New expectations to soliton arising from the (2+1)-dimensional generalized coupled nonlinear Schrödinger equation with four waves mixing via three different techniques

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April 16, 2024

Abstract

From the point of view of three famous and important techniques we will achieve new expectations for the soliton configurations to the (2+1)-dimensional generalized nonlinear Schrödinger equation (GNLSE) with four waves mixing (FWM). The suggested model describes propagation of solitons in birefringent fiber. The FWM governed effectively the performance of the resultant soliton amplitudes. The three famous methods candidates for this purpose are the extended direct algebraic method (EDAM), the extended simple equation method(ESEM) and the solitary wave ansatz method (SWAM). The three techniques are implemented successively for the suggested model successfully. Surprise expectations for solitons via these three techniques to this model which weren't achieved previously by any other authors who used other techniques have been demonstrated.

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ABSTRACT

From the point of view of three famous and important techniques we will achieve new expectations for the soliton configurations to the (2+1)-dimensional generalized nonlinear Schrödinger equation (GNLSE) with four waves mixing (FWM). The suggested model describes propagation of solitons in birefringent fiber. The FWM governed effectively the performance of the resultant soliton amplitudes. The three famous methods candidates for this purpose are the extended direct algebraic method (EDAM), the extended simple equation method(ESEM) and the solitary wave ansatz method (SWAM). The three techniques are implemented successively for the suggested model successfully. Surprise expectations for solitons via these three techniques to this model which weren't achieved previously by any other authors who used other techniques have been demonstrated.

Keywords : The generalized coupled Schrödinger equation, the SWAM, the EDAM, the ESEM, Solitons.

1-Introduction

The soliton solutions of the GNLS with FWM which describes effectively the propagations of the solitons in birefringent fiber has been proposed for the first once in the framework of three important and impressive techniques. The FWM and the other parameters arising in the proposed model will governs the speed and propagation direction of the resultant solitons, as well as reduce the interactions between them. Moreover, we shall discuss the energy transfer of solitons during elastic collision and separation. When dispersion term and the nonlinearity term reach the equilibrium the resultant soliton is special case of the occurred ultra-short pulse. The optical solitons have been widely studied either experimentally or theoretically via many authors through [1–18]. Some various forms of Schrödinger equation have been studied see for example Yang, et al [19] who studied the dynamics of soliton propagation in nonlinear fibers. In related subject, the GCNLS which gives good description of the solitons propagate in the multiple fields with various frequencies or polarizations at the same time has been discussed for through different branches of physics see for example, Agrawal [20] who utilized that, Som, et al. [21] who observed this behavior for plasma physics, Alwyn [22] who observed this behavior for biophysics, [23] Dalfovo, et al. [23] who observed this behavior for Bose-Einstein condensation and Lazarides, et al. [24]who observed this behavior for meta-material technology [24]. The main responsibility for the nonlinear effects for the GCNLS emerged from the four-wave mixing (FWM), cross phase modulation (XPM), and self-phase modulation (SPM). Among them, FWM has the greatest effect on the optical system [25]. The GCNLSE with a FWM effect to describe the optical solitons in a birefringent fiber is demonstrated through [26–30]. According to [30] the GCNLSE with four-wave mixing (FWM) can be written as

$$\begin{aligned} &iu_t + u_{xx} + 2(a\left|u^2\right| + c\left|v^2\right| + buv^* + b^*u^*v)u = 0 \\ &iv_t + v_{xx} + 2(a\left|u^2\right| + c\left|v^2\right| + buv^* + b^*u^*v)v = 0 \end{aligned} \tag{1}$$

In the above equation if $a = c, b = 0,$ it will be converted to Manakov system and if $a = -c, b = 0,$ it will be converted to mixed coupled NLS

The generalize form of Eq.(1) to a (2+1)-dimensional GCNLSE which will extract more abundant physical phenomena can be written according to [30-33] as,

$$\begin{aligned} &iU_t + U_{xx} + U_{yy} + 2(a\left|U^2\right| + c\left|V^2\right| + bUV^* + b^*U^*V)U = 0 \\ &iV_t + V_{xx} + V_{yy} + 2(a\left|U^2\right| + c\left|V^2\right| + bUV^* + b^*U^*V)V = 0 \end{aligned} \tag{2}$$

The functions U, V appearing in this equation are in terms of x, y, t and describe the envelopes of the two circularly polarized waves.

Let us consider the transformations

$$U(x,t) = Q_1(\eta)Exp[i\varphi(x,t)], V(x,t) = Q_2(\eta)Exp[i\varphi(x,t)].$$

where $\eta = x - vt, \varphi(x,t) = -k_1x - k_2y + wt + \theta_0, v, w, k$ and θ_0 denotes respectively to velocity, frequency, wave number and phase constants. With the aid of these transformations the above system will split into the following real and imaginary systems, the real parts are,

$$\begin{aligned} &2Q_1'' - 2(k_1 + k_1)Q_1' + 2(k_1^2 + k_1^2)Q_1 + 2(aQ_1^3 + cQ_1Q_2^2 + bQ_2Q_1^2 + b^*Q_2Q_1^2) = 0 \\ &2Q_2'' - 2(k_1 + k_1)Q_2' + 2(k_1^2 + k_1^2)Q_2 + 2(aQ_1^2Q_2 + cQ_2^3 + bQ_1Q_2^2 + b^*Q_1Q_2^2) = 0 \end{aligned} \tag{3}$$

While the imaginary parts are,

$$\begin{aligned} &vQ_1' - wQ_1 = 0 \\ &vQ_2' - wQ_2 = 0 \end{aligned} \tag{4}$$

When one insert each one of the imaginary part into the corresponding one of the real part and use the transformation $\psi_2 = \mu\psi_1$ the system of real parts will be reduced to the same equation which is,.

$$Q_1'' + [(k_1^2 + k_1^2) - 2(k_1 + k_1)\frac{w}{v}]Q_1 + [a + c + b + b^*]Q_1^3 = 0 \tag{5}$$

When the homogeneous balance implemented between Q_1'' and Q_1^3 in equation (5) it implies $m = 1$.

