Fuzzy Laplace-Adomian Decomposition method for solving Fuzzy Klein-Gordan equations

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Abstract

In this article, we have use Fuzzy Laplace-Adomian Decomposition method to evaluate the approximate solution of non linear and linear Time Fractional Klein-Gordan equations with suitable initial conditions in Fuzzy environment. We apply Fuzzy Laplace Transform and iterative method under caputo fractional derivative

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RESEARCH ARTICLE

Fuzzy Laplace-Adomian Decomposition method for solving Fuzzy Klein-Gordan equations.

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Summary

In this article, we have use Fuzzy Laplace-Adomian Decomposition method to evaluate the approximate solution of non linear and linear Time Fractional Klein-Gordan equations with suitable initial conditions in Fuzzy environment. We apply Fuzzy Laplace Transform and iterative method under caputo fractional derivative.

KEYWORDS:
Fuzzy partial differential equations, Fractional derivatives and integrals, Fuzzy Laplace Transform,Fuzzy Fractional Klein- Gordan equation

1 | INTRODUCTION

For describing uncertainty and interpreting ambiguous or subjective data in mathematical models, fuzzy set theory is a useful tool [1,2,3,4,5]. Fuzzy differential equations (FDEs) are an interesting issue from a theoretical standpoint; Fei [6] studied fuzzy random differential equations (FRDEs) with non-Lipschitz coefficients. 1986's Kaleva [7] presented fuzzy differential equations. Kaleva [8] introduced the FDE Cauchy issue. Lopez [9] compared FDEs. Friedman and Kandel [10] solved FDEs numerically. Mizukoshi et al. [11] explored FDEs. Chang and Zadeh [12] introduced fuzzy derivatives in 1972. Dubois and Prade [13] introduced extension.

This work develops a method for approximating nonlinear PDE solutions. LADM combines ADM and Laplace transformations. [14,15,16,17,18] successfully employed this strategy. Adomian's [19,20] ADM has been used in biology, physics, and chemical processes. The method provides a convergent series that may be computed quickly. To solve the differential equation apply the Laplace transformation and decomposing nonlinear terms into Adomian polynomials generates a recursive, iterative technique.

In recent decades there has appeared a big number of nonlinear evolution equations [21,22]. To solve the fuzzy heat equation, Allahviranloo used the Adomian decomposition technique (ADM) [23]. Alikhani and Bahrami [24] presented Fuzzy number cross products for FPDEs. Jameel et al. [25] proposed a new approximation method for fuzzy heat equations. To solve partial differential equations, Stepnicka and Valasek were the first to employ a fuzzy transform (PDEs) [26]. Finally, Jameel studied the heat equation's semianalytical solution in a fuzzy environment [27].

The Fuzzy Fractional Klein-Gordan equation [28]

$$\frac{\partial^\alpha \tilde{\aleph}(\flat,\wp;\delta)}{\partial \wp^\alpha} + \frac{\partial^2 \tilde{\aleph}(\flat,\wp;\delta)}{\partial \flat^2} + a\tilde{\aleph}(\flat,\wp;\delta) + b\tilde{\aleph}^2(\flat,\wp;\delta) + c\tilde{\aleph}^3(\flat,\wp;\delta) = \tilde{k}(\delta)\aleph(\flat,\wp;\delta) \tag{1}$$

[0]**Abbreviations:** FDE, Fuzzy Differential equations; FRDE, Fuzzy Random Differential equations ; PDE, Partial Differential Equations; LADM, Laplace Adomian Decomposition Method; FPDE, Fuzzy Partial Differential equations

subject to the fuzzy initial condition as

$$\tilde{\aleph}(\flat, 0; \delta) = \tilde{\aleph}_0 \quad \text{and} \quad \frac{\partial \tilde{\aleph}(\flat, 0; \delta)}{\partial \wp} = \tilde{\aleph}_1 \tag{2}$$

where $\aleph(\flat, \wp; \delta)$ be function of fuzzy number value and α is a parameter that describes the order of the fractional derivatives in Caputo sense. a,b,c are constants, $\tilde{k}(\delta)$ is a fuzzy number.

2 | IMPORTANT TERMINOLOGY

Here, we present important definitions, theorems, and Lemmas of fractional calculus and popular operators in fuzzy environments.

Definition 1. [29] The order $\alpha > 0$ Riemann-Liouville integral operator $\mathcal{J}^\alpha$ is defined as

$$\mathcal{J}^\alpha g(\wp) = \frac{1}{\Gamma(\alpha)} \int_0^\wp (\wp - \mu)^{\alpha-1} g(\mu) d\mu, \quad \wp > 0. \tag{3}$$

Definition 2. [30,31] In the case of the fuzzy number-valued function g, the R-L fractional integral of order α can be represented as follows:

$$\left[\mathcal{J}^\alpha \tilde{\aleph}(\wp; \delta)\right] = \left[\mathcal{J}^\alpha \underline{\aleph}(\wp; \delta), \mathcal{J}^\alpha \overline{\aleph}(\wp; \delta)\right], \quad \wp > 0, \tag{4}$$

where

$$\mathcal{J}^\alpha \underline{\aleph}(\wp) = \frac{1}{\Gamma(\alpha)} \int_0^\wp (\wp - \mu)^{\alpha-1} \underline{\aleph}(\mu) d\mu, \quad \wp > 0,$$

$$\mathcal{J}^\alpha \overline{\aleph}(\wp) = \frac{1}{\Gamma(\alpha)} \int_0^\wp (\wp - \mu)^{\alpha-1} \overline{\aleph}(\mu) d\mu, \quad \wp > 0.$$

Definition 3. [29] The fractional derivative of $g(\wp)$ in the Caputo sense is defined as follows:

$${}^c\mathcal{D}^\alpha_\wp g(\wp) = \mathcal{J}^{\zeta-\alpha} \mathcal{D}^\zeta_\wp g(\wp) = \begin{cases} \frac{1}{\Gamma(\zeta-\alpha)} \int_0^\wp \frac{g^\zeta(\mu) d\mu}{(\wp-\mu)^{\alpha+1-\zeta}} & \zeta - 1 < \alpha < \zeta, \zeta \in \mathbb{N} \\ \frac{d^\zeta}{d\wp^\zeta} g(\wp) & \alpha = \zeta, \quad \zeta \in \mathbb{N} \end{cases} \tag{5}$$

Definition 4. [30,31] Let $\left[\tilde{\aleph}(\wp; \delta)\right] = \left[\underline{\aleph}(\wp; \delta), \overline{\aleph}(\wp; \delta)\right]$ be a parametric representation of fuzzy valued function $\tilde{\aleph}(\wp; \delta)$, where $\delta \in [0, 1], 0 < \alpha < 1$ and $\wp \in (a, b)$.

