak{I}_p+\varpi_p\mathfrak{A}_p-\varphi\mathfrak{M}],
\end{aligned}
$$

On rearranging

$$
\begin{aligned}
S_h(\mathfrak{S}_p) &= \left(\frac{u}{s}\right)\mathfrak{S}_p(0)+\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h\left[\coprod_p -\omega_p\mathfrak{S}_p-\zeta_p\mathfrak{S}_p(\mathfrak{I}_p+\Psi\mathfrak{A}_p)-\omega_w\mathfrak{S}_p\mathfrak{M}\right],\\
S_h(\mathfrak{E}_p) &= \left(\frac{u}{s}\right)\mathfrak{E}_p(0)+\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h\Big[\zeta_p\mathfrak{S}_p(\mathfrak{I}_p+\Psi\mathfrak{A}_p)+\omega_w\mathfrak{S}_p\mathfrak{M}-(1-\Phi_p)\eta_p\mathfrak{E}_p\\
&\quad -\Phi_p\varrho_p\mathfrak{E}_p-\omega_p\mathfrak{E}_p\Big],\\
S_h(\mathfrak{I}_p) &= \left(\frac{u}{s}\right)\mathfrak{I}_p(0)+\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h[(1-\Phi_p)\eta_p\mathfrak{E}_p-(\tau_p+\omega_p)\mathfrak{I}_p],\\
S_h(\mathfrak{A}_p) &= \left(\frac{u}{s}\right)\mathfrak{A}_p(0)+\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h[\Phi_p\varrho_P\mathfrak{E}_p-(\tau_{ap}+\omega_p)\mathfrak{A}_p],\\
S_h(\mathfrak{R}_p) &= \left(\frac{u}{s}\right)\mathfrak{R}_p(0)+\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h[\tau_p\mathfrak{I}_p+\tau_{ap}\mathfrak{A}_p-\omega_p\mathfrak{R}_p],\\
S_h(\mathfrak{M}_p) &= \left(\frac{u}{s}\right)\mathfrak{M}_p(0)+\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h[\phi_p\mathfrak{I}_p+\varpi_p\mathfrak{A}_p-\varphi\mathfrak{M}],
\end{aligned}
\tag{21}
$$

Operating S_h^{-1} on both sides of (21) and taking into account that $S_h^{-1}\left(\frac{u}{s}\right)=1$, we get

$$
\begin{aligned}
\mathfrak{S}_p &= \mathfrak{S}_p(0)+S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h\left[\coprod_p-\omega_p\mathfrak{S}_p-\zeta_p\mathfrak{S}_p(\mathfrak{I}_p+\Psi\mathfrak{A}_p)-\omega_w\mathfrak{S}_p\mathfrak{M}\right]\right\},\\
\mathfrak{E}_p &= \mathfrak{E}_p(0)+S_h^{-1}\Bigg\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h\Bigg[\zeta_p\mathfrak{S}_p(\mathfrak{I}_p+\Psi\mathfrak{A}_p)+\omega_w\mathfrak{S}_p\mathfrak{M}-(1-\Phi_p)\eta_p\mathfrak{E}_p\\
&\quad-\Phi_p\varrho_p\mathfrak{E}_p-\omega_p\mathfrak{E}_p\Bigg]\Bigg\},\\
\mathfrak{I}_p &= \mathfrak{I}_p(0)+S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h[(1-\Phi_p)\eta_p\mathfrak{E}_p-(\tau_p+\omega_p)\mathfrak{I}_p]\right\},\\
\mathfrak{A}_p &= \mathfrak{A}_p(0)+S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h[\Phi_p\varrho_P\mathfrak{E}_p-(\tau_{ap}+\omega_p)\mathfrak{A}_p]\right\},\\
\mathfrak{R}_p &= \mathfrak{R}_p(0)+S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h[\tau_p\mathfrak{I}_p+\tau_{ap}\mathfrak{A}_p-\omega_p\mathfrak{R}_p]\right\},\\
\mathfrak{M}_p &= \mathfrak{M}_p(0)+S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h[\phi_p\mathfrak{I}_p+\varpi_p\mathfrak{A}_p-\varphi\mathfrak{M}]\right\},
\end{aligned}
\tag{22}
$$

