Accurate perceptions of the bright and dark soliton solutions to the modified nonlinear Schrödinger equation

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Abstract

In this study, we will implement new perceptions for the bright and dark soliton solutions to the modified nonlinear Schrödinger equation (MNLSE)or forms of the rogue wave modes for a derivative nonlinear Schrodinger model with positive linear dispersion which describe the propagation of rogue waves in Ocean engineering as well as all similar waves such as dynamics waveguides that have unexpected large displacements, the waves which occur only in the regime of positive cubic nonlinearity, regime that coincides exactly with the existence of instabilities of plane waves , long-wave limit of a breather (a pulsing mode). Two famous different schemas are involved for this purpose. The first schema is the solitary wave ansatze method (SWAM), while the second scheme is the extended simple equation method (ESEM). The two schemas are implemented in the same vein and parallel to construct new perceptions to the soliton solutions of this model. A comparison between the obtained new perceptions with the old perceptions that achieved previously by other authors has been demonstrated.

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ABSTRACT

In this study, we will implement new perceptions for the bright and dark soliton solutions to the modified nonlinear Schrödinger equation (MNLSE)or forms of the rogue wave modes for a derivative nonlinear Schrodinger model with positive linear dispersion which describe the propagation of rogue waves in Ocean engineering as well as all similar waves such as dynamics waveguides that have unexpected large displacements, the waves which occur only in the regime of positive cubic nonlinearity, regime that coincides exactly with the existence of instabilities of plane waves , long-wave limit of a breather (a pulsing mode). Two famous different schemas are involved for this purpose. The first schema is the solitary wave ansatze method (SWAM), while the second scheme is the extended simple equation method (ESEM). The two schemas are implemented in the same vein and parallel to construct new perceptions to the soliton solutions of this model. A comparison between the obtained new perceptions with the old perceptions that achieved previously by other authors has been demonstrated.

Keywords: The modified nonlinear Schrödinger equation; the solitary wave ansatze method, the extended simple equation method; soliton solutions.

1-Introduction

The Schrödinger equation is considered as the main base of many phenomena arising in different branches of physics such as atomic and nuclear physics, optics, plasma physics, fluid-dynamics, etc. Recently several different forms of this equations have been discovered to represent many phenomena in different branches of physics. This article focused on the famous one of these forms of this equation which is the MNLSE that represents the propagation of random waves in Ocean engineering as long-wave which widely occurs in fluid dynamics and optical waveguides that have unexpected large displacements. Two different perceptions for the accurate solution to this model have been established via two different techniques. The first technique is implemented through the SWAM [1-3], while the second technique is implemented via the ESEM [4-6]**.** These two perceptions are implemented successfully by these two distinct schemas which are invited for this purpose.

Several different forms of the Schrödinger equation have been studied via big number of authors through their suggested methods which treat many phenomenon behavior in various branches of sciences to achieve the exact and hence the solitary solutions of these phenomenon. See for example, Bekir and Zahran [6] who achieved the bright and dark soliton solutions to the complex Kundu-Eckhaus equation which represents a general form of integrable system that is governed by the equivalent to the mixed nonlinear Schrodinger equation, Bekir and Zahran [7] who extracted three distinct and impressive visions for the soliton solutions to the higher-order nonlinear Schrodinger equation, Mirzazadeh et al. [8] who obtained the optical soliton solutions to the Kundu–Eckhaus equation with general coefficients using the Riccati–Bernoulli's sub-ODE method as well as Kudryashov's scheme, Biswas et al. [9] who extracted the soliton solutions from the Lakshmanan–Porsezian–Daniel model by the aid of the modified simple equation method, Biswas [10] who extracted the optical soliton cooling with polynomial law of nonlinear refractive index via the perturbation theory, Seadawy et al. [11] who achieved the bright and dark solitary wave soliton solutions for the generalized higher order nonlinear Schrödinger equation and its stability, Vinita and Ray [12] who used the Lie symmetry analysis to achieve the invariant solution and similarity reduction of the resonance nonlinear Schrödinger equation, Raza et al. [13] who established the optical solitons and stability analysis for the generalized second-order nonlinear Schrödinger equation in an optical fiber. Moreover, big number of manners which were applied to solve many forms of the NLPDE arising in different nonlinear phenomenons was listed through references [14-31].

Specially, few tries were constructed through some authors to demonstrate the soliton solutions to the MNLSE using different methods namely, Stéphane et al. [32] who apply the extended (G'/G) method to the modified nonlinear Schrodinger equation in the case of ocean rogue waves, Chan et al. [33] who calculated the rogue waves of a derivative non-linear Schrödinger equation as a long-wave limit of a breather (a pulsing mode) which widely occurs in fluid dynamics and optical waveguides that have unexpected large displacements, Yu and Yan [34] who constructed explicit rouge wave solutions and dark-bright solutions for the inhomogeneous coupled nonlinear Schrödinger equation with variable coefficients by means of similarity transformations and Younis et al. [35] who used the extended Fan sub-equation method with five parameters to achieve new families of exact traveling wave solutions for the modified nonlinear Schrödinger equation.

According to [32-35], the MNLSE can be proposed in the form,

$$iQ_t + \gamma_1 Q_{xx} + \gamma_2 Q\left|Q^2\right| = i\alpha_1 Q_{xxx} + i\alpha_2 Q^2 Q_x^* - i\alpha_3 Q_x \left|Q^2\right| + \alpha_4 Q \tag{1}$$

Where the slightly changes in the boundary region of the random waves in Ocean engineering are governed by the complex function $Q(x,t)$,

$$\gamma_1 = \frac{w}{8k^2(2-3\operatorname{Cos}\theta)}, \gamma_2 = \frac{-wk^2}{2}, \alpha_1 = \frac{-w\operatorname{Cos}\theta}{16k^3(6+5\operatorname{Cos}^2\theta)},$$
$$\alpha_2 = \frac{-wk\operatorname{Cos}\theta}{4}, \alpha_3 = \frac{3wk}{2}, \alpha_4 = k\left|Q\right|_x^2\Big|_{x=0}.$$

And k, w are the wave number and frequency of the carrier wave respectively.

This paper is organized as follow, in sections two and three we will give description of the SWAM and its application to find the soliton solution of MNLSE respectively. In sections four and five the ESEM schema and its application to find other new perceptions of soliton solution of this model respectively. In section six brief conclusion of our work has been established.