The main purpose of this work extracting new expectations to the soliton arising from equation (5) in the framework three distinct techniques, the three techniques candidates for this target are the EDAM [34], the ESEM [35-38] and SWAM [39-43]. The suggested methods are examined previously to many other nonlinear differential equations which appearing in different branches of science and usually achieved good results.

This paper designed as follow, in section two the algorithm of the EDAM and its applications to constructing the soliton solutions of this model. In section three, the algorithm of the ESEM and its applications to generate the soliton solutions of this model. In section four the algorithm of the SWAM and its application to extracting the soliton solutions of the suggested model and in section five the conclusion of this article presented.

2. The EDAM algorithm

To discuss this technique let us firstly discuss the NLPDE algorithm by propose the function Υ as a function in R(x, t) and its partial derivatives as,

$$\Upsilon(R, R_x, R_t, R_{xx}, R_{tt}.........) = 0 \tag{6}$$

That contain the highest order derivatives and nonlinear terms.

With the aid of the transformation $R(x,t) = R(\zeta), \zeta = wx + kt$ equation (34) can be reduced to the following ODE:

$$Z(R, R', R'', R''', ...) = 0 \tag{7}$$

where Z in terms of $R(\zeta)$, its total derivatives, while $' = \dfrac{d}{d\zeta}$.

The solution of equation (7) can be written in the form [34],

$$Q(\eta) = \sum_{i=1}^{M} a_i \ \varphi^i(\eta), \qquad \varphi'^2 = \alpha\varphi^2 + \beta\varphi^3 + \gamma\varphi^4. \tag{8}$$

Now, we will apply this technique to the suggested model equation (5) mentioned above, since the balance is one thus the solution is

$$Q(\eta) = a_0 + a_1\varphi. \tag{9}$$

Via substituting about Q'', Q, Q^3 into equation (5)mentioned above which is

$$2Q'' + \left((k_1^2 + k_1^2) - 2(k_1 + k_1)\frac{w}{v}\right)Q + 2\left(a + c + b + b^*\right)Q^3 = 0 \ . \tag{10}$$

Via equating the coefficients of various power of Q^i to zero leads to system of equations from which the following results will be detected.

$$\begin{aligned}
&(1)\alpha = \frac{2w(k_1 + k_2) - v(k_1^2 + k_2^2)}{2v}, \gamma = -\frac{a_1^2}{2}(a + b + c + b^*), a_0 = \beta = 0 \\
&(2)\alpha = (k_1^2 + k_2^2) - \frac{2w}{v}(k_1 + k_2), \beta = -a_1 \frac{\sqrt{-2v(a+b+c+b^*)}\sqrt{v(k_1^2 + k_2^2) - 2w(k_1 + k_2)}}{v}, \\
&\quad \gamma = -\frac{a_1^2}{2}(a + b + c + b^*), a_0 = -\frac{\sqrt{v(k_1^2 + k_2^2) - 2w(k_1 + k_2)}}{\sqrt{-2v(a+b+c+b^*)}} \\
&(3)\alpha = (k_1^2 + k_2^2) - \frac{2w}{v}(k_1 + k_2), \beta = a_1 \frac{\sqrt{-2v(a+b+c+b^*)}\sqrt{v(k_1^2 + k_2^2) - 2w(k_1 + k_2)}}{v}, \\
&\quad \gamma = -\frac{a_1^2}{2}(a + b + c + b^*), a_0 = \frac{\sqrt{v(k_1^2 + k_2^2) - 2w(k_1 + k_2)}}{\sqrt{-2v(a+b+c+b^*)}}
\end{aligned} \tag{11}$$

These three results will generate three solutions, for simplicity and similarity we will implement only the first and the third one.

Firstly, for the first result which is:

$$(1)\alpha = \frac{2w(k_1 + k_2) - v(k_1^2 + k_2^2)}{2v}, \gamma = -\frac{a_1^2}{2}(a + b + c + b^*), a_0 = \beta = 0$$

From which we can obtain

$$\alpha = \frac{2w(k_1 + k_2) - v(k_1^2 + k_2^2)}{2v}, \gamma = -\frac{a_1^2}{2}(a + b + c + b^*), a_0 = \beta = 0 \tag{12}$$

This can be simplified to be

$\alpha = 1, \beta = 0, \gamma = 1, c = -3, a = -1, a_1 = w = v = k_1 = k_2 = 1, a_0 = 0, b = 1 + i.$

In the framework of this values and the relation $\varphi'^2 = \alpha\varphi^2 + \beta\varphi^3 + \gamma\varphi^4$, we can easily obtain $\varphi = \left(\frac{1+e^{\eta}}{1-e^{\eta}}\right)^2 - 1$, the solution in the framework of the proposed method is $Q(\eta) = a_0 + a_1\varphi$

$$Q(\eta) = \left(\frac{1+e^{\eta}}{1-e^{\eta}}\right)^2 - 1 \tag{13}$$

$$U(x,t) = \left\{\left(\frac{1+e^{\eta}}{1-e^{\eta}}\right)^2 - 1\right\} Expi(-k_1x - k_2y + wt + \theta_0) \tag{14}$$

$$\operatorname{Re}U(x,t) = \left\{\left(\frac{1+e^{\eta}}{1-e^{\eta}}\right)^2 - 1\right\} \cos(-k_1x - k_2y + wt + \theta_0) \tag{15}$$

$$\operatorname{Im}U(x,t) = \left\{\left(\frac{1+e^{\eta}}{1-e^{\eta}}\right)^2 - 1\right\} \sin(-k_1x - k_2y + wt + \theta_0) \tag{16}$$

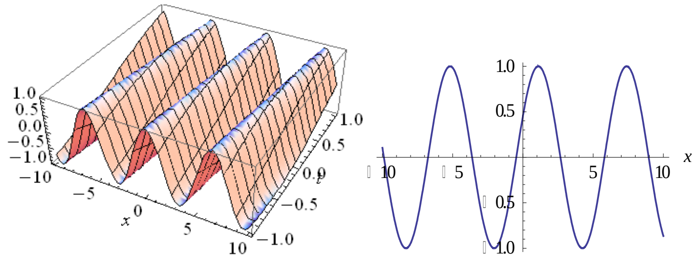

Fig.1: The Re. part Eq. (15) in 2-nd and 3-th dimensions when:
$\alpha = 1, \beta = 0, \gamma = 1, c = -3, a = -1, a_1 = w = v = k_1 = k_2 = 1, a_0 = 0, b = 1+i.$