The recursive formula is given by

$$
\begin{aligned}
\mathfrak{S}_p^{n+1} &= \mathfrak{S}_p^n(0)+S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h\left[\coprod_p-\omega_p\mathfrak{S}_p^n-\zeta_p\mathfrak{S}_p^n(\mathfrak{I}_p+\Psi\mathfrak{A}_p^n)-\omega_w\mathfrak{S}_p^n\mathfrak{M}^n\right]\right\},\\
\mathfrak{E}_p^{n+1} &= \mathfrak{E}_p^n(0)+S_h^{-1}\Bigg\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h\Bigg[\zeta_p\mathfrak{S}_p^n(\mathfrak{I}_p^n+\Psi\mathfrak{A}_p^n)+\omega_w\mathfrak{S}_p^n\mathfrak{M}^n-(1-\Phi_p)\eta_p\mathfrak{E}_p^n\\
&\quad-\Phi_p\varrho_p\mathfrak{E}_p^n-\omega_p\mathfrak{E}_p^n\Bigg]\Bigg\},\\
\mathfrak{I}_p^{n+1} &= \mathfrak{I}_p^n(0)+S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h[(1-\Phi_p)\eta_p\mathfrak{E}_p^n-(\tau_p+\omega_p)\mathfrak{I}_p^n]\right\},\\
\mathfrak{A}_p^{n+1} &= \mathfrak{A}_p^n(0)+S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h[\Phi_p\varrho_P\mathfrak{E}_p^n-(\tau_{ap}+\omega_p)\mathfrak{A}_p^n]\right\},\\
\mathfrak{R}_p^{n+1} &= \mathfrak{R}_p^n(0)+S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h[\tau_p\mathfrak{I}_p^n+\tau_{ap}\mathfrak{A}_p^n-\omega_p\mathfrak{R}_p^n]\right\},\\
\mathfrak{M}_p^{n+1} &= \mathfrak{M}_p^n(0)+S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h[\phi_p\mathfrak{I}_p^n+\varpi_p\mathfrak{A}_p^n-\varphi\mathfrak{M}^n]\right\},
\end{aligned}
\tag{23}
$$

The approximate solution of (23) is given by

$$
\begin{aligned}
\mathfrak{S}_p &= \lim_{n\to\infty}\mathfrak{S}_p^n, \quad \mathfrak{E}_p=\lim_{n\to\infty}\mathfrak{E}_p^n, \quad \mathfrak{I}_p=\lim_{n\to\infty}\mathfrak{I}_p^n,\\
\mathfrak{A}_p &= \lim_{n\to\infty}\mathfrak{A}_p^n, \quad \mathfrak{R}_p=\lim_{n\to\infty}\mathfrak{R}_p^n, \quad \mathfrak{M}_p=\lim_{n\to\infty}\mathfrak{M}_p^n.
\end{aligned}
$$

5. Stability Analysis and Iterative Solution

Consider a Banach space $\mathcal{X}$ together with norm $\|x\| = \max_{t\in[a,b]} |x(t)|$, $x \in \mathcal{X}$ and $\mathcal{F}$ a self map on $\mathcal{X}$. The recursive procedure is

$$S_{n+1} = h(\mathcal{F}, S_n). \tag{24}$$

The set of fixed points $Fix(\mathcal{F})$ of $\mathcal{F}$ is nonempty and S_n converges to a point of $Fix(\mathcal{F})$. Choose a sequence (f_n) in $\mathcal{X}$ and $e_n = \|f_{n+1} - h(\mathcal{F}, S_n)\|$. The recursive procedure (24) is $\mathcal{F}$-stable if $\lim_{n\to\infty} e_n = 0$. We suppose that the sequence (f_n) is bounded above, else it will diverge. Under these conditions, $S_{n+1} = \mathcal{F}S_n$ is Picard's iteration as described in [**26**], implies it is $\mathcal{F}$-stable.

