Game theoretic computing of producer's and consumer's risks, α & β, for acceptance sampling using cost and utility

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KEYWORDS

non-linear optimization, quality control inspection, type-I error, type-II error

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ABSTRACT

When establishing a hypothesis testing procedure to ensure its credibility, the most significant step is unquestionably to select and/or compute the optimal type-I and type-II error probabilities, namely the producer's and consumer's risks, or α & β errors, respectively if the research hypothesis is set to be a good product vs bad. This article is fundamentally opposed to conventionally and judgmentally picking at best a subjective type-I error probability (α error) and it therefore outlines a game theoretic approach, i.e. that of von Neumann, to this historically century-old unresolved paradigm to justify optimal choices when relevant market-centric factors such as cost and utility are incorporated for input data. A game theory-based algorithmic methodology and several detailed numerical examples of practical nature with specific emphasis to company-specific acceptance sampling plans (including a simple hospital scenario) for quality control are studied. A side benefit of this method, in addition to improving the enterprise acceptance sampling plans, is to transform the traditional hypothesis testing procedure so as to make sound engineering decisions from a "subjective" to an "objective" stance, provided that the monetary cost and utility values as consequences to committing error and non-error combinations are available*.

KEYWORDS

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1. INTRODUCTION AND MOTIVATION

The objective of this article lies in optimizing a procedure to statistically test the credibility of a stated hypothesis in today's quality control-conscious and business-savvy world. Aside from the usual rule-of-thumb or best-guess or judgment-call-based choices of such as 1-out-of-20 or 1-out-of-50 etc., there have been alternative attempts to compute α (type-I error probability) by deriving the first and second derivatives of the standard normal distribution curve. This is performed by determining the second derivative to reach maximum at $z=\pm1.732$ which corresponds to a p-value of 0.083. An alternative approach has been to find a point where the concavity in the normal distribution curve is maximal to the first derivative. That is, the maximal curvature $k(z)$ occurs when $z = \pm1.749$ corresponding to a p-value of 0.08. The p-value is used to reject H_0 for a given α. The calculus-based algebraic approaches have been recently studied by Kelley[1] (2013) and Grant[2] (2014). Kelley[1] (2013) quoted, "No one therefore has come up with an objective statistically based reasoning behind choosing the now ubiquitous 5% level, although there are objective reasons for levels above and below it. And no one is forcing us to choose 5% either." The issue with these approaches is that they are detached from the market realities such as cost (loss) or utility (profit) associated with varying error values (α and β), or non-error values (1-α, and 1-β) and their cross products, namely [$\alpha * \beta$], [$\alpha * (1-\beta)$], [(1- α) $* \beta$] and [(1- α) $*$ (1- β)] that manifest themselves in the form of producer's or consumer's risks, or both or none. Not only are the usual judgment-call based selections subjective, those are also not attached to any joint treatment of producer's and consumer's risks that may occur. Simply because, thousands of products are subject to: i) Producer's risk, i.e. underappreciated or declared to be bad by the consumers while 'H_0: Good Product' is true, hence costing the producer a financial loss, or ii) Consumer's risk, i.e. over appreciated or declared to be good by the disadvantaged consumer while 'H_1: Bad Product' is true, hence costing the consumer a financial loss. None may have incurred with no financial loss for the producers and consumers with a complete market satisfaction due to the 'power: (1-β)' and 'confidence: (1-α)'.

Game theory is a branch of mathematical sciences devoted to the logic of decision-making in social or managerial interactions, and concerns the behaviour of decision-makers whose decisions influence each other (Sahinoglu et al.[3], 2012; Blackwell and Girschik[4], 1954). Each decision maker has only partial limited control. Game theory is a generalization of decision theory where two or more decision makers compete by selecting each of the several optimal strategies, whereas decision theory is essentially a one-person game theory. A common practice is to select type-I error probability (*alpha*) by an existing best-judgement call, and then, given an alternative H_1 values, to compute the related type-II error probabilities (*beta*). The plan is to build power curve ("1.0-beta" vs "population mean: mu") or operating characteristics curve ("beta" vs "population mean: mu") for the business-critical acceptance sampling procedure. This paper therefore outlines a game theoretic application to a century-old, unresolved statistical procedure toward the optimal estimation of *alpha* (type-I error probability) and *beta* (type-II error probability) while maximizing the overall sales net income. This proposed method becomes feasible to implement when market-based cost and utility input for composite errors (or lack-of) are deliverables. The optimal estimates, given the input cost (and utility) parameters, can be implemented to business enterprises. The objective of this research is to develop computationally intensive math-statistical algorithms such as Neumann's game theoretic approach to estimate *alpha* and *beta* error probabilities, i.e. producer's and consumer's risks, respectively, toward the statistical hypothesis testing of a *wellness* of a given product such as that of a piece of a cyberware (or chip) vs its *non-wellness.* The end goal is to objectively compute the operating characteristic (*OC*) or power curves toward the process of acceptance sampling. For sentencing incoming batches or lots of items without doing an 100% inspection, acceptance sampling is a well-known statistical sampling procedure to determine the quality level of an incoming shipment in order to judge whether quality level is within those limits predetermined to be desirable. However, the acceptance sampling gives one no idea about the quality process that is producing those items. Therefore, acceptance sampling is a form of sampling inspection applied to lots or batches of items before or after a process to judge conformance with predetermined standards. Sampling plans are those plans that specify a lot or batch size (large N), sample size (small n) and acceptance or rejection criteria. This article will deal with single-sampling plans where double- and multiple- or sequential-sampling plans are excluded because out of scope. Within the *single-sampling* plans of the *incoming (vs outgoing) reception* and *non-rectifying (vs rectifying)* classifications, this article will examine scenarios or examples to study sampling by *variables*, usually *gauge-measured for mean and standard deviation* (with continuous Normal pdf). However, batch sampling for *attributes counting the number of defectives* will also be adapted for large sample size $n \geq 100$ with Normal to approximate Binomial pdf. Acceptance sampling has two critical levels: 1) *Acceptance quality level or limit* (*AQL*) that denotes the probability or percentage of defects at which consumers are willing to accept lots or batches as "good". 2) *Rejectable quality level or limit* (*RQL*) that denotes the upper limit on the percentage or probability of defects that a consumer is willing to accept lots or batches as "good". Therefore, *RQL* or *LTPD* (lot tolerance percent defective) is the poorest quality level that the consumer is willing to accept in an individually inspected lot. This brings us to the two essential concepts: A) *Producer's risk*: The probability that a lot or batch containing defectives, either through gauge-measurement or head-counting of defectives, between the allowed limits of *AQL* and *RQL* will wrongly be rejected. B) *Consumer's risk*: The probability that a lot containing defectives, either through gauge-measurement or head-counting defectives outside-the-range of the limits of *AQL* and *RQL*, will be wrongly accepted. Briefly, type-I error of the producer's risk (5% is common) is the probability of rejecting a good lot (or batch), whereas type-II error of the consumer's risk (10% is typical) is the probability of not-rejecting a bad lot by Baghci[5] (2012). Lastly, the 5% (=α) or 10% (=β) cited are subjective takes but not scientific. MIL-STD-105E's (among others) standard assumptions on α and β are what this article challenges like those of Kelley[1] (2013) and Grant[2] (2015).

2. OBJECTIVES, AND METHODS: DECISION TABLES, RISKS, AND ERRORS

The concept of game theory has been brought to the attention of hypothesis testing in the past but at strictly theoretical albeit not at a pragmatic level by Schlag[6] (2008) involving the establishment of finite sample bounds on the general theme of statistical inference where he expressed," Let the data speak!" and compared tests based on sequential sampling. Schlag[6] (2008) employed Nash-induced game theory to establish the minimal *type-II* error (β: *beta)* whereby the associated randomized test was characterized as part of Nash[7] (1950) equilibrium, as stated in Osborne and Rubinstein[8] (1994). However, these attempts did not lead to an algorithmically and ubiquitously simple, and a practical formulation usable by the practicing statistician, routinely dealing with hypothesis testing at an elementary level. Sahinoglu et al.[9,10,11] (2015, 2016, 2017) instead followed up with a pragmatic approach, respectively, with a published ASA'15 proceedings paper and a Wiley Inc. textbook, and an *ISI*'17 proceedings paper. As pointed out by Savage[12] (1954), game theory can also be used to solve problems in statistics. The underlying idea is to solve worst-case problems by invoking the minimax theorem for zero-sum games developed by von Neumann[13] (1928) before the WWII, and further improved with the contributions of Neumann and Morgenstern[14] (1944) at Princeton. However, game theoretical methods have not yet been used in hypothesis testing curriculum in layman's terms to teach the fundamental concepts at an elementary statistics level. Mainly because the applications to everyday routine hypothesis tests with pertinent costs associated to *type-I* (=α) and *type-II* errors (=β) and their cross products, and additionally, utility or profit with respect to non-errors (confidence = 1-α, and power = 1-β) were not properly formulated. However, in this applied research article, the author deals with von Neumann's game theoretic equilibrium approach. In a hypothesis testing scenario, one associates a variety of costs (money lost due to the decision errors) or a utility (profit for the non-error) and observe what the optimal α and β will turn out to be by employing the principles of game theory. This is an alternative method to concurring with the usual rule of thumb such as $\alpha \approx 0.05$ or $\alpha \approx 0.08$ etc. by calculus algebra as pointed out and outlined by Grant[2] (2015). The proposed new approach vs the previous one based on a subjective rule-of-thumb with no econometric parameters and devoid of cost factors is empirical, data-consequential and market-friendlier. To determine whether to reject a null hypothesis based on a sample data, statistical hypothesis testing with various steps is outlined in the statistical literature (Ostle and Mensing[15], 1975; Hogg and Ledolter[16], 1992). The two types of errors can result from testing a statistical hypothesis H_0 as follow: *Type-I* error occurs when the analyst rejects a null hypothesis when it is true. The probability of committing a *type-I* error is called the significance level. This probability is conventionally denoted by α. This is also known in industrial quality control as the *producer's risk,* where H_0*: Good product* vs H_1*: Bad product.* The probability of not committing a *type-I* error is called the confidence of the test (1- α). Note, if "|" denotes "given that", then the *producer's risk* is given by,

$$\boldsymbol{\alpha = P\{\textit{Type-I}\text{ error}\} = P\{\text{reject } H_0 \mid H_0 \text{ is true}\}} \quad (1)$$