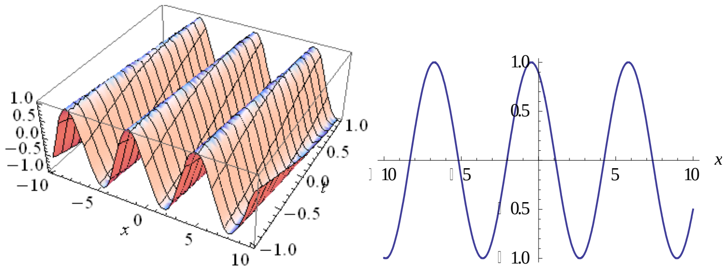

Fig.2: The Im. part Eq. (16) in 2-nd and 3-th dimensions when: $\zeta_0 = 1$
$\alpha = 1, \beta = 0, \gamma = 1, c = -3, a = -1, a_1 = w = v = k_1 = k_2 = 1, a_0 = 0, b = 1+i.$

Secondly, for the third result which is

$$\alpha = (k_1^2 + k_2^2) - \frac{2w}{v}(k_1 + k_2), \beta = a_1 \frac{\sqrt{-2v(a+b+c+b^*)}\sqrt{v(k_1^2+k_2^2) - 2w(k_1+k_2)}}{v},$$

$$\gamma = -\frac{a_1^2}{2}(a+b+c+b^*), a_0 = \frac{\sqrt{v(k_1^2+k_2^2) - 2w(k_1+k_2)}}{\sqrt{-2v(a+b+c+b^*)}}$$

This can be simplified to be
$\alpha = -2, \beta = -4, \gamma = 1, c = -3, a = -1, a_1 = w = v = k_1 = k_2 = 1, a_0 = 0.7, b = 1+i.$
In the framework of this values and the relation $\varphi'^2 = \alpha\varphi^2 + \beta\varphi^3 + \gamma\varphi^4$, we can easily obtain

$\varphi = \dfrac{-2}{2\sqrt{5}\sin\eta + 4}$, the solution in the framework of the proposed method is $Q(\eta) = a_0 + a_1\varphi$

$$Q(\eta) = 0.7 + \frac{-2}{2\sqrt{5}\sin\eta + 4} \tag{17}$$

$$U(x,t) = \left\{0.7 + \frac{-2}{2\sqrt{5}\sin\eta + 4}\right\} Expi(-k_1x - k_2y + wt + \theta_0) \tag{18}$$

$$\operatorname{Re}U(x,t) = \left\{0.7 + \frac{-2}{2\sqrt{5}\sin\eta + 4}\right\} \cos(-k_1x - k_2y + wt + \theta_0) \tag{19}$$

$$\operatorname{Im}U(x,t) = \left\{0.7 + \frac{-2}{2\sqrt{5}\sin\eta + 4}\right\} \sin(-k_1x - k_2y + wt + \theta_0) \tag{20}$$

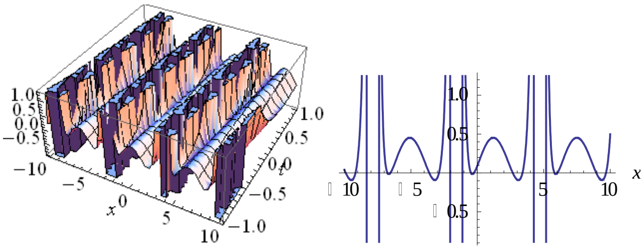

Fig.3: The Re. part Eq. (19) in 2-nd and 3-th dimensions when:
$\alpha = -2, \beta = -4, \gamma = 1, c = -3, a = -1, a_1 = w = v = k_1 = k_2 = 1, a_0 = 0.7, b = 1 + i.$

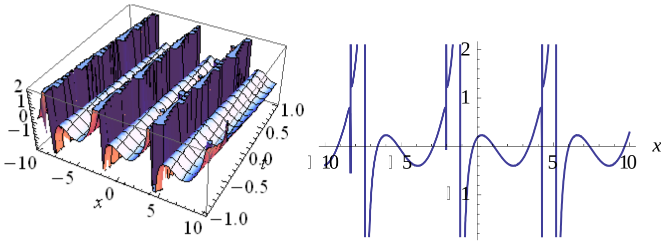

Fig.4: The Im. part Eq. (20) in 2-nd and 3-th dimensions when: $\zeta_0 = 1$
$\alpha = -2, \beta = -4, \gamma = 1, c = -3, a = -1, a_1 = w = v = k_1 = k_2 = 1, a_0 = 0.7, b = 1 + i.$

3. The ESEM algorithm

The solution of equation (7) in the framework of the ESEM is

$$Q(\zeta) = \sum_{i=-M}^{M} A_i\ \psi^i(\zeta). \tag{21}$$

Where $\psi(\zeta)$ achieves the equation,

$$\psi'(\zeta) = B_0 + B_1\psi + B_2\psi^2 \tag{22}$$

The parameters A_i will be located later, while the other parameters $B_0, B_1\ and\ B_2$ will established the following forms of solutions,

(1) If $B_1 = B_3 = 0$, Eq. (22) will be Riccati equation whose solutions are

$$\psi(\zeta)=\frac{\sqrt{B_0B_2}}{B_2}\tan(\sqrt{B_0B_2}(\zeta+\zeta_0),B_0B_2\succ 0 \tag{23}$$

$$\psi(\zeta)=\frac{\sqrt{-B_0B_2}}{B_2}\tanh(\sqrt{-B_0B_2}\zeta-\frac{\rho\ln\zeta_0}{2}),B_0B_2\prec 0,\zeta\succ 0,\rho=\pm 1 \tag{24}$$

