Modeling Complex Fluid Flow in Rough-Fractures: a Lubrication-Based Approach

Alessandro Lenci[1], Yves Meheust[2], Mario Putti[3], and Vittorio Di Federico[1]

[1]University of Bologna
[2]Géosciences Rennes
[3]University of Padova

November 23, 2022

Abstract

Flow in formations having both low-porosity and low-permeability prevalently occurs in fracture networks. Fracture connectivity and permeability are the main features governing flow in these media, and affect the effectiveness of any subsurface operation (e.g. enhanced oil recovery, enhanced geothermal systems). Moreover, the rheology of the fluid is often non-Newtonian due to their complex microstructure, resulting in a non-linear constitutive law at the continuum scale. Modeling flow of complex fluids in a fracture is challenging, when both medium heterogeneity and a realistic fluid rheology, typically shear-thinning, are considered. Full 3-D simulations are computationally expensive and time-consuming; alternatively, a lubrication-based approach can be adopted to implement an efficient 2-D flow solver able to produce vast statistics in reasonable time. We propose a numerical code that solves the 2-D generalized non-linear Reynolds equation for Ellis fluid flow in a single variable aperture fracture, adopting the finite volume method. The inexact Newton-Krylov method solves the associated non-linear system of equations, while a preconditioned conjugate gradient algorithm is used to solve the linear system at each Newton iteration. A parameter continuation strategy is also introduced to handle strongly non-linear cases. A synthetic fracture aperture field $w(x)$ is generated and the flow problem is solved. The numerical scheme is particularly efficient and robust for most input parameters of practical interest. Numerical parameters of the Inexact Newton-Krylov method have been optimized to simulate flow on large mesh (e.g. $10^{10} \times 10^{10}$), considering a range of fracture closure ($\sigma_w/<w>$) from 0 to 1, a shear-thinning index (n) of the Ellis rheologic model varying from 0.1 (strongly shear-thinning behavior) to 1 (Newtonian case). Pressure gradients ($[?]P$) typical of subsurface flow, either natural or forced, have been considered to study natural phenomena and industrial applications.

1

Alma Mater Studiorum Università di Bologna
Department of Civil, Chemical, Environmental and Materials Engineering

MODELING COMPLEX FLUID FLOW IN ROUGH-FRACTURES:
A LUBRICATION-BASED APPROACH

Alessandro Lenci[1,2], Yves Méheust[2], Mario Putti[3], Vittorio Di Federico[1]

[1] Università di Bologna, Department of Civil, Chemical, Environmental and Materials Engineering, 40126 Bologna, Italy
[2] Université de Rennes 1, CNRS, Géosciences Rennes UMR 6118, 35042 Rennes, France
[3] Università di Padova, Department of Mathematics, 35131 Padua, Italy

✉ alessandro.lenci@unibo.it

AGU FALL MEETING
New Orleans, LA & Online Everywhere
13–17 December 2021

Session: H35R. Non-linearity in Subsurface Flow and Transport: Modeling, Experiments, and Applications

1 FLOW OF SHEAR-THINNING FLUIDS IN GEOLOGICAL FRACTURES

The **hydraulic behavior of geological formations** is mainly governed by the **fractures connectivity** and **permeability**, Fracture heterogeneity strongly affects flow and transport, with fluid rheology playing an important role, often oversimplified.

- Typically, unconventional and deep geothermal reservoirs present both **low porosity** and **low permeability**
- Operations in gas shale or hot rock require **hydraulic stimulation** to enhance productivity and become cost effective

Fluids involved in subsurface industrial activities present a **shear-thinning behaviour** at **continuum scale,** due to their complex microstructure.
- muds
- foams
- water-based suspensions

Subsurface industrial activities:
- Enhanced Oil Recovery (EOL)
- Enhanced Geothermal Systems (EGS)
- Carbon sequestration

Enhanced Geothermal Systems

Fourier Spectrum

$-2(1+H) = -2.60$

Aperture Fields

$w(\mathbf{x}) \times 10^{-3}$ (m)