(i) If $\tilde{\aleph}(\wp; \delta)$ is Caputo-type fuzzy fractional differential function in the first form can be expressed in the following way:

$$\left[{}^c\mathcal{D}^\alpha_\wp \tilde{\aleph}(\wp; \delta)\right] = \left[{}^c\mathcal{D}^\alpha_\wp \underline{\aleph}(\wp; \delta), {}^c\mathcal{D}^\alpha_\wp \overline{\aleph}(\wp; \delta)\right].$$

(ii) If $\tilde{\aleph}(\wp; \delta)$ is Caputo-type fuzzy fractional differential function in the second form can be expressed in the following way:

$$\left[{}^c\mathcal{D}^\alpha_\wp \tilde{\aleph}(\wp; \delta)\right] = \left[{}^c\mathcal{D}^\alpha_\wp \overline{\aleph}(\wp; \delta), {}^c\mathcal{D}^\alpha_\wp \underline{\aleph}(\wp; \delta)\right].$$

where,

$${}^c\mathcal{D}^\alpha_\wp \underline{\aleph}(\wp; \delta) = \frac{1}{\Gamma(\zeta - \alpha)} \int_0^\wp \frac{\underline{\aleph}^\zeta(\mu) d\mu}{(\wp - \mu)^{\alpha+1-\zeta}}, \qquad \zeta - 1 < \alpha < \zeta, \quad \zeta \in \mathbb{N}$$

$${}^c\mathcal{D}^\alpha_\wp \overline{\aleph}(\wp; \delta) = \frac{1}{\Gamma(\zeta - \alpha)} \int_0^\wp \frac{\overline{\aleph}^\zeta(\mu) d\mu}{(\wp - \mu)^{\alpha+1-\zeta}}, \qquad \zeta - 1 < \alpha < \zeta, \quad \zeta \in \mathbb{N}$$

Definition 5. [32,31]The Laplace transform of fuzzy function g is expressed as

$$\aleph(p) = \mathfrak{L}\Big[\tilde{\aleph}(\wp;\delta)\Big](p) = \int_0^\infty Exp(-p\wp) \odot \tilde{\aleph}(\wp)d\wp, \quad p > 0 \text{ and integer.} \tag{6}$$

Definition 6. [31] The transform Fuzzy Laplace for Caputo Type Fuzzy Fractional Derivative is

$$\mathfrak{L}\Big[{}^c\mathcal{D}^\alpha_\wp \tilde{\aleph}(\wp;\delta)\Big](p) = p^\alpha \aleph(p) - \sum_{q=0}^{\zeta-1} p^{\alpha-q-1}\tilde{\aleph}^q(\flat, 0;\delta), \quad \alpha \in (\zeta-1, \zeta] \tag{7}$$

Definition 7. The "Mittag-Leffler"operator $E_\alpha(\wp)$ is

$$E_\alpha(\wp) = \sum_{n=0}^\infty \frac{\wp^n}{\Gamma(1+n\alpha)}, \tag{8}$$

where, $\alpha > 0$.

3 | PROCESS OF FUZZY LAPLACE ADOMIAN DECOMPOSITION METHOD

For a rough estimate, we'll look at our conceptual model in this part. An Adomian decomposition[19,20] techniques is used in conjunction with the fractional Caputo derivative

$$\begin{aligned}\mathfrak{L}\Big[{}^c\mathcal{D}^\alpha_\wp \tilde{\aleph}(\flat,\wp;\delta)\Big] &= \mathfrak{L}\Big[\mathcal{D}^2_\flat \tilde{\aleph}(\flat,\wp;\delta) + a\tilde{\aleph}(\flat,\wp;\delta) \\ &\quad + b\tilde{\aleph}^2(\flat,\wp;\delta) + c\tilde{\aleph}^3(\flat,\wp;\delta)\Big] + \mathfrak{L}\Big[\tilde{k}(\delta)\aleph(\flat,\wp;\delta)\Big]\end{aligned} \tag{9}$$

where $\alpha \in (\zeta-1,\zeta]$; therefore the Laplace Transform of equation (9) is

$$\begin{aligned}p^\alpha \mathfrak{L}\Big[\tilde{\aleph}(\flat,\wp;\delta)\Big] - \sum_{q=0}^{\zeta-1} p^{\alpha-q-1}\tilde{\aleph}^q(\flat,0;\delta) &= \mathfrak{L}\Big[\mathcal{D}^2_\flat \tilde{\aleph}(\flat,\wp;\delta) + a\tilde{\aleph}(\flat,\wp;\delta) + b\tilde{\aleph}^2(\flat,\wp;\delta) \\ &\quad + c\tilde{\aleph}^3(\flat,\wp;\delta)\Big] + \mathfrak{L}\Big[\tilde{k}(\delta)\aleph(\flat,\wp;\delta)\Big]\end{aligned} \tag{10}$$

$$\begin{aligned}\mathfrak{L}\Big[\tilde{\aleph}(\flat,\wp;\delta)\Big] &= \Bigg\{ \frac{1}{p^\alpha}\sum_{q=0}^{\zeta-1} p^{\alpha-q-1}\tilde{\aleph}^q(\flat,0;\delta) + \frac{1}{p^\alpha}\mathfrak{L}\Big[\mathcal{D}^2_\flat \tilde{\aleph}(\flat,\wp;\delta) \\ &\quad + a\tilde{\aleph}(\flat,\wp;\delta) + b\tilde{\aleph}^2(\flat,\wp;\delta) + c\tilde{\aleph}^3(\flat,\wp;\delta)\Big] + \frac{1}{p^\alpha}\mathfrak{L}\Big[\tilde{k}(\delta)\aleph(\flat,\wp;\delta)\Big]\Bigg\}\end{aligned} \tag{11}$$

Apply inverse Laplace Transform to the equation (11) we get,

$$\begin{aligned}\tilde{\aleph}(\flat,\wp;\delta) &= \mathfrak{L}^{-1}\Bigg\{ \frac{1}{p^\alpha}\sum_{q=0}^{\zeta-1} p^{\alpha-q-1}\tilde{\aleph}^q(\flat,0;\delta) + \frac{1}{p^\alpha}\mathfrak{L}\Big[\tilde{k}(\delta)\aleph(\flat,\wp;\delta)\Big]\Bigg\} \\ &\quad + \mathfrak{L}^{-1}\Bigg\{\frac{1}{p^\alpha}\mathfrak{L}\Big[\mathcal{D}^2_\flat \tilde{\aleph}(\flat,\wp;\delta) + a\tilde{\aleph}(\flat,\wp;\delta) + b\tilde{\aleph}^2(\flat,\wp;\delta) + c\tilde{\aleph}^3(\flat,\wp;\delta)\Big]\Bigg\}.\end{aligned} \tag{12}$$

We supoose series solution of function $\tilde{\aleph}(\flat,\wp;\delta)$ as

$$\tilde{\aleph}(\flat,\wp;\delta) = \sum_{n=0}^\infty \tilde{\aleph}_n(\flat,\wp;\delta) \tag{13}$$

Here, note that $\tilde{\aleph}(\flat,\wp;\delta)$ is a fuzzy number-valued function that can be represented in terms of r-cut representations.

