A family of novel exact solutions to $(2+1)$-dimensional Boiti-Leon-Manna-Pempinelli equation

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Abstract

In this manuscript, some novel exact traveling wave solutions are constructed for (2 + 1)-dimensional Boiti-Leon-Manna-Pempinelli(BLMP) equation. The analytical techniques, namely extended rational sine-cosine method and extended rational sinh-cosh method are utilized for constructing the new solitary wave solutions of BLMP equation. The proposed techniques provides different types of solutions which are expressed in terms of singular periodic wave, solitary waves, bright solitons, dark solitons, periodic wave and kink wave solutions with specific values of parameters.

Keywords: New extended rational trigonometric methods , Nonlinear partial differential equations, Exact solutions, Traveling wave solutions.
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1 Introduction

Nonlinear dynamics is an engine of modern sciences which describes nonlinear phenomena. Many nonlinear phenomena in engineering, physics, biology, economics, chemistry and other fields are described by nonlinear partial differential equations (NPDEs). Nonlinear PDE is one of the main research area and interesting issue in mathematics and physics. Their exact and numerical traveling wave solutions in the type of soliton solutions have essential significance since they create a strong relation between mathematics and physics. Exact traveling wave solutions are considered best to understand the phenomena of natural sciences. A better deal of applications of NPDEs therefore appealed numerous researchers to look for their exact solutions. Many methods have been applied to find exact solutions of NPDEs such as, generalized exponential rational function [1], tanh method [2], the $\exp(-\phi\xi)$-expansion method [3], the extended rational sine-cosine approach and extended rational sinh-cosh approach [4, 5], exp-function method [6], F-expansion method [7], Hirotas method [8], extended Fan sub-equation method [9], sine-cosine method [10], the $(\frac{G'}{G})-$expansion method [11], the first integral method [12, 13], the unified method [14], the extended $(\frac{G'}{G^2})-$expansion [15], and so on.
However, the present work focus on the adoption of two novel approaches: the extended rational

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sine-cosine approach and extended rational sinh-cosh approach to seek exact traveling wave solutions of the (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation. The BLMP equation is firmly identified with the Korteweg-de Vries (KdV) equation. Boiti *et al.* [16] first derived the system of equations

$$\phi_x - \psi_y = 0$$

$$\phi_t - 3(\phi\psi)_x + \phi_{xxx} = 0,$$

and the Asymmetric-Nizhnik-Novikov-Veselov equation (ANNV) is described by the above system of equations. The ANNV equation is, in fact, a two-dimensional KdV equation and by inserting the transformation: $= \Phi_x$ and $\phi = \Phi_y$, this system of equations yields

$$\Phi_{yt} + \Phi_{xxxy} - 3\Phi_{xx}\Phi_y - 3\Phi_{xy}\Phi_x = 0, \tag{1.1}$$

where $\Phi = \Phi(x, y, t)$ and this equation is called as Boiti-Leon-Manna-Pempinelli (BLMP) equation, which was derived by Gilson et al.[17] during their researched a (2 + 1)-dimensional generalization of the AKNS shallow-water wave equation using the bilinear method. This equation was utilized to depict the (2+1)-dimensional interaction of the Riemann wave propagated along the y-axis with a long wave propagated along the x-axis. Nowadays, many researchers are focusing to extract exact solutions of BLMP equation using various different methods such as, based on the binary Bell polynomials [18], Wronskian formalism and the Hirota method [19, 20], the extended homoclinic test approach [21] and so on.
The strategy of the paper is summarized as follows: Demarcation of extended rational sine-cosine and extended rational sinh-cosh approaches are presented, in Section 2. In Section 3, application of these methods on the BLMP equation is investigated and graphs of some obtained solutions are drawn. Conclusion is given in Section 4.

