Stability properties of a crack inverse problem in half space

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July 24, 2020

Abstract

We show in this paper a Lipschitz stability result for a crack inverse problem in half space. The direct problem is a Laplace equation with zero Neumann condition on the top boundary. The forcing term is a discontinuity across the crack. This formulation can be related to geological faults in elastic media or to irrotational incompressible flows in a half space minus an inner wall. The direct problem is well posed in an appropriate functional space. We study the related inverse problem where the jump across the crack is unknown, and more importantly, the geometry and the location of the crack are unknown. The data for the inverse problem is of Dirichlet type over a portion of the top boundary. We prove that this inverse problem is uniquely solvable under some assumptions on the geometry of the crack. The highlight of this paper is showing a stability result for this inverse problem. Assuming that the crack is planar, we show that reconstructing the plane containing the crack is Lipschitz stable despite the fact that the forcing term for the underlying PDE is unknown. This uniform stability result holds under the assumption that the forcing term is bounded above and the Dirichlet data is bounded below away from zero in appropriate norms.

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July 21, 2020

Abstract

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1 Introduction

We consider a PDE model derived from a problem in geophysics where seismic or displacement data is collected by surface sensors and then processed to reconstruct a source and an unknown fault. Recently, there has been some progress in the understanding of the underlying mathematics. In particular, well posedness of the forward problem for the three dimensional linear elasticity model and uniqueness for the related inverse problem were shown in [12], and in [2, 1] where more general fault geometries and elasticity tensors were studied. A stability result in the case of planar faults was achieved in [11].
Here, we examine the case of a model involving the Laplace equation. This model is also relevant to geophysics: in dimension two, it relates to the so called anti-plane strain configuration. This configuration has already attracted much attention from geophysicists and

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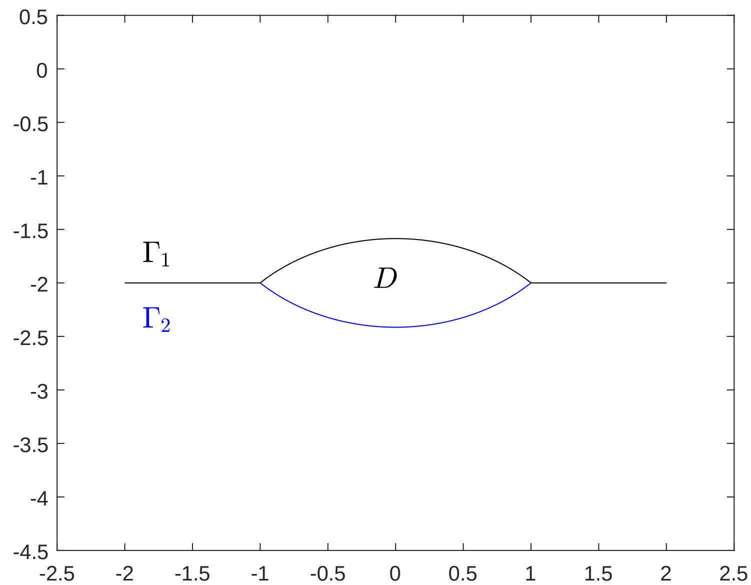
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