Mathematical modelling of oscillating patterns for chronic autoimmune diseases

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Abstract

Many autoimmune diseases are chronic in nature, so that in general patients experience periods of recurrence and remission of the symptoms characterizing their specific autoimmune ailment. In order to describe this very important feature of autoimmunity, we construct a mathematical model of kinetic type describing the immune system cellular interactions in the context of autoimmunity exhibiting recurrent dynamics. The model equations constitute a non-linear system of integro-differential equations with quadratic terms that describe the interactions between self-antigen presenting cells, self-reactive T cells and immunosuppressive cells. We consider a constant input of self-antigen presenting cells, due to external environmental factors that are believed to trigger autoimmunity in people with predisposition for this condition. We also consider the natural death of all cell populations involved in our model, caused by their interaction with cells of the host environment. We derive the macroscopic analogue and show positivity and well-posedness of the solution, and then we study the equilibria of the corresponding dynamical system and their stability properties. By applying dynamical system theory, we prove that steady oscillations may arise due to the occurrence of a Hopf bifurcation. We perform some numerical simulations for our model, and we observe a recurrent pattern in the solutions of both the kinetic description and its macroscopic analogue, which leads us to conclude that this model is able to capture the chronic behaviour of many autoimmune diseases.

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