2 Algorithms

Consider the nonlinear partial differential equation (NPDE):

$$F(\Phi, \Phi_x, \Phi_t, \Phi_{xx}, \Phi_{xt}, ...) = 0, \tag{2.2}$$

where $\Phi = \Phi(x, t)$ and inserting the following traveling wave transformation

$$\Phi(x, t) = \Phi(\psi), \qquad \psi = x - ct, \tag{2.3}$$

where c refers the wave speed, which converts the NPD Eq.(2.2)into an ODE:

$$G(\Phi, -c\Phi^{'}, \Phi^{'}, c^2\Phi^{''}, \Phi^{''}, -c\Phi^{''}, ...) = 0, \tag{2.4}$$

where $^{'}$ denotes the derivative with respect to ψ

2.1 Extended rational sine-cosine method

Step 1. To obtain the solutions of Eq.(2.4), extended rational sine-cosine method asserts the general solution in the form

$$\Phi(\psi) = \frac{\xi_0 \sin(\mu\psi)}{\xi_2 + \xi_1 \cos(\mu\psi)}, \quad \cos(\mu\psi) \neq -\frac{\xi_2}{\xi_1}, \tag{2.5}$$

or,

$$\Phi(\psi) = \frac{\xi_0 \cos(\mu\psi)}{\xi_2 + \xi_1 \sin(\mu\psi)}, \quad \sin(\mu\psi) \neq -\frac{\xi_2}{\xi_1}, \tag{2.6}$$

where the unknown parameters ξ_0, ξ_1, ξ_2 and μ is the wave number can be determined later.
Step 2. By substituting Eq.(2.5) or Eq.(2.6) into Eq.(2.4), polynomials in $\cos(\mu\psi)$ or $\sin(\mu\psi)$ are obtained. Then collecting all coefficients with like powers of $\cos(\mu\psi)^z$ or $\sin(\mu\psi)^z$, (where z is a positive integer) and equating them to zero. A set of algebraic equations can be obtained. The resulting equations are solved with the aid of Maple to get the values of unknown constants ξ_0, ξ_1, ξ_2, c and μ.
Step 3. Substituting the obtained unknown values from Step 2 into Eq.(2.5) or Eq.(2.6), the solution of Eq.(2.4) can be found.

2.2 Extended rational sinh-cosh method

Step 1. To obtain the solutions of Eq.(2.4), extended rational sinh-cosh method asserts the general solution in the form

$$\Phi(\psi) = \frac{\xi_0 \sinh(\mu\psi)}{\xi_2 + \xi_1 \cosh(\mu\psi)}, \quad \cosh(\mu\psi) \neq -\frac{\xi_2}{\xi_1}, \tag{2.7}$$

or,

$$\Phi(\psi) = \frac{\xi_0 \cosh(\mu\psi)}{\xi_2 + \xi_1 \sinh(\mu\psi)}, \quad \sinh(\mu\psi) \neq -\frac{\xi_2}{\xi_1}, \tag{2.8}$$

where the unknown parameters ξ_0, ξ_1, ξ_2 and μ refers the wave number can be determined later.
Step 2. By substituting Eq.(2.7) or Eq.(2.8) into Eq.(2.4), polynomials in $\cosh(\mu\psi)$ or $\sinh(\mu\psi)$ are obtained. Then collecting all coefficients with like powers of $\cosh(\mu\psi)^z$ or $\sinh(\mu\psi)^z$, (where z is a positive integer) and equating them to zero. A set of equations can be obtained. The resulting equations are solved with the aid of Maple to get the values of unknown constants ξ_0, ξ_1, ξ_2, c and μ.
Step 3. Substituting the obtained unknown values from Step 2 into Eq.(2.7) or Eq.(2.8), the solution of Eq.(2.4) can be found.