(2) If $B_0=B_3=0,$ Eq. (22) will be the Bernoulli equation whose solutions are

$$\psi(\zeta)=\frac{B_1\,Exp[B_1(\zeta+\zeta_0)]}{1-B_2Exp[B_1(\zeta+\zeta_0)]}\ ,B_1\succ 0 \tag{25}$$

$$\psi(\zeta)=\frac{-B_1\,Exp[B_1(\zeta+\zeta_0)]}{1+B_2Exp[B_1(\zeta+\zeta_0)]}\ ,B_1\prec 0 \tag{26}$$

And the above solutions have the general forms which are;

$$\psi(\zeta)=-\frac{1}{B_2}\left(B_1-\sqrt{4B_1B_2-B_1^2}\tan\left(\frac{\sqrt{4B_1B_2-B_1^2}}{2}(\zeta+\zeta_0)\right)\right),4B_1B_2\succ B_1^2,B_2\succ 0, \tag{27}$$

$$\psi(\zeta)=\frac{1}{B_2}\left(B_1+\sqrt{4B_1B_2-B_1^2}\tanh\left(\frac{\sqrt{4B_1B_2-B_1^2}}{2}(\zeta+\zeta_0)\right)\right),4B_1B_2\succ B_1^2,B_2\prec 0, \tag{28}$$

Where the integer ζ_0 is the integration constancy

Lastly, by inserting Eq. (22) into Eq. (21), equating the coefficients of various powers of ψ^i to zero implies system of equations from which the values of the unknown variables will be detected. Furthermore, the required solutions can be detected by inserting the obtained values of these unknown variables into equations (21).
Now we will apply this technique to converted Eq. (10) mentioned above,

$$2Q''+\left((k_1^2+k_1^2)-2(k_1+k_1)\frac{w}{v}\right)Q+2\left(a+c+b+b^*\right)Q^3=0 \tag{29}$$

Since the balance is $M=1$ thus the solution is,

$$Q(\zeta)=\frac{A_{-1}}{\psi}+A_0+A_1\psi \tag{30}$$

Where $\psi'=B_0+B_1\psi+B_2\psi^2+B_3\psi^3$

Case 1: The 1st family in which $B_1=B_3=0\Rightarrow\psi'=B_0+B_2\psi^2$

$$Q'=-\frac{B_0A_{-1}}{\psi^2}+A_1B_0+A_1B_2\psi^2-B_2A_{-1} \tag{31}$$

$$Q''=\frac{2B_0^2A_{-1}}{\psi^3}+\frac{2B_0B_2A_{-1}}{\psi}+2A_1B_0B_2\psi+2A_1B_2^2\psi^3 \tag{32}$$

$$Q^3 = A_1^3\psi^3 + 3A_0A_1^2\psi^2 + (3A_1A_0^2 + 3A_{-1}A_1^2)\psi + (A_0^3 + 6A_{-1}A_0A_1) + \frac{A_{-1}^3}{\psi^3} + \frac{3A_0A_{-1}^2}{\psi^2} + \frac{3A_{-1}A_0^2 + 3A_1A_{-1}^2}{\psi}. \quad (33)$$

Via inserting the relations (30-33) into Eq. (29), equating the coefficients of various powers of ψ^i to zero implies a system of equations in terms of unknown variables and by solves it the following results will be achieved,

$$(1) w = \frac{v[4A_{-1}A_1(a+c+b+b^*) + (k_1^2+k_2^2)]}{2(k_1+k_2)}, B_0 = \frac{-iA_{-1}\sqrt{a+c+b+b^*}}{\sqrt{2}}, B_2 = \frac{-iA_1\sqrt{a+c+b+b^*}}{\sqrt{2}}, A_0 = 0. \quad (34)$$

$$(2) w = \frac{v[4A_{-1}A_1(a+c+b+b^*) + (k_1^2+k_2^2)]}{2(k_1+k_2)}, B_0 = \frac{-iA_{-1}\sqrt{a+c+b+b^*}}{\sqrt{2}}, B_2 = \frac{iA_1\sqrt{a+c+b+b^*}}{\sqrt{2}}, A_0 = 0. \quad (35)$$

$$(3) w = \frac{v[4A_{-1}A_1(a+c+b+b^*) + (k_1^2+k_2^2)]}{2(k_1+k_2)}, B_0 = \frac{iA_{-1}\sqrt{a+c+b+b^*}}{\sqrt{2}}, B_2 = \frac{-iA_1\sqrt{a+c+b+b^*}}{\sqrt{2}}, A_0 = 0. \quad (36)$$

$$(4) w = \frac{v[4A_{-1}A_1(a+c+b+b^*) + (k_1^2+k_2^2)]}{2(k_1+k_2)}, B_0 = \frac{iA_{-1}\sqrt{a+c+b+b^*}}{\sqrt{2}}, B_2 = \frac{iA_1\sqrt{a+c+b+b^*}}{\sqrt{2}}, A_0 = 0. \quad (37)$$

$$(5) B_0 = \frac{i[2w(k_1+k_2) - v(k_1^2+k_2^2)]}{2\sqrt{2}vA_1\sqrt{a+c+b+b^*}}, B_2 = \frac{-iA_1\sqrt{a+c+b+b^*}}{\sqrt{2}}, A_0 = A_{-1} = 0. \quad (38)$$

$$(6) B_0 = \frac{-i[2w(k_1+k_2) - v(k_1^2+k_2^2)]}{2\sqrt{2}vA_1\sqrt{a+c+b+b^*}}, B_2 = \frac{iA_1\sqrt{a+c+b+b^*}}{\sqrt{2}}, A_0 = A_{-1} = 0. \quad (39)$$

$$(7) B_0 = \frac{iA_{-1}\sqrt{a+c+b+b^*}}{\sqrt{2}}, B_2 = \frac{-i[2w(k_1+k_2) - v(k_1^2+k_2^2)]}{2\sqrt{2}vA_{-1}\sqrt{a+c+b+b^*}}, A_0 = A_1 = 0. \quad (40)$$

$$(8) B_0 = \frac{-iA_{-1}\sqrt{a+c+b+b^*}}{\sqrt{2}}, B_2 = \frac{i[2w(k_1+k_2) - v(k_1^2+k_2^2)]}{2\sqrt{2}vA_{-1}\sqrt{a+c+b+b^*}}, A_0 = A_1 = 0. \quad (41)$$