$$\tilde{\aleph}(\flat,\wp;\delta) = \Big[\underline{\aleph}(\flat,\wp;\delta), \overline{\aleph}(\flat,\wp;\delta)\Big] \tag{14}$$

and the operator L is linear;

$$\therefore \qquad L\Bigg(\sum_{n=0}^\infty \tilde{\aleph}_n(\flat,\wp;\delta)\Bigg) = \sum_{n=0}^\infty L\Big[\tilde{\aleph}_n(\flat,\wp;\delta)\Big] \tag{15}$$

and the operator N is nonlinear; then equation (12) becomes,

$$\begin{aligned}\sum_{n=0}^{\infty}\tilde{\aleph}_n(\flat,\wp;\delta) &= \mathfrak{L}^{-1}\left\{\frac{1}{p^{\alpha}}\sum_{q=0}^{\zeta-1}p^{\alpha-q-1}\tilde{\aleph}^{q}(\flat,0;\delta)+\frac{1}{p^{\alpha}}\mathfrak{L}\left[\tilde{k}(\delta)\aleph(\flat,\wp;\delta)\right]\right\} \\ &+\mathfrak{L}^{-1}\left\{\frac{1}{p^{\alpha}}\mathfrak{L}\left[\mathcal{D}_{\flat}^{2}\sum_{n=0}^{\infty}\tilde{\aleph}_n(\flat,\wp;\delta)+a\sum_{n=0}^{\infty}\tilde{\aleph}_n(\flat,\wp;\delta)+b\sum_{n=0}^{\infty}\tilde{\aleph}_n^{2}(\flat,\wp;\delta)+c\sum_{n=0}^{\infty}\tilde{\aleph}_n^{3}(\flat,\wp;\delta)\right]\right\}.\end{aligned} \tag{16}$$

and the fuzzy nonlinear term $N(\tilde{\aleph}) = a\tilde{\aleph}_n(\flat,\wp;\delta) + b\tilde{\aleph}^2(\flat,\wp;\delta) + c\tilde{\aleph}^3(\flat,\wp;\delta)$ is decomposed as follow:

$$\begin{aligned} N(\underline{\aleph}) &= \sum_{n=0}^{\infty}\underline{A}_n \\ N(\overline{\aleph}) &= \sum_{n=0}^{\infty}\overline{A}_n \end{aligned} \tag{17}$$

where $\tilde{A}_n = \left[\underline{A}_n, \overline{A}_n\right]$ is the set polynomials of Adomian . The $\tilde{A}_n$'s has the formula

$$\begin{aligned} \underline{A}_n &= \frac{1}{n!}\frac{\partial^n}{\partial\aleph^n}\left[N\Big(\sum_{i=0}^{n}\aleph^i\underline{\aleph}_i\Big)\right]_{\aleph=0}, \quad n=0,1,2,3,\ldots. \\ \overline{A}_n &= \frac{1}{n!}\frac{\partial^n}{\partial\aleph^n}\left[N\Big(\sum_{i=0}^{n}\aleph^i\overline{\aleph}_i\Big)\right]_{\aleph=0}, \quad n=0,1,2,3,\ldots. \end{aligned} \tag{18}$$

some $\tilde{A}_i, (i\to n)$ terms are as follow:

$$\begin{aligned} \underline{A}_0 &= N\left(\underline{\aleph}_0\right) \\ \underline{A}_1 &= \underline{\aleph}_1\,(N)'\left(\underline{\aleph}_0\right) \\ \underline{A}_2 &= \tfrac{1}{2}\,(N)''\left(\underline{\aleph}_0\right)\underline{\aleph}_1^2 + \underline{\aleph}_2\,(N)'\left(\underline{\aleph}_0\right) \\ \underline{A}_3 &= \tfrac{1}{6}\,(N)'''\left(\underline{\aleph}_0\right)\underline{\aleph}_1^3 + \underline{\aleph}_2\,(N)''\left(\underline{\aleph}_0\right)\underline{\aleph}_1 + \underline{\aleph}_3\,(N)'\left(\underline{\aleph}_0\right) \\ &\vdots \end{aligned}$$

and

$$\begin{aligned} \overline{A}_0 &= N\left(\overline{\aleph}_0\right) \\ \overline{A}_1 &= \overline{\aleph}_1\,(N)'\left(\overline{\aleph}_0\right) \\ \overline{A}_2 &= \tfrac{1}{2}\,(N)''\left(\overline{\aleph}_0\right)\overline{\aleph}_1^2 + \overline{\aleph}_2\,(N)'\left(\overline{\aleph}_0\right) \\ \overline{A}_3 &= \tfrac{1}{6}\,(N)'''\left(\overline{\aleph}_0\right)\overline{\aleph}_1^3 + \overline{\aleph}_2\,(N)''\left(\overline{\aleph}_0\right)\overline{\aleph}_1 + \overline{\aleph}_3\,(N)'\left(\overline{\aleph}_0\right) \\ &\vdots \end{aligned}$$

substituting equation (17) and equation (18) into (16) we get, parametric form of $\tilde{\aleph}(\flat,\wp;\delta)$ as follows:

$$\begin{aligned} \underline{\aleph}(\flat,\wp;\delta) = \sum_{n=0}^{\infty}\underline{\aleph}_n(\flat,\wp;\delta) &= \begin{pmatrix} \mathfrak{L}^{-1}\left\{\frac{1}{p^{\alpha}}\sum_{q=0}^{\zeta-1}p^{\alpha-q-1}\underline{\aleph}^{q}(\flat,0;\delta)+\frac{1}{p^{\alpha}}\mathfrak{L}\left[\underline{k}(\delta)\aleph(\flat,\wp;\delta)\right]\right\} \\ +\mathfrak{L}^{-1}\left\{\frac{1}{p^{\alpha}}\mathfrak{L}\left[\mathcal{D}_{\flat}^{2}\sum_{n=0}^{\infty}\underline{\aleph}_n(\flat,\wp;\delta)+\frac{1}{p^{\alpha}}\mathfrak{L}\left[\sum_{n=0}^{\infty}\underline{A}_n\right]\right\} \end{pmatrix} \\ \overline{\aleph}(\flat,\wp;\delta) = \sum_{n=0}^{\infty}\overline{\aleph}_n(\flat,\wp;\delta) &= \begin{pmatrix} \mathfrak{L}^{-1}\left\{\frac{1}{p^{\alpha}}\sum_{q=0}^{\zeta-1}p^{\alpha-q-1}\overline{\aleph}^{q}(\flat,0;\delta)+\frac{1}{p^{\alpha}}\mathfrak{L}\left[\overline{k}(\delta)\aleph(\flat,\wp;\delta)\right]\right\} \\ +\mathfrak{L}^{-1}\left\{\frac{1}{p^{\alpha}}\mathfrak{L}\left[\mathcal{D}_{\flat}^{2}\sum_{n=0}^{\infty}\overline{\aleph}_n(\flat,\wp;\delta)+\frac{1}{p^{\alpha}}\mathfrak{L}\left[\sum_{n=0}^{\infty}\overline{A}_n\right]\right\} \end{pmatrix} \end{aligned} \tag{19}$$

apply the iterative technique we find the recursive equation in the following form as,