3 Exact solutions of the Proposed PDE

The transformation:

$$\Phi(x, y, t) = U(\psi), \quad \psi = \lambda_1 x + \lambda_2 y - ct, \tag{3.9}$$

where λ_1, λ_2 and c are constants, is inserting into Eq.(1.1) and the resulting ODE can be written as

$$-c\lambda_2 U^{''} + \lambda_1^3 \lambda_2 U^{''''} - 6\lambda_1^2 \lambda_2 U^{'} U^{''} = 0. \tag{3.10}$$

Integrating Eq.(3.10) and setting the constant of integration equals to zero which leads

$$-c\lambda_2 U^{'} + \lambda_1^3 \lambda_2 U^{'''} - 3\lambda_1^2 \lambda_2 (U^{'})^2 = 0. \tag{3.11}$$

3.1 Exact solutions by extended rational sine-cosine method

Suppose that solution of Eq.(3.11) has the form

$$U(\psi) = \frac{\xi_0 \sin(\mu\,\psi)}{\xi_2 + \xi_1 \cos(\mu\,\psi)}. \tag{3.12}$$

Substituting Eq.(3.12) into Eq.(3.11), we get a polynomial in $\cos(\mu\psi)$ and then collecting all coefficient of the like powers of $\cos(\mu\psi)^z$ and setting them to zero. The following algebraic equations are obtained:

$$
\begin{aligned}
\cos(\mu\psi)^3 &: \quad c\lambda_2\xi_2{\xi_1}^2 + {\lambda_1}^3\mu^2\xi_2{\xi_1}^2 = 0,\\
\cos(\mu\psi)^2 &: \quad c\lambda_2{\xi_1}^3 + 2\,c\lambda_2{\xi_2}^2\xi_1 + 4\,{\lambda_1}^3\mu^2{\xi_1}^3 - 4\,{\lambda_1}^3\mu^2{\xi_2}^2\xi_1 + 3\,{\lambda_1}^2\xi_0\mu\,{a_2}^2 = 0,\\
\cos(\mu\psi)^1 &: \quad c\lambda_2{\xi_2}^3 + 2\,c\lambda_2\xi_2{\xi_1}^2 - 4\,{\lambda_1}^3\mu^2\xi_2{a_1}^2 + {\lambda_1}^3\mu^2{\xi_2}^3 + 6\,{\lambda_1}^2\xi_0\mu\,\xi_2\xi_1 = 0,\\
\cos(\mu\psi)^0 &: \quad c\lambda_2{\xi_2}^2\xi_1 - 6\,{\lambda_1}^3\mu^2{\xi_1}^3 + 4\,{\lambda_1}^3\mu^2{\xi_2}^2\xi_1 + 3\,{\lambda_1}^2\xi_0\mu\,{\xi_1}^2 = 0.
\end{aligned}
$$

The solutions of above equations are classified as

$$\textbf{Case}\quad 1.\quad \mu = \pm\frac{1}{2\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}},\ \xi_0 = \pm\sqrt{-\frac{c\lambda_2}{\lambda_1}}\xi_1,\ \xi_1 = \xi_1,\ \xi_2 = 0.$$

For the case 1 the solutions of Eq.(1.1):

$$
\begin{aligned}
\Phi_{1_1}(x,y,t) &= \sqrt{-\frac{c\lambda_2}{\lambda_1}}\tan\left[\frac{1}{2\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right].\\
\Phi_{1_2}(x,y,t) &= -\sqrt{-\frac{c\lambda_2}{\lambda_1}}\tan\left[\frac{1}{2\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right].
\end{aligned}
\tag{3.13}
$$

$$\textbf{Case}\quad 2.\quad \mu = \pm\frac{1}{\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}},\ \xi_0 = \pm\sqrt{-\frac{c\lambda_2}{\lambda_1}}\xi_1,\ \xi_1 = \pm\xi_2,\ \xi_2 = \xi_2.$$