Ellis Fluid Model

Carreau-Yasuda Model

n

F1 -0.28
F2 -0.49
F3 -0.60
F4 $n-1 = -0.90$

2 NUMERICAL MODELING

2.1 Synthetic fracture generator

A synthetic aperture field is estimated by mating two **isotropic self-affine surfaces** $h_i(\mathbf{x})$

$$w(\mathbf{x}) = h_1(\mathbf{x}) - h_2(\mathbf{x}) + \langle w \rangle$$

A rough surface can be generated as a 2D white noise and introducing spatial correlations: multiplying the modulus of the Fourier transform by the modulus of the wave numbers $|k| = (k_{x_1}^2 + k_{x_2}^2)^{1/2}$ to the power $-1 - H$

$$|h(\mathbf{k})| \to |k|^{-1-H} |h(\mathbf{k})|$$

$h_1(\mathbf{x})$
$h_2(\mathbf{x})$

Fracture Geometry

$\sigma_w / \langle w \rangle$	1.0
$\langle w \rangle$	10^{-3} m
H	0.8
L_c	0.1 m
L	0.4 m

2.2 Fluid Rheology: Ellis model
The Ellis rheology is a **three-parameter model**

$$\eta = \frac{\mu_0}{1 + \left(\dfrac{\tau_{xz}}{\tau_{1/2}}\right)^{\frac{1}{n}-1}}$$

μ_0 low-shear rate viscosity ($\eta \to \mu_0$ for $\dot{\gamma} \to 0$)
n shear-thinning index
$\tau_{1/2}$ characteristic shear stress: $\eta(\tau_{1/2}) = \mu_0/2$

Fluids		n (-)	μ_0 (Pa·s)	$\tau_{1/2}$ (Pa)
F1	CMC at 0.3 wt%	0.72	0.05	4.07
F2	CMC at 0.5 wt%	0.51	0.22	2.50
F3	CMC at 1.0 wt%	0.40	2.99	5.14
F4	VES	0.10	49.00	1.07

2.3 Generalized Non-linear Reynolds Equation

- **Lubrication theory** holds ($\nabla w \ll 1$ and $\mathrm{Re} \ll 1$)

$$-\nabla \cdot \left[\frac{w(\mathbf{x})^3}{12\mu_0} + \frac{n}{(2n+1)} \left(\frac{1}{2^{1+n} \mu_0^n \tau_{1/2}^{1-n}} \right)^{\frac{1}{n}} w(\mathbf{x})^{\frac{2n+1}{n}} |\nabla P|^{\frac{1}{n}-1} \right] \nabla P = 0$$

Numerical modeling via finite volume method:
- The non-linear system of equations is solved via **inexact Newton-Krylov** method
- **Variable-fill-in Cholesky preconditioned conjugate gradient** → linear problem
- A **parameter continuation strategy** is adopted to handle strongly non-linear cases

Boundary Conditions

Five-point Stencil

3 RESULTS

3.1 Experimental Convergence

- Starting from a 2x2 aperture field (level mesh 0), we estimate the error at different mesh levels. The **convergence of this sequence of errors** together with the **scheme consistency** implies the **convergence to the true solution** of the problem.

- The adopted strategy aims at **reducing the pre-asymptotic phase of the Newton method**, thus engaging efficient quadratic convergence as quickly as possible.

3.2 Impact of Rheology

- The fluid shear-thinning behaviour **promote flow localization**, with parts of the fractures characterized by high velocities (channels) and others where the flow is almost stagnant.

- **Flow mainly occurs in channels of low apparent viscosity and high velocity**, resulting from flow localization.

- **Shear-thinning behaviour reduces the impact of the closure**, increasing the apparent transmissivity.

Apparent Viscosity

n
F1
F2
F3
$\frac{\overline{\nabla P}}{\overline{\nabla P_c}} = 100$

$\log_{10}(\mu/\mu_c)$

Velocity Magnitude

F3 $\frac{\overline{\nabla P}}{\overline{\nabla P_c}} = 100$
F4 $\frac{\overline{\nabla P}}{\overline{\nabla P_c}} = 5$

$\log_{10}(\|\overline{\mathbf{u}}\|/\langle \overline{\mathbf{u}} \rangle)$

Experimental Convergence

$\log_2 M$

F4

d

Impact of the Closure (1000 fractures)

$\frac{T_0}{T}$

F1 $\frac{\overline{\nabla P}}{\overline{\nabla P_c}} = 100$
F4 $\frac{\overline{\nabla P}}{\overline{\nabla P_c}} = 5$