Theorem 5.1. *Let $(\mathcal{X}, \|\cdot\|)$ be a Banach space and $\mathcal{F}$ be a self map on $\mathcal{X}$ satisfying*

$$\|\mathcal{F}_x - \mathcal{F}_y\| \leq R\|x - \mathcal{F}_x\| + r\|x - y\| \tag{25}$$

for all $x, y \in \mathcal{X}$, where $R \geq 0$ and $0 \leq r < 1$. Then $\mathcal{F}$ is Picard $\mathcal{F}$-stable.

Theorem 5.2. *A self map $\mathcal{F}$ given by*

$$\begin{aligned}
\mathcal{F}(\mathfrak{S}_p^n(t)) &= \mathfrak{S}_p^{n+1}(t)\\
&= \mathfrak{S}_p^n(t) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h\left[\coprod_p -\omega_p\mathfrak{S}_p^n - \zeta_p\mathfrak{S}_p^n(\mathfrak{I}_p^n + \Psi\mathfrak{A}_p^n) - \omega_w\mathfrak{S}_p^n\mathfrak{M}^n\right]\right\}\\
\mathcal{F}(\mathfrak{E}_p^n(t)) &= \mathfrak{E}_p^{n+1}(t)\\
&= \mathfrak{E}_p^n(t) + S_h^{-1}\Bigg\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h\Bigg[\zeta_p\mathfrak{S}_p^n(\mathfrak{I}_p^n + \Psi\mathfrak{A}_p^n) + \omega_w\mathfrak{S}_p^n\mathfrak{M}^n - (1-\Phi_p)\eta_p\mathfrak{E}_p^n\\
&\quad -\Phi_p\varrho_p\mathfrak{E}_p^n - \omega_p\mathfrak{E}_p^n\Bigg]\Bigg\}\\
\mathcal{F}(\mathfrak{I}_p^n(t)) &= \mathfrak{I}_p^{n+1}(t)\\
&= \mathfrak{I}_p^n(t) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h[(1-\Phi_p)\eta_p\mathfrak{E}_p^n - (\tau_p + \omega_p)\mathfrak{I}_p^n]\right\}\\
\mathcal{F}(\mathfrak{A}_p^n(t)) &= \mathfrak{A}_p^{n+1}(t)\\
&= \mathfrak{A}_p^n(t) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h[\Phi_p\varrho_P\mathfrak{E}_p^n - (\tau_{ap} + \omega_p)\mathfrak{A}_p^n]\right\}\\
\mathcal{F}(\mathfrak{R}_p^n(t)) &= \mathfrak{R}_p^{n+1}(t)\\
&= \mathfrak{R}_p^n(t) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h[\tau_p\mathfrak{I}_p^n + \tau_{ap}\mathfrak{A}_p^n - \omega_p\mathfrak{R}_p^n]\right\}\\
\mathcal{F}(\mathfrak{M}^n(t)) &= \mathfrak{M}^{n+1}(t)\\
&= \mathfrak{M}_p^n(t) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h[\phi_p\mathfrak{I}_p^n + \varpi_p\mathfrak{A}_p^n - \varphi\mathfrak{M}^n]\right\}
\end{aligned}$$

is $\mathcal{F}$-stable in $H^1(a,b)$ if the following conditions holds:

$$\begin{cases} 1-\omega_p f_1(\kappa) - \frac{\zeta_p}{\mathfrak{N}_p}(L_1+L_3'+\Psi L_1+\Psi L_4')f_2(\kappa) - \omega_w(L_1+L_6')\}f_3(\kappa) < 1 \\ 1+\frac{\zeta_p}{\mathcal{N}_p}(L_1+L_3'+\psi L_1+\psi L_4')f_4(\kappa)+\omega_w(L_1+L_6')f_5(\kappa) - \{(1-\Phi_p)\eta_p+\Phi_p\varrho_p+\omega_p\}f_6(\kappa) < 1 \\ 1+(1-\Phi_p)\eta_p f_7(\kappa) - (\tau_p+\omega_p)f_8(\kappa) < 1 \\ 1+\Phi_p\varrho_P f_9(\kappa) - (\tau_{ap}+\omega_p)f_{10}(\kappa) < 1 \\ 1+\tau_p f_{11}(\kappa)+\tau_{ap}f_{12}(\kappa) - \omega_p f_{13}(\kappa) < 1 \\ 1+\phi_p f_{14}(\kappa)+\varpi_p f_{15}(\kappa) - \varphi_p(\kappa)f_{16}(\kappa) < 1. \end{cases}$$