The main purpose of this study is to implement new different perceptions of the optical soliton solution to the MNLSE (in terms of some parameters) using these two various schemas which are introduced above. If we give definite values for the appearing parameters in these exact solutions, new perceptions of the solitary solutions could be demonstrated.

2. Description of the SWAM

According to [1, 3] the SWAM solutions can be proposed as follows,

Consider the wave transformation,

$$Q(x,t)=\psi(x,t)e^{iR(x,t)} \tag{2}$$

Where $\psi(x,t)$ and $R(x,t)$ are the amplitude portion and the phase portion of soliton respectively. Hence, via simple calculation of Eq.(2) we get the following relations,

$$Q_t=(\psi_t+i\psi R_t)e^{iR} \tag{3}$$

$$Q_x=(\psi_x+i\psi R_x)e^{iR} \tag{4}$$

$$Q_{xx}=(\psi_{xx}+2i\psi_x R_x+i\psi R_{xx}-\psi R_x^2)e^{iR} \tag{5}$$

$$Q_{xxx}=(\psi_{xxx}+3i\psi_{xx}R_x-i\psi R_x^3-3\psi_x R_x^2+i\psi R_{xxx}+3i\psi_x R_{xx}-3\psi R_x R_{xx})e^{iR} \tag{6}$$

Consequently, the bright and dark soliton solutions can be implemented as follows,

(I) The bright soliton solutions

$$\psi(x,t)=A_1\operatorname{sech}^{R_1} t_1,\ where\ t_1=B(x-w_1t)\ and\ R_1(x,t)=kx-\Omega t \tag{7}$$

$$\psi_t=-A_1Bw_1R_1\operatorname{sech}^{R_1} t_1\tanh t_1 \tag{8}$$

$$\psi_x=A_1BR_1\operatorname{sech}^{R_1} t_1\tanh t_1 \tag{9}$$

$$\psi_{xx}=A_1B^2R_1(1+R_1)\operatorname{sech}^{R_1+2} t_1-A_1B^2R_1^2\operatorname{sech}^{R_1} t_1 \tag{10}$$

$$\psi_{xxx}=A_1B^3R_1(R_1+1)(R_1+2)\operatorname{sech}^{R_1+2} t_1\tanh t_1-A_1B^3R_1^3\operatorname{sech}^{R_1} t_1\tanh t_1 \tag{11}$$

(II) The dark soliton solutions

$$\psi(x,t)=A_2\tanh^{R_2} t_2,\ where\ t_2=B(x-w_2t)\ and\ R_2(x,t)=kx-\Omega t \tag{12}$$

$$\psi_t = A_2 w_2 B R_2[\tanh^{R_2+1} t_2 - \tanh^{R_2-1} t_2] \tag{13}$$

$$\psi_x = A_2 B R_2[\tanh^{R_2-1} t_2 - \tanh^{R_2+1} t_2] \tag{14}$$

$$\psi_{xx} = A_2 R_2 (R_2-1) B^2 \tanh^{R_2-2} t_2 - 2A_2 R_2^2 B^2 \tanh^{R_2} t_2 + A_2 R_2 (R_2+1) B^2 \tanh^{R_2+2} t_2 \tag{15}$$

$$\begin{aligned}\psi_{xxx} &= A_1 R_1 (R_1-1)(R_1-2) B^3 \tanh^{R_1-3}(t_1) - [2A_1 B^3 R_1^3 + A_1 R_1 (R_1-1)(R_1-2) B^3] \tanh^{R_1-1}(t_1) \\ &+ A_1 B^3 R_1 [2R_1^2 + (R_1+1)(R_1+2)] \tanh^{R_1+1}(t_1) - A_1 B^3 R_1 (R_1+1)(R_1+2) \tanh^{R_1+3}(t_1)\end{aligned} \tag{16}$$

3. The bright and dark soliton solutions to MNLSE

Substituting about $Q, Q_t, Q_x, Q_{xx}, Q_{xxx}$ the relations (2-6) at equation (1), we get,

$$\begin{aligned}& i(\psi_t + i\psi R_t) e^{iR} + \gamma_1[\psi_{xx} + 2i\psi_x R_x + i\psi R_{xx} - \psi R_x^2] e^{iR} \\ & = i\alpha_1[\psi_{xxx} + 3i\psi_{xx} R_x - i\psi R_x^3 - 3\psi_x R_x^2 + i\psi R_{xxx} + 3i\psi_x R_{xx} - 3\psi R_x R_{xx}] e^{iR} + \\ & i\alpha_2 \psi^2 e^{2iR} (\psi_x - i\psi R_x) e^{-iR} - i\alpha_3 \psi^2 (\psi_x + i\psi R_x) e^{iR} + \alpha_4 \psi e^{iR}\end{aligned} \tag{17}$$

This can be splits into two parts one real and the other is imaginary which are given respectively as,

$$(\gamma_1 + 3k\alpha_1)\psi_{xx} - k(\alpha_2 + \alpha_3)\psi^3 + (\Omega - \gamma_1 - \alpha_1 - \alpha_4)\psi = 0, \tag{18}$$

$$\alpha_1 \psi_{xxx} + (\alpha_2 + \alpha_3)\psi^2 \psi_x - (3\alpha_1 k^2 + 2\gamma_1 k)\psi_x - \psi_t = 0. \tag{19}$$

3.1 The bright soliton solution

Now; use the constructed relations (7-11) into the real part equation (18) we obtain:

$$\begin{aligned}&(\gamma_1 + 3k\alpha_1) A_1 B^2 R_1 (R_1+1) \operatorname{sech}^{R_1+2} t_1 + k(\alpha_2 + \alpha_3) A_1^3 \operatorname{sech}^{3R_1} t_1 \\ & -[(\gamma_1 + 3k\alpha_1) B^2 R_1^2 + (\Omega - \gamma_1 - \alpha_1 - \alpha_4)] A_1 \operatorname{sech}^{R_1} t_1 = 0,\end{aligned} \tag{20}$$

$$\begin{aligned}&\alpha_1 A_1 B^3 R_1 (R_1+1)(R_1+2) \operatorname{sech}^{R_1+2}(t_1) \tanh t_1 - B R_1 (\alpha_2 + \alpha_3) A_1^3 \operatorname{sech}^{3R_1} t_1 \tanh t_1 \\ & + A_1 B R_1 [(3\alpha_1 k^2 + 2\gamma_1 k) - \alpha_1 B^2 R_1^2 - w_1] \operatorname{sech}^{R_1} t_1 \tanh t_1 = 0.\end{aligned} \tag{21}$$