A *type-II* error occurs when the analyst fails to reject a null hypothesis that is false. This becomes a grave error in dealing with medical tests when a false H_{0P} : *Well-patient* vs true H_{1P} : *Sick patient* is erroneously not rejected, while P denotes patient. The probability of committing a *type-II* error is β. This is known in quality control as the *consumer's risk*:

$$\boldsymbol{\beta = P\{\textit{Type-II}\text{ error}\} = P\{\text{fail to reject } H_0 \mid H_0 \text{ is false}\}} \quad (2)$$

The probability of not committing a *type-II* error is called the power of the test (1- β):

$$\boldsymbol{(1-\beta) = P\{\text{reject } H_0 \mid H_0 \text{ is false}\}} \quad (3)$$

The power of hypothesis testing is represented as [1- $\beta(\Theta)$], where Θ denotes the true parameter value, i.e. population mean: μ. The $\beta(\Theta)$, the complement of power, is known as the operating characteristic (*OC*) function, popularly used in quality control. Observe Table 1 for the types of errors and their cross-products accompanied by associated costs.

TABLE 1 Costs (C_{11}, C_{12}, C_{21}) and Utility (C_{22}) for the cross-products of types of errors.

	β↓	(1-β)↓
α →	*C11*	*C12*
(1-α) →	*C21*	*C22*

Cost (opposite of utility) matrix is a function of α, β and C_{ij} related to the cross product of *type-I* and *type-II* errors. If cost bears a negative value, then cost denotes utility. Also:

$$\boldsymbol{\alpha * \beta + \alpha * (1-\beta) + (1-\alpha) * \beta + (1-\alpha) * (1-\beta) = 1.0;\ 0< \alpha, \beta <1} \quad (4)$$

$$\boldsymbol{\Pi(\alpha, \beta, C_{ij}) = \alpha*\beta*C_{11} + \alpha*(1-\beta)*C_{12} + (1-\alpha)*\beta*C_{21} + (1-\alpha)*(1-\beta)*C_{22};\ 0< \alpha, \beta <1} \quad (5)$$

where Π (α, β, C_{ij}) is the expected total cost. Let $P_{11} = \alpha*\beta$, $P_{12} = \alpha*(1-\beta)$, $P_{21} = (1-\alpha)*\beta$, $P_{22} = (1-\alpha)*(1-\beta)$ where C_{11}, C_{12}, and C_{21} are assigned individual costs respectively due to products of errors, or C_{22} due to non-errors in the case of in Table 1. Next in line follow:

$$\boldsymbol{\alpha = P_{11} + P_{12}} \tag{6}$$

$$\boldsymbol{\beta = P_{11} + P_{21}} \tag{7}$$

3. COMPOSITE-, PARTIAL- AND NON-RISKINESS, AND AN EXAMPLE

Example 1: Given the following input data e.g. about the diameter of a circular integrated circuit (IC) board of a computer chip critical to automobile manufacturing, let's test:

Null hypothesis: H_0: Cyberware is functional (good, operating),

i.e. H_0: μ_0: *Population mean* of an integrated circuit (IC) board's diameter = 5 cm.

One-sided Alternative hypothesis: H_1: Cyberware is dysfunctional (bad, not-operating), i.e. H_1: μ_1: *Population mean* of an integrated circuit (IC) board's diameter $\geq$ 5 cm.

Given the input sample costs**;** C_{11} = +\$800K (cost incurred), C_{12} = +\$70K (cost incurred), C_{21} = +\$200K (cost incurred), and C_{22} = -\$400K (utility profited) are the cost and utility coefficients, respectively, with the following expressions where K=1,000:

Composite riskiness (*CR*) = $P_{11} = \alpha * \beta$ (8.A)

Partial riskiness *(PR₁)* due to *type-I error probability* only = $P_{12} = \alpha * (1-\beta)$ (8.B)

Partial riskiness *(PR₂)* due to *type-II error probability only* = $P_{21} = (1-\alpha) * \beta$ (8.C)

Composite non-riskiness due to *power and confidence* = $P_{22} = (1-\alpha) * (1-\beta)$ (8.D)

It remains to cost-optimize type-I (α) and type-II (β) error probabilities, *producer's* and *consumer's risks* respectively with game theoretic mixed-strategy solution by Neumann et al.[13,14] (1928, 1944) and Sahinoglu et al.[9,10,11] (2015; 2016, pp. 6-13; 2017). Note:

CR (composite riskiness) + PR (partial riskinesses) + NR (non-riskiness)=1.0 (9)

It is timely to formulate Neumann's two-player optimal mixed strategy zero-sum game with the objective function: *Min LOSS* (which is the defensive gamer's objective function readily transformable to *Max GAIN* when eyed through the rivalling offensive gamer's perspective) by Sahinoglu et al.[3] (2012) subject to constraints from equations 10 to 23:

$$P_{11} * C_{11} - LOSS < 0 \quad (10)$$

$$P_{12} * C_{12} - LOSS < 0 \quad (11)$$

$$P_{21} * C_{21} - LOSS < 0 \quad (12)$$

$$P_{22} * C_{22} - LOSS < 0 \quad (13)$$

$$P_{22} \geq P_{11} \quad (14)$$

$$P_{22} \geq P_{12} \quad (15)$$

$$P_{22} \geq P_{21} \quad (16)$$

$$P_{11} < 1 \quad (17)$$

$$P_{12} < 1 \quad (18)$$

$$P_{21} < 1 \quad (19)$$

$$P_{22} < 1 \quad (20)$$

$$LOSS > LOSS_{\mathrm{min}} \quad (21)$$

$$P_{11} + P_{12} + P_{21} + P_{22} = 1 \quad (22)$$

$$\Pi\ (\alpha, \beta, C_{ij}) = P_{11} * C_{11} + P_{21} * C_{21} + P_{12} * C_{12} + P_{22} * C_{22} < 0 \quad (23)$$

Observe the general representation theorem (*GRT*) behind the linear programming for an outline of the forward and backwards proofs given by Lewis[17] (2008, pp. 17-22). The following spreadsheets show the data entry and outputs with *NLP*: Non-Linear programming algorithm, whereas equation (23) denoting total cost ($) units accrued, for instance, will show an overall positive utility or profit. If the minimum or at-least utility assumed is $-LOSS \leq -\$5$ or $LOSS \geq \$5$ (Equations 10 to 13 and 21) as follows per cells in Table 2, one sets up the *NLP* (non-linear programming) problem given the game-theoretic constraints of equations 10 to 23 for the objective function of *Min LOSS*. The following spreadsheets show the data entry and outputs with the game theoretic *NLP* for Example 1. Nonlinear means not necessarily linear by Rapcsak[18] (1997).

TABLE 2 Game theoretic input spreadsheet for Example 1.

Enter/Edit data: Objective function coefficients. For each constraint, enter constraint coefficients, constraint relationship (<, =, >), and constraint right-hand-side value. Do not enter nonnegativity constraints.

Optimization Type: Minimize

Variable Names: (Change if Desired)	P11	P21	P12	P22	LOSS
Objective Function Coefficients:					1

	Coefficients				
Subject To:	P11	P21	P12	P22	LOSS
Constraint 1	1	0	0	0	0
Constraint 2	0	1	0	0	0
Constraint 3	0	0	1	0	0
Constraint 4	0	0	0	1	0
Constraint 5	1	1	1	1	0
Constraint 6	800	0	0	0	-1
Constraint 7	0	70	0	0	-1
Constraint 8	0	0	200	0	-1
Constraint 9	0	0	0	-400	-1
Constraint 10	0	0	0	0	1
Constraint 11	800	70	200	-400	0
Constraint 12	-1			1	0
Constraint 13		-1		1	0
Constraint 14			-1	1	0

TABLE 3 Feasible Vector Solution for Example 1.