For the case 2 the solutions of Eq.(1.1):

$$
\begin{aligned}
\Phi_{2_1}(x,y,t) &= \sqrt{-\frac{c\lambda_2}{\lambda_1}}\frac{\sin\left[\frac{1}{\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right]}{1+\cos\left[\frac{1}{\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right]}.\\
\Phi_{2_2}(x,y,t) &= \sqrt{-\frac{c\lambda_2}{\lambda_1}}\frac{\sin\left[\frac{1}{\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right]}{1-\cos\left[\frac{1}{\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right]}.\\
\Phi_{2_3}(x,y,t) &= -\sqrt{-\frac{c\lambda_2}{\lambda_1}}\frac{\sin\left[\frac{1}{\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right]}{1+\cos\left[\frac{1}{\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right]}.\\
\Phi_{2_4}(x,y,t) &= -\sqrt{-\frac{c\lambda_2}{\lambda_1}}\frac{\sin\left[\frac{1}{\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right]}{1-\cos\left[\frac{1}{\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right]}.
\end{aligned}
\tag{3.14}
$$

OR

Suppose that Eq.(3.11) has solution in the form, as

$$U(\psi) = \frac{\xi_0\cos(\mu\,\psi)}{\xi_2 + \xi_1\sin(\mu\,\psi)}. \tag{3.15}$$

Substituting Eq.(3.15) into Eq.(3.11), we get a polynomial in $\sin(\mu\psi)$ and collecting all coefficients of the like powers of $\sin(\mu\psi)$ and setting them to zero. The following algebraic equations are obtained:

$$
\begin{aligned}
\sin(\mu\psi)^3 &: \quad -c\lambda_2{\xi_1}^2\xi_2 + {\lambda_1}^3\mu^2\xi_2{\xi_1}^2 = 0,\\
\sin(\mu\psi)^2 &: \quad -2\,c\lambda_2\xi_1{\xi_2}^2 - c\lambda_2{\xi_1}^3 + 4\,{\lambda_1}^3\mu^2{\xi_1}^3 - 4\,{\lambda_1}^3\mu^2{\xi_2}^2\xi_1 + 3\,{\lambda_1}^2\xi_0\mu\,{d_2}^2 = 0,\\
\sin(\mu\psi)^1 &: \quad -c\lambda_2{\xi_2}^3 - 2\,c\lambda_2{\xi_1}^2\xi_2 + {\lambda_1}^3\mu^2{\xi_2}^3 - 4\,{\lambda_1}^3\mu^2\xi_2{\xi_1}^2 + 6\,{\lambda_1}^2\xi_0\mu\,\xi_2\xi_1 = 0,\\
\sin(\mu\psi)^0 &: \quad -c\lambda_2\xi_1{\xi_2}^2 - 6\,{\lambda_1}^3\mu^2{\xi_1}^3 + 4\,{\lambda_1}^3\mu^2{\xi_2}^2\xi_1 + 3\,{\lambda_1}^2\xi_0\mu\,{\xi_1}^2 = 0.
\end{aligned}
$$

The solutions of above equations are classified as

$$\textbf{Case} \quad 3. \quad \mu=\pm\frac{1}{2\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}, \quad \xi_0=\xi_0, \quad \xi_1=\pm\sqrt{\frac{\lambda_1}{c\lambda_2}}\xi_0, \quad \xi_2=0.$$

For the case 3 the solutions of Eq.(3.11):

$$\begin{aligned}
\Phi_{3_1}(x,y,t) &= \sqrt{\frac{c\lambda_2}{\lambda_1}}\cot\left[\frac{1}{2\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x+\lambda_2 y-ct)\right].\\
\Phi_{3_2}(x,y,t) &= -\sqrt{\frac{c\lambda_2}{\lambda_1}}\cot\left[\frac{1}{2\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x+\lambda_2 y-ct)\right].
\end{aligned} \tag{3.16}$$

$$\textbf{Case} \quad 4. \quad \mu=\pm\frac{1}{2\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}, \quad \xi_0=\pm\sqrt{\frac{c\lambda_2}{\lambda_1}}\xi_2, \quad \xi_1=\pm\xi_2, \quad \xi_2=\xi_2.$$