$$
\begin{aligned}
\underline{\aleph}_0(\flat,\wp;\delta) &= \mathfrak{L}^{-1}\left\{\frac{1}{p^\alpha}\sum_{q=0}^{\zeta-1} p^{\alpha-q-1}\underline{\aleph}^q(\flat,0;\delta) + \frac{1}{p^\alpha}\mathfrak{L}\left[\underline{k}(\delta)\aleph(\flat,\wp;\delta)\right]\right\}\\
\underline{\aleph}_1(\flat,\wp;\delta) &= \mathfrak{L}^{-1}\left\{\tfrac{1}{p^\alpha}\mathfrak{L}\left[\mathcal{D}_\flat^2\underline{\aleph}_0(\flat,\wp;\delta)\right] + \tfrac{1}{p^\alpha}\mathfrak{L}\left[\underline{A}_0\right]\right\}\\
\underline{\aleph}_2(\flat,\wp;\delta) &= \mathfrak{L}^{-1}\left\{\tfrac{1}{p^\alpha}\mathfrak{L}\left[\mathcal{D}_\flat^2\underline{\aleph}_1(\flat,\wp;\delta)\right] + \tfrac{1}{p^\alpha}\mathfrak{L}\left[\underline{A}_1\right]\right\}\\
&\vdots\\
\underline{\aleph}_{n+1}(\flat,\wp;\delta) &= \mathfrak{L}^{-1}\left\{\tfrac{1}{p^\alpha}\mathfrak{L}\left[\mathcal{D}_\flat^2\underline{\aleph}_n(\flat,\wp;\delta)\right] + \tfrac{1}{p^\alpha}\mathfrak{L}\left[\underline{A}_n\right]\right\}
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
\overline{\aleph}_0(\flat,\wp;\delta) &= \mathfrak{L}^{-1}\left\{\frac{1}{p^\alpha}\sum_{q=0}^{\zeta-1} p^{\alpha-q-1}\overline{\aleph}^q(\flat,0;\delta) + \frac{1}{p^\alpha}\mathfrak{L}\left[\overline{k}(\delta)\aleph(\flat,\wp;\delta)\right]\right\}\\
\overline{\aleph}_1(\flat,\wp;\delta) &= \mathfrak{L}^{-1}\left\{\tfrac{1}{p^\alpha}\mathfrak{L}\left[\mathcal{D}_\flat^2\overline{\aleph}_0(\flat,\wp;\delta)\right] + \tfrac{1}{p^\alpha}\mathfrak{L}\left[\overline{A}_0\right]\right\}\\
\overline{\aleph}_2(\flat,\wp;\delta) &= \mathfrak{L}^{-1}\left\{\tfrac{1}{p^\alpha}\mathfrak{L}\left[\mathcal{D}_\flat^2\overline{\aleph}_1(\flat,\wp;\delta)\right] + \tfrac{1}{p^\alpha}\mathfrak{L}\left[\overline{A}_1\right]\right\}\\
&\vdots\\
\overline{\aleph}_{n+1}(\flat,\wp;\delta) &= \mathfrak{L}^{-1}\left\{\tfrac{1}{p^\alpha}\mathfrak{L}\left[\mathcal{D}_\flat^2\overline{\aleph}_n(\flat,\wp;\delta)\right] + \tfrac{1}{p^\alpha}\mathfrak{L}\left[\overline{A}_n\right]\right\}
\end{aligned}
\tag{21}
$$

Thus, the solution becomes

$$\underline{\aleph}(\flat,\wp;\delta) = \underline{\aleph}_0(\flat,\wp;\delta) + \underline{\aleph}_1(\flat,\wp;\delta) + \dots \tag{22}$$

$$\overline{\aleph}(\flat,\wp;\delta) = \overline{\aleph}_0(\flat,\wp;\delta) + \overline{\aleph}_1(\flat,\wp;\delta) + \dots \tag{23}$$

The Solution of the given series is convergent rapidly and the classical approach to converge to this series has been represented by Y. Cherruault and G. Adomian[3]

4 | NUMERICAL EXAMPLES

Example 1. We consider the fuzzy fractional Klein-Gordan equation as

$$\frac{\partial^\alpha\tilde{\aleph}(\flat,\wp;\delta)}{\partial\wp^\alpha} - \frac{\partial^2\tilde{\aleph}(\flat,\wp;\delta)}{\partial\flat^2} + \tilde{\aleph}(\flat,\wp;\delta) = 0, \quad 1 < \alpha \leq 2, \tag{24}$$

With the fuzzy initial conditions

$$\tilde{\aleph}(\flat,0;\delta) = 0 \quad \text{and} \quad \tilde{\aleph}_\wp(\flat,0;\delta) = \tilde{k}(\delta)\odot\flat \tag{25}$$

where $\tilde{k}(\delta) = [\delta-1, 1-\delta]$

Using the scheme of equation (20) and (21), we obtain

$$
\begin{aligned}
\underline{\aleph}_0(\flat, \wp; \delta) &= \flat(\delta - 1)\wp \\
\overline{\aleph}_0(\flat, \wp; \delta) &= \flat(1 - \delta)\wp \\
\underline{\aleph}_1(\flat, \wp; \delta) &= -\flat(\delta - 1)\frac{\wp^{\alpha+1}}{\Gamma(\alpha + 2)} \\
\overline{\aleph}_1(\flat, \wp; \delta) &= -\flat(1 - \delta)\frac{\wp^{\alpha+1}}{\Gamma(\alpha + 2)} \\
\underline{\aleph}_2(\flat, \wp; \delta) &= \flat(\delta - 1)\frac{\wp^{2\alpha+1}}{\Gamma(2\alpha + 2)} \\
\overline{\aleph}_2(\flat, \wp; \delta) &= \flat(1 - \delta)\frac{\wp^{2\alpha+1}}{\Gamma(2\alpha + 2)} \\
\underline{\aleph}_3(\flat, \wp; \delta) &= -\flat(\delta - 1)\frac{\wp^{3\alpha+1}}{\Gamma(3\alpha + 2)} \\
\overline{\aleph}_3(\flat, \wp; \delta) &= -\flat(1 - \delta)\frac{\wp^{3\alpha+1}}{\Gamma(3\alpha + 2)} \\
\underline{\aleph}_4(\flat, \wp; \delta) &= \flat(\delta - 1)\frac{\wp^{4\alpha+1}}{\Gamma(4\alpha + 2)} \\
\overline{\aleph}_4(\flat, \wp; \delta) &= \flat(1 - \delta)\frac{\wp^{4\alpha+1}}{\Gamma(4\alpha + 2)} \\
\underline{\aleph}_5(\flat, \wp; \delta) &= -\flat(\delta - 1)\frac{\wp^{5\alpha+1}}{\Gamma(5\alpha + 2)} \\
\overline{\aleph}_4(\flat, \wp; \delta) &= -\flat(1 - \delta)\frac{\wp^{5\alpha+1}}{\Gamma(5\alpha + 2)} \\
&\vdots
\end{aligned}
$$

and so on. Thus the lower and upper forms of a solution are given as

$$
\begin{aligned}
\underline{\aleph}(\flat, \wp; \delta) &= \underline{\aleph}_0(\flat, \wp; \delta) + \underline{\aleph}_1(\flat, \wp; \delta) + \underline{\aleph}_2(\flat, \wp; \delta) + \underline{\aleph}_3(\flat, \wp; \delta) + \dots \\
\overline{\aleph}(\flat, \wp; \delta) &= \overline{\aleph}_0(\flat, \wp; \delta) + \overline{\aleph}_1(\flat, \wp; \delta) + \overline{\aleph}_2(\flat, \wp; \delta) + \overline{\aleph}_3(\flat, \wp; \delta) + \dots \\
\underline{\aleph}(\flat, \wp; \delta) &= \flat(\delta - 1)\wp \begin{bmatrix} 1 - \frac{\wp^{\alpha}}{\Gamma(\alpha+2)} + \frac{\wp^{2\alpha}}{\Gamma(2\alpha+2)} - \frac{\wp^{3\alpha}}{\Gamma(3\alpha+2)} \\ + \frac{\wp^{4\alpha}}{\Gamma(4\alpha+2)} - \frac{\wp^{5\alpha}}{\Gamma(5\alpha+2)} + \dots \end{bmatrix} \\
\overline{\aleph}(\flat, \wp; \delta) &= \flat(1 - \delta)\wp \begin{bmatrix} 1 - \frac{\wp^{\alpha}}{\Gamma(\alpha+2)} + \frac{\wp^{2\alpha}}{\Gamma(2\alpha+2)} - \frac{\wp^{3\alpha}}{\Gamma(3\alpha+2)} \\ + \frac{\wp^{4\alpha}}{\Gamma(4\alpha+2)} - \frac{\wp^{5\alpha}}{\Gamma(5\alpha+2)} + \dots \end{bmatrix}
\end{aligned} \tag{26}
$$