Proof. We first show that $\mathcal{F}$ has a fixed point. For $m,n\in\mathbb{N}$, we have

$$\begin{aligned}
&\mathcal{F}(\mathfrak{S}_p^n(t)) - \mathcal{F}(\mathfrak{S}_p^m(t)) \\
&= \mathfrak{S}_p^n(t) - \mathfrak{S}_p^m(t) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h\left[\coprod_p -\omega_p\mathfrak{S}_p^n - \zeta_p\mathfrak{S}_p^n(\mathfrak{I}_p^n+\Psi\mathfrak{A}_p^n) - \omega_w\mathfrak{S}_p^n\mathfrak{M}^n\right]\right\} \\
&\quad - S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h\left[\coprod_p -\omega_p\mathfrak{S}_p^m - \zeta_p\mathfrak{S}_p^m(\mathfrak{I}_p^m+\Psi\mathfrak{A}_p^m) - \omega_w\mathfrak{S}_p^m\mathfrak{M}^m\right]\right\} \\
&\mathcal{F}(\mathfrak{E}_p^n(t)) - \mathcal{F}(\mathfrak{E}_p^m(t)) \\
&= \mathfrak{E}_p^n(t) - \mathfrak{E}_p^m(t) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h\left[\zeta_p\mathfrak{S}_p^n(\mathfrak{I}_p^n+\Psi\mathfrak{A}_p^n)+\omega_w\mathfrak{S}_p^n\mathfrak{M}^n - (1-\Phi_p)\eta_p\mathfrak{E}_p^n\right.\right. \\
&\left.\left. - \Phi_p\varrho_p\mathfrak{E}_p^n - \omega_p\mathfrak{E}_p^n\right]\right\} \\
&\quad - S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h\left[\zeta_p\mathfrak{S}_p^m(\mathfrak{I}_p^m+\Psi\mathfrak{A}_p^m)+\omega_w\mathfrak{S}_p^m\mathfrak{M}^m - (1-\Phi_p)\eta_p\mathfrak{E}_p^m\right.\right. \\
&\left.\left. - \Phi_p\varrho_p\mathfrak{E}_p^m - \omega_p\mathfrak{E}_p^m\right]\right\} \\
&\mathcal{F}(\mathfrak{I}_p^n(t)) - \mathcal{F}(\mathfrak{I}_p^m(t)) \\
&= \mathfrak{I}_p^n(t) - \mathfrak{I}_p^m(t) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h[(1-\Phi_p)\eta_p\mathfrak{E}_p^n - (\tau_p+\omega_p)\mathfrak{I}_p^n]\right\} \\
&\quad - S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h[(1-\Phi_p)\eta_p\mathfrak{E}_p^m - (\tau_p+\omega_p)\mathfrak{I}_p^m]\right\} \\
&\mathcal{F}(\mathfrak{A}_p^n(t)) - \mathcal{F}(\mathfrak{A}_p^m(t)) \\
&= \mathfrak{A}_p^n(t) - \mathfrak{A}_p^m(t) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h[\Phi_p\varrho_P\mathfrak{E}_p^n - (\tau_{ap}+\omega_p)\mathfrak{A}_p^n]\right\} \\
&\quad - S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h[\Phi_p\varrho_P\mathfrak{E}_p^m - (\tau_{ap}+\omega_p)\mathfrak{A}_p^m]\right\}
\end{aligned}$$