These eight results will generate eight solutions, for simplicity we will draw only one say the first identical to the first result which is

$$w = \frac{v[4A_{-1}A_1(a+c+b+b^*) + (k_1^2+k_2^2)]}{2(k_1+k_2)}, B_0 = \frac{-iA_{-1}\sqrt{a+c+b+b^*}}{\sqrt{2}}, B_2 = \frac{-iA_1\sqrt{a+c+b+b^*}}{\sqrt{2}}, A_0 = 0.$$

This result can be simplified to be

$$v = k_1 = k_2 = A_{-1} = A_1 = 1, a = -1, c = -3, w = -1.5, B_0 = 1, B_2 = 1, A_0 = 0, b = 1+i, b^* = 1-i \quad (42)$$

According to these values the proposed solution is

$$Q(\zeta) = \frac{A_{-1}}{\psi} + A_0 + A_1\psi\,,\ \psi(\zeta) = \frac{\sqrt{B_0B_2}}{B_2}\tan(\sqrt{B_0B_2}(\zeta+\zeta_0)), B_0B_2 \succ 0$$

$$\psi(\zeta) = \tan(x - t + 1) \quad (43)$$

$$Q(\zeta) = \cot(x - t + 1) + \tan(x - t + 1) \quad (44)$$

$$U(x,t)=\left(\cot(x-t+1)+\tan(x-t+1)\right)Expi(-x+1.5t+1.1) \tag{45}$$

$$\mathrm{Re}\,U(x,t)=\left(\cot(x-t+1)+\tan(x-t+1)\right)\times\cos(-x+1.5t+1.1) \tag{46}$$

$$\mathrm{Im}\,U(x,t)=\left(\cot(x-t+1)+\tan(x-t+1)\right)\times\sin(-x+1.5t+1.1) \tag{47}$$

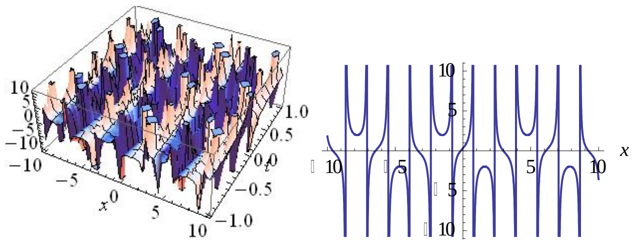

Fig. 5: The Re. part Eq. (46) in **2D** and **3D** with values: $\zeta_0=1$
$v=k_1=k_2=A_{-1}=A_1=1, a=-1, c=-3, w=-1.5, B_0=1, B_2=1, A_0=0, b=1+i, b^*=1-i$

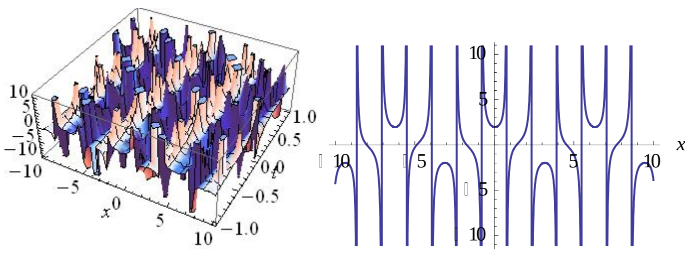

Fig.6: The Im. part Eq. (47) in **2D** and **3D** with values: $\zeta_0=1$
$v=k_1=k_2=A_{-1}=A_1=1, a=-1, c=-3, w=-1.5, B_0=1, B_2=1, A_0=0, b=1+i, b^*=1-i$

By the same steps we can draw the other cases

Case 2: The 2nd family in which $B_0=B_3=0 \Rightarrow \psi'=B_1+B_2\psi^2$

$$Q(\zeta)=\frac{A_{-1}}{\psi}+A_0+A_1\psi, \tag{48}$$

$$Q'=A_1B_2\psi^2+B_1A_1\psi-\frac{A_{-1}B_1}{\psi}-A_{-1}B_2, \tag{49}$$

$$Q''=2A_1B_2^2\psi^3+3A_1B_1B_2\psi^2+A_1B_1^2\psi+A_{-1}B_1B_2+\frac{B_1^2A_{-1}}{\psi}. \tag{50}$$

$$\begin{aligned}Q^3&=A_1^3\psi^3+3A_0A_1^2\psi^2+(3A_1A_0^2+3A_{-1}A_1^2)\psi+(A_0^3+6A_{-1}A_0A_1)\\&+\frac{A_{-1}^3}{\psi^3}+\frac{3A_0A_{-1}^2}{\psi^2}+\frac{3A_{-1}A_0^2+3A_1A_{-1}^2}{\psi}.\end{aligned} \tag{51}$$

Via inserting the relations (48-51) into Eq. (26), equating the coefficients of various powers of ψ^i to zero implies a system of equations in terms of unknown variables and by solves it the following results will be achieved

$$(1)\, w=\frac{v[2A_0^2(a+c+b+b^*)+(k_1^2+k_2^2)]}{2(k_1+k_2)}, B_1=-\sqrt{2}\,iA_0\sqrt{a+c+b+b^*}, B_2=\frac{-iA_1\sqrt{a+c+b+b^*}}{\sqrt{2}}, A_{-1}=0. \tag{52}$$

$$(2) w=\frac{v[2A_0^2(a+c+b+b^*)+(k_1^2+k_2^2)]}{2(k_1+k_2)}, B_1=\sqrt{2}\, iA_0\sqrt{a+c+b+b^*}, B_2=\frac{iA_1\sqrt{a+c+b+b^*}}{\sqrt{2}}, A_{-1}=0. \tag{53}$$