$\frac{\sigma_w}{\langle w \rangle}$

Apparent Transmissivity

$\frac{1-n}{n} = 9$
1.5
0.96
0.39

$\frac{T}{T_0}$

$\frac{\overline{\nabla P}}{\overline{\nabla P_c}}$

4 CONCLUSIONS AND FUTURE PERSPECTIVES
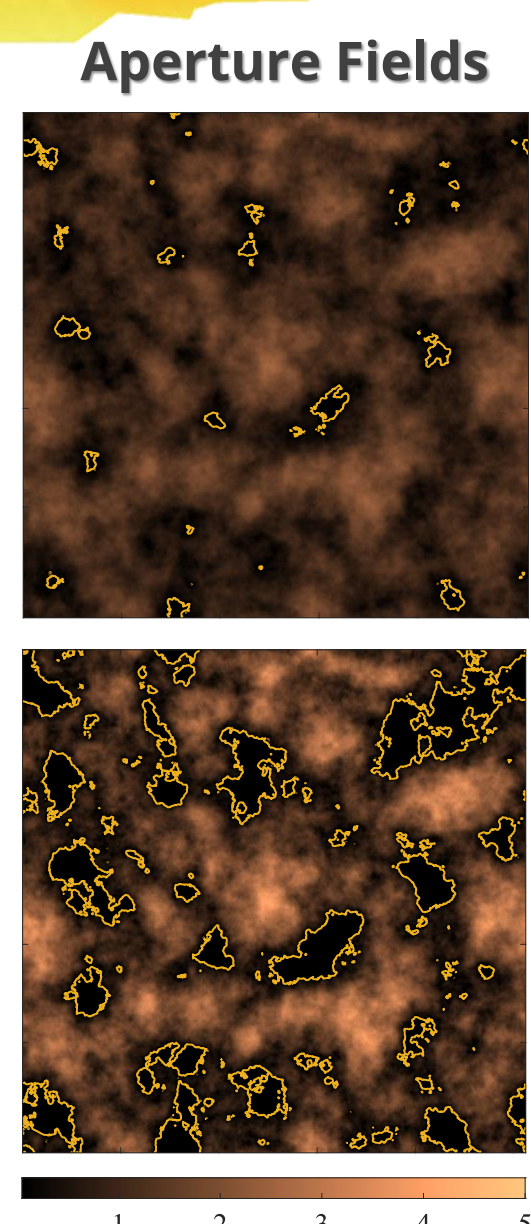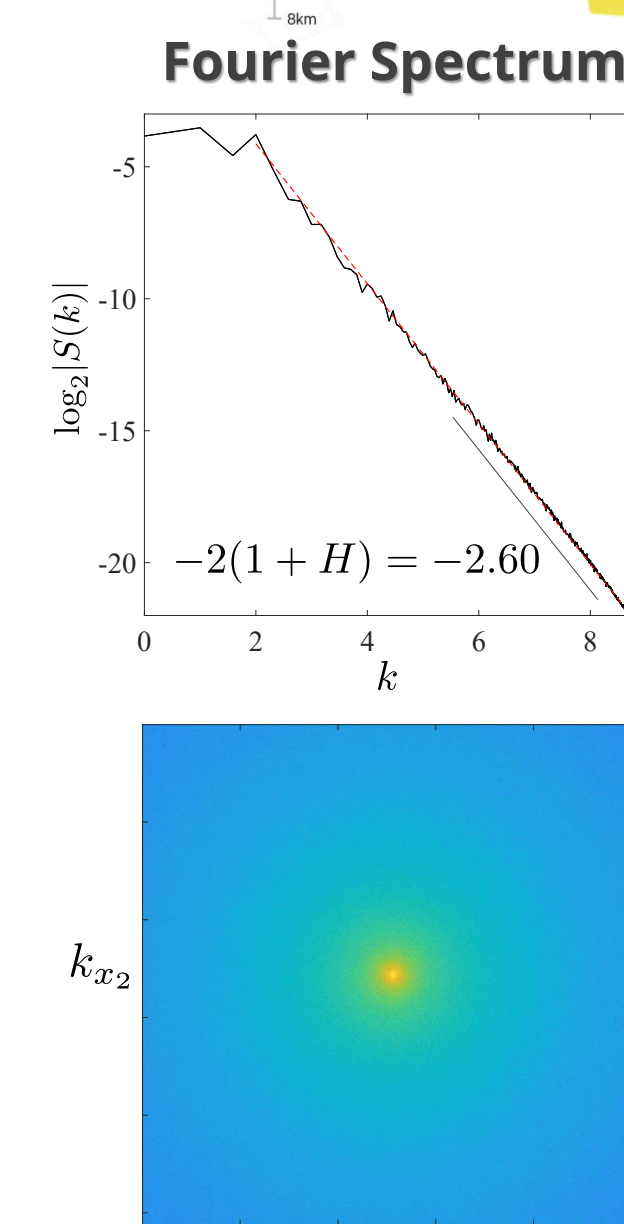