Optimal Solution

Objective Function Value = 5.000

Variable	Value
P11	0.006
P21	0.071
P12	0.025
P22	0.897
LOSS	5.000

Note in the *EXCEL* Solver, the *GRG* nonlinear engine is selected for constrained minimization problems with *differentiable* (where partial derivatives of order k are continuous), *nonlinear*, *convex* functions that are *smooth of order k* by Rapcsak[18] (1997). This includes the case where all functions are linear, i.e. the linear programming problem.

TABLE 4 Input $\{C_{ij}, i,j=1,2\}$and Output *EXCEL* Solution of $\{P_{ij}, i,j=1,2\}$ for Example 1.

C11	800
C21	70
C12	200
C22	-400

MIN LOSS				
P11	P21	P12	P22	LOSS
0.00625	0.071428571	0.025	0.897322	5
P11	0.0062500000		<	1
P21	0.071428571		<	1
P12	0.025		<	1
P22	0.897322429		<	1
Constraint 1	-343.9289714		<	0
Constraint 2	1.00000		equal	1
Constraint 3	2.04281E-14		<	0
Constraint 4	0		<	0
Constraint 5	7.30971E-13		<	0
Constraint 6	-363.9289714		<	0
Constraint 7	5		>	5

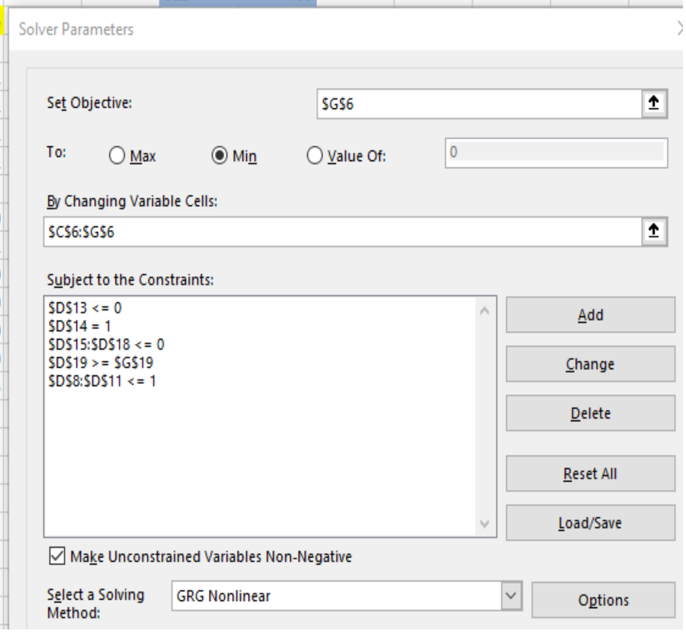
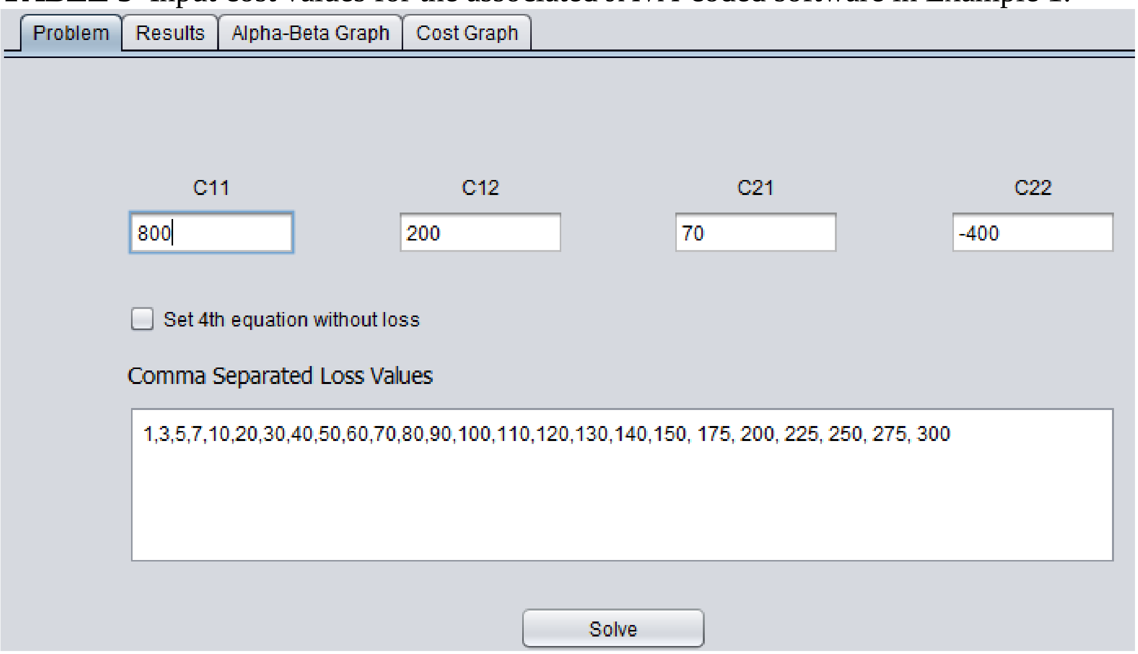


TABLE 5 Input cost values for the associated *JAVA*-coded software in Example 1.



The results corresponding to the horizontal axis epoch of *LOSS*: \$5:

P_{11} = 0.006249, P_{12} = 0.024999, P_{21} = 0.071428, P_{22} = 0.897321

Alpha ≈ 0.0313, *Beta* ≈ 0.0777 and Expected Total Cost ≈ -\$343.92

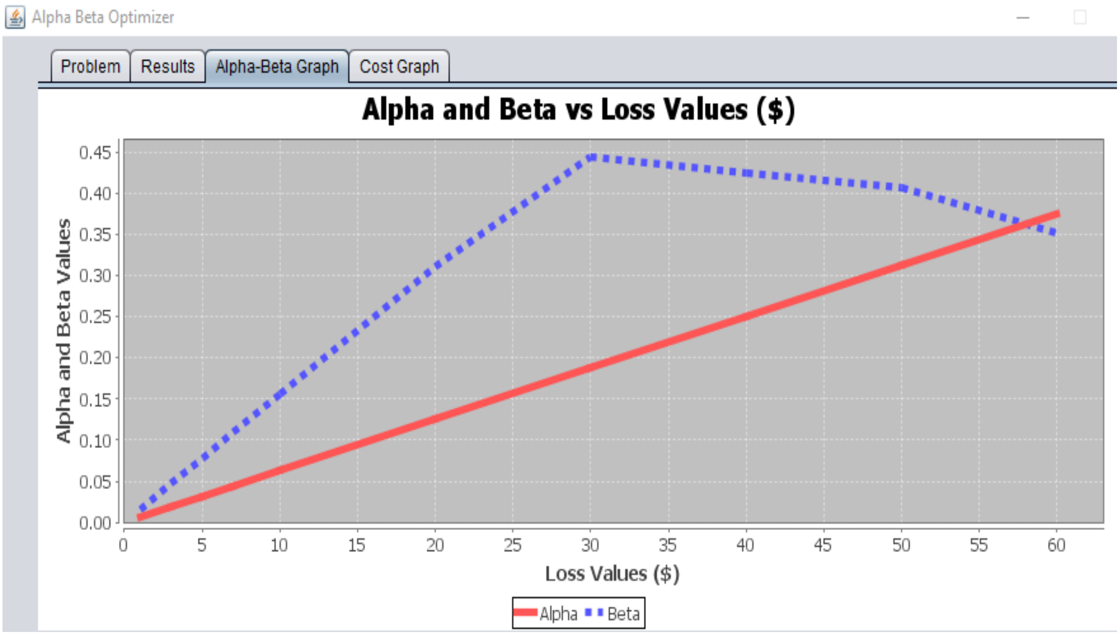


FIGURE 1 Game theoretic *Alpha (≈.0313), Beta* (≈.0777) vs *LOSS*=\$5 in Example 1.