From equations (20), (21), by equating the highest exponents of $\operatorname{sech}^i t_1$ we get $R_1 = 1$, hence, we can establish these relations,

$$\begin{aligned}& A_1^2 = \frac{-2B^2(\gamma_1 + 3k\alpha_1)}{k(\alpha_2 + \alpha_3)}, \\ & B^2(\gamma_1 + 3k\alpha_1) = -(\Omega - \gamma_1 - \alpha_1 - \alpha_4), \\ & 6\alpha_1 B^2 = (\alpha_2 + \alpha_3) A_1, \\ & A_1 B[(2\gamma_1 k + 3\alpha_1 k^2) - \alpha_1 B^2 - w_1] = 0.\end{aligned} \tag{22}$$

From which we can easily obtain,

$$
\begin{aligned}
A_1 &= \frac{2(\Omega-\gamma_1-\alpha_1-\alpha_4)}{k(\alpha_2+\alpha_3)},\\
B^2 &= \frac{(\Omega-\gamma_1-\alpha_1-\alpha_4)}{3k\alpha_1},\\
w_1 &= \alpha_1 B^2-(2\gamma_1 k+3\alpha_1 k^2).
\end{aligned}
\tag{23}
$$

Hence we will achieve these results $A_1=-1.5, B=\pm\sqrt{0.6}\,i, w_1=-16$ under the same values used in [33], hence the solution is,

$$Q(x,t)=-1.5\,\mathrm{sech}(\sqrt{0.6}i(x+16t)e^{i(x-t)} \tag{24}$$

$$\mathrm{Re}\,Q(x,t)=\frac{-3\,\mathrm{Cos}(x-t)}{2\,\mathrm{Cos}\sqrt{0.6}(x+16t)} \tag{25}$$

$$\mathrm{Im}\,Q(x,t)=\frac{-3\,\mathrm{Sin}(x-t)}{2\,\mathrm{Cos}\sqrt{0.6}(x+16t)} \tag{26}$$

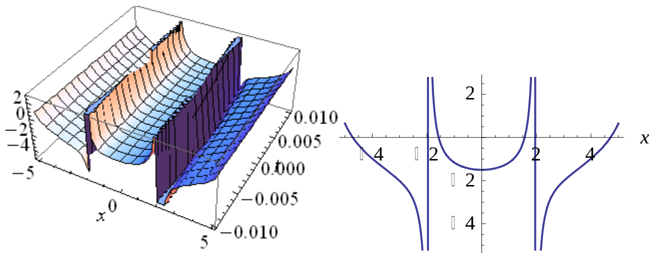

Figure 1. The bright soliton solution of the real part Eq.(25) in two and three dimensions when:
$A_1=-1.5, B=\pm\sqrt{0.6}\,i, w_1=-16, k_0=0.3, \gamma_1=2.2, \gamma_2=3, \alpha_1=3.2, \alpha_2=4.5, \alpha_3=3.6, \alpha_4=1.8, k=1$

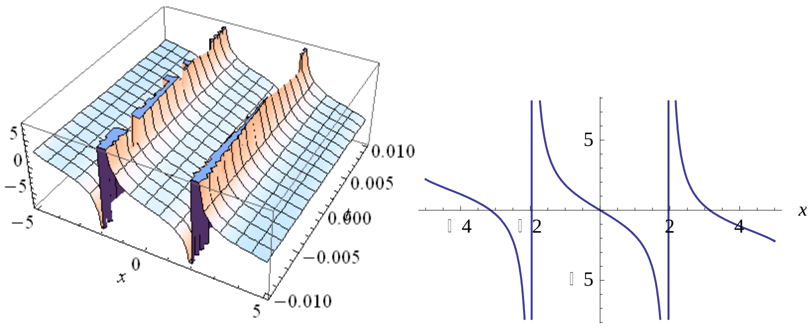

Figure 2. The bright soliton solution of the imaginary part Eq.(26) in two and three dimensions when:
$A_1=-1.5, B=\pm\sqrt{0.6}\,i, w_1=-16, k_0=0.3, \gamma_1=2.2, \gamma_2=3, \alpha_1=3.2, \alpha_2=4.5, \alpha_3=3.6, \alpha_4=1.8, k=1$

3.2 The dark soliton solution

Now; via inserting the relations (12-17) into the real and imaginary parts Eq's (18), (19) respectively we get,

$$
\begin{aligned}
&(\gamma_1+3k\alpha_1)A_2R_2(R_2-1)B^2\tanh^{R_2-2}t_2+A_2[(\Omega-\gamma_1-\alpha_1-\alpha_4)-2(\gamma_1+3k\alpha_1)R_2^2B^2]\tanh^{R_2}t_2\\
&+A_2R_2(R_2+1)B^2(\gamma_1+3k\alpha_1)\tanh^{R_2+2}t_2-k(\alpha_2+\alpha_3)A_2^3\tanh^{3R_2}t_2=0,
\end{aligned}
\tag{27}
$$

$$\begin{aligned}&\alpha_1[A_2R_2(R_2-1)(R_2-2)B^3\tanh^{R_2-3}t_2+\left(2A_2B^3R_2^3-A_2R_2(R_2-1)(R_2-2)B^3\right)\tanh^{R_2-1}t_2\\&+A_2B^3R_2\left(2R_2^2+(R_2+1)(R_2+2)\right)\tanh^{R_2+1}t_2-A_2B^3R_2(R_2+1)(R_2+2)\tanh^{R_2+3}t_2]\\&+(\alpha_2+\alpha_3)A_2^2BR_2[\tanh^{3R_2-1}t_2-\tanh^{3R_2+1}t_2]\\&-(3\alpha_1k^2+2\gamma_1k)A_2BR_2[\tanh^{R_2-1}t_2-\tanh^{R_2+1}t_2]-A_2w_2BR_2[\tanh^{R_2+1}t_2-\tanh^{R_2-1}t_2]=0.\end{aligned} \tag{28}$$