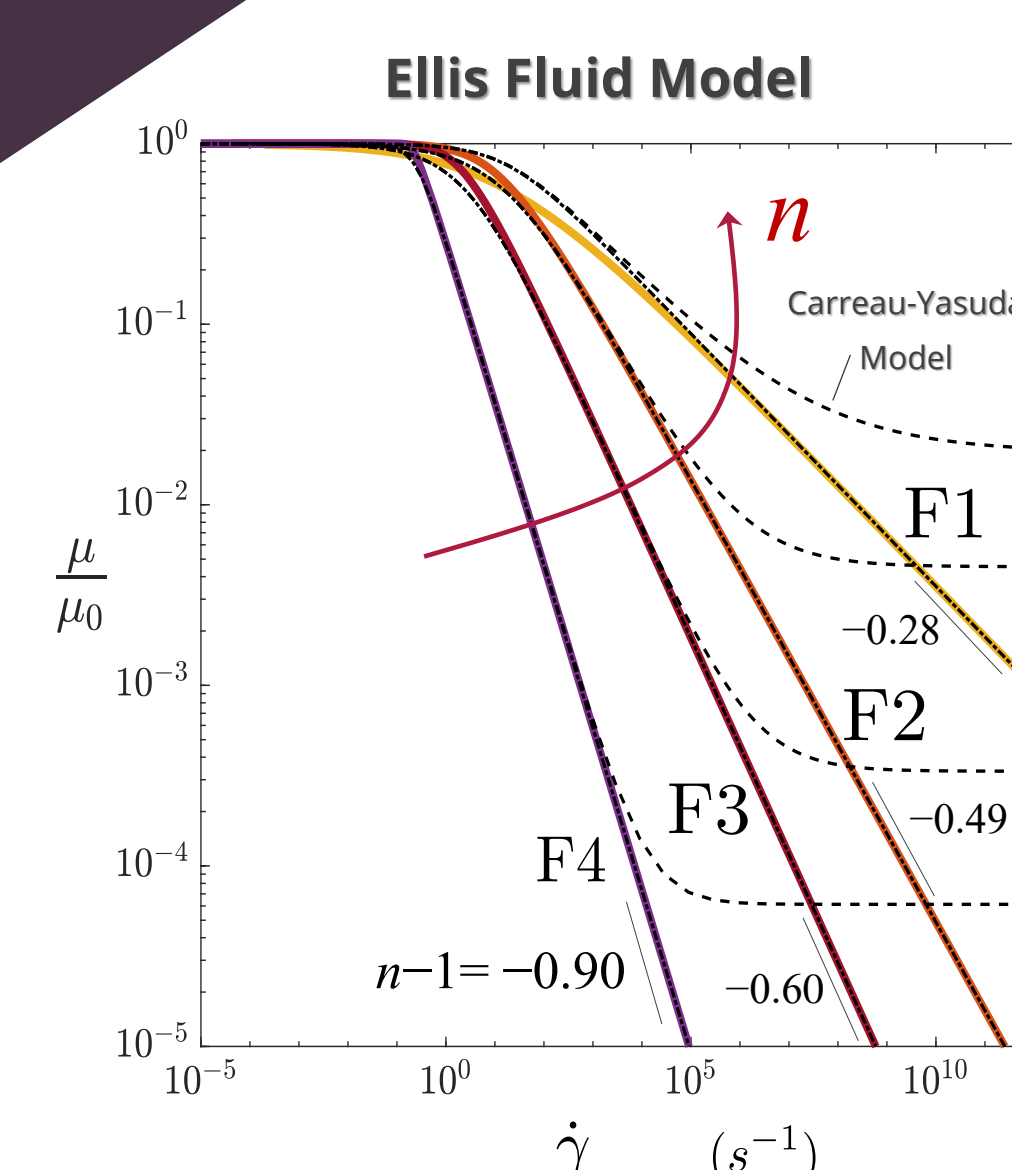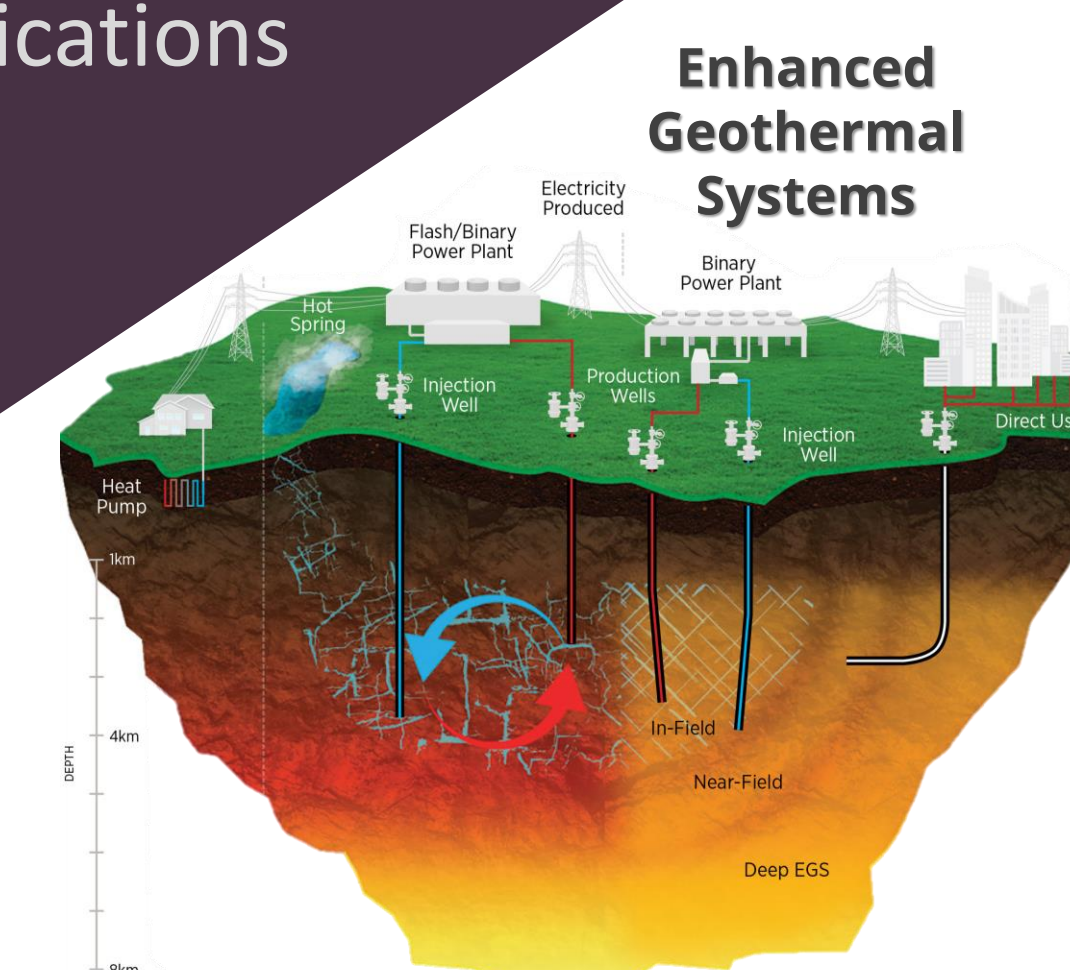
4.1 Conclusions
- Transmissivity attenuation due to fracture closure is mitigated by the shear-thinning rheology.
- High $\overline{\nabla P}$ and low n increases shear-thinning behaviour, favouring flow localization