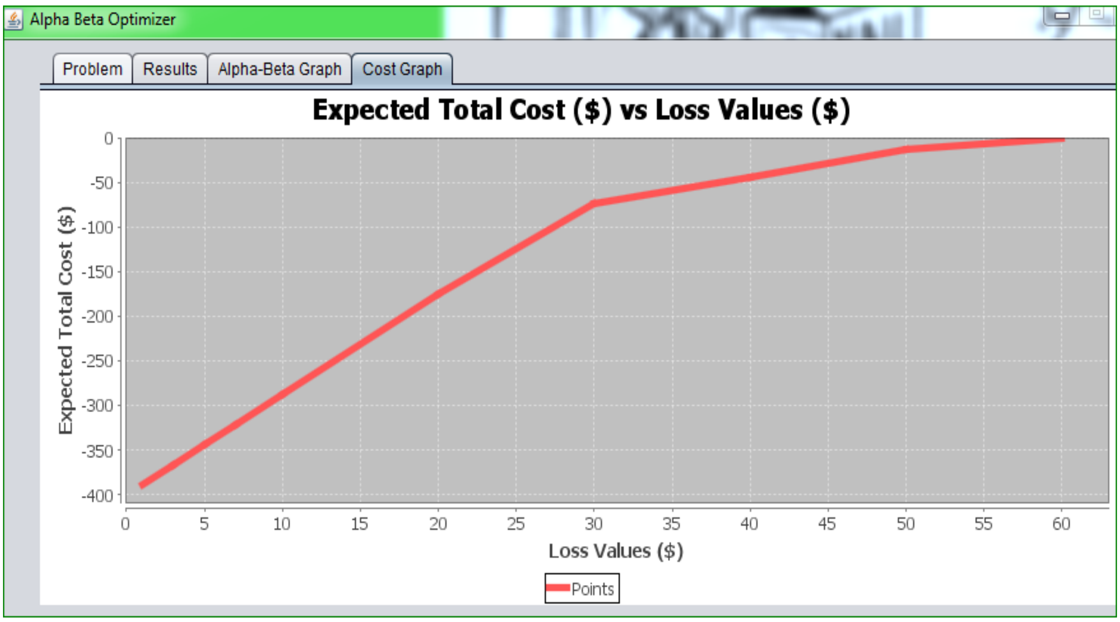


FIGURE 2 Game theoretic Expected Total Cost ≈ -\$344 vs *LOSS*=\$5 in Example 1.

Optimal Cost-Optimized Results: Utilizing Equations 6 to 9, and 10 to 23, software solutions to the unknown vector, [P_{ij}] = [P_{11}=*.0062499978*, P_{21}=*.07142863*,

P_{12}=.02499999, P_{22}=.89732146]; one derives, $\alpha = P_{11} + P_{12} = .0062499978 + .02499999 = 0.03124499$, and $\beta = P_{11} + P_{21} = .0062499978+.07142863 = 0.07767862$. For a given small n (sample size from batches) =100, σ (standard deviation) = 9; the optimized α = 0.03124999 (≈3.13%) and β = 0.07767862 (≈7.77%) as computed by the input data in Table 5, demonstrates the following plan in a software-enabled Figure 3 below after Figures 1 and 2. Then, $0.03124999 = \alpha = P(Z \geq Z_C \mid H_{0:}\ \mu_0 = 5)$ and $0.07767862 = \beta = P(Z \leq Z_C \mid H_{1:}\ \mu = \mu_1)$ yield critical Z values: $(Z_C \mid H_0 : \mu_0 = 5) = 1.86$ and $(Z_C \mid H_1 : \mu_1 \geq 5) = -1.42$.

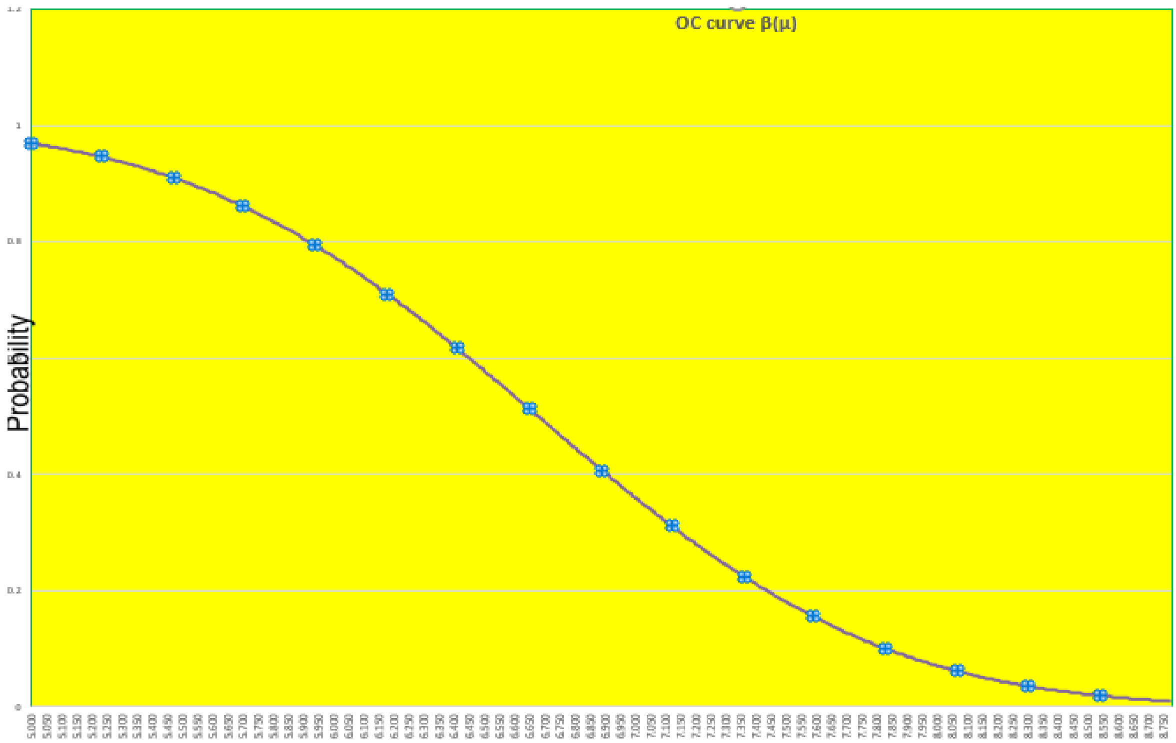
The preceding calculations will further result, assuming the standard deviation to be σ=9, in the critical value of C. That is, $C = 5+1.86*9/\sqrt{100}=6.674$ under H_0, and hence leading to $\mu_1 = 6.674+1.42*\ 9/\sqrt{100}=7.95$ under H_1. Note, $H_{1:}\ \mu_1 = 7.95$ is formulated by $C-Z(\beta)*\sigma/\sqrt{n} = 6.674 - (-1.42) *(9/10) =7.95$. Hence, one is testing $H_{0:}\ \mu_0 = 5$ cm. vs $H_{1:}\ \mu_1 = 7.95$ cm. for the IC chip's mean diameter length. Therefore, the decision plan becomes as in Figure 3 followed by its *OC* Curve in Figures 4 A. and B. Therefore, reject H_o if $\bar{x}$ (sample mean) > $C \approx 6.67$ when $H_{0:}\ \mu = 5$ to commit type-I error, and fail to reject H_o when $\bar{x} < C \approx 6.67$. This implies under $H_{1:}\ \mu = 7.95$ to commit type-II error to attain optimal outcomes as dictated by the acceptance sampling plan's *OC* curve designed in Figures 3, and 4 A and B. Therefore, executing input Table 5 subject to C_{11} = \$800K (unit cost incurred), C_{12} = \$70K (unit cost incurred), C_{21} = \$200K (unit cost incurred) and C_{22} = -\$400K (unit utility credited) under the *LOSS* constraint of Equation (21), the overall process cost without the K (=1,000) multiplier in Figure 2 follows in Equation (24):

$$\mathbf{\Sigma\{P_{ij}C_{ij}\} = .006245*800 + .07143*70 + .0245*200 + .897321*(-400) = -\$343.93} \quad (24)$$

Thus, the total negative cost (utility) that the planner is expected to profit is ≈ -\$344 given *LOSS* (max) limited to \$5. *LOSS* varies for sensitivity from \$1 to \$60 in Figures 1 and 2.

Significance Level	α =	0.0313			
	Z(α) =	1.86			
Defective Proportion p =	0.05	Std Dev(#Def) σ = SQRT[np(1-p)]=	2.179449472		
Sample Size (n) =	100				
Expected Value of Variable or # Defectives =	5		QUALITY ATTRIBUTES (#Defectives)↓		
Standard Deviation of Variable =	9			H1 : μ = μ1 > μ0	9.76881825
			MEASURABLE VARIABLES (Mean) ↓		
Ho : μ =	μ0 =	5		H1 : μ = μ1 > μ0	7.952
Critical Value C	=	6.674			
Z(β) =	-1.42				
β =	0.0777				

FIGURE 3 Optimal plan with α, β, from Tables 2-5 of Example 1 for n=100, σ=9, C=6.67 for both i) variables and ii) attributes where np≥5 with Binomial to Normal approximation.



μ1		OC(μ1)
AQL=5.00	Z(α) = 1.86	0.9687(=1-α=1-0.0313)
LQL =7.95	Z(β) = -1.42	0.0777(=β)

FIGURES 4 A and B Example 1's OC curve by Tables 1-5 and Figures 1-3 show results for n = 100 and σ = 9 give *RQL:* Rejectable quality and *AQL*: Acceptable quality levels.

Conclusive Outcomes: In Tables 1-5, Figures 1, 2, 3 and 4 A and B, observe H_0: $\mu = 5$ (≈*AQL*) vs H_1: μ_0= 7.95 (≈*RQL*). Therefore, the *OC curve* plots H_1: μ_1 =7.95 on the x-axis with its y-axis ≈ 0.078 vs H_0: μ_1=5.0 on the x-axis with its y-axis ≈ 1 - 0.0313 = 0.9687. Note, *RQL:* Rejectable quality level, whereas *AQL*: Acceptable quality level. Unfortunately, the ideal *OC* Curve can almost never be obtained in practice; whereas in theory, it could be realized by 100% inspection if the inspection were error free. This implies that the ideal *OC* curve can be approached by increasing the small sample size = n → N = batch or lot population size (Montgomery[20], 2009; pp. 631-642, pp. 670-676). For attributes, *AQL*= 5 defectives vs *RQL*= 9.77≈10 defectives from Figure 3 for large n.