$$\mathcal{F}(\mathfrak{R}_p^n(t)) - \mathcal{F}(\mathfrak{R}_p^m(t))$$
$$= \mathfrak{R}_p^n(t) - \mathfrak{R}_p^m(t) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)} S_h[\tau_p \mathfrak{I}_p^n + \tau_{ap}\mathfrak{A}_p^n - \omega_p \mathfrak{R}_p^n]\right\}$$
$$- S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)} S_h[\tau_p \mathfrak{I}_p^m + \tau_{ap}\mathfrak{A}_p^m - \omega_p \mathfrak{R}_p^m]\right\}$$
$$\mathcal{F}(\mathfrak{M}^n(t)) - \mathcal{F}(\mathfrak{M}^m(t))$$
$$= \mathfrak{M}_p^n(t) - \mathfrak{M}_p^m(t) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)} S_h[\phi_p \mathfrak{I}_p^n + \varpi_p\mathfrak{A}_p^n - \varphi \mathfrak{M}^n]\right\}$$
$$- S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)} S_h[\phi_p \mathfrak{I}_p^m + \varpi_p\mathfrak{A}_p^m - \varphi \mathfrak{M}^m]\right\}$$

Taking norm, we have

$$\begin{aligned}
& \|\mathcal{F}(\mathfrak{S}_p^n(t)) - \mathcal{F}(\mathfrak{S}_p^m(t))\| \\
\leq\ & \|\mathfrak{S}_p^n(t) - \mathfrak{S}_p^m(t)\| + \Bigg\| S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)} S_h\left[\coprod_p -\omega_p\mathfrak{S}_p^n - \zeta_p\mathfrak{S}_p^n(\mathfrak{I}_p^n + \Psi\mathfrak{A}_p^n) - \omega_w\mathfrak{S}_p^n\mathfrak{M}^n\right]\right\} \\
& - S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)} S_h\left[\coprod_p -\omega_p\mathfrak{S}_p^m - \zeta_p\mathfrak{S}_p^m(\mathfrak{I}_p^m + \Psi\mathfrak{A}_p^m) - \omega_w\mathfrak{S}_p^m\mathfrak{M}^m\right]\right\}\Bigg\| \\
\leq\ & \|\mathfrak{S}_p^n(t) - \mathfrak{S}_p^m(t)\| + S_h^{-1}\Bigg\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)} S_h\Bigg[- \|\omega_p(\mathfrak{S}_p^n - \mathfrak{S}_p^m)\| - \|\zeta_p\mathfrak{S}_p^n(\mathfrak{I}_p^n - \mathfrak{I}_p^m)\| \\
& - \|\zeta_p\mathfrak{I}_p^m(\mathfrak{S}_p^n - \mathfrak{S}_p^m)\| - \|\Psi\zeta_p\mathcal{S}_p^n(\mathfrak{A}_p^n - \mathfrak{A}_p^m)\| - \|\Psi\zeta_p\mathcal{A}_p^m(\mathfrak{S}_p^n - \mathfrak{S}_p^m)\| - \|\omega_w\mathfrak{S}_p^n(\mathfrak{M}^n - \mathfrak{M}^m\| \\
& - \|\omega_w\mathfrak{M}^m(\mathfrak{S}_p^n - \mathfrak{S}_p^m\|\Bigg]\Bigg\}.
\end{aligned} \tag{26}$$

Due to similar functioning of both solutions, we have

$$\begin{aligned}
\|\mathfrak{S}_p^n(t) - \mathfrak{S}_p^m(t)\| &\cong \|\mathfrak{E}_p^n(t) - \mathfrak{E}_p^m(t)\| \\
&\cong \|\mathfrak{I}_p^n(t) - \mathfrak{I}_p^m(t)\| \\
&\cong \|\mathfrak{A}_p^n(t) - \mathfrak{A}_p^m(t)\| \\
&\cong \|\mathfrak{R}_p^n(t) - \mathfrak{R}_p^m(t)\| \\
&\cong \|\mathfrak{M}^n(t) - \mathfrak{M}^m(t)\|.
\end{aligned} \tag{27}$$