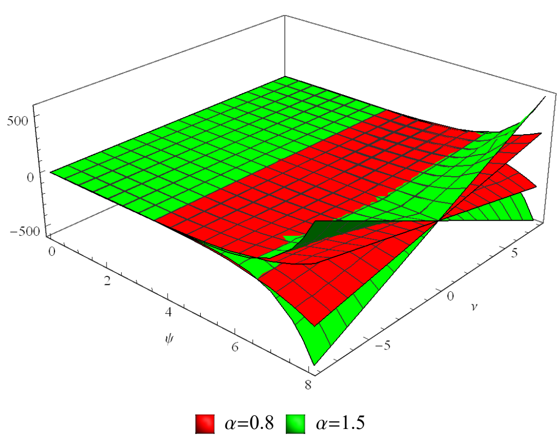

Figure 1 : 3-Dimensional simulation of Example (1) at $\alpha = 0.8, 1.5$ and uncertainty $\delta \in [0, 1]$

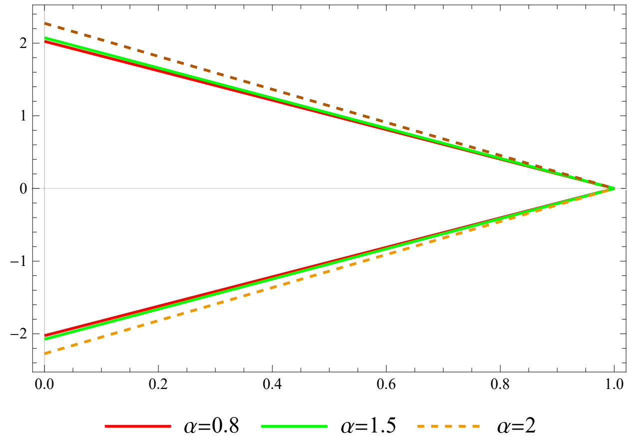

Figure 2 : 2-Dimensional simulation of Example (1) at $\alpha = 0.8, 1.5, 2$ and uncertainty $\delta \in [0, 1]$

If we put $\alpha = 2$ in equation (26) then we have

$$\tilde{\aleph}(\flat, \wp; \delta) = \tilde{k}(\delta)\flat \sin(\wp)$$

which is exact solution.

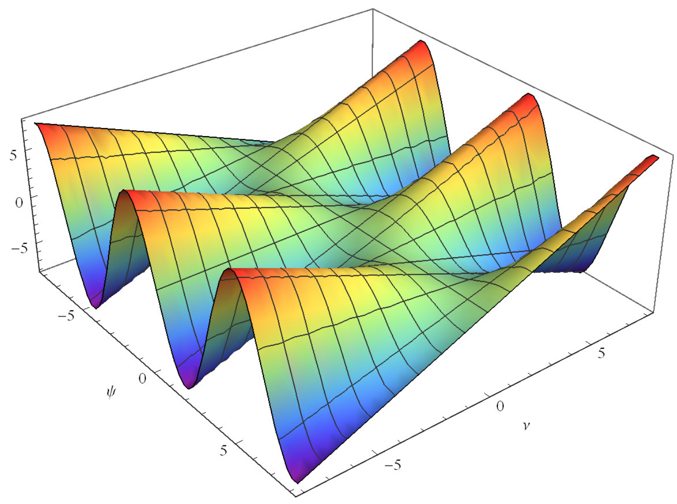

Figure 3 : Exact Geometrical representation of equation (26) at $\alpha = 2$

$\alpha = 2$						
	Lower			Upper		
r	Approximate	Exact	Error	Approimate	Exact	Error
0	-1.81859	-1.81859	2.58173×10^{-6}	1.81859	1.81859	2.58173×10^{-6}
0.2	-1.45487	-1.45488	2.06538×10^{-6}	1.45487	1.45488	2.06538×10^{-6}
0.4	-1.09116	-1.09116	1.54904×10^{-6}	1.09116	1.09116	1.54904×10^{-6}
0.6	-0.727437	-0.727438	1.03269×10^{-6}	7.27437×10^{-1}	7.27438×10^{-1}	1.03269×10^{-6}
0.8	-0.363718	-0.363719	5.16345×10^{-7}	3.63718×10^{-1}	3.63719×10^{-1}	5.16345×10^{-7}
1	0	0	0.00000	0.00000	0.00000	0.00000

Example 2. We consider the non-homogeneous fuzzy fractional Klein-Gordan equation as

$$\frac{\partial^\alpha \tilde{\aleph}(\flat, \wp; \delta)}{\partial \wp^\alpha} - \frac{\partial^2 \tilde{\aleph}(\flat, \wp; \delta)}{\partial \flat^2} + \tilde{\aleph}(\flat, \wp; \delta) = 2\sin(\flat), \quad 1 < \alpha \leq 2, \tag{27}$$

With the fuzzy initial conditions

$$\tilde{\aleph}(\flat, 0; \delta) = \tilde{k}(\delta)\sin(\flat) \quad \text{and} \quad \tilde{\aleph}_\wp(\flat, 0; \delta) = \tilde{k}(\delta) \tag{28}$$