4. STEPS FOR INPUT DATA COLLECTION ABOUT COST PARAMETERS

How to obtain the input data: C_{11}, C_{12}, C_{21}, C_{22}, and *LOSS* from the corporate world's actual sales data recording with hypothetical examples keeping the input Tables 2, 4, 5:

C_{11}: Incurred losses due to penalty arising from both the consumer's and producer's risks intersected on the product (PC or automobile or washing machine, etc.) while the product sold was recalled or returned either due to a faulty production or a misinterpreting a good product as bad. Say, $800K as a huge cost of penalty due to this confusion was recorded.
C_{12}: Incurred losses experienced by the producer due to producer's risk while the product was returned by the consumer due to misconception of a hypothetical fault, but it was not a faulty product. Say, $200K cost due to incorrect or unjustifiable returns was recorded.
C_{21}: Incurred losses by the consumer due to consumer's risk while the product sold was recalled or returned due to a sheer defective production. Say, $70K cost of penalty due to recall action or returns officially recorded.

C_{22}: Credited net income (profit) due to entirety of sold products uncontested, i.e. not returned. Say, \$400K as utility, not cost-incurred or penalized, but a profit (net income) was recorded. The extreme cost data show that it is not a sound company with cash issues.

LOSS: If the minimum *LOSS* assumed is $-LOSS \leq -\$5K$ or $LOSS \geq \$5K$; then this is the tolerable, or company-paid indemnity to circumvent or intercept the damage incurred after the deductibles due to each of the three risk related constraints. They were entered for each of the three cubicles in Tables 2, 4 and 5 to be \$5K. Each term in the constraints of Equations 10 to 13, $P_{ij}*C_{i\,j}<LOSS$ for $i,j=1,2$ where these four constraints are bound not to exceed $\$LOSS=\$5K$, including the one where the negative cost (i.e. profit) value obeys the constraints. The *\$LOSS*, is a company-paid compensation after the deductibles, a factor which is being targeted to minimize with the objective function of *Min LOSS*.

In every statistical experiment, the statement of the problem goal is the first step. The goal for input data is to estimate the C_{ij}, $i,j=1,2$ from the company's historical accounting data. *\$LOSS* parameter to be estimated (after the deductions) is a company policy, and therefore, it is a constant to be dictated by the associated company. To determine how large a sample is needed, there are three questions to answer by Ostle and Mensing[15] (1975): i) How large a shift (i.e. range) in the parameter do you expect to detect? ii) Based on experience, how much variability do you wish to detect? iii) What sizes of risk (i.e. *alpha and beta*) you are willing to take. Note that these risks are what one is going to optimize, before one can determine sample size. A composite metric that considers these queries can be found in the *CV*: Coefficient of Variation= (Sample Standard Deviation / Sample Mean). The *CV* is an ideal device for comparing the variation in the two *Cause* and *Effect* series of data that are measured in two different units, e.g. a comparison of variation in height with a variation in weight (Hicks[19], 1973). Hence for sample size (n), we can a tabulate a list of *CV*100%* vs *n* to justify where to stop. The lesser the *CV*, the better the outcomes stand to halt sampling.

5. COSTS AND UTILITY: C_{11}, C_{12}, C_{21} AND C_{22} IN THE BUSINESS WORLD

Many of the larger merchandisers will break the returns down into four distinct groups: http://businessecon.org/2014/12/returns-allowances-and-discounts-in-accounting/ [12/24/2014]:

A) The first group reflects customer-based mistakes. It is solely a customer's mistake. This is therefore attributed to the producer's risk indicating the cost associated with C_{12}. As a merchandiser, you would only need to monitor the growth rate for this group. If this ratio begins to increase, it might be a sign that the sales staff is unethically forcing the wrong product onto the market and hence, the customer.

B) The other form of a return is a type of merchandise that is broken or has a warranty issue. It is generally important to track this information since this form of return can be a clear sign of a quality issue with a particular brand or product line. That is, it's not the consumer's mistake but that of the producer. This is therefore attributed to the consumer's risk indicating the cost associated with C_{21}. If the issue is brand-related, the merchant may want to consider discontinuing the brand or substituting the brand with a higher quality product. Most popular examples are large repair-recalls from the automotive industry.

C) The two adjustments above in the business world are followed by another described as allowances, or incentives or write-offs. These are adjustments to normal sales reflecting defective items or courtesy calls for failure in delivering the product or service in a timely fashion (merchandise fault). Then, re-education of the sales representatives is required if customers' erroneous returns increase since this relates to the wrong kind of purchase, or consumers are not educated for what they bought assuming that the product is defective while it truly is not; thus, leading to a customer fault but producer's risk. The combination is attributed to the intersection of the producer's and consumer's risk to define the C_{11}.

D) *C22* is the uncontested profit (utility) not returned with 100% customer satisfaction.

In Figure 5, Venn Diagram constituting all four sample elements (sets) of *V* are indicated. Note that [(1-α) *(1-β)], $\alpha*\beta + \alpha*(1-\beta) + (1-\alpha)*\beta + (1-\alpha)*(1-\beta)] = 1.0$ per constraint (22).

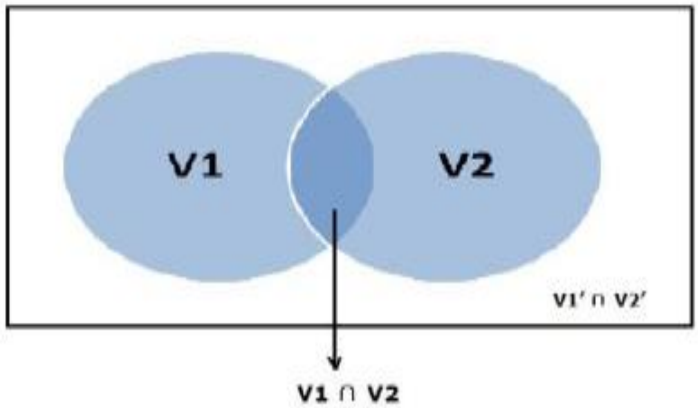


FIGURE 5 Take V_1 = Producer's Risk, V_2 = Consumer's Risk, $V_1{}' = V_1$ complement and $V_2{}' = V_2$ complement, where P ($V_1 \cap V_2{}'$) = Producer's Risk only by $\alpha*(1-\beta)$; P ($V_2 \cap V_1{}'$) = Consumer's Risk only by $(1-\alpha)*\beta$; P ($V_1 \cap V_2$) = Intersection of both risks by α*β, and lastly P ($V_1{}' \cap V_2{}'$) = No producer's and/or consumer's risks concurrently.

Example 2: How to collect the C_{ij} costs and utility, and *LOSS* limitation for Acceptance Sampling by a company for the hypothesis test H_0: *Good product vs* H_1: *Bad product*, or H_{0P}: *Well-patient* vs H_{1P}: *Sick patient*, *P* for patient after Equation (1) in Section 3.

A hypothetical *Large Automobile Production (LAP)* plant (or *ABC* Hospital) statement for the recent year ending with company (hospital) discounts excluded after 15 year-data:

Total Auto Revenue: $1,000,000 (for *ABC,* total revenue received from patient therapy)

No Adjustments-Sale: $800,000 (for *ABC,* net income or profit from patient therapy)

Adjustments:

Returns: \$150,000 (for *ABC*: amount of revenue lost per erroneous actions arising from patient non-compliance or hospital's wrongdoings exasperating the patient's illness)

Customer Based: \$110,000 (for *ABC*: patient non-compliance causing court litigation)

Plant-Merchandise Based: \$40,000 (for *ABC*: hospital malpractice litigation penalty)

Allowances: \$50,000 (for *ABC*: therapy non-adherence or malpractice expenses in both)

Based on this tabulation, one seeks what kind of an *OC* curve to facilitate an acceptance sampling plan that the *LAP* plant is at best to undertake. The data as follow are the without *K*(=1000) values for the following designed example of input Table 6, Table 7 and Table 8, Figures 6 and 7. The analyst may plan to explore e.g. 15 such large companies with their accounting history back to year 2000. Note these figures are updated each year. C_{ij} are: C_{22} = -\$800; C_{12}= \$110; C_{21}= \$40 and C_{11}= \$50 recorded, as company-specific input in Table 6. The *LAP* plant has selected *LOSS* (after deductibles) =\$5 as a company policy.

TABLE 6 Input cost values for the associated *JAVA*-coded software in Example 2.