- The smaller the fracture length, the higher the dispersions of the velocities
- Shear-thinning behaviour enhances fracture transmissivity, leading to non-darcian flow regime for sufficiently high pressure gradients
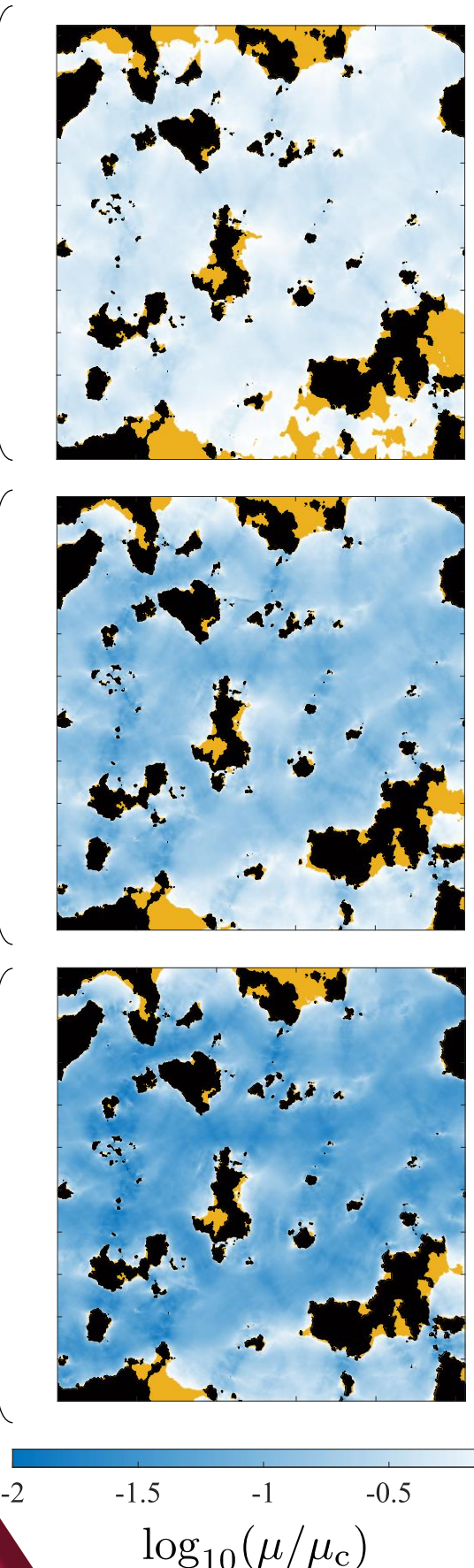
4.2 Future Perspectives
- Comparison with full **3-D CFD simulations** to investigate the **limits of the lubrication approximation**.
- Implement a **transport solver** to study the **impact of the shear-thinning rheology on breakthrough curves**.