From equations (27), (28), by equating the highest exponents of $\tanh^i t_2$ we get $R_2=1$, hence, we can establish these relations,

$$\begin{aligned}&A_2^2=\frac{2B^2(\gamma_1+3k\alpha_1)}{k(\alpha_2+\alpha_3)},\\&B^2=\frac{(\Omega-\gamma_1-\alpha_1-\alpha_4)}{2(\gamma_1+3k\alpha_1)},\\&6B^2=-(\alpha_2+\alpha_3)A_2,\\&2A_2B^3+A_2B(2\gamma_1k+3\alpha_1k^2)-A_2Bw_2=0.\end{aligned} \tag{29}$$

From which we get,

$$\begin{aligned}&A_1=\frac{(\Omega-\gamma_1-\alpha_1-\alpha_4)}{(\alpha_2+\alpha_3)(\gamma_1+3k\alpha_1)},\\&B^2=\frac{k(\Omega-\gamma_1-\alpha_1-\alpha_4)}{2(\gamma_1+3k\alpha_1)(\alpha_2+\alpha_3)},\\&w_1=\frac{2(\Omega-\gamma_1-\alpha_1-\alpha_4)}{3k\alpha_1}+2\gamma_1k+3\alpha_1k^2.\end{aligned} \tag{30}$$

Hence we will achieve these results $A_2=-0.1, B=\pm\sqrt{0.03}\,i, w_2=12.7$ under the same values used in [33], hence the solution is,

$$Q(x,t)=-0.1\tanh(\sqrt{0.03}i(x-12.7t)e^{i(x-t)} \tag{31}$$

$$\operatorname{Re}Q(x,t)=0.1\tan(\sqrt{0.03}i(x-12.7t)\times\operatorname{Sin}(x-t) \tag{32}$$

$$\operatorname{Im}Q(x,t)=-0.1\tan(\sqrt{0.03}i(x-12.7t)\times\operatorname{Cos}(x-t) \tag{33}$$

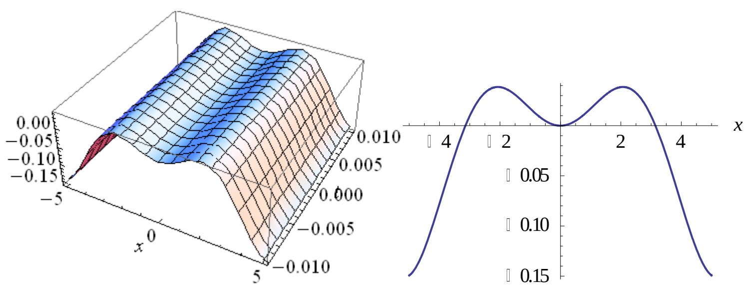

Figure 3. The dark soliton solution of the real part Eq.(32) in two and three dimensions when: $A_2=-0.1, B=\pm\sqrt{0.03}\,i, w_2=12.7, k_0=0.3, \gamma_1=2.2, \gamma_2=3, \alpha_1=3.2, \alpha_2=4.5, \alpha_3=3.6, \alpha_4=1.8, R=k=1$

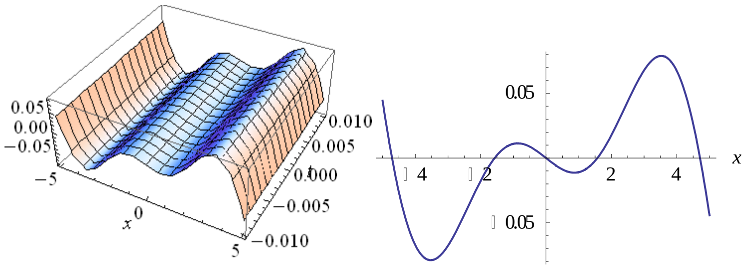

Figure 4. The dark soliton solution of the imaginary part Eq.(33) in two and three dimensions when: $A_2=-0.1, B=\pm\sqrt{0.03}\,i, w_2=12.7, k_0=0.3, \gamma_1=2.2, \gamma_2=3, \alpha_1=3.2, \alpha_2=4.5, \alpha_3=3.6, \alpha_4=1.8, R=k=1$

4. The second schema: the ESEM

First of all to introduce the form of ESEM [4-6], let us firstly introduce the general form of the MNLSE by propose the function R as a function of E(x, t) and its partial derivatives as,

$$R(E, E_x, E_t, E_{xx}, E_{tt}..........)=0 \tag{34}$$

That involves the highest order derivatives and nonlinear terms.

With the aid of the transformation $E(x,t)=E(\zeta), \zeta=wx+kt$ equation (33) can be reduced to the following ODE:

$$S(E, E', E''...........)=0 \tag{35}$$

Where S is a function in $E(\zeta)$ and its total derivatives, while $'=\dfrac{d}{d\zeta}$

The solution in the framework of this method is:

$$\phi(\zeta)=\sum_{i=-M}^{M} A_i\ \psi^i(\zeta). \tag{36}$$

Where $\psi(\zeta)$ achieves the equation,

$$\psi'(\zeta)=B_0+B_1\psi+B_2\psi^2 \tag{37}$$

The constant M appearing in Eq. (36) can be defined by applying the homogeneous balance between the orders of highest derivative and the nonlinear terms, while the other parameters A_i will be located later, while the other parameters B_0, B_1 *and* B_2 will propose the following facts.

(1) If $B_1=B_3=0$ it will admit to the Riccati equation [34–36], whose solutions are;

$$\psi(\zeta)=\frac{\sqrt{B_0B_2}}{B_2}\tan(\sqrt{B_0B_2}(\zeta+\zeta_0), B_0B_2\succ 0 \tag{38}$$

$$\psi(\zeta)=\frac{\sqrt{-B_0B_2}}{B_2}\tanh(\sqrt{-B_0B_2}\zeta-\frac{\rho\ln\zeta_0}{2}),B_0B_2\prec 0,\zeta\succ 0,\rho=\pm 1 \quad (39)$$

(2) If $B_0=B_3=0,$ it will admit the Bernoulli equation [34], whose solutions are;

$$\psi(\zeta)=\frac{B_1\,Exp[B_1(\zeta+\zeta_0)]}{1-B_2Exp[B_1(\zeta+\zeta_0)]}\;,B_1\succ 0 \quad (40)$$

$$\psi(\zeta)=\frac{-B_1\,Exp[B_1(\zeta+\zeta_0)]}{1+B_2Exp[B_1(\zeta+\zeta_0)]}\;,B_1\prec 0 \quad (41)$$

And the above solutions have the general forms which are:

$$\psi(\zeta)=-\frac{1}{B_2}\left(B_1-\sqrt{4B_1B_2-B_1^2}\tan\left(\frac{\sqrt{4B_1B_2-B_1^2}}{2}(\zeta+\zeta_0)\right)\right),4B_1B_2\succ B_1^2,B_2\succ 0, \quad (42)$$

$$\psi(\zeta)=\frac{1}{B_2}\left(B_1+\sqrt{4B_1B_2-B_1^2}\tanh\left(\frac{\sqrt{4B_1B_2-B_1^2}}{2}(\zeta+\zeta_0)\right)\right),4B_1B_2\succ B_1^2,B_2\prec 0, \quad (43)$$

Where the integer ζ_0 is the constancy of integration.