$$(3) A_0=\frac{-ik_2}{\sqrt{a+c+b+b^*}}, B_1=-\sqrt{2}k_2, B_2=\frac{-iA_1\sqrt{a+c+b+b^*}}{\sqrt{2}}, A_{-1}=0, k_1=-k_2. \tag{54}$$

$$(4) A_0=\frac{-ik_2}{\sqrt{a+c+b+b^*}}, B_1=\sqrt{2}k_2, B_2=\frac{iA_1\sqrt{a+c+b+b^*}}{\sqrt{2}}, A_{-1}=0, k_1=-k_2. \tag{55}$$

$$(5) A_0=\frac{ik_2}{\sqrt{a+c+b+b^*}}, B_1=-\sqrt{2}k_2, B_2=\frac{iA_1\sqrt{a+c+b+b^*}}{\sqrt{2}}, A_{-1}=0, k_1=-k_2. \tag{56}$$

$$(6) A_0=\frac{ik_2}{\sqrt{a+c+b+b^*}}, B_1=\sqrt{2}k_2, B_2=\frac{-iA_1\sqrt{a+c+b+b^*}}{\sqrt{2}}, A_{-1}=0, k_1=-k_2. \tag{57}$$

These six results will generate six solutions, for simplicity we will draw only one say the first identical to the first result which is

$$w=\frac{v[2A_0^2(a+c+b+b^*)+(k_1^2+k_2^2)]}{2(k_1+k_2)}, B_1=-\sqrt{2}\, iA_0\sqrt{a+c+b+b^*}, B_2=\frac{-iA_1\sqrt{a+c+b+b^*}}{\sqrt{2}}, A_{-1}=0.$$

This result can be simplified to be

$$v=k_1=k_2=A_0=A_1=1, w=-0.5, a=-1, c=-3, B_1=2, B_2=1, A_{-1}=0, b=1+i, b^*=1-i \tag{58}$$

According to these values the solution from point of view of the proposed method is

$$Q(\zeta)=\frac{A_{-1}}{\psi}+A_0+A_1\psi\,,\ \psi(\zeta)=\frac{B_1\, Exp[B_1(\zeta+\zeta_0)]}{1-B_2 Exp[B_1(\zeta+\zeta_0)]}\,,\ B_1\succ 0$$

$$\psi(\zeta)=\frac{2Exp[2(x-t+1)]}{1-Exp[2(x-t+1)]} \tag{59}$$

$$Q(\zeta)=1+\frac{2Exp[2(x-t+1)]}{1-Exp[2(x-t+1)]} \tag{60}$$

$$U(x,t)=\left(1+\frac{2Exp[2(x-t+1)]}{1-Exp[2(x-t+1)]}\right)Expi(-x+0.5t+1.1) \tag{61}$$

$$\mathrm{Re}U(x,t)=\left(1+\frac{2Exp[2(x-t+1)]}{1-Exp[2(x-t+1)]}\right)\times\cos(-x+0.5t+1.1) \tag{62}$$

$$\mathrm{Im}U(x,t)=\left(1+\frac{2Exp[2(x-t+1)]}{1-Exp[2(x-t+1)]}\right)\times\sin(-x+0.5t+1.1) \tag{63}$$

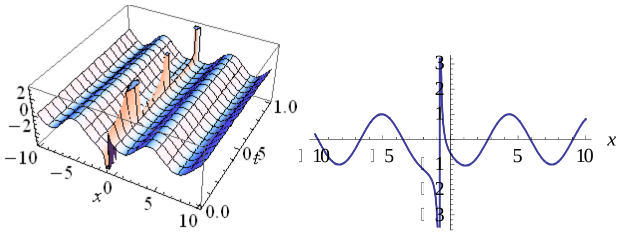

Fig. 7: The Re. part Eq. (62) in **2D** and **3D** with values: $\zeta_0 = 1$

$$v = k_1 = k_2 = A_0 = A_1 = 1, w = -0.5, a = -1, c = -3, B_1 = 2, B_2 = 1, A_{-1} = 0, b = 1+i, b^* = 1-i$$

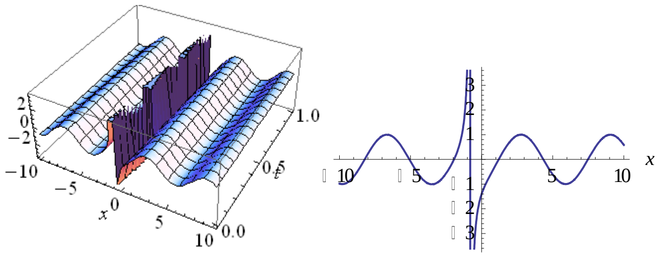

Fig. 8: The Im. part Eq. (63) in **2D** and **3D** with values: $\zeta_0 = 1$

$$v = k_1 = k_2 = A_0 = A_1 = 1, w = -0.5, a = -1, c = -3, B_1 = 2, B_2 = 1, A_{-1} = 0, b = 1+i, b^* = 1-i$$

By the same steps we can draw the other solutions.

4. The SWAM algorithm

From point of view of the SWAM [38, 42] the solution can be proposed in the form,

$$U(x,t) = \psi(x,t) e^{iR_1(x,t)} \tag{64}$$

Where $\psi(x,t)$ and $R(x,t)$ are the amplitude portion and the phase portion of soliton respectively. Hence, via simple calculation of Eq.(2) we get the following relations,

$$U_t = (\psi_t + i\psi R_{1t}) e^{iR_1} \tag{65}$$

$$U_x = (\psi_x + i\psi R_{1x}) e^{iR_1} \tag{66}$$

$$U_{xx} = (\psi_{xx} + 2i\psi_x R_{1x} + i\psi R_{1xx} - \psi R_{1x}^2) e^{iR_1} \tag{67}$$

$$U_{yy} = (\psi_{yy} + 2i\psi_y R_{1y} + i\psi R_{1yy} - \psi R_{1y}^2) e^{iR_1} \tag{68}$$

The two parts in equation (2) are the same when $U = \mu V$ and can be converted to

$$iU_t + U_{xx} + U_{yy} + 2(a|U^2| + c|U^2| + b|U^2| + b^*|U^2|)U = 0 \tag{69}$$