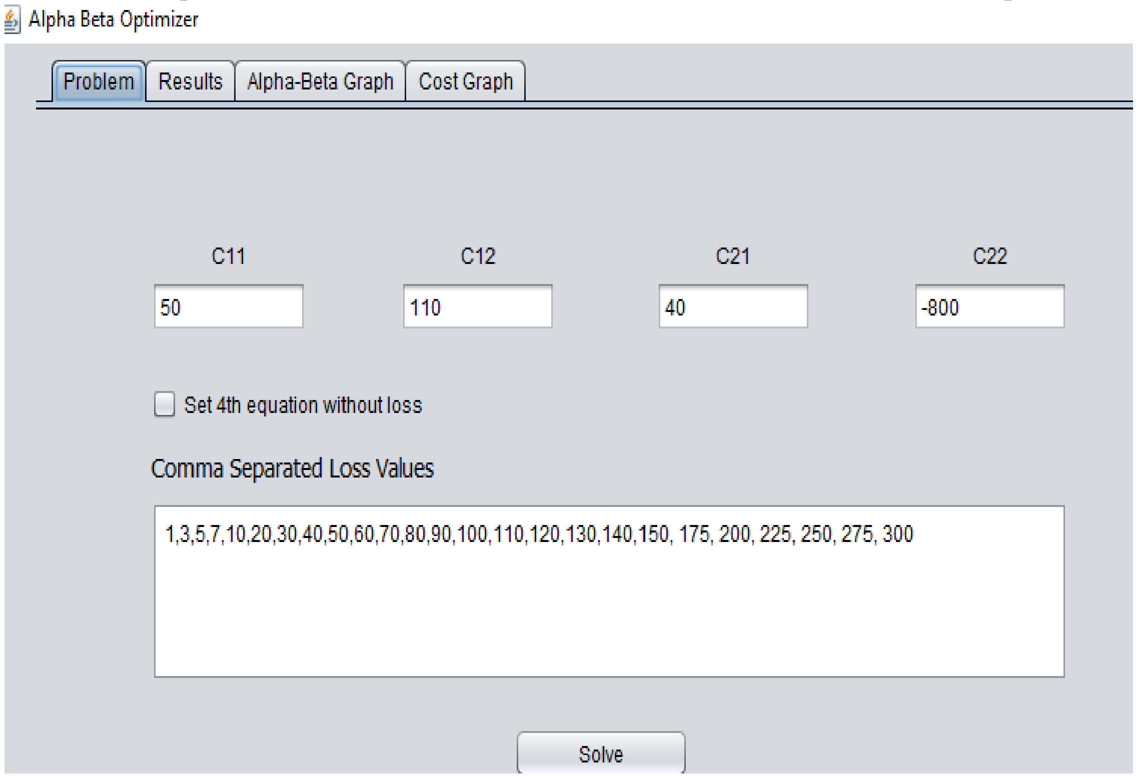


TABLE 7 Game theoretic input spreadsheet for Example 2.

Enter/Edit data: Objective function coefficients. For each constraint, enter constraint coefficients, constraint relationship (<, =, >), and constraint right-hand-side value. Do not enter nonnegativity constraints.

Optimization Type: Minimize

Variable Names: (Change if Desired)	P11	P21	P12	P22	LOSS
Objective Function Coefficients:					1

	Coefficients				
Subject To:	P11	P21	P12	P22	LOSS
Constraint 1	1				
Constraint 2		1			
Constraint 3			1		
Constraint 4				1	
Constraint 5	1	1	1	1	
Constraint 6	50				-1
Constraint 7		40			-1
Constraint 8			110		-1
Constraint 9				-800	-1
Constraint 10					1
Constraint 11	50	40	110	-800	
Constraint 12	-1			1	
Constraint 13		-1		1	
Constraint 14			-1	1	

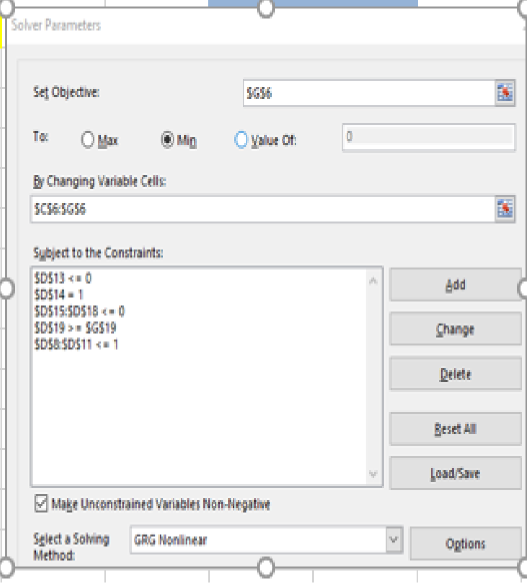
TABLE 8 Feasible vector solution for Example 2.

```
Optimal Solution
Objective Function Value =                5.000

        Variable                  Value
     ---------------       ---------------
           P11                    0.100
           P21                    0.125
           P12                    0.045
           P22                    0.730
           LOSS                   5.000
```

TABLE 9 Input $\{C_{ij}, i,j=1,2\}$ and output *EXCEL* solution of $\{P_{ij}, i,j=1,2\}$ for Example 2.

					C11	50
MIN LOSS					C21	40
					C12	110
P11	P21	P12	P22	LOSS	C22	-800
0.1	0.125	0.045455	0.729546	5		
P11	0.1000000000		<	1		
P21	0.125		<	1		
P12	0.045454545		<	1		
P22	0.729546455		<	1		
Constraint 1	-568.6371636		<	0		
Constraint 2	1.00000		equal	1		
Constraint 3	0		<	0		
Constraint 4	0		<	0		
Constraint 5	0		<	0		
Constraint 6	-588.6371636		<	0		
Constraint 7	5		>	5		



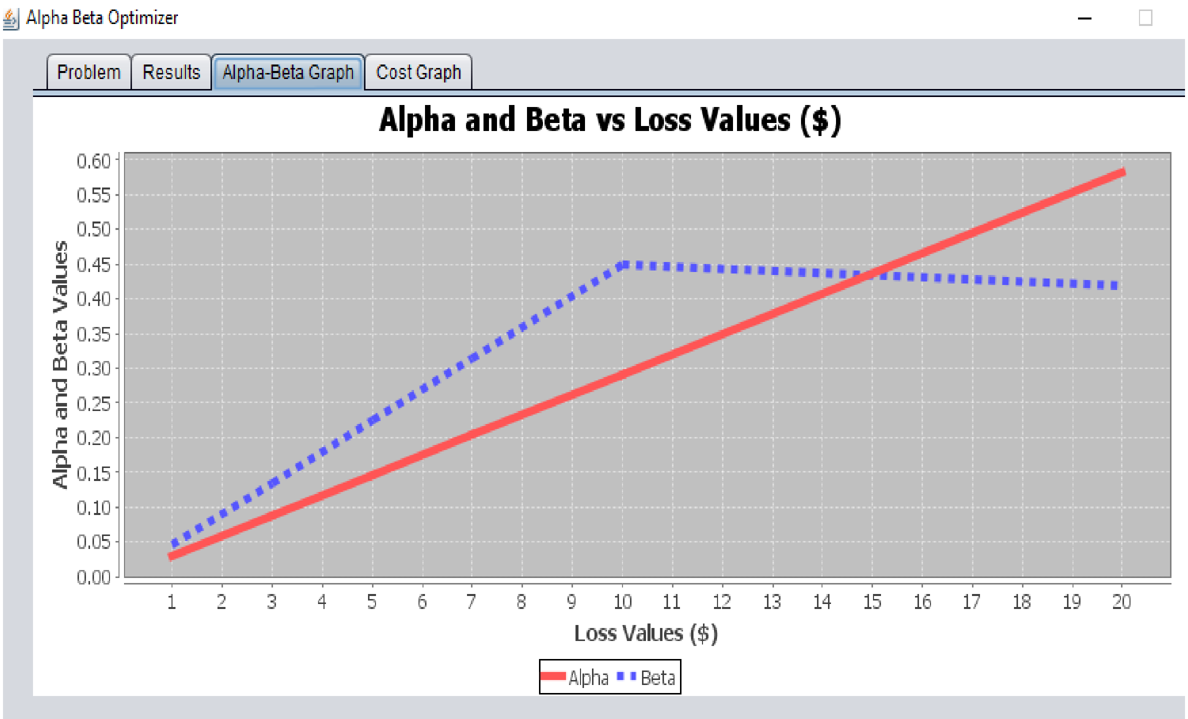


FIGURE 6 *LAP* Company: *Alpha* ≈ 0.145 (14.5%) and *Beta* ≈ 0.225 (22.5%) for *LOSS*=$5.