For the case 4 the solutions of Eq.(3.11):

$$\begin{aligned}
\Phi_{4_1}(x,y,t) &= \sqrt{\frac{c\lambda_2}{\lambda_1}}\frac{\cos\left[\frac{1}{2\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x+\lambda_2 y-ct)\right]}{1+\sin\left[\frac{1}{2\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x+\lambda_2 y-ct)\right]}.\\
\Phi_{4_2}(x,y,t) &= \sqrt{\frac{c\lambda_2}{\lambda_1}}\frac{\cos\left[\frac{1}{2\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x+\lambda_2 y-ct)\right]}{1-\sin\left[\frac{1}{2\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x+\lambda_2 y-ct)\right]}.\\
\Phi_{4_3}(x,y,t) &= -\sqrt{\frac{c\lambda_2}{\lambda_1}}\frac{\cos\left[\frac{1}{2\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x+\lambda_2 y-ct)\right]}{1+\sin\left[\frac{1}{2\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x+\lambda_2 y-ct)\right]}.\\
\Phi_{4_4}(x,y,t) &= -\sqrt{\frac{c\lambda_2}{\lambda_1}}\frac{\cos\left[\frac{1}{2\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x+\lambda_2 y-ct)\right]}{1-\sin\left[\frac{1}{2\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x+\lambda_2 y-ct)\right]}.
\end{aligned} \tag{3.17}$$

3.2 Exact solutions by extended rational sinh-cosh method

Suppose that the traveling wave solution of Eq.(3.11) has the form,

$$U(\psi)=\frac{\xi_0\sinh(\mu\,\psi)}{\xi_2+\xi_1\cosh(\mu\,\psi)}. \tag{3.18}$$

Substituting Eq.(3.18) into Eq.(3.11), we get a polynomial in $\cosh(\mu\psi)$ and collecting all terms with the like powers of $\cosh(\mu\psi)^z$ and setting them to zero. The following algebraic equations are obtained:

$$\begin{aligned}
\cosh(\mu\psi)^3 &: \; c\lambda_2\xi_2{\xi_1}^2-{\lambda_1}^3\mu^2\xi_2{\xi_1}^2=0,\\
\cosh(\mu\psi)^2 &: \; 2\,c\lambda_2{\xi_2}^2\xi_1+c\lambda_2{\xi_1}^3-4\,{\lambda_1}^3\mu^2{\xi_1}^3+4\,{\lambda_1}^3\mu^2{\xi_2}^2\xi_1+3\,{\lambda_1}^2\xi_0\mu\,{a_2}^2=0,\\
\cosh(\mu\psi)^1 &: \; c\lambda_2{\xi_2}^3+2\,c\lambda_2\xi_2{\xi_1}^2-{\lambda_1}^3\mu^2{\xi_2}^3+4\,{\lambda_1}^3\mu^2\xi_2{\xi_1}^2+6\,{\lambda_1}^2\xi_0\mu\,\xi_2\xi_1=0,\\
\cosh(\mu\psi)^0 &: \; c\lambda_2{\xi_2}^2\xi_1-4\,{\lambda_1}^3\mu^2{\xi_2}^2\xi_1+6\,{\lambda_1}^3\mu^2{\xi_1}^3+3\,{\lambda_1}^2\xi_0\mu\,{\xi_1}^2=0.
\end{aligned}$$

The solutions of above equations are classified as

$$\textbf{Case} \quad 5. \quad \mu=\pm\frac{1}{2\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}, \quad \xi_0=\pm\sqrt{\frac{c\lambda_2}{\lambda_1}}\xi_1, \quad \xi_1=\xi_1, \quad \xi_2=0.$$

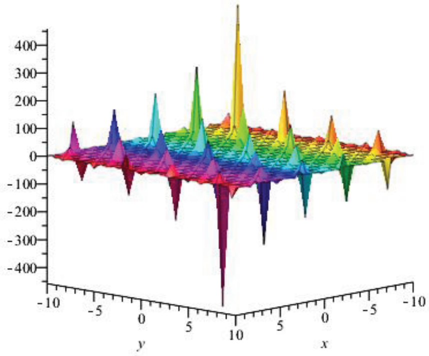


Figure 1: 3D graphics of Φ_{1_1} with $\lambda_1 = 0.09$, $\lambda_2 = -1.78$ and $c = 0.078$.