where $\tilde{k}(\delta) = [\delta - 1, 1 - \delta]$

Using the scheme of equation (20) and (21), we obtain

$$\begin{aligned}
\underline{\aleph}_0(\flat, \wp; \delta) &= \frac{2\sin(\flat)\wp^\alpha}{\Gamma(\alpha+1)} - \sin(\flat) + \sin(\flat)\delta + \wp\delta - \wp \\
\overline{\aleph}_0(\flat, \wp; \delta) &= \frac{2\sin(\flat)\wp^\alpha}{\Gamma(\alpha+1)} + \sin(\flat) - \sin(\flat)\delta - \wp\delta + \wp \\
\underline{\aleph}_1(\flat, \wp; \delta) &= -\frac{4\sin(\flat)\wp^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{2\wp^\alpha\sin(\flat)}{\Gamma(\alpha+1)} + \frac{\wp^{\alpha+1}}{\Gamma(\alpha+2)} - \frac{2\sin(\flat)\delta\wp^\alpha}{\Gamma(\alpha+1)} - \frac{\delta\wp^{\alpha+1}}{\Gamma(\alpha+2)} \\
\overline{\aleph}_1(\flat, \wp; \delta) &= -\frac{4\sin(\flat)\wp^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{2\wp^\alpha\sin(\flat)}{\Gamma(\alpha+1)} - \frac{\wp^{\alpha+1}}{\Gamma(\alpha+2)} + \frac{2\sin(\flat)\delta\wp^\alpha}{\Gamma(\alpha+1)} + \frac{\delta\wp^{\alpha+1}}{\Gamma(\alpha+2)} \\
\underline{\aleph}_2(\flat, \wp; \delta) &= -\frac{4\wp^{2\alpha}\sin(\flat)}{\Gamma(2\alpha+1)} + \frac{8\wp^{3\alpha}\sin(\flat)}{\Gamma(3\alpha+1)} - \frac{\wp^{2\alpha+1}}{\Gamma(2\alpha+2)} + \frac{4\delta\wp^{2\alpha}\sin(\flat)}{\Gamma(2\alpha+1)} + \frac{\delta\wp^{2\alpha+1}}{\Gamma(2\alpha+2)} \\
\overline{\aleph}_2(\flat, \wp; \delta) &= \frac{4\wp^{2\alpha}\sin(\flat)}{\Gamma(2\alpha+1)} + \frac{8\wp^{3\alpha}\sin(\flat)}{\Gamma(3\alpha+1)} + \frac{\wp^{2\alpha+1}}{\Gamma(2\alpha+2)} - \frac{4\delta\wp^{2\alpha}\sin(\flat)}{\Gamma(2\alpha+1)} - \frac{r\wp^{2\alpha+1}}{\Gamma(2\alpha+2)} \\
\underline{\aleph}_3(\flat, \wp; \delta) &= \frac{8\wp^{3\alpha}\sin(\flat)}{\Gamma(3\alpha+1)} - \frac{16\wp^{4\alpha}\sin(\flat)}{\Gamma(4\alpha+1)} + \frac{\wp^{3\alpha+1}}{\Gamma(3\alpha+2)} - \frac{8\delta\wp^{3\alpha}\sin(\flat)}{\Gamma(3\alpha+1)} - \frac{\delta\wp^{3\alpha+1}}{\Gamma(3\alpha+2)} \\
\overline{\aleph}_3(\flat, \wp; \delta) &= -\frac{8\wp^{3\alpha}\sin(\flat)}{\Gamma(3\alpha+1)} - \frac{16\wp^{4\alpha}\sin(\flat)}{\Gamma(4\alpha+1)} - \frac{\wp^{3\alpha+1}}{\Gamma(3\alpha+2)} + \frac{8\delta\wp^{3\alpha}\sin(\flat)}{\Gamma(3\alpha+1)} + \frac{\delta\wp^{3\alpha+1}}{\Gamma(3\alpha+2)} \\
\underline{\aleph}_4(\flat, \wp; \delta) &= \left[\begin{array}{c} -\frac{4\wp^{2\alpha}\sin(\flat)}{\Gamma(2\alpha+1)} + \frac{8\wp^{3\alpha}\sin(\flat)}{\Gamma(3\alpha+1)} - \frac{32\wp^{4\alpha}\sin(\flat)}{\Gamma(4\alpha+1)} + \frac{2\wp^\alpha\sin(\flat)}{\Gamma(\alpha+1)} - \frac{\wp^{4\alpha+1}}{\Gamma(4\alpha+2)} \\ + \frac{16\delta\wp^{4\alpha}\sin(\flat)}{\Gamma(4\alpha+1)} + \frac{\delta\wp^{4\alpha+1}}{\Gamma(4\alpha+2)} \end{array}\right] \\
\overline{\aleph}_4(\flat, \wp; \delta) &= \left[\begin{array}{c} -\frac{4\wp^{2\alpha}\sin(\flat)}{\Gamma(2\alpha+1)} + \frac{8\wp^{3\alpha}\sin(\flat)}{\Gamma(3\alpha+1)} - \frac{32\wp^{4\alpha}\sin(\flat)}{\Gamma(4\alpha+1)} + \frac{2\wp^\alpha\sin(\flat)}{\Gamma(\alpha+1)} - \frac{\wp^{4\alpha+1}}{\Gamma(4\alpha+2)} \\ + \frac{16r\wp^{4\alpha}\sin(\flat)}{\Gamma(4\alpha+1)} + \frac{\delta\wp^{4\alpha+1}}{\Gamma(4\alpha+2)} \end{array}\right] \\
&\vdots
\end{aligned}$$

and so on. Thus the lower and upper forms of a solution are given as

$$\begin{aligned}
\underline{\aleph}(\flat, \wp; \delta) &= \underline{\aleph}_0(\flat, \wp; \delta) + \underline{\aleph}_1(\flat, \wp; \delta) + \underline{\aleph}_2(\flat, \wp; \delta) + \underline{\aleph}_3(\flat, \wp; \delta) + \ldots \\
\overline{\aleph}(\flat, \wp; \delta) &= \overline{\aleph}_0(\flat, \wp; \delta) + \overline{\aleph}_1(\flat, \wp; \delta) + \overline{\aleph}_2(\flat, \wp; \delta) + \overline{\aleph}_3(\flat, \wp; \delta) + \ldots
\end{aligned}$$

$$\begin{aligned}
\underline{\aleph}(\flat,\wp;\delta) &= \left[\begin{array}{c}
-\frac{12\wp^{2\alpha}\sin(\flat)}{\Gamma(2\alpha+1)}+\frac{24\wp^{3\alpha}\sin(\flat)}{\Gamma(3\alpha+1)}-\frac{48\wp^{4\alpha}\sin(\flat)}{\Gamma(4\alpha+1)}+\frac{6\wp^{\alpha}\sin(\flat)}{\Gamma(\alpha+1)}+\frac{\wp^{\alpha+1}}{\Gamma(\alpha+2)}-\frac{\wp^{2\alpha+1}}{\Gamma(2\alpha+2)}\\
+\frac{\wp^{3\alpha+1}}{\Gamma(3\alpha+2)}-\frac{\wp^{4\alpha+1}}{\Gamma(4\alpha+2)}-\sin(\flat)+\frac{4r\wp^{2\alpha}\sin(\flat)}{\Gamma(2\alpha+1)}-\frac{8r\wp^{3\alpha}\sin(\flat)}{\Gamma(3\alpha+1)}+\frac{16r\wp^{4\alpha}\sin(\flat)}{\Gamma(4\alpha+1)}\\
-\frac{2r\wp^{\alpha}\sin(\flat)}{\Gamma(\alpha+1)}-\frac{r\wp^{\alpha+1}}{\Gamma(\alpha+2)}+\frac{r\wp^{2\alpha+1}}{\Gamma(2\alpha+2)}-\frac{r\wp^{3\alpha+1}}{\Gamma(3\alpha+2)}+\frac{r\wp^{4\alpha+1}}{\Gamma(4\alpha+2)}+r\sin(\flat)+r\wp-\wp
\end{array}\right]\\
\overline{\aleph}(\flat,\wp;\delta) &= \left[\begin{array}{c}
-\frac{4\wp^{2\alpha}\sin(\flat)}{\Gamma(2\alpha+1)}+\frac{8\wp^{3\alpha}\sin(\flat)}{\Gamma(3\alpha+1)}-\frac{48\wp^{4\alpha}\sin(\flat)}{\Gamma(4\alpha+1)}+\frac{2\wp^{\alpha}\sin(\flat)}{\Gamma(\alpha+1)}-\frac{\wp^{\alpha+1}}{\Gamma(\alpha+2)}+\frac{\wp^{2\alpha+1}}{\Gamma(2\alpha+2)}\\
-\frac{\wp^{3\alpha+1}}{\Gamma(3\alpha+2)}-\frac{\wp^{4\alpha+1}}{\Gamma(4\alpha+2)}+\sin(\flat)-\frac{4r\wp^{2\alpha}\sin(\flat)}{\Gamma(2\alpha+1)}+\frac{8r\wp^{3\alpha}\sin(\flat)}{\Gamma(3\alpha+1)}+\frac{16r\wp^{4\alpha}\sin(\flat)}{\Gamma(4\alpha+1)}\\
+\frac{2r\wp^{\alpha}\sin(\flat)}{\Gamma(\alpha+1)}+\frac{r\wp^{\alpha+1}}{\Gamma(\alpha+2)}-\frac{r\wp^{2\alpha+1}}{\Gamma(2\alpha+2)}+\frac{r\wp^{3\alpha+1}}{\Gamma(3\alpha+2)}+\frac{r\wp^{4\alpha+1}}{\Gamma(4\alpha+2)}-r\sin(\flat)-r\wp+\wp
\end{array}\right]
\end{aligned} \tag{29}$$