Finally, via inserting Eq. (37) into Eq. (36), collecting and equating the coefficients of various powers of ψ^i to zero implies system of equations through which we can calculate the values of the unknown variables. Moreover, via inserting these variables into equations (36) then we can establish the required solutions.

5. The exact solutions in the framework of the ESEM

We will implement this technique to the Eq. (1) mentioned above,

$$iQ_t+\gamma_1Q_{xx}+\gamma_2Q\left|Q^2\right|=i\alpha_1Q_{xxx}+i\alpha_2Q^2Q_x^*-i\alpha_3Q_x\left|Q^2\right|+\alpha_4Q.$$

The solution according to the ESEM is,

$$Q(x,t)=\phi(\zeta)\,e^{i\mu(x,t)},\zeta=kx+wt,\mu=qx+\delta t \quad (44)$$

$$Q_t=i\delta\phi\,e^{i\mu}+w\phi' e^{i\mu}, \quad (45)$$

$$Q_x=iq\phi\,e^{i\mu}+k\phi' e^{i\mu}, \quad (46)$$

$$Q_{tt}=-\delta^2\phi\,e^{i\mu}+2i\delta w\phi'\,e^{i\mu}+w^2\phi'' e^{i\mu}, \quad (47)$$

$$Q_{ttt}=-i\delta^3\phi\,e^{i\mu}-3\delta^2w\phi'\,e^{i\mu}+3i\delta w^2\phi''\,e^{i\mu}+w^3\phi''' e^{i\mu}, \quad (48)$$

$$\left|Q^2\right| = \varphi^2, \left(\left|Q^2\right|\right)_t = 2w\phi\phi', Q\left(\left|Q^2\right|\right)_t = 2w\phi^2\phi' e^{i\mu}. \tag{48}$$

Substituting about the above relations at the MNLSE we get,

$$\begin{aligned} & i\left(i\delta\phi e^{i\mu} + w\phi' e^{i\mu}\right) + \gamma_1\left(-q^2\phi e^{i\mu} + 2ikq\phi' e^{i\mu} + k^2\phi'' e^{i\mu}\right) + \gamma_2\phi^3 e^{i\mu} \\ & = i\alpha_1\left(-iq^3\phi e^{i\mu} - 3q^2k\phi' e^{i\mu} + 3iqk^2\phi'' e^{i\mu} + k^3\phi''' e^{i\mu}\right) \\ & \quad + i\alpha_2\phi^2 e^{2i\mu}\left(-iq\phi e^{-i\mu} + k\phi' e^{-i\mu}\right) - i\alpha_3\phi^2\left(iq\phi e^{i\mu} + k\phi' e^{i\mu}\right) + \alpha_4\phi e^{i\mu} \end{aligned} \tag{49}$$

This splits into the following real and imaginary parts respectively,

$$\text{Re} \quad (\gamma_1 + 3\alpha_1 q)k^2\phi'' + \left(\gamma_2 - \alpha_2 q - \alpha_3 q\right)\phi^3 + \left(\alpha_1 q^3 + \alpha_4 - \delta - \gamma_1 q^2\right)\phi = 0, \tag{50}$$

$$\text{Im} \quad \alpha_1 k^3\phi''' + (\alpha_2 - \alpha_3)k\phi^2\phi' - (w + 2kq + 3\alpha_1 kq^2)\phi' = 0. \tag{51}$$

We will firstly implement the ESEM to the real part

$$(\gamma_1 + 3\alpha_1 q)k^2\phi'' + \left(\gamma_2 - \alpha_2 q - \alpha_3 q\right)\phi^3 + \left(\alpha_1 q^3 + \alpha_4 - \delta - \gamma_1 q^2\right)\phi = 0. \tag{52}$$

Via balancing φ'', φ^3 appearing at Eq. (52) lead to $3M = 2M + 1$ which implies $M = 1$, hence the solution is,

$$\phi(\zeta) = \frac{A_{-1}}{\psi} + A_0 + A_1\psi \tag{53}$$

Where $\psi' = B_0 + B_1\psi + B_2\psi^2 + B_3\psi^3$

Case 1: The 1st family which suppose $B_1 = B_3 = 0 \Rightarrow \psi' = B_0 + B_2\psi^2$, consequently

$$\phi' = -\frac{B_0 A_{-1}}{\psi^2} + A_1 B_0 + A_1 B_2\psi^2 - B_2 A_{-1} \tag{54}$$

$$\phi'' = \frac{2B_0^2 A_{-1}}{\psi^3} + \frac{2B_0 B_2 A_{-1}}{\psi} + 2A_1 B_0 B_2\psi + 2A_1 B_2^2\psi^3 \tag{55}$$

$$\phi^2 = A_1^2\psi^2 + 2A_0 A_1\psi + (A_0^2 + 2A_{-1}A_1) + \frac{A_{-1}^2}{\psi^2} + \frac{2A_{-1}A_0}{\psi}, \tag{56}$$

$$\begin{aligned} \phi^3 = {} & A_1^3\psi^3 + 3A_0 A_1^2\psi^2 + (3A_1 A_0^2 + 3A_{-1}A_1^2)\psi + (A_0^3 + 6A_{-1}A_0 A_1) \\ & + \frac{A_{-1}^3}{\psi^3} + \frac{3A_0 A_{-1}^2}{\psi^2} + \frac{3A_{-1}A_0^2 + 3A_1 A_{-1}^2}{\psi}. \end{aligned} \tag{57}$$