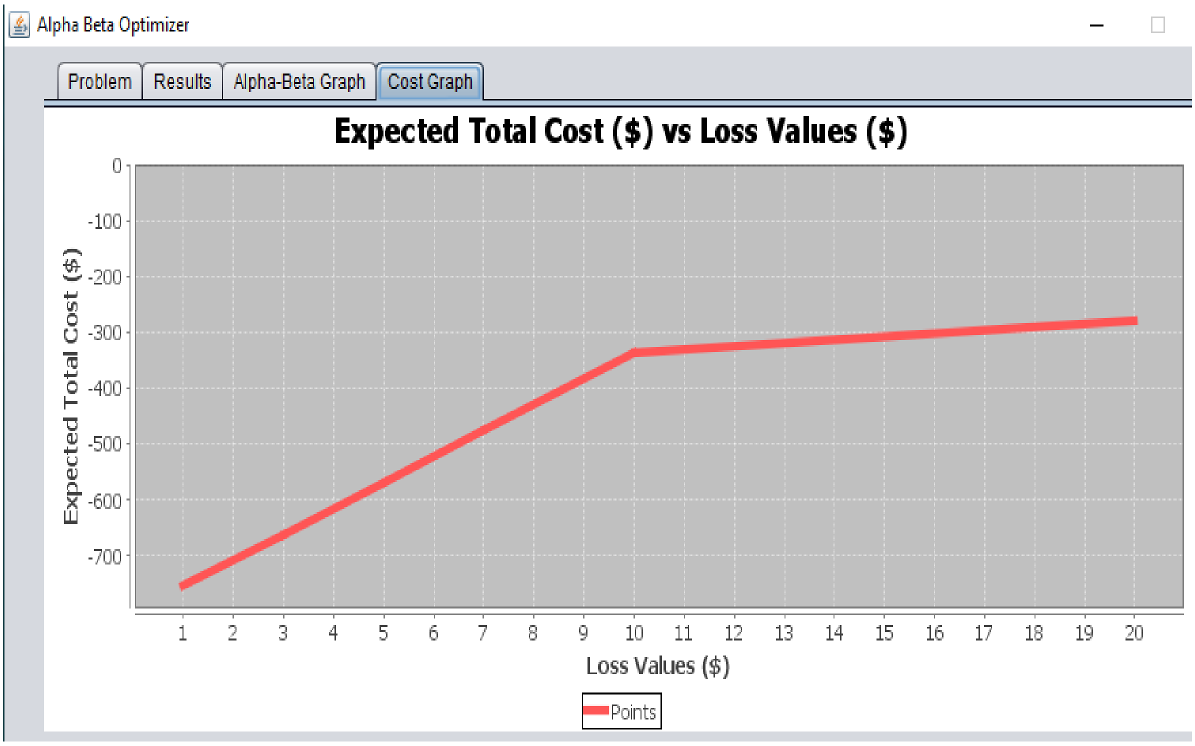


FIGURE 7 *LAP* Company: Expected Total Cost ≈ -$569 for *LOSS*=$5.

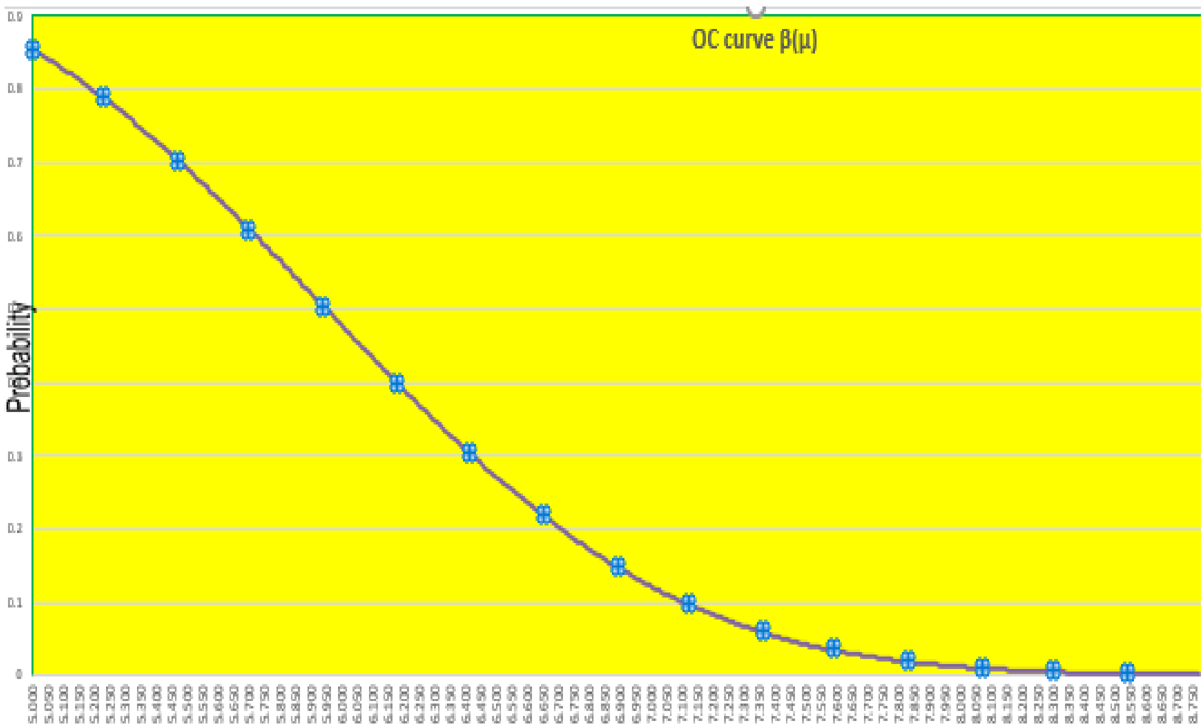
The results for the horizontal axis of *LOSS*: $5 based on Tables 6-9, and Figures 6-7:

P_{11} = 0.1, P_{12} = 0.045454, P_{21} = 0.125, P_{22} = 0.729545 are

Alpha ≈ 0.145, *Beta* ≈ 0.225 and Expected Total Cost ≈ -$568.63

Significance Level	α =	0.145		
	Z(α) =	1.05		
Defective Proportion p =	0.05	Std Dev(#Def)	σ = SQRT[np(1-p)]=	2.179449472
Sample Size (n) =	100			
Expected Value of Variable or # Defectives =	5	QUALITY ATTRIBUTES (#Defectives)↓		
Standard Deviation of Variable =	9		H1 : μ = μ1 > μ0	7.579587104
		MEASURABLE VARIABLES (Mean) ↓		
Ho : μ =	μ0 =	5	H1 : μ = μ1 > μ0	6.62
Critical Value C =		5.945		
Z(β) =	-0.75			
β =	0.225			

FIGURE 8 Optimal plan with α, β from Tables 6-9 in Example 2 for n=100, σ=9, C=5.95 for both i) variables and ii) attributes where np≥5 for Binomial to Normal approximation.



μ1		OC(μ1)
AQL= 5.00	Z(α) = 1.05	0.855(=1-α=1-0.145)
RQL = 6.62	Z(β) = -0.75	0.225(=β)

FIGURES 9 A and B Example 2's OC Curve with *AQL*, *RQL* by Tables 6-9 and Figures 6-8 for n = 100, σ = 9 give *RQL:* Rejectable quality, and *AQL*: Acceptable quality levels.

Conclusive Outcomes: In Tables 1 and 5-7, Figures 6-8 and 9 A and B; Observe H_0: $\mu_0 = 5$ ($\approx AQL$) vs H_1: $\mu_1 = 6.62$ ($\approx RQL$). The *OC* curve plots H_1: $\mu_1 = 6.62$ on the x-axis with its y-axis = 0.225 and H_0: $\mu_0 = 5.0$ on x-axis with its y-axis=1-0.145 = 0.855. For attributes, *AQL* =5 vs *RQL* =7.58 ≈8 defectives from Figure 8 for n=100.

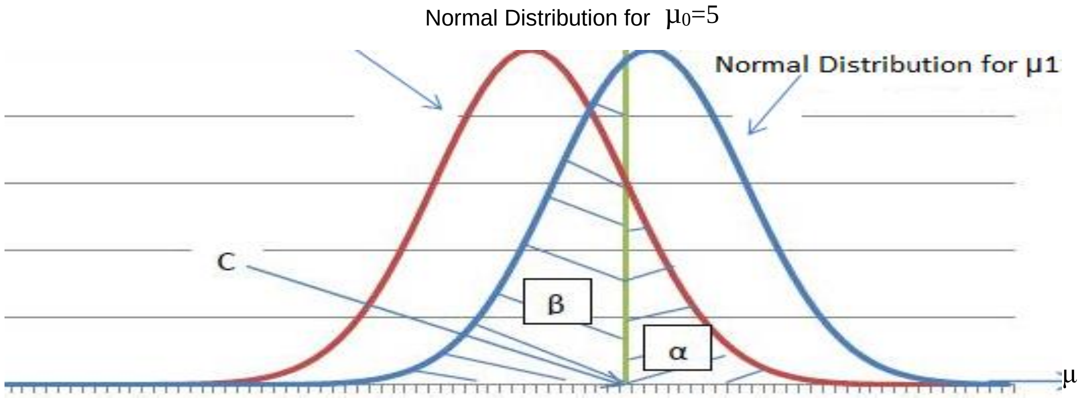


FIGURE 10 Generalized graph of the acceptance and rejection regions, and type-I and type-II error probabilities where α (hatched ascending) and β (hatched descending) for the one-sided hypothesis tests of *IC* Example 1 for H_0: μ_0 or np_0=5 vs H_1: μ_1=7.95 or $np_1 \approx 9.77 = 10$ with α = .0313, β = .077, C = 6.67 and of *LAP* Example 2 for H_0: μ_0 or np_0 =5 vs H_1: $\mu_1 = 6.62$ or $np_1 \approx 8$ (*RQL*) with α = .145, β = .225, C =5.95. For *ABC*: *RQL*≈8 is the maximum allowed *defective* patients tolerated for an acceptable lot of sample, n=100.