Méheust, Y., Schmittbuhl, J. (2000) Flow enhancement of rough fracture. *Geophys. Res. Lett.*, 27(18), 2989-2992. doi: 10.1029/1888gl100464
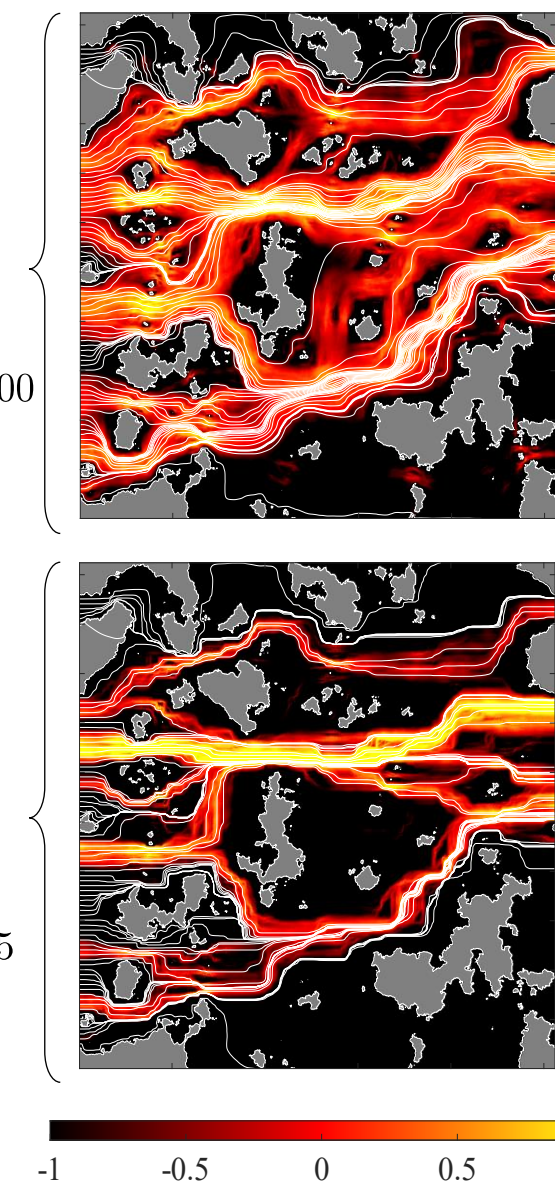
Lenci, A., Méheust, Y., Putti, M., Di Federico, V. (2021) Monte Carlo Simulations of Shear-thinning Flow in Geological Fractures, *Water Resour. Res.* (preprint)
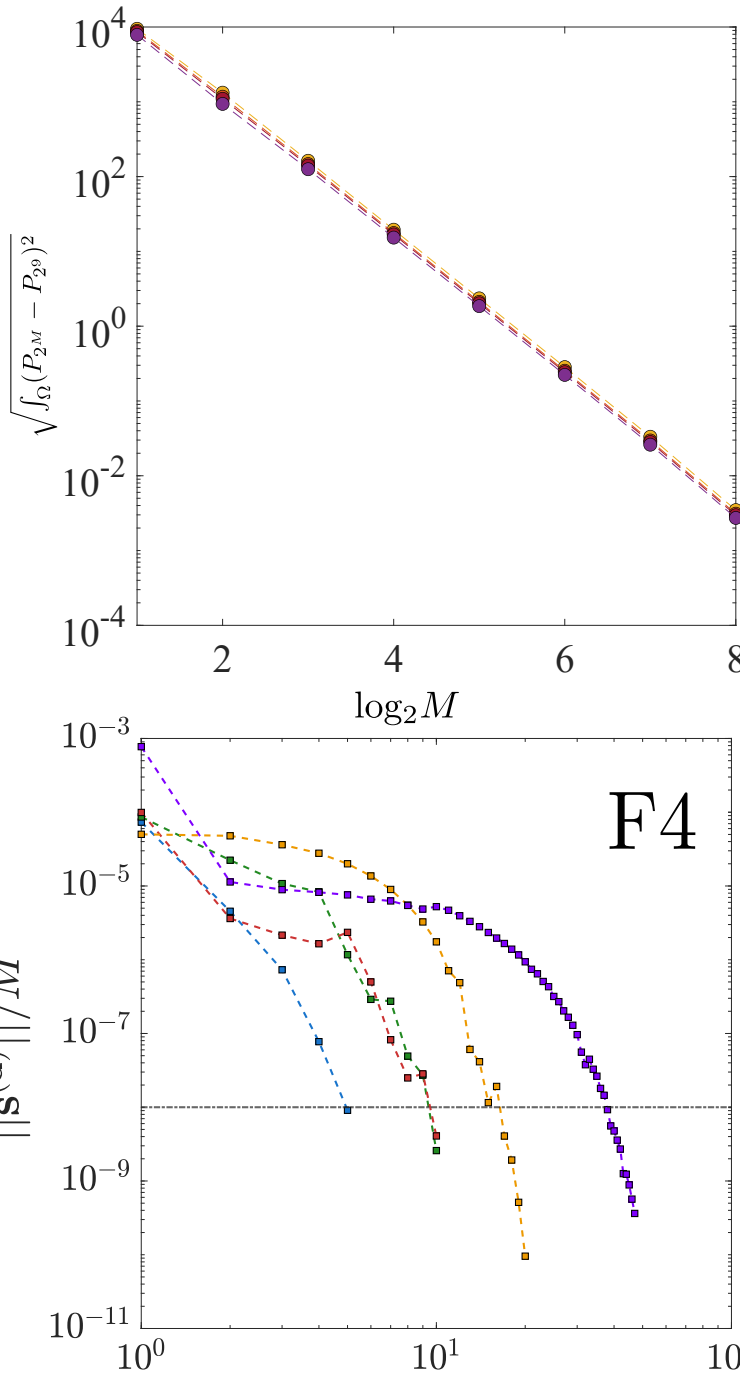
GEOSCIENCES Rennes
UNIVERSITÀ DEGLI STUDI DI PADOVA
ALMA MATER STUDIORUM UNIVERSITÀ DI BOLOGNA