Supporting Information for "The conductive cooling of planetesimals with temperature-dependent properties"

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Additional Supporting Information

1. Captions for Data Sets S1 to S2

Introduction

This document provides additional information on the availability of our model source code and output data, extended description of our analytical verification of our model,

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extended tabulated results referenced in the main text, and captions for Data Sets S1 and S2 hosted by the National Geoscience Data Centre (NGDC).

Text S1: Data and model availability

The code used in this paper to model the thermal evolution of a planetesimal is available for public download at `https://github.com/murphyqm/pytesimal` (Murphy Quinlan & Walker, 2020). The software is written in Python 3 under the MIT license. The repository contains instructions on installation and use, and contains examples which reproduce the cases described in this paper. This paper uses version 1.0.0, archived at DOI:10.5281/zenodo.4321772.

The Pytesimal repository also contains a simple Python script to download and plot the Data Sets S1 and S2 which are archived at the National Geoscience Data Centre (`http://data.bgs.ac.uk/id/dataHolding/13607679`; Murphy Quinlan, 2020).

Tables S1 and S2 in this document are also available to download at `https://github.com/murphyqm/pytesimal-test-results` (Murphy Quinlan, 2021).

Text S2: Analytical verification

As discussed in the main text, the model using constant material properties was compared to the analytical solution for a sphere given by equation 6.18 in Crank (1979) with an initial uniform temperature T_i and a constant surface temperature T_s:

$$\frac{T - T_i}{T_s - T_i} = 1 + \frac{2r_{\mathrm{p}}}{\pi r} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi r}{r_{\mathrm{p}}} \exp\left(-\kappa n^2 \pi^2 t / r_{\mathrm{p}}^2\right) \tag{1}$$

where $r = 0$ at the centre of the sphere and κ is a constant diffusivity, given by $\kappa = \frac{k}{\rho C_{\mathrm{p}}}$.

To allow the numerical spherical shell model to approximate a sphere for comparison, the core was minimized and a zero flux boundary condition was applied across the centre to ensure symmetry. While this is not a perfect sphere, it highlights whether the numerical model is producing sensible results.

Average Root-Mean-Squared-Deviation (RMSD) in the temperature and between the analytical solution and the numerical model was calculated for a range of radii through time. The error increases at the near surface, with an RMSD = 13.56 K at r = 245 km in contrast to RMSD = 5.28 at r = 1 km. At r = 125 km, RMSD = 5.89. The peak cooling rates for the shallow case are reached almost instantaneously, while the analytical solution is not accurate for small times, resulting in larger RMSD values.

Table S1

This table is also available for download in PDF, CSV and spreadsheet format at `https://github.com/murphyqm/pytesimal-test-results` (Murphy Quinlan, 2021).

Table S1: *Input parameter variation*

Varied parameter	**Value**	**Core starts** Myr	**Core ends** Myr	**Duration** Myr	**Esquel depth** km	**Imilac depth** km
r_p	600 km	1022	1201	179	31	22
r_p	150 km	61	86	25	52	51
r_c	200 km	95	157	62	27	25
r_c	50 km	199	240	41	93	69
d_reg	20 km	245	326	81	44	30
d_reg	0 km	159	230	70	64	57
k_m	4 W m^{-1} K^{-1}	132	185	53	77	67
k_m	1.5 W m^{-1} K^{-1}	330	400	70	42	36
c_m	2000 J kg^{-1} K^{-1}	293	383	90	37	32
c_m	600 J kg^{-1} K^{-1}	148	215	67	71	65
ρ_m	3560 kg m−3	177	249	71	62	55
ρ_m	2500 kg m−3	149	216	67	71	64
c_c	850 J kg^{-1} K^{-1}	172	242	71	64	57
c_c	780 J kg^{-1} K^{-1}	166	237	71	65	58
ρ_c	7800 kg m−3	172	242	71	64	57
ρ_c	7011 kg m−3	164	229	65	65	58
T_init	1820 K	213	283	70	57	51
T_init	1450 K	138	210	72	70	62
T_surf	300 K	176	250	74	58	52
T_surf	150 K	164	228	65	75	67
l_c	2.56×10^5 J K^{-1} kg^{-1}	172	239	67	64	57
T_L	1213 K	168	238	70	64	57

Note: Model results with maximised and minimised constant values for parameters. References for parameter choices given in Table 1 in the main text.

Table S2

This table is also available for download in PDF, CSV and spreadsheet format at `https://github.com/murphyqm/pytesimal-test-results` (Murphy Quinlan, 2021).