Via inserting the relations (61-65) into the above equation the following real and imaginary parts,

$$\Omega\psi + \psi_{xx} + \psi_{yy} - (R_{1x}^2 + R_{1y}^2)\psi + 2(a + c + b + b^*)\psi^3 = 0 \tag{70}$$

$$\psi_t + 2R_{1x}\psi_x + 2R_{1y}\psi_y = 0 \tag{71}$$

The bright solutions according to the proposed method [35-38] can be extracted as follow,

$$\psi(x,t) = A_1 \operatorname{sech}^{R_1} t_1,\ where\ t_1 = B(x + y - wt)\ and\ R_1(x,t) = kx + \delta y - \Omega t \tag{72}$$

$$\psi_t = -A_1 B w_1 R_1 \operatorname{sech}^{R_1} t_1 \tanh t_1 \tag{73}$$

$$\psi_x = A_1 B R_1 \operatorname{sech}^{R_1} t_1 \tanh t_1 \tag{74}$$

$$\psi_{xx} = A_1 B^2 R_1 (1 + R_1) \operatorname{sech}^{R_1+2} t_1 - A_1 B^2 R_1^2 \operatorname{sech}^{R_1} t_1 \tag{75}$$

$$\psi_{yy} = A_1 B^2 R_1 (1 + R_1) \operatorname{sech}^{R_1+2} t_1 - A_1 B^2 R_1^2 \operatorname{sech}^{R_1} t_1 \tag{76}$$

By inserting the relations (72-76) into the real and imaginary parts equations (70), (71) we obtain

$$(\Omega - k^2 - \delta^2)A_1 \operatorname{sech}^{R_1} t_1 + 2A_1 B^2 R_1(1+R_1)\operatorname{sech}^{R_1+2} t_1$$
$$-2A_1 B^2 R_1^2 \operatorname{sech}^{R_1} t_1 + 2(a+c+b+b^*)A_1^3 \operatorname{sech}^{3R_1} t = 0 \tag{77}$$

$$-A_1 B w_1 R_1 \operatorname{sech}^{R_1} t_1 \tanh t_1 + 2(k++\delta)A_1 B R_1 \operatorname{sech}^{R_1} t_1 \tanh t_1 = 0 \tag{78}$$

When equivalence operation is implemented for the higher of $\operatorname{sech}^{nR_1} t_1$ in the real part this implies $R_1 = 1$

$$(\Omega - k^2 - \delta^2 - 2B^2)A_1 \operatorname{sech} t_1 + [4A_1 B^2 + 2A_1^3(a+c+b+b^*)]\operatorname{sech}^3 t = 0 \tag{79}$$

From which we obtain these relations $\Omega = k^2 + \delta^2 + 2B^2, A_1^2 = \dfrac{-2B^2}{a+c+b+b^*}$

Moreover, the imaginary part implies $w_1 = 2(k++\delta)$. When we take the same values of parameters like that chooses for the above two methods we get,

$$\Omega = 4, A_1 = \pm 1, w = 4, B = \delta = k = 1, a = -1, c = -3 \tag{80}$$

Thus the bright solution in the framework of these parameters is

$$U(x,t) = \pm \operatorname{sech}[x+y-4t]e^{i(x+y-4t)} \tag{81}$$

$$\operatorname{Re}U(x,t) = \left(\pm \operatorname{sech}[x+y-4t]\right) \times \cos(x+y-4t) \tag{82}$$

$$\operatorname{Im}U(x,t) = \left(\pm \operatorname{sech}[x+y-4t]\right) \times \sin(x+y-4t) \tag{83}$$

We will draw only the positive one

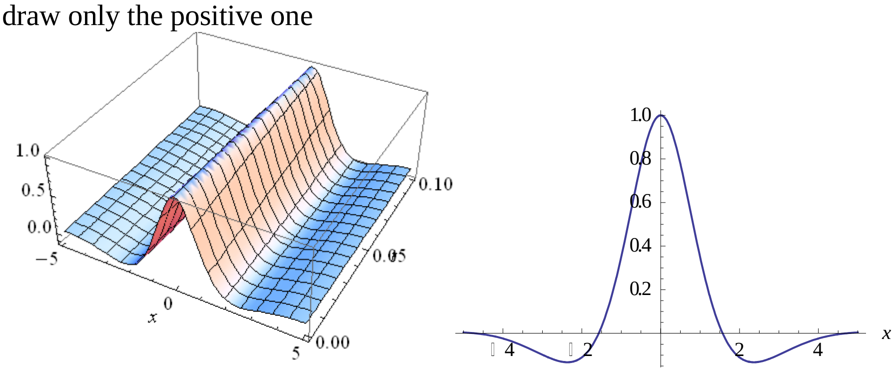

Fig. 9: The Bright soliton Re. part Eq. (82) in **2D** and **3D** with values:
$v = k_1 = k_2 = B = \delta = k = 1, \Omega = 4, A_1 = 1, w = 4, a = -1, c = -3a = -1, c = -3, b = 1+i, b^* = 1-i$

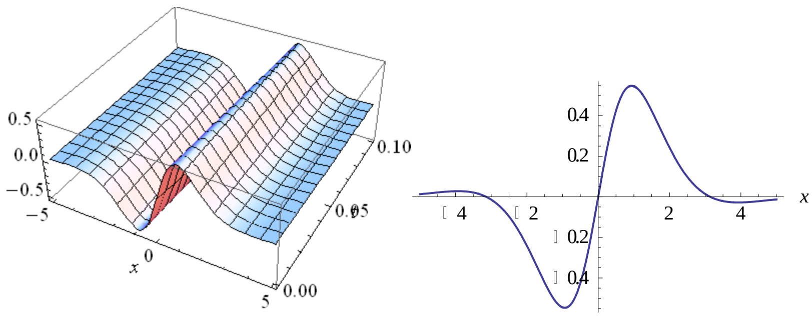

Fig. 10: The Bright soliton Im. part Eq. (83) in **2D** and **3D** with values:
$v = k_1 = k_2 = B = \delta = k = 1, \Omega = 4, A_1 = 1, w = 4, a = -1, c = -3a = -1, c = -3, b = 1+i, b^* = 1-i$