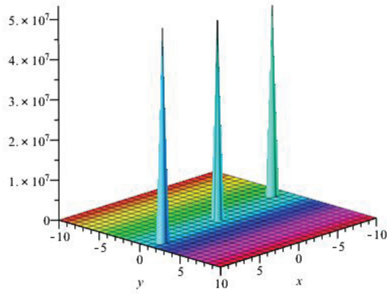


Figure 2: 3D graphics of Φ_{2_4} with $\lambda_1 = 0.09$, $\lambda_2 = -0.6$ and $c = 0.78$.

For the case 5 the solutions of Eq.(3.11):

$$\begin{aligned}\Phi_{5_1}(x,y,t) &= \sqrt{\frac{c\lambda_2}{\lambda_1}}\tanh\left[\frac{1}{2\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right].\\ \Phi_{5_2}(x,y,t) &= -\sqrt{\frac{c\lambda_2}{\lambda_1}}\tanh\left[\frac{1}{2\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right].\end{aligned} \tag{3.19}$$

Case 6. $\mu = \pm\frac{1}{\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}, \quad \xi_0 = \pm\sqrt{\frac{c\lambda_2}{\lambda_1}}\xi_2, \quad \xi_1 = \pm\xi_2, \quad \xi_2 = \xi_2.$

For the case 6 the solutions of Eq.(3.11):

$$\begin{aligned}\Phi_{6_1}(x,y,t) &= \sqrt{\frac{c\lambda_2}{\lambda_1}}\frac{\sinh\left[\frac{1}{\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right]}{1+\cosh\left[\frac{1}{\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right]}.\\ \Phi_{6_2}(x,y,t) &= -\sqrt{\frac{c\lambda_2}{\lambda_1}}\frac{\sinh\left[\frac{1}{\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right]}{1+\cosh\left[\frac{1}{\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right]}.\\ \Phi_{6_3}(x,y,t) &= \sqrt{\frac{c\lambda_2}{\lambda_1}}\frac{\sinh\left[\frac{1}{\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right]}{1-\cosh\left[\frac{1}{\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right]}.\\ \Phi_{6_4}(x,y,t) &= -\sqrt{\frac{c\lambda_2}{\lambda_1}}\frac{\sinh\left[\frac{1}{\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right]}{1-\cosh\left[\frac{1}{\lambda_1}\sqrt{\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right]}.\end{aligned} \tag{3.20}$$

OR

Suppose that Eq.(3.11) has solution in the form, as

$$U(\psi) = \frac{\xi_0 \cosh(\mu\psi)}{\xi_2 + \xi_1 \sinh(\mu\psi)}. \tag{3.21}$$

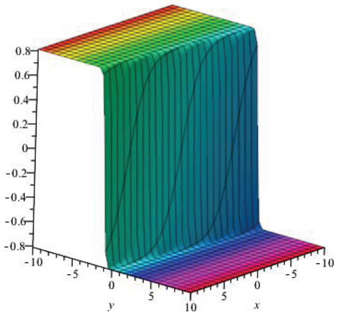


Figure 3: 3D graphics of Φ_{5_2} with $\lambda_1 = 0.09$, $\lambda_2 = 0.76$ and $c = 0.078$.

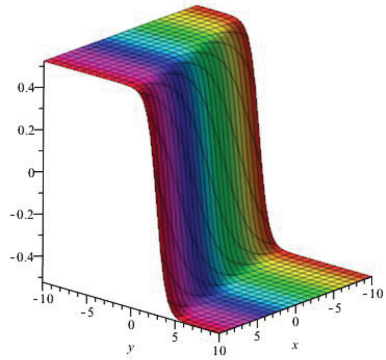


Figure 4: 3D graphics of Φ_{6_1} with $\lambda_1 = 0.5$, $\lambda_2 = -1.76$ and $c = 0.078$.

