Supporting Information for "The conductive cooling of planetesimals with temperature-dependent properties"

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Introduction

Additional detail on numerical stability and analytical verification of the numerical method. Tabulated results referenced in the main text are included in the file `tables01`, with captions listed below. Representative model output data are openly available to download from the National Geoscience Data Centre (NGDC), the Natural Environment Research Council (UK) data centre for geoscience data, with the data ID: 138605. The corresponding captions are included here.

Text S1: Numerical method and stability criteria

We solve the conduction equation numerically using an explicit finite difference scheme, FTCS (Forward-Time Central-Space). FTCS gives first-order convergence in time and second-order in space. Due to its explicit nature, it is computationally inexpensive. It is conditionally stable when applied to the heat equation. In 1D, it must satisfy Von Neumann stability analysis:

$$\frac{\kappa \delta t}{\delta r^2} \leq \frac{1}{2},$$

with the largest κ of the scheme being chosen for the most restrictive conditions (Crank & Nicolson, 1947; Charney et al., 1950). For a constant spatial grid with $\delta r = 1000$ m, $\delta t = 1 \times 10^{11}$ s was sufficient to meet this criterion for the most restrictive cases with large κ. An adaptive grid was not used due to the first-order nature of the problem being addressed.

Text S2: Analytical verification

In order to assess accuracy, the constant model was compared to the analytical solution for a sphere given by equation 6.18 in Crank (1979) with an initial uniform temperature T_i and a constant surface temperature T_s:

$$\frac{T - T_i}{T_s - T_i} = 1 + \frac{2r_{\mathrm{p}}}{\pi r} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi r}{r_{\mathrm{p}}} \exp\left(-\kappa n^2 \pi^2 t / r_{\mathrm{p}}^2\right) \tag{1}$$

where $r = 0$ at the centre of the sphere and κ is a constant diffusivity, given by $\kappa = \frac{k}{\rho C_{\mathrm{p}}}$.

To allow the numerical spherical shell model to approximate a sphere for comparison, the core radius was reduced to 0.001 km and a zero flux boundary condition was applied across the centre to ensure symmetry. While this is not a perfect sphere, it highlights whether the numerical model is producing sensible results.

Average Root-Mean-Squared-Deviation (RMSD) in the temperature and between the analytical solution and the numerical model was calculated for a range of radii through time. The error increases at the near surface, with an RMSD = 13.56 K at $r = 245$ km in contrast to RMSD = 5.28 at $r = 1$ km. At $r = 125$ km, RMSD = 5.89. The peak cooling rates for the shallow case are reached almost instantaneously, while the analytical solution is not accurate for small times, resulting in larger RMSD values.

Table S1: file `tables01`

Note: Model results with parameters varied to ±10 % of the default value. References for parameter choices given in Table X in the main text. [a]Regolith thickness increased or decreased by 1 km as 10 % (0.8 km) is smaller than δr. [b]Increasing or decreasing C_{m} or ρ_{m} by 10 % in effect results in a change in ρc by 10 %. [c] As for [b] with core properties.

Table S2: file `tables01`

Note: Model results with parameters varied to ±10 % of the default value. References for parameter choices given in Table X in the main text. [a]Regolith thickness increased or decreased by 1 km as 10 % (0.8 km) is smaller than δr. [b]Increasing or decreasing c_{m} or ρ_{m} by 10 % in effect results in a change in ρc by 10 %. [c] As for [b] with core properties.

Data Set S1.

Data Set S1 model output data is available from the National Geoscience Data Centre (NGDC), the Natural Environment Research Council (UK) data centre for geoscience data. The data are available through the ID 138605, and filename `constant_properties.dat`.

File `constant_properties.dat` is a compressed NumPy array of temperatures and cooling rates for a conductively cooling planetesimal with constant material properties and other reference parameters given in Table 1 in the main text. The data can be loaded with a simple Python script:

```
import numpy as np


data_file = "constant_properties"


with np.load(str(data_file)+".dat") as data:
    temperatures=data['temperatures'] # mantle temperatures
    coretemp=data['coretemp'] #core temperatures
    dT_by_dt=data['dT_by_dt']  # mantle cooling rates
    dT_by_dt_core=data['dT_by_dt_core'] # core cooling rates
```

Data Set S2.

Data Set S2 model output data is available from the National Geoscience Data Centre (NGDC), the Natural Environment Research Council (UK) data centre for geoscience data. The data are available through the ID 138605, and filename `variable_properties.dat`.

File `variable_properties.dat` is a compressed NumPy array of temperatures and cooling rates for a conductively cooling planetesimal with temperature-dependent material properties as described in section 2.4 in the main text. The data can be loaded with a simple Python script as for Data Set S1, with `data_file = "variable_properties"`.

Data Set S3.

File `ds03.py`, included as additional supplementary materials, is a Python script that loads and plots a heatmap of the data included in Data Set S1. This script requires Python 3.7, and uses the NumPy module to load the data and the Matplotlib library to visualise the data. To load Data Set S2, line 19 in the script can be changed from `data_file = "constant_properties"` to `data_file = "variable_properties"`.

References

Charney, J. G., Fjörtoft, R., & Neumann, J. (1950). Numerical integration of the barotropic vorticity equation. *Tellus*, *2*(4), 237–254.

Crank, J. (1979). *The mathematics of diffusion.* Clarendon Press.

Crank, J., & Nicolson, P. (1947). A practical method for numerical evaluation of solutions of partial differential equations of the heat-conduction type. *Mathematical Proceedings of the Cambridge Philosophical Society*, *43*(1), 50–67.