Conclusive outcomes on Examples 1 and 2 can be illustrated respectively in Figure 10. The *LAP* in Example 2, if not satisfied with this plan, will for the next year try to reduce the customer-based C_{12}= \$110*K* by re-educating the customer base and increasing the smooth-sale margin to experience less adjustments. Similar remedies for C_{21}= \$40*K* and C_{11}= \$50*K* can be undertaken to improve the cost-free profit, C_{22}= -\$800*K*. A data management program may have to define two sets of large company-based data contributors. The first group are the ones that have just begun to provide data (yet to meet the 15-year threshold) and the second group are those (having met the threshold) which are using the algorithms on the historical data, and they could agree to report the results on their current operations by updating data. Without the incentives to participate, what would be the motivating factors for companies to join in the effort and publish information which eventually their competitors will see? Those companies will act within their company-specific and unique only to them, by identifying batch or lot acceptance sampling plans rather than an unjustified and subjective assumption of the popular *type-I* and *type-II* error probabilities. This article in brief suggests a game theoretic predictive solution provided the cost parameters, rather than endorsing a set of given producer's and consumer's risks. It is critical that each enterprise will monitor their acceptance sampling plans to improve quality control inspections for overall productivity.

6. CONCLUSIONS

An innovative, game theoretic, business-savvy and market-centric method with optimal algorithmic solution for *type-I* and *type-II* error probabilities (also known as *producer's* and *consumer's risks, or alpha* and *beta* errors respectively with an aside implementation to hospital patient therapy) is proposed in contrast to merely selecting these parameters using subjective judgment calls conventionally practiced as Kelley[1] (2013) so remarked. One may label the past practices as habitual guess-work procedures devoid of cost or utility constraints in a business-plan state-of-mind. It falls upon the author to further state that the most challenging task in this game theoretic proposition is to generate the most-fitting rightful and authentic market-centric input data, i.e. C_{ij} for the firmware, cyber-ware or any other commodity-based market about which the tests of hypotheses are being conducted. This will lead to a necessary series of econometric data collection challenges to generate the most compatible input data sets for the innovative problem solution proposed in this research so as to give life and meaning to the cost (and utility) factors explained. The designed *OC* Curve depending upon choice, can be a business standard for a new enterprise's market-entry acceptance plan, such as a smart phone company in quest for an opening e.g. in Europe. See Tables 1-4, Figures 1, 2, 3 and 4. A and B, and similarly, Tables 1 and 5, Figures 6, 7, 8 and 9 A and B for Examples 1 and 2 respectively. The goal is to follow up with a business plan, e.g. *MIL-STD-105E* found at https://variation.com/wp-content/uploads/standards/mil-std-105e.pdf [10 May 1989] that uses popular standards such as α=0.05 and β=0.10. For more, see Sahinoglu et al.[9,10,11] (2015, 2016, 2017). In this approach to calculate the cost-optimized type-I and type-II error probabilities, the author follows a game theoretic algorithm where the probabilistic and cost-related *LP* constraints as well as the five input monetary parameters, C_{ij} and *LOSS*, must be incorporated by the analyst to reflect the market realities indispensable for a profitable business model. The solutions for two illustrating and clarifying examples follow with a clarifying Figure 10 in sight:

In *Example 1* with C_{ij} = [\$800, \$200, \$70, -\$400] from the input Tables 1-5, Figures 1-3, and Figures 4 A and B, and a given *LOSS* = \$5, the game theoretic algorithm generates $\alpha \approx 0.0313$ and $\beta \approx 0.0777$ resulting in an expected total cost ≈ -\$343.92 (utility) using the company-specific input parameters. Final currency solutions are to be multiplied by K=1,000. See Figure 10 to place the findings on a plot with a clear illustration.

In *Example 2* from the input Tables 1 and 6, 7, and Figures 6-8, and Figures 9 A and B with C_{ij} = [\$50, \$110, \$40, -\$800], the Game theoretic algorithm results in an Expected Total Cost: -\$568.64 (utility) with the company-specific optimal $\alpha \approx 0.0145$ and $\beta \approx 0.0225$ for a given *LOSS*=\$5. Final solutions are multiplied by K=1,000. See Figure 10.

The producer establishes a sampling plan for a continued supply of components with reference to *AQL*, which represents the acceptable level of quality for the supplier's process that the consumer would consider acceptable as a process average. The consumer may also be interested in the other end of the *OC* Curve, i.e. *RQL* or *LQL* (*rejectable or limiting quality level*), as the poorest level of quality that the consumer is willing to accept with a low probability of acceptance in an individual lot (Montgomery[20], 2009). The publications by Sahinoglu et al.[9,10,11] (2015, 2016, 2017) significantly improved this research. Therefore, it is emphasized that the business entities were traditionally not able to design their company-specific quality control goals by computing and embedding their own producer's and consumer's risk onto their acceptance sampling plans. This was previously done by randomly assuming or best-guessing a type-I error probability and continue executing a sensitivity analysis. Now, they can include and benefit by quality-managing their process from the very first step on without expecting to be given any type-I error probability. Utilizing the game theoretic results, they can invest smarter mindfully as opposed to practicing the conventional with a subjective state of mind and therefore, appreciate a meaningful quote by Kelley[1] (2013). The author believes that this technique is appropriate for pragmatic uses when batch- or lot-sampling regarding *acceptance sampling* plans, because the article relates to the statistical computing and numerical optimization of hypothesis testing parameters (α, β), including an empirical, substantively data-scientific and user-friendly albeit business-savvy application. Lastly, it would be surprising if any one theory could address such an enormous range of "games," and in fact there is no single game theory. Several theories have been proposed, each applicable to different situations and each with its own concepts of a solution by Davis[21] (1997). The reader is recommended to refer to Figure 5, which clarifies the Venn Diagram's sample sets, and to Figure 10, which illustrates and summarizes the nature of the Figures 3 and 8 for Examples 1 and 2 in an aggregated composite diagram. Last but not least, Figures 3 and 8 display tests for i) variables (continuous measurement), and ii) attributes (defective counts), where $np \geq 5$ with Binomial to Normal approximation for relatively large sample sizes such as $n=100$ from batches with e.g. $N \approx 10K$ (Sahinoglu[10], 2016, pp. 20-21; Ostle and Mensing[15], 1975, pp. 84-85) in Examples 1 and 2 of sections 3 and 5 respectively.

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AUTOBIOGRAPHICAL SKETCH

M. Sahinoglu served as the founding director of the Informatics Institute and the founder coordinator of the Cybersystems and Information Security Graduate Program (2008-18) in Auburn University at Montgomery(AUM). Formerly, the Eminent Scholar and Chair-Professor at Troy University's Computer Science Department (1999-08), he holds a BSEE from METU, Turkey (1969-73), and MSEE from University of Manchester, UK, as a British Council scholar (1974-75). He completed a joint Ph.D. jointly in Statistics and EE from Texas A&M University at College Station, TX (1977-81). He served as the founder Dean of the College of Arts & Sciences at Izmir's DEU (1992-97) and founded the Statistics Dept in 1993-94 which celebrated its 25th anniversary in 2019. He conducts research on Cyber-Risk Informatics-Quantitative Risk Assesment and Management. He authored Trustworthy Computing (2007) and Cyber-Risk Informatics: Engineering Evaluation with Data Science (2016) both published by Wiley Inc. Dr. Sahinoglu, a Professor Emeritus from Auburn University System as of June 1, 2018, is currently teaching Cybersecurity curriculum at Troy University. Dr. Sahinoglu is an ASA (1980-), ISI Elected (1995-), IEEE Senior Life (1978-) and SDPS: Society of Design & Process Science Fellow Member (2003-). He taught at TAMU (1978-81), METU (1977-92), Purdue (1989-90, 1997-98; Fulbright & NATO) and CWRU (1998-99). One of the 14 Microsoft Trustworthy Computing Awardees (2006) and silver medalist for the DAU's Hirsch Paper Competition on Sofware Assurance (2015) and Digital Forensics (2016); Mehmet originated, i) S&L: Sahinoglu-Libby pdf jointly with D. Libby, PhD of the University of Iowa on repairable hardware (1981), ii) CPSRM: Compound Poisson Software Reliability Prediction Model (1992), iii) MESAT: Cost-Optimal Stopping-Rule in Reliability/Security Testing (2002), iv) SM: Security and Privacy Risk Meter (2005), v) CODING/DECODING of large complex networks using Polish Algorithms (2006), vi) OVERLAP ingress-egress solution for large Complex Block Diagrams jointly with Benjamin Rice (2007), and vii) CLOURAM: Cloud Computing Risk Assessor & Manager (2017).