The dark solutions according to the proposed method [35-38] can be extracted as follow,

$$\psi(x,t) = A_2 \tanh^{R_2} t_2 \text{ where } t_2 = B(x+y-w_2 t) \text{ and } R_2(x,y,t) = kx + \delta y - \Omega t \tag{84}$$

$$\psi_t = A_2 w_2 B R_2 [\tanh^{R_2+1} t_2 - \tanh^{R_2-1} t_2] \tag{85}$$

$$\psi_x = A_2 B R_2 [\tanh^{R_2-1} t_2 - \tanh^{R_2+1} t_2] \tag{86}$$

$$\psi_{xx} = A_2 R_2 (R_2-1) B^2 \tanh^{R_2-2} t_2 - 2A_2 R_2^2 B^2 \tanh^{R_2} t_2 + A_2 R_2 (R_2+1) B^2 \tanh^{R_2+2} t_2 \tag{87}$$

$$\psi_{yy} = A_2 R_2 (R_2-1) B^2 \tanh^{R_2-2} t_2 - 2A_2 R_2^2 B^2 \tanh^{R_2} t_2 + A_2 R_2 (R_2+1) B^2 \tanh^{R_2+2} t_2 \tag{88}$$

By inserting the relations (84-88) into the real and imaginary parts equations (70), (71) we obtain

$$\begin{gathered}(\Omega - k^2 - \delta^2 - 4R_2^2 B^2) A_2 \tanh^{R_2} t_2 + 2A_2 R_2 (R_2-1) B^2 \tanh^{R_2-2} t_2 \\ +2A_2 R_2 (R_2+1) B^2 \tanh^{R_2+2} t_2 + 2(a+c+b+b^*) A_2^3 \tanh^{3R_2} t_2 = 0\end{gathered} \tag{89}$$

$$A_2 w_2 B R_2 [\tanh^{R_2+1} t_2 - \tanh^{R_2-1} t_2] + 2(k+\delta) A_2 B R_2 [\tanh^{R_2-1} t_2 - \tanh^{R_2+1} t_2] = 0 \tag{90}$$

When equivalence operation is implemented for the higher of $\tanh^{nR_2} t_2$ in the real part this implies $R_2 = 1$ hence the real and imaginary parts will be

$$(\Omega - k^2 - \delta^2 - 4B^2) A_2 \tanh t_2 + 4A_2 B^2 \tanh^3 t_2 + 2(a+c+b+b^*) A_2^3 \tanh^3 t_2 = 0 \tag{91}$$

$$A_2 w_2 B [\tanh^2 t_2 - 1] + 2(k+\delta) A_2 B [1 - \tanh^2 t_2] = 0 \tag{92}$$

Consequently, from the real part we get $\Omega = k^2 + \delta^2 + 4B^2$, $A_2^2 = \dfrac{-2B^2}{(a+c+b+b^*)}$ and the imaginary part implies $w_2 = 2(k+\delta)$.

From which we can get $w_2 = 4$, $\Omega = 6$, $A_2^2 = 1$ and the dark solution is

$$U(x,t) = \pm \tanh[x+y-4t] e^{i(x+y-6t)} \tag{93}$$

$$\operatorname{Re} U(x,t) = \left(\pm \tanh[x+y-4t]\right) \times \cos(x+y-6t) \tag{94}$$

$$\operatorname{Im} U(x,t) = \left(\pm \tanh[x+y-4t]\right) \times \sin(x+y-6t) \tag{95}$$

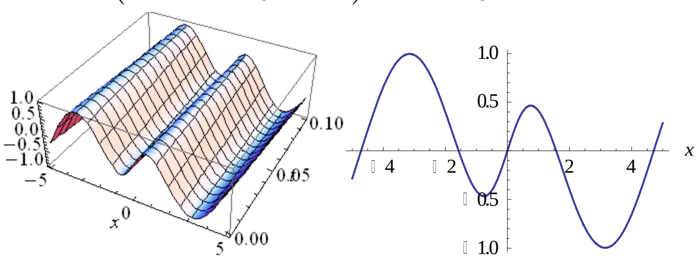

Fig. 11: The dark soliton Re. part Eq. (94) in **2D** and **3D** with values:
$v = k_1 = k_2 = B = \delta = k = 1, \Omega = 6, A_1 = 1, w = 4, a = -1, c = -3a = -1, c = -3, b = 1+i, b^* = 1-i$

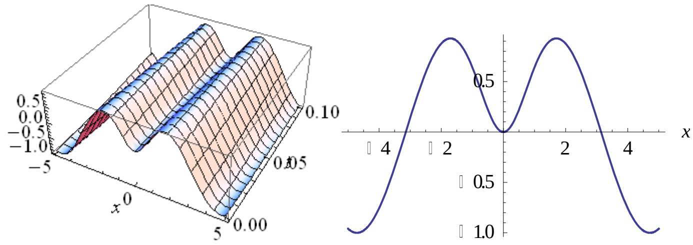

Fig. 12: The dark soliton Im. part Eq. (95) in **2D** and **3D** with values:
$v = k_1 = k_2 = B = \delta = k = 1, \Omega = 6, A_1 = 1, w = 4, a = -1, c = -3a = -1, c = -3, b = 1+i, b^* = 1-i$

5. Conclusion

From the point of view for three various manners which are the EDAM, the ESEM and the SWAM we detected new impressive expectations of solitons for the generalized (2+1) nonlinear Schrödinger equation with four waves mixing. The three manners have been applied for the first time to construct these new various solitons of this model. We success to determine the speed and propagation direction of the resultant solitons, reduce the interactions between two or more of two propagating waves via the four waves mixing. Furthermore, through this article we establish many new impressive visions for solitons arising from the suggested model via EDAM figures [1-4], ESEM figures [5-8] and the SWAM figures [9-12]. The novelty of our new achieved solitons for this model is clear when it compared with that obtained previously [26-30] who applied different techniques.

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