Via inserting the relations (53-57) into Eq. (52), collecting and equating the coefficients of various powers of ψ^i to zero, we get the following system,

$$2k^2(\gamma_1+3\alpha_1 q)B_2^2+(\gamma_2-\alpha_2 q-\alpha_3 q)A_1^2=0, \tag{58}$$

$$3(\gamma_2-\alpha_2 q-\alpha_3 q)A_0 A_1^2=0, \tag{59}$$

$$2B_0B_2k^2(\gamma_1+3\alpha_1 q)+3(\gamma_2-\alpha_2 q-\alpha_3 q)(A_0^2+A_{-1}A_1)+(\alpha_1 q^3+\alpha_4-\delta+\gamma_1 q^2)=0, \tag{60}$$

$$(\gamma_2-\alpha_2 q-\alpha_3 q)(A_0^2+6A_{-1}A_1)+(\alpha_1 q^3+\alpha_4-\delta+\gamma_1 q^2)=0, \tag{61}$$

$$2k^2(\gamma_1+3\alpha_1 q)B_0^2+(\gamma_2-\alpha_2 q-\alpha_3 q)A_{-1}^2=0, \tag{62}$$

$$3(\gamma_2-\alpha_2 q-\alpha_3 q)A_0 A_{-1}^2=0, \tag{63}$$

$$2B_0B_2k^2(\gamma_1+3\alpha_1 q)+3(\gamma_2-\alpha_2 q-\alpha_3 q)(A_0^2+A_{-1}A_1)+(\alpha_1 q^3+\alpha_4-\delta+\gamma_1 q^2)=0. \tag{64}$$

It is clear that equations (59), (63) imply that $A_0=0$, in addition equations (60), (64) are the same and by substitute from equation (61) at equation (60) and put $A_0=0$ we can reduced the above system to,

$$2k^2(\gamma_1+3\alpha_1 q)B_2^2+(\gamma_2-\alpha_2 q-\alpha_3 q)A_1^2=0,$$

$$2B_0B_2k^2(\gamma_1+3\alpha_1 q)-3(\gamma_2-\alpha_2 q-\alpha_3 q)A_{-1}A_1=0,$$

$$2k^2(\gamma_1+3\alpha_1 q)B_0^2+(\gamma_2-\alpha_2 q-\alpha_3 q)A_{-1}^2=0. \tag{65}$$

Via solving this system the following results will be achieved,

$$\begin{aligned}&(1)B_2=0, B_0=\frac{-iA_{-1}\sqrt{\gamma_2-\alpha_2 q-\alpha_3 q}}{\sqrt{6k^2\alpha_1 q+2k^2\gamma_1}}, A_1=0,\\ &(2)B_2=0, B_0=\frac{iA_{-1}\sqrt{\gamma_2-\alpha_2 q-\alpha_3 q}}{\sqrt{6k^2\alpha_1 q+2k^2\gamma_1}}, A_1=0.\end{aligned} \tag{66}$$

These results lead to that there are no solution because $B_2=0$.

Case 2: The 2nd family which suppose $B_0=B_3=0 \Rightarrow \psi'=B_1+B_2\psi^2$, consequently

$$\phi(\zeta)=\frac{A_{-1}}{\psi}+A_0+A_1\psi, \tag{67}$$

$$\phi'=A_1B_2\psi^2+B_1A_1\psi-\frac{A_{-1}B_1}{\psi}-A_{-1}B_2, \tag{68}$$

$$\phi''=2A_1B_2^2\psi^3+3A_1B_1B_2\psi^2+A_1B_1^2\psi+A_{-1}B_1B_2+\frac{B_1^2A_{-1}}{\psi}. \tag{69}$$

$$\phi^3 = A_1^3\psi^3 + 3A_0A_1^2\psi^2 + (3A_1A_0^2 + 3A_{-1}A_1^2)\psi + (A_0^3 + 6A_{-1}A_0A_1) + \frac{A_{-1}^3}{\psi^3} + \frac{3A_0A_{-1}^2}{\psi^2} + \frac{3A_{-1}A_0^2 + 3A_1A_{-1}^2}{\psi}. \tag{70}$$

Substituting for equations (67-70) at equation (52) and collecting the coefficients of different powers of ψ^i and equating them to zero, we can easily obtain this system of algebraic

$$2k^2(\gamma_1 + 3\alpha_1 q)B_2^2 + (\gamma_2 - \alpha_2 q - \alpha_3 q)A_1^2 = 0, \tag{71}$$

$$k^2(\gamma_1 + 3\alpha_1 q)B_1B_2 + (\gamma_2 - \alpha_2 q - \alpha_3 q)A_0A_1 = 0, \tag{72}$$

$$B_1^2k^2(\gamma_1 + 3\alpha_1 q) + 3(\gamma_2 - \alpha_2 q - \alpha_3 q)(A_0^2 + A_{-1}A_1) + (\alpha_1 q^3 + \alpha_4 - \delta + \gamma_1 q^2) = 0, \tag{73}$$

$$k^2(\gamma_1 + 3\alpha_1 q)A_{-1}B_1B_2 + (\gamma_2 - \alpha_2 q - \alpha_3 q)(A_0^3 + 6A_0A_{-1}A_1) + (\alpha_1 q^3 + \alpha_4 - \delta + \gamma_1 q^2)A_0 = 0, \tag{74}$$

$$k^2(\gamma_1 + 3\alpha_1 q)A_{-1}^3 = 0, \tag{75}$$

$$3(\gamma_2 - \alpha_2 q - \alpha_3 q)A_0A_{-1}^2 = 0, \tag{76}$$

$$B_1^2k^2(\gamma_1 + 3\alpha_1 q) + 3(\gamma_2 - \alpha_2 q - \alpha_3 q)(A_0^2 + A_{-1}A_1) + (\alpha_1 q^3 + \alpha_4 - \delta + \gamma_1 q^2) = 0. \tag{77}$$

It is clear that (75, 76) will lead to $A_{-1} = 0$, use this value of A_{-1} and substituting by (74) at (73) and collecting the coefficients of different powers of ψ^i and equating them to zero, then the above system will be reduced to,

$$\begin{gathered} 2k^2(\gamma_1 + 3\alpha_1 q)B_2^2 + (\gamma_2 - \alpha_2 q - \alpha_3 q)A_1^2 = 0, \\ k^2(\gamma_1 + 3\alpha_1 q)B_1B_2 + (\gamma_2 - \alpha_2 q - \alpha_3 q)A_0A_1 = 0, \\ B_1^2k^2(\gamma_1 + 3\alpha_1 q) + 2(\gamma_2 - \alpha_2 q - \alpha_3 q)A_0^2 = 0. \end{gathered} \tag{78}$$