Table S2: *Sensitivity test of constant model*

Varied parameter	**Value**	**Core starts** Myr	**Core ends** Myr	**Duration** Myr	**Esquel depth** km	**Imilac depth** km
r_p +10%	275 km	210	296	86	64	56
r_p −10%	225 km	146	204	58	66	58
r_c +10%	138 km	167	241	74	58	53
r_c −10%	113 km	185	252	67	70	61
d_reg +1 km[a]	9 km	172	242	71	64	57
d_reg −1 km[a]	7 km	165	236	70	64	57
k_m +10%	3.3 W $\mathrm{m^{-1}\ K^{-1}}$	157	221	64	68	60
k_m −10%	2.7 W $\mathrm{m^{-1}\ K^{-1}}$	189	268	78	61	54
C_m +10%[b]	901 J $\mathrm{kg^{-1}\ K^{-1}}$	180	252	72	61	54
C_m −10%[b]	737 J $\mathrm{kg^{-1}\ K^{-1}}$	163	232	69	67	60
ρ_m +10%[b]	3675 kg $\mathrm{m^{-3}}$	180	252	72	61	54
ρ_m −10%[b]	3007 kg $\mathrm{m^{-3}}$	163	232	69	67	60
C_c +10%[c]	935 J $\mathrm{kg^{-1}\ K^{-1}}$	179	248	70	63	57
C_c −10%[c]	765 J $\mathrm{kg^{-1}\ K^{-1}}$	164	236	71	65	58
ρ_c +10%[c]	8580 kg $\mathrm{m^{-3}}$	179	248	70	63	57
ρ_c −10%[c]	7020 kg $\mathrm{m^{-3}}$	164	236	71	65	58
T_init +10%	1760 K	202	272	70	59	53
T_init −10%	1440 K	135	208	72	70	63
T_surf +10%	275 K	174	246	72	61	55
T_surf −10%	225 K	169	238	69	67	60
l_c +10%	2.97×10^5 J $\mathrm{K^{-1}\ kg^{-1}}$	172	249	77	64	57
l_c −10%	2.43×10^5 J $\mathrm{K^{-1}\ kg^{-1}}$	172	236	64	64	57
T_L +10%	1320 K	137	202	65	64	57
T_L −10%	1080 K	209	288	79	64	57

Note: Model results with parameters varied to ±10 % of the default value. References for parameter choices given in Table 1 in the main text. [a]Regolith thickness increased or decreased by 1 km as 10 % (0.8 km) is smaller than δr. [b]Increasing or decreasing C_m or ρ_m by 10 % in effect results in a change in ρc by 10 %. [c] As for [b] with core properties.

Data Set S1.

Data Set S1 model output data is available from the National Geoscience Data Centre (NGDC), the Natural Environment Research Council (UK) data centre for geoscience data. The data are available available for download at `http://data.bgs.ac.uk/id/dataHolding/13607679` with the filename `constant_properties.dat` (Murphy Quinlan, 2020).

File `constant_properties.dat` is a compressed NumPy array of temperatures and cooling rates for a conductively cooling planetesimal with constant material properties and other reference parameters given in Table 1 in the main text. The data can be downloaded and plotted using a simple Python script available at `https://github.com/murphyqm/pytesimal` (Murphy Quinlan & Walker, 2020).

Data Set S2.

Data Set S2 model output data is available from the National Geoscience Data Centre (NGDC), the Natural Environment Research Council (UK) data centre for geoscience data. The data are available available for download at `http://data.bgs.ac.uk/id/dataHolding/13607679` with the filename `variable_properties.dat` (Murphy Quinlan, 2020).

File `variable_properties.dat` is a compressed NumPy array of temperatures and cooling rates for a conductively cooling planetesimal with temperature-dependent material properties as described in section 2.4 in the main text. The data can be loaded with a simple Python script as for Data Set S1 (`https://github.com/murphyqm/pytesimal`; Murphy Quinlan & Walker, 2020).

References

Crank, J. (1979). *The mathematics of diffusion.* Clarendon Press.

Murphy Quinlan, M. (2020). *Temperature and cooling rate data for a conductively cooling 250km radius planetesimal with and without temperature-dependent material properties.* `http://data.bgs.ac.uk/id/dataHolding/13607679`. British Geological Survey.

Murphy Quinlan, M. (2021). *The conductive cooling of planetesimals with temperature-dependent properties: tabulated results.* `https://github.com/murphyqm/pytesimal-test-results`. doi: 10.5281/zenodo.4439180

Murphy Quinlan, M., & Walker, A. (2020). *Pytesimal: v1.0.0.* `https://github.com/murphyqm/pytesimal`. doi: 10.5281/zenodo.4321772