Substituting Eq.(3.21) into Eq.(3.11), we get a polynomial in $\sinh(\mu\psi)$ and collecting all terms of the like powers of $\sinh(\mu\psi)^z$ and setting them to zero. The following algebraic equations are obtained:

$$
\begin{aligned}
\sinh(\mu\psi)^3 \quad &: \quad c\lambda_2{\xi_1}^2\xi_2 + {\lambda_1}^3\mu^2{\xi_1}^2\xi_2 = 0,\\
\sinh(\mu\psi)^2 \quad &: \quad 2\,c\lambda_2{\xi_2}^2\xi_1 - c\lambda_2{\xi_1}^3 - 4\,{\lambda_1}^3\mu^2{\xi_1}^3 - 4\,{\lambda_1}^3\mu^2{\xi_2}^2\xi_1 - 3\,{\lambda_1}^2\xi_0\mu\,{a_2}^2 = 0,\\
\sinh(\mu\psi)^1 \quad &: \quad c\lambda_2{\xi_2}^3 - 2\,c\lambda_2{\xi_1}^2\xi_2 + {\lambda_1}^3\mu^2{\xi_2}^3 + 4\,{\lambda_1}^3\mu^2{\xi_1}^2\xi_2 + 6\,{\lambda_1}^2\xi_0\mu\,\xi_2\xi_1 = 0,\\
\sinh(\mu\psi)^0 \quad &: \quad -c\lambda_2{\xi_2}^2\xi_1 - 6\,{\lambda_1}^3\mu^2{\xi_1}^3 - 4\,{\lambda_1}^3\mu^2{\xi_2}^2\xi_1 - 3\,{\lambda_1}^2\xi_0\mu\,{\xi_1}^2 = 0.
\end{aligned}
$$

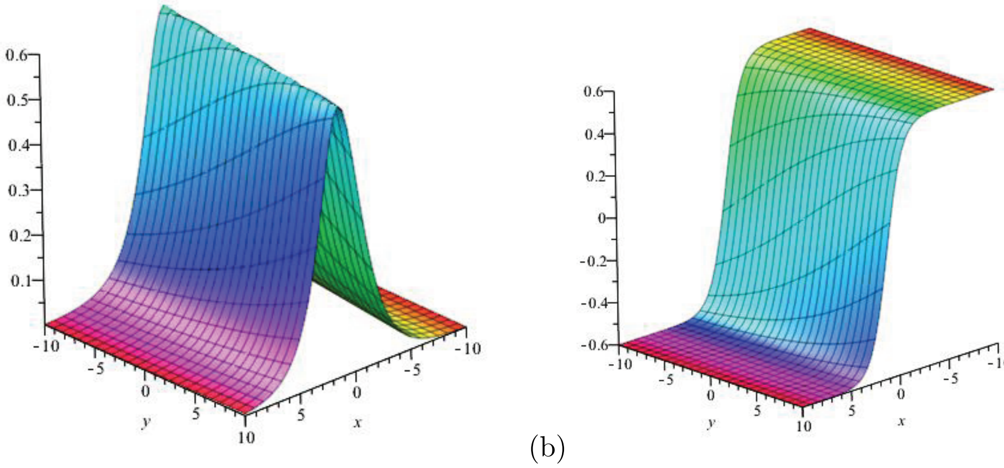
The solutions of above equations are classified as

$$
\textbf{Case} \quad 7. \quad \mu = \pm\frac{1}{2\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}, \quad \xi_0 = \xi_0, \quad \xi_1 = \pm\iota\sqrt{\frac{\lambda_1}{c\lambda_2}}\xi_0,\ \xi_2 = 0.
$$