By solving this system we get,

$$\begin{gathered} (1) B_1 = \frac{-i\sqrt{2}A_0\sqrt{\gamma_2 - \alpha_2 q - \alpha_3 q}}{\sqrt{3k^2\alpha_1 q + k^2\gamma_1}}, A_1 = \frac{iS\left(\dfrac{(3\sqrt{2}\,k^2\alpha_1 q + k^2)\sqrt{\gamma_2 - \alpha_2 q - \alpha_3 q}}{\sqrt{3k^2\alpha_1 q + k^2\gamma_1}}\right)}{\gamma_2 - \alpha_2 q - \alpha_3 q}, \\ (2) B_1 = \frac{i\sqrt{2}A_0\sqrt{\gamma_2 - \alpha_2 q - \alpha_3 q}}{\sqrt{3k^2\alpha_1 q + k^2\gamma_1}}, A_1 = \frac{-iS\left(\dfrac{(3\sqrt{2}\,k^2\alpha_1 q + k^2)\sqrt{\gamma_2 - \alpha_2 q - \alpha_3 q}}{\sqrt{3k^2\alpha_1 q + k^2\gamma_1}}\right)}{\gamma_2 - \alpha_2 q - \alpha_3 q}. \end{gathered} \tag{79}$$

By substituting about the values of the parameters these solutions becomes,

$$(1) B_1 = 0.7, B_2 = -0.1,$$
$$(2) B_1 = -0.7, B_2 = 0.1. \tag{80}$$

In the framework of the suggested method these two results implies only one solution which is,

$$\psi(\zeta) = \frac{0.7\, Exp[0.7(x+t+1)]}{1+0.1 Exp[0.7(x+t+1)]}\,, \tag{81}$$

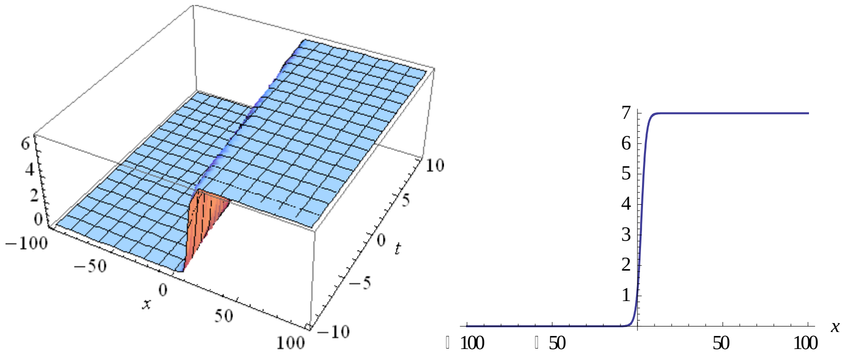

Figure 5. The soliton solution Eq.(81) in 2D and 3D with values: $q=1, k=1, \zeta_1 = 1$ $A_1 = 1, A_0 = 1, B_1 = \pm 0.7, B_2 = \pm 0.1, k_0 = 0.3, \gamma_1 = 2.2, \gamma_2 = 3, \alpha_1 = 3.2, \alpha_2 = 4.5, \alpha_3 = 3.6, \alpha_4 = 1.8,$

We will secondly implement the ESEM to imaginary part

$$\alpha_1 k^3 \phi''' + (\alpha_2 - \alpha_3) k \phi^2 \phi' - (w + 2kq + 3\alpha_1 k q^2)\phi' = 0.$$

By integrating once we get,

$$\alpha_1 k^3 \phi'' + \frac{1}{3}(\alpha_2 - \alpha_3) k \phi^3 - (w + 2kq + 3\alpha_1 k q^2)\phi = 0. \tag{82}$$

Case 1: The first family, in which $B_1 = B_3 = 0 \Rightarrow \psi' = B_0 + B_2 \psi^2$ **.**

Via inserting the relations (53-57) into Eq. (82), collecting and equating the coefficients of various powers of ψ^i to zero, we get the following system,

$$6\alpha_1 k^2 B_2^2 + (\alpha_2 - \alpha_3) A_1^2 = 0, \tag{83}$$

$$k(\alpha_2 - \alpha_3) A_0 A_1^2 = 0, \tag{84}$$

$$2\alpha_1 k^3 B_0 B_2 + k(\alpha_2 - \alpha_3)(A_0^2 + A_{-1} A_1) - (w + 2kq + 3\alpha_1 k q^2) = 0, \tag{85}$$

$$6\alpha_1 k^2 B_0^2 + (\alpha_2 - \alpha_3) A_{-1}^2 = 0, \tag{86}$$

$$k(\alpha_2 - \alpha_3) A_0 A_{-1}^2 = 0, \tag{87}$$

$$2\alpha_1 k^3 B_0 B_2 + k(\alpha_2 - \alpha_3)(A_0^2 + A_{-1} A_1) - (w + 2kq + 3\alpha_1 k q^2) = 0, \tag{88}$$

$$\frac{1}{3}k(\alpha_2-\alpha_3)(A_0^2+A_{-1}A_1)-(w+2kq+3\alpha_1 kq^2)=0. \tag{89}$$

It is clear that equations (84), (87) imply that $A_0=0$, in addition equations (85), (88) are the same thus this system could be reduced to,

$$6\alpha_1 k^2 B_2^2+(\alpha_2-\alpha_3)A_1^2=0,$$

$$\alpha_1 k^2 B_0 B_2-(\alpha_2-\alpha_3)A_{-1}A_1=0,$$

$$6\alpha_1 k^2 B_0^2+(\alpha_2-\alpha_3)A_{-1}^2=0, \tag{90}$$

By solving this system we get,

$$\begin{aligned}&(1)B_0=0, B_2=\frac{-A_1\sqrt{\alpha_3-\alpha_2}}{k\sqrt{6}\sqrt{\alpha_1}}, A_1=0,\\&(2)B_0=0, B_2=\frac{A_1\sqrt{\alpha_3-\alpha_2}}{k\sqrt{6}\sqrt{\alpha_1}}, A_1=0.\end{aligned} \tag{91}$$

From which we conclude that there are no solution because $B_0=0$.