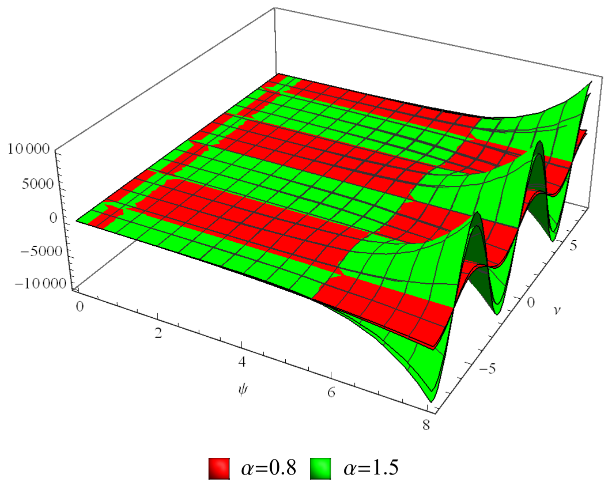

Figure 4 : 3-Dimensional simulation of (2) at $\alpha = 0.8, 1.5$ and uncertainty $\delta \in [0, 1]$

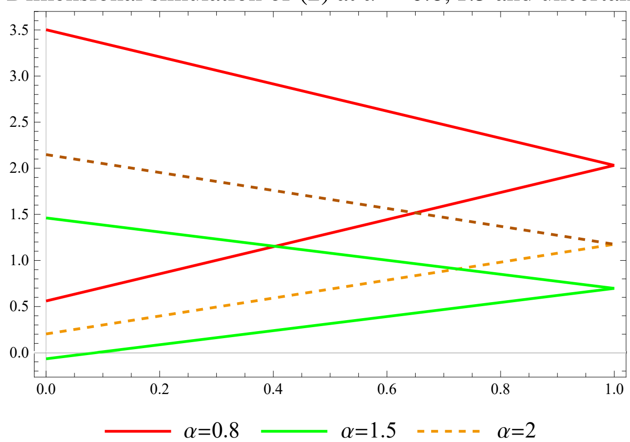

Figure 5 : 2-Dimensional simulation of (2) at $\alpha = 0.8, 1.5, 2$ and uncertainty $\delta \in [0, 1]$

If we put $\alpha = 2$ in equation (29) then we have

$$\tilde{\aleph}(\flat,\wp;\delta) = \tilde{k}(\delta)\Big(\sin(\flat)+\sin(\wp)\Big)$$

which is exact solution.

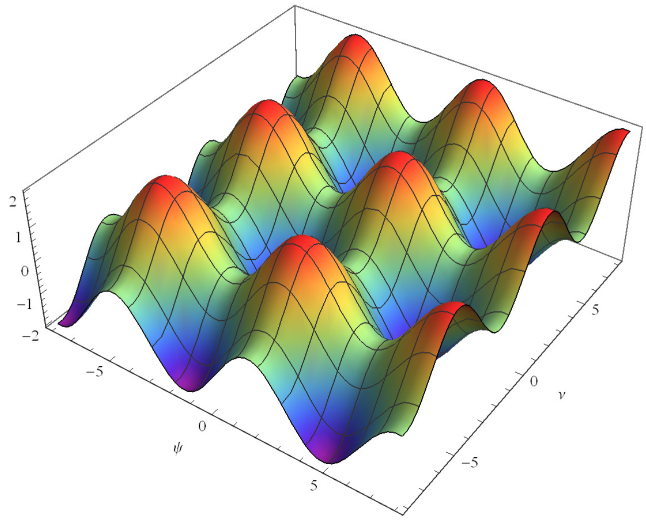

Figure 6 : Exact Geometrical representation of equation (26) at $\alpha = 2$

$\alpha = 2$						
	Lower			Upper		
r	Approximate	Exact	Error	Approimate	Exact	Error
0	3.4699×10^{-1}	-1.81859	2.16559	4.5101×10^{-1}	1.81859	1.36758
0.2	3.57392×10^{-1}	-1.45488	1.81227	4.40608×10^{-1}	1.45488	1.01427
0.4	3.67794×10^{-1}	-1.09116	1.45895	4.30206×10^{-1}	1.09116	6.60951×10^{-1}
0.6	3.78196×10^{-1}	-0.727438	1.10563	4.19804×10^{-1}	7.27438×10^{-1}	3.07634×10^{-1}
0.8	3.88598×10^{-1}	-0.363719	7.52317×10^{-1}	4.09402×10^{-1}	3.63719×10^{-1}	4.56834×10^{-2}
1	3.99×10^{-1}	0	3.99×10^{-1}	3.99×10^{-1}	0	3.99×10^{-1}

Example 3. We consider the fuzzy fractional Klein-Gordan equation as

$$\frac{\partial^{\alpha}\tilde{\aleph}(\flat,\wp;\delta)}{\partial\wp^{\alpha}} - \frac{\partial^{2}\tilde{\aleph}(\flat,\wp;\delta)}{\partial\flat^{2}} - \left(\frac{\partial\tilde{\aleph}(\flat,\wp;\delta)}{\partial\flat}\right)^{2} - \tilde{\aleph}^{2}(\flat,\wp;\delta) = 0, \quad 0 < \alpha \leq 1, \tag{30}$$