Replacing (27) in (26), we get

$$\begin{aligned}
& \|\mathcal{F}(\mathfrak{S}_p^n(t)) - \mathcal{F}(\mathfrak{S}_p^m(t))\| \\
\leq\ & \|\mathfrak{S}_p^n(t) - \mathfrak{S}_p^m(t)\| + S_h^{-1}\Bigg\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)} S_h\Bigg[- \|\omega_p(\mathfrak{S}_p^n - \mathfrak{S}_p^m)\| - \|\zeta_p\mathfrak{S}_p^n(\mathfrak{S}_p^n - \mathfrak{S}_p^m)\| \\
& - \|\zeta_p\mathfrak{I}_p^m(\mathfrak{S}_p^n - \mathfrak{S}_p^m)\| - \|\Psi\zeta_p\mathfrak{S}_p^n(\mathfrak{S}_p^n - \mathfrak{S}_p^m)\| - \|\Psi\zeta_p\mathfrak{A}_p^m(\mathfrak{S}_p^n - \mathfrak{S}_p^m)\| - \|\omega_w\mathfrak{S}_p^n(\mathfrak{S}^n - \mathfrak{S}^m\| \\
& - \|\omega_w\mathfrak{M}^m(\mathfrak{S}_p^n - \mathfrak{S}_p^m\|\Bigg]\Bigg\}.
\end{aligned} \tag{28}$$

The sequences $\mathfrak{S}_p^n, \mathfrak{I}_p^m, \mathfrak{A}_p^m, \mathfrak{M}^m$ are bounded being convergent, so there exist L_1, L_3', L_4', L_6' for all t such that

$$\|\mathfrak{S}_p^n\| < L_1, \|\mathfrak{I}_p^m\| < L_3', \|\mathfrak{A}_p^m\| < L_4', \|\mathfrak{M}^m\| < L_6'.$$

Together with this, (28) become

$$\begin{aligned} & \|\mathcal{F}(\mathfrak{S}_p^n(t)) - \mathcal{F}(\mathfrak{S}_p^m(t))\| \\ \leq\ & \{1 - \omega_p f_1(\kappa) - \zeta_p(L_1 + L_3' + \Psi L_1 + \Psi L_4') f_2(\kappa) - \omega_w(L_1 + L_6')\} f_3(\kappa) \|\mathfrak{S}_p^n - \mathfrak{S}_p^m\|, \end{aligned} \tag{29}$$

where f_i are the functions obtained by $S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h[\cdot]\right\}$. In a similar fashion, we can have

$$\begin{aligned} \|\mathcal{F}(\mathfrak{E}_p^n(t)) - \mathcal{F}(\mathfrak{E}_p^m(t))\| \ \leq\ & \Big[1 + \zeta_p(L_1 + L_3' + \Psi L_1 + \Psi L_4') f_4(\kappa) + \omega_w(L_1 + L_6') f_5(\kappa) \\ & - \{(1 - \Phi_p)\eta_p + \Phi_p \varrho_p + \omega_p\} f_6(\kappa)\Big] \|\mathfrak{E}_p^n - \mathfrak{E}_p^m\| \end{aligned} \tag{30}$$

$$\|\mathcal{F}(\mathfrak{I}_p^n(t)) - \mathcal{F}(\mathfrak{I}_p^m(t))\| \ \leq\ \{1 + (1 - \Phi_p)\eta_p f_7(\kappa) - (\tau_p + \omega_p) f_8(\kappa)\} \|\mathfrak{I}_p^n - \mathfrak{I}_p^m\| \tag{31}$$

$$\|\mathcal{F}(\mathfrak{A}_p^n(t)) - \mathcal{F}(\mathfrak{A}_p^m(t))\| \ \leq\ \{1 + \Phi_p \varrho_p f_9(\kappa) - (\tau_{ap} + \omega_p) f_{10}(\kappa)\} \|\mathfrak{A}_p^n - \mathfrak{A}_p^m\| \tag{32}$$

$$\|\mathcal{F}(\mathfrak{R}_p^n(t)) - \mathcal{F}(\mathfrak{R}_p^m(t))\| \ \leq\ \{1 + \tau_p f_{11}(\kappa) + \tau_{ap} f_{12}(\kappa) - \omega_p f_{13}(\kappa)\} \|\mathfrak{R}_p^n - \mathfrak{R}_p^m\| \tag{33}$$