Case 2: The 2nd family which suppose $B_0=B_3=0 \Rightarrow \psi'=B_1+B_2\psi^2$, hence,

Via inserting the relations (67-70) into Eq. (82), collecting and equating the coefficients of various powers of ψ^i to zero, we get the following system,

$$6\alpha_1 k^2 B_2^2+(\alpha_2-\alpha_3)A_1^2=0, \tag{92}$$

$$3\alpha_1 k^2 B_1 B_2+(\alpha_2-\alpha_3)A_0 A_1=0, \tag{93}$$

$$\alpha_1 k^3 B_1^2+k(\alpha_2-\alpha_3)(A_0^2+A_{-1}A_1)-(w+2kq+3\alpha_1 kq^2)=0, \tag{94}$$

$$\frac{1}{3}k(\alpha_2-\alpha_3)A_{-1}^3=0, \tag{95}$$

$$k(\alpha_2-\alpha_3)A_0 A_{-1}^2=0, \tag{96}$$

$$\alpha_1 k^3 B_1^2+k(\alpha_2-\alpha_3)(A_0^2+A_{-1}A_1)-(w+2kq+3\alpha_1 kq^2)=0, \tag{97}$$

$$\alpha_1 k^3 A_{-1}B_1 B_2+\frac{1}{3}k(\alpha_2-\alpha_3)(A_0^3+6A_0 A_{-1}A_1)-(w+2kq+3\alpha_1 kq^2)A_0=0. \tag{98}$$

The equations (95), (96) imply that $A_{-1}=0$, in addition equations (94), (97) are the same thus this system could be reduced to,

$$6\alpha_1 k^2 B_2^2 + (\alpha_2 - \alpha_3) A_1^2 = 0,$$

$$3\alpha_1 k^2 B_1 B_2 + (\alpha_2 - \alpha_3) A_0 A_1 = 0,$$

$$\alpha_1 k^2 B_1^2 + \frac{2}{3}(\alpha_2 - \alpha_3) A_0^2 = 0. \tag{99}$$

By solving this system we get,

$$\begin{aligned}
&(1) B_2 = \frac{-A_1\sqrt{\alpha_3-\alpha_2}}{k\sqrt{6}\sqrt{\alpha_1}}, B_1 = \frac{\sqrt{\frac{2}{3}}A_0\sqrt{\alpha_3-\alpha_2}}{k\sqrt{\alpha_1}},\\
&(2) B_2 = \frac{A_1\sqrt{\alpha_3-\alpha_2}}{k\sqrt{6}\sqrt{\alpha_1}}, B_1 = \frac{-\sqrt{\frac{2}{3}}A_0\sqrt{\alpha_3-\alpha_2}}{k\sqrt{\alpha_1}}.
\end{aligned} \tag{100}$$

By substituting about the values of the parameters these solutions become,

$$\begin{aligned}
&(1) B_1 = 0.4i, B_2 = -0.2i,\\
&(2) B_1 = -0.4i, B_2 = 0.2i.
\end{aligned} \tag{101}$$

In the framework of the suggested method these two results implies only one solution which is,

$$\psi(\zeta) = \frac{0.4i\, Exp[0.4i(x+t+1)]}{1+0.2iExp[0.4i(x+t+1)]}, \tag{102}$$

$$\mathrm{Re}\,\psi(\zeta) = \frac{0.8 - 0.4\,\mathrm{Sin}\,(0.4x+0.4t+0.4)}{1.04 - 0.4\times\mathrm{Sin}\,(0.4x+0.4t+0.4)}. \tag{103}$$

$$\mathrm{Im}\,\psi(\zeta) = \frac{0.4\,\mathrm{Cos}(0.4x+0.4t+0.4)}{1.04 - 0.4\times\mathrm{Sin}\,(0.4x+0.4t+0.4)}. \tag{104}$$

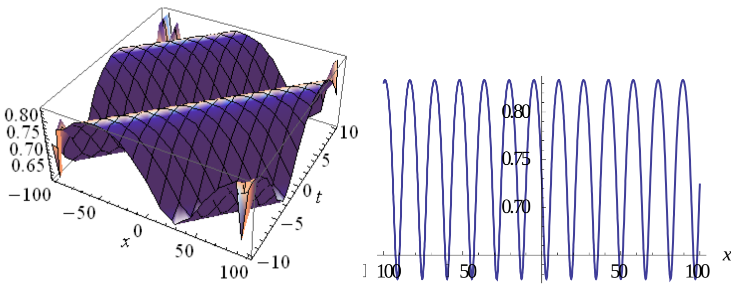

Figure 6. The soliton solution the real part Eq.(103) in 2D and 3D with values: $k=1, \zeta_1 = 1$ $A_1 = 1, A_0 = 1, B_1 = 0.4i, B_2 = -0.2i, k_0 = 0.3, \alpha_1 = 3.2, \alpha_2 = 4.5, \alpha_3 = 3.6, \alpha_4 = 1.8,$

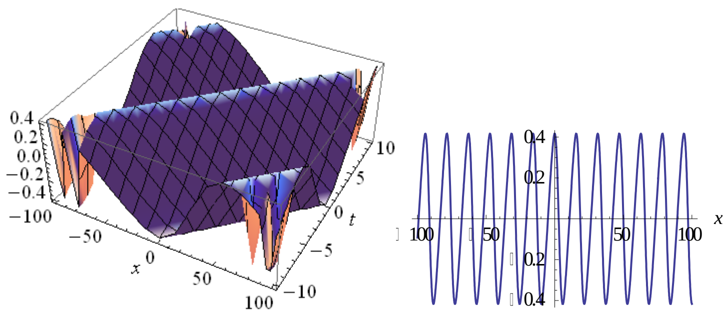

Figure 7. The soliton solution the imaginary part Eq.(104) in 2D and 3D with values: $k=1,\zeta_1=1$

$$A_1=1, A_0=1, B_1=0.4i, B_2=-0.2i, k_0=0.3, \alpha_1=3.2, \alpha_2=4.5, \alpha_3=3.6, \alpha_4=1.8,$$

6. Conclusion

This study has success to establish multiple impressive accurate perceptions of the optical solution to MNLSE through two important various algorithms. The first one which is regrestsed with the name the SWAM which achieves new perceptions of the soliton solution to the suggested equation figures (1-4).While the second one is the ESEM which has been applied effectively to establish other new visions to the soliton solutions to the suggested equation figures (5-7). Our achieved solutions are new and demonstrate new distinct perceptions to the soliton solutions of this model compared with that obtained previously [32-36] who applied different techniques.

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