With the fuzzy initial condition

$$\tilde{\aleph}(\flat,0;\delta) = 0 \quad \text{and} \quad \tilde{\aleph}_{\wp}(\flat,0;\delta) = \tilde{k}(\delta)e^{\flat} \tag{31}$$

where $\tilde{k}(\delta) = \left[\delta - 1, 1 - \delta\right]$

Using the scheme of equation(18), (20) and (21), we get

$$\begin{aligned}
\underline{\aleph}_0(\flat, \wp; \delta) &= \wp(\delta - 1)e^{\flat}\\
\overline{\aleph}_0(\flat, \wp; \delta) &= \wp(1 - \delta)e^{\flat}\\
\underline{\aleph}_1(\flat, \wp; \delta) &= e^{\flat}(\delta - 1)\frac{\wp^{\alpha+1}}{\Gamma(\alpha + 2)}\\
\overline{\aleph}_1(\flat, \wp; \delta) &= e^{\flat}(1 - \delta)\frac{\wp^{\alpha+1}}{\Gamma(\alpha + 2)}\\
\underline{\aleph}_2(\flat, \wp; \delta) &= e^{\flat}(\delta - 1)\frac{\wp^{2\alpha+1}}{\Gamma(2\alpha + 2)}\\
\overline{\aleph}_2(\flat, \wp; \delta) &= e^{\flat}(1 - \delta)\frac{\wp^{2\alpha+1}}{\Gamma(2\alpha + 2)}\\
\underline{\aleph}_3(\flat, \wp; \delta) &= e^{\flat}(\delta - 1)\frac{\wp^{3\alpha+1}}{\Gamma(3\alpha + 2)}\\
\overline{\aleph}_3(\flat, \wp; \delta) &= e^{\flat}(1 - \delta)\frac{\wp^{3\alpha+1}}{\Gamma(3\alpha + 2)}\\
&\vdots
\end{aligned}$$

and so on. Thus the lower and upper forms of a solution are given as

$$\begin{aligned}
\underline{\aleph}(\flat, \wp; \delta) &= e^{\flat}(\delta - 1)\wp\left(1 + \frac{\wp^{\alpha}}{\Gamma(\alpha + 2)} + \frac{\wp^{2\alpha}}{\Gamma(2\alpha + 2)} + \frac{\wp^{3\alpha}}{\Gamma(3\alpha + 2)} + \frac{\wp^{4\alpha}}{\Gamma(4\alpha + 2)} + \dots\right)\\
\overline{\aleph}(\flat, \wp; \delta) &= e^{\flat}(1 - \delta)\wp\left(1 + \frac{\wp^{\alpha}}{\Gamma(\alpha + 2)} + \frac{\wp^{2\alpha}}{\Gamma(2\alpha + 2)} + \frac{\wp^{3\alpha}}{\Gamma(3\alpha + 2)} + \frac{\wp^{4\alpha}}{\Gamma(4\alpha + 2)} + \dots\right)
\end{aligned} \tag{32}$$

If we put $\alpha = 2$ in equation (32) then we have

$$\tilde{\aleph}(\flat, \wp; \delta) = \tilde{k}(\delta)\Big(e^{\flat}\sinh(\wp)\Big)$$

which is exact solution.

$\alpha = 2$						
	Lower			Upper		
r	Approximate	Exact	Error	Approimate	Exact	Error
0	-26.7987	-26.7991	3.89017×10^{-4}	2.67987×10^{1}	2.67991×10^{1}	3.89017×10^{-4}
0.2	-21.4389	-21.4393	3.11213×10^{-4}	2.14389×10^{1}	2.14393×10^{1}	3.11213×10^{-4}
0.4	-16.0792	-16.0794	2.3341×10^{-4}	1.60792×10^{1}	1.60794×10^{1}	2.3341×10^{-4}
0.6	-10.7195	-10.7196	1.55607×10^{-4}	1.07195×10^{1}	1.07196×10^{1}	1.55607×10^{-4}
0.8	-5.35974	-5.35982	7.78034×10^{-5}	5.35974	5.35982	7.78034×10^{-5}
1	0	0	0	0	0	0

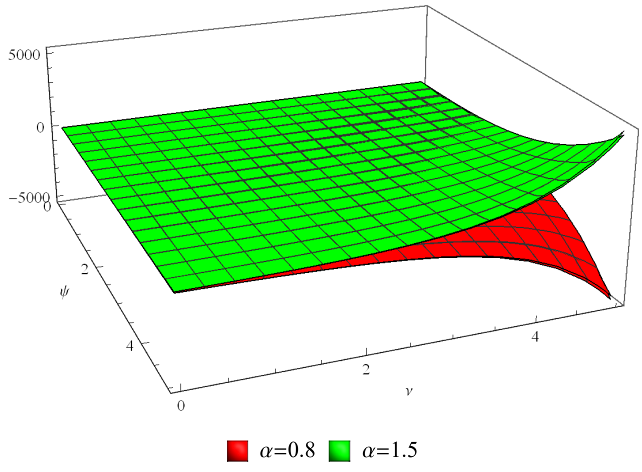

Figure 7 : 3-Dimensional simulation of Example (3) at $\alpha = 0.6, 0.8$ and uncertainty $r \in [0, 1]$

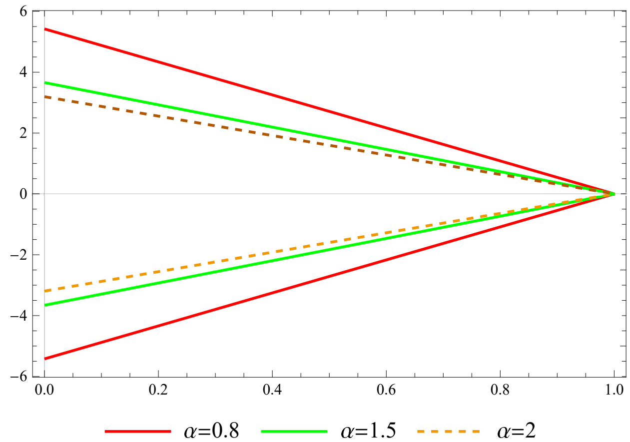

Figure 8 : 2-Dimensional simulation of Example (3) at $\alpha = 0.6, 0.8, 1$ and uncertainty $r \in [0, 1]$

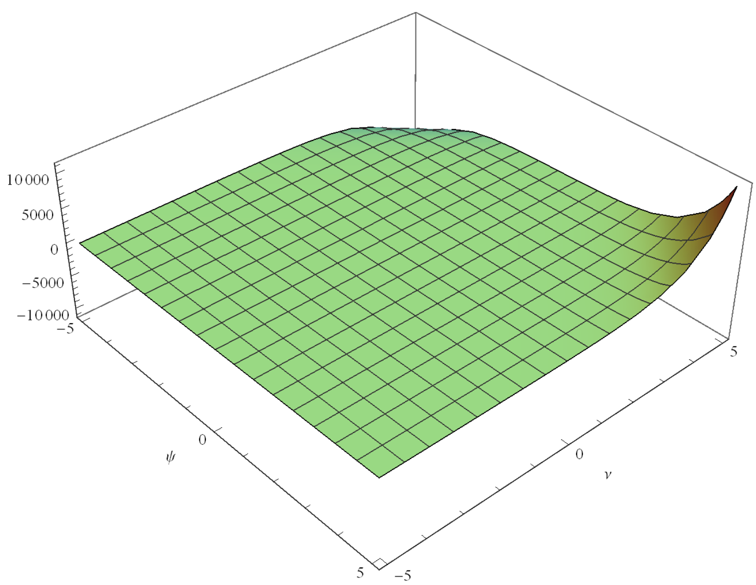

Figure 9 : Exact Geometrical representation of equation (26) at $\alpha = 2$

CONCLUSION

An approximation to the solution of the fuzzy time fractional telegraph equations was the primary focus of this article's efforts. The authors were successful in accomplishing their mission thanks to the application of the Fuzzy Laplace-Adomian decomposition approach. The Adomian decomposition method is a powerful method that has presented a possibility for effective efficiency in the solution of nonlinear differential equations for physical applications. The authors conducted their computations with the help of the Wolfram Mathematica 11.3 programme.

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Conflict of interest

The authors declare no potential conflict of interests.

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