$$\|\mathcal{F}(\mathfrak{M}^n(t)) - \mathcal{F}(\mathfrak{M}^m(t))\| \ \leq\ \{1 + \phi_p f_{14}(\kappa) + \varpi_p f_{15}(\kappa) - \varphi_p(\kappa) f_{16}(\kappa)\} \|\mathfrak{M}^n - \mathfrak{M}^m\|, \tag{34}$$

where

$$\begin{cases} 1 - \omega_p f_1(\kappa) - \zeta_p(L_1 + L_3' + \Psi L_1 + \Psi L_4') f_2(\kappa) - \omega_w(L_1 + L_6')\} f_3(\kappa) < 1 \\ 1 + \zeta_p(L_1 + L_3' + \Psi L_1 + \Psi L_4') f_4(\kappa) + \omega_w(L_1 + L_6') f_5(\kappa) - \{(1 - \Phi_p)\eta_p + \Phi_p \varrho_p + \omega_p\} f_6(\kappa) < 1 \\ 1 + (1 - \Phi_p)\eta_p f_7(\kappa) - (\tau_p + \omega_p) f_8(\kappa) < 1 \\ 1 + \Phi_p \varrho_P f_9(\kappa) - (\tau_{ap} + \omega_p) f_{10}(\kappa) < 1 \\ 1 + \tau_p f_{11}(\kappa) + \tau_{ap} f_{12}(\kappa) - \omega_p f_{13}(\kappa) < 1 \\ 1 + \phi_p f_{14}(\kappa) + \varpi_p f_{15}(\kappa) - \varphi_p(\kappa) f_{16}(\kappa) < 1. \end{cases}$$

Hence, $\mathcal{F}$ possesses a fixed point. Thus to prove that the assumptions of Theorem 5.1 are satisfied by $\mathcal{F}$, we assume inequalities (29)-(34) holds, denote $r = (0, 0, 0, 0, 0, 0)$ and

$$R = \begin{cases} 1 - \omega_p f_1(\kappa) - \zeta_p(L_1 + L_3' + \Psi L_1 + \Psi L_4') f_2(\kappa) - \omega_w(L_1 + L_6')\} f_3(\kappa) < 1 \\ 1 + \zeta_p(L_1 + L_3' + \Psi L_1 + \Psi L_4') f_4(\kappa) + \omega_w(L_1 + L_6') f_5(\kappa) - \{(1 - \Phi_p)\eta_p + \Phi_p \varrho_p + \omega_p\} f_6(\kappa) < 1 \\ 1 + (1 - \Phi_p)\eta_p f_7(\kappa) - (\tau_p + \omega_p) f_8(\kappa) < 1 \\ 1 + \Phi_p \varrho_P f_9(\kappa) - (\tau_{ap} + \omega_p) f_{10}(\kappa) < 1 \\ 1 + \tau_p f_{11}(\kappa) + \tau_{ap} f_{12}(\kappa) - \omega_p f_{13}(\kappa) < 1 \\ 1 + \phi_p f_{14}(\kappa) + \varpi_p f_{15}(\kappa) - \varphi_p(\kappa) f_{16}(\kappa) < 1. \end{cases}$$

Hence all the conditions of Theorem 5.1 are satisfied, therefor $\mathcal{F}$ is Picard $\mathcal{F}$-stable. $\square$

6. Conclusion

In this paper, considering fractional order derivative due to Atangana and Baleanu we have studied mathematical model of novel coronavirus proposed by Chen *et al.* [23]. We presented the existence and uniqueness of the related fractional fractional differential equation of the model utilizing Schaefer's and Banach fixed point theorems respectively. Making use of Shehu transform and Picard iterative procedure, we presented iterative solutions and proved the stability of iterative method.

Competing interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Author's contributions

All authors jointly worked on the results and they read and approved the final manuscript.

Funding

There is no funding for this work.

Acknowledgements

The authors express their gratitude to the referees for their helpful suggestions.

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