For the case 7 the solutions of Eq.(3.11):

$$
\begin{aligned}
\Phi_{7_1}(x,y,t) = &\quad \sqrt{-\frac{c\lambda_2}{\lambda_1}}\coth\left[\frac{1}{2\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right].\\
\Phi_{7_2}(x,y,t) = &\ -\sqrt{-\frac{c\lambda_2}{\lambda_1}}\coth\left[\frac{1}{2\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x + \lambda_2 y - ct)\right].
\end{aligned}
\tag{3.22}
$$

$$
\textbf{Case} \quad 8. \quad \mu = \pm\frac{1}{\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}, \quad \xi_0 = \pm\sqrt{\frac{c\lambda_2}{\lambda_1}}\xi_2, \quad \xi_1 = \pm\iota\xi_2,\ \xi_2 = \xi_2.
$$



(a) (b)

Figure 5: 3D graphics of real and imaginary parts of Φ_{8_1} with $\lambda_1 = -0.5$, $\lambda_2 = 0.06$ and $c = 3$.

For the case 8 the solutions of Eq.(3.11):

$$\begin{aligned}
\Phi_{8_1}(x,y,t) &= \sqrt{-\frac{c\lambda_2}{\lambda_1}}\frac{\cosh\left[\frac{1}{\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x+\lambda_2 y-ct)\right]}{1+\iota\sinh\left[\frac{1}{\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x+\lambda_2 y-ct)\right]}.\\
\Phi_{8_2}(x,y,t) &= -\sqrt{-\frac{c\lambda_2}{\lambda_1}}\frac{\cosh\left[\frac{1}{\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x+\lambda_2 y-ct)\right]}{1+\iota\sinh\left[\frac{1}{\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x+\lambda_2 y-ct)\right]}.\\
\Phi_{8_3}(x,y,t) &= \sqrt{-\frac{c\lambda_2}{\lambda_1}}\frac{\cosh\left[\frac{1}{\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x+\lambda_2 y-ct)\right]}{1-\iota\sinh\left[\frac{1}{\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x+\lambda_2 y-ct)\right]}.\\
\Phi_{8_4}(x,y,t) &= -\sqrt{-\frac{c\lambda_2}{\lambda_1}}\frac{\cosh\left[\frac{1}{\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x+\lambda_2 y-ct)\right]}{1-\iota\sinh\left[\frac{1}{\lambda_1}\sqrt{-\frac{c\lambda_2}{\lambda_1}}(\lambda_1 x+\lambda_2 y-ct)\right]}.
\end{aligned} \tag{3.23}$$

Numerical simulations: Some obtained exact rational trigonometric solutions of (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation are shown by graphs along with the physical explanations which are plotted only for $-10 \leq x \leq 10$, $-10 \leq y \leq 10$ and $t = 0$. Fig.1 illustrates the evolution of singular periodic wave solutions for $\Phi_{1_1}(x,y,t)$. Fig.(2) shows periodic solutions for $\Phi_{2_4}(x,y,t)$. Fig.(3) describes kink and dark soliton solutions for $\Phi_{5_2}(x,y,t)$. Fig.(4) also represents kink type wave solutions for $\Phi_{6_1}(x,y,t)$. Part (a) of Fig.(5) illustrates the graph of the real part of $\Phi_{8_1}(x,y,t)$ which represents bright soliton wave solutions, while part (b) of Fig.(5) shows the graph of the imaginary part of $\Phi_{8_1}(x,y,t)$ which describes kink and dark soliton solutions.

4 Conclusion

Exact rational trigonometric solutions of (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation have been constructed via extended rational sine-cosine and extended rational sinh-cosh methods.

The obtained solutions are expressed as solitary waves, bright soltiton, dark soliton, periodic wave and kink wave solutions. Some new graphical representations are obtained with the help of these methods. It is found that from the two proposed techniques the former provides a variety of different solutions as compared to later.

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