# Estimating radiative forcing with a nonconstant feedback parameter and linear response

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#### Abstract

A new algorithm is proposed for estimating time-evolving global forcing in climate models. The method is a further development of the work of Forster et al. (2013), taking into account the non-constancy of the global feedbacks. We assume that the nonconstancy of this global feedback can be explained as a time-scale dependence, associated with linear temperature responses to the forcing on different time scales. With this method we obtain stronger forcing estimates than previously assumed for the representative concentration pathway experiments in the Coupled Model Intercomparison Project Phase 5 (CMIP5). The reason for the higher future forcing is that the global feedback parameter is more negative at shorter time scales than at longer time scales, consistent with the equilibrium climate sensitivity increasing with equilibration time. Our definition of forcing provides a clean separation of forcing and response, and we find that linear temperature response functions estimated from experiments with abrupt quadrupling of CO\$\_2\$ can be used to predict responses also for future scenarios. In particular, we demonstrate that for most models, the response to our new forcing estimate applied on the 21st century scenarios provides a global surface temperature up to year 2100 consistent with the output of coupled model versions of the respective model.

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## Key Points:

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8	•	We present a new method for estimating radiative forcing and apply it to $abrupt4xCO_2$ ,
9		1%CO <sub>2</sub> , historical, and future scenario experiments
10	•	Including a time-scale dependent feedback parameter results in stronger forcing
11		estimates for the 21st century
12	•	The temperature responses to the new forcing are well described by a linear re-
13		sponse

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#### 14 Abstract

A new algorithm is proposed for estimating time-evolving global forcing in climate mod-15 els. The method is a further development of the work of Forster et al. (2013), taking into 16 account the non-constancy of the global feedbacks. We assume that the non-constancy 17 of this global feedback can be explained as a time-scale dependence, associated with lin-18 ear temperature responses to the forcing on different time scales. With this method we 19 obtain stronger forcing estimates than previously assumed for the representative con-20 centration pathway experiments in the Coupled Model Intercomparison Project Phase 21 5 (CMIP5). The reason for the higher future forcing is that the global feedback param-22 eter is more negative at shorter time scales than at longer time scales, consistent with 23 the equilibrium climate sensitivity increasing with equilibration time. Our definition of 24 forcing provides a clean separation of forcing and response, and we find that linear tem-25 perature response functions estimated from experiments with abrupt quadrupling of  $CO_2$ 26 can be used to predict responses also for future scenarios. In particular, we demonstrate 27 that for most models, the response to our new forcing estimate applied on the 21st cen-28 tury scenarios provides a global surface temperature up to year 2100 consistent with the 29 output of coupled model versions of the respective model. 30

### 31 1 Introduction

Diagnosing the magnitude of a climate forcing is necessary to determine the climate responses to this forcing. However, defining a clear separation between forcing and response is challenging, and no clear distinction exists (Sherwood et al., 2015). In this paper we attempt to apply a separation within a linear temperature response framework, incorporating also the possibility of globally nonconstant atmospheric feedbacks. We test this method on models participating in the Coupled Model Intercomparison Project Phase 5 (CMIP5).

In the most common forcing-feedback framework, the radiative imbalance at the top of the atmosphere (N) is described as

$$N = \lambda T + F,\tag{1}$$

where T is the temperature response,  $\lambda$  is the feedback parameter, and F is the radia-39 tive forcing, all evaluated as global means. According to this equation, forcing is the ini-40 tial radiative imbalance, before the global mean surface temperature starts to respond. 41 However, as discussed by Hansen et al. (2005); Richardson et al. (2019), there are many 42 ways of defining the forcing, allowing various rapid adjustments before diagnosing the 43 radiative imbalance. Forcing estimates are therefore method and model dependent. Some 44 studies even consider multi-annual adjustments associated with ocean inertia (Williams 45 et al., 2008; M. Rugenstein, Gregory, et al., 2016; Menzel & Merlis, 2019). A motivation 46 for this study is therefore to find an estimation method aiming for a clean separation be-47 tween forcing and response. By design, our method aims at finding the forcing estimates 48 that are the most predictable for the surface temperature responses. 49

The uncertainties associated with forcing estimates are large, not only due to the 50 different rapid adjustments between models (Smith et al., 2018), but also due to differ-51 ences in the parameterizations of the radiative transfer (Soden et al., 2018). The instan-52 taneous forcing spread contributes to about half of the total intermodel spread in forc-53 ing (Chung & Soden, 2015), and the remaining spread is largely due to fast cloud ad-54 justments (Zelinka et al., 2013). These uncertainties have led to an effort aiming at bet-55 ter characterizing the forcing used for the new CMIP6 model versions (Forster et al., 2016; 56 Pincus et al., 2016). These studies recommend using fixed-SST forcing, largely due to 57 the reduced level of noise by this method as compared to regression-based estimates. Fixed-58 SST forcing estimates are made by diagnosing the top of atmosphere radiative imbal-59 ance after fixing the sea-surface temperatures and letting the atmosphere adjust. These 60

effective forcings include rapid adjustments, e.g. atmospheric temperature and cloud adjustments, and are found to be better predictors of global surface temperature responses

Justments, and are found to be better predictors of global surface temperature responses than instantaneous forcing estimates (Richardson et al., 2019). There is, however, sub-

than instantaneous forcing estimates (Richardson et al., 2019). There is, however, substantial land warming in these simulations. Our main motivation is to improve forcing

estimates based on already existing simulations, which can be used for models where fixed-

66 SST forcing is unavailable, and to circumvent the problem of land warming in fixed-SST

67 simulations.

In experiments with a time-varying forcing, forcing estimates may be even more uncertain than in idealized experiments with constant forcing. Forster et al. (2013), hereafter F13, computes forcing time series F(t) by rearranging Eq. (1). Their method consists of first determining  $\lambda$  following the regression method of Gregory et al. (2004) using idealized step-forcing simulations, and then using time series of N(t) and T(t) from any experiment to compute what they call adjusted forcing:

$$F(t) = N(t) - \lambda T(t) \tag{2}$$

We note that adjusted forcing in F13 does not mean the same as adjusted forcing in Hansen et al. (2005), where the latter allows only fast stratospheric adjustments to take place before the forcing is estimated from the top of the atmosphere imbalance in an idealized step-forcing experiment. Forcing estimates based on regressions in a Gregory plot, such as in Andrews et al. (2012) and F13 are what Forster et al. (2016) refers to as regression-based methods, assuming a constant feedback parameter.

However, several recent studies have pointed out that  $\lambda$  is not a constant (Armour 74 et al., 2013; Geoffroy, Saint-Martin, Bellon, et al., 2013; Andrews et al., 2015; Gregory 75 & Andrews, 2016; Proistosescu & Huybers, 2017; M. Rugenstein et al., 2020). Armour 76 et al. (2013) demonstrate that locally constant feedbacks can result in a globally time-77 dependent feedback parameter because the pace of sea surface temperatures (SST) equi-78 libration depends on the location, weighting the local feedbacks differently with time. 79 Other studies demonstrated that also locally, feedbacks change magnitude with equili-80 bration time (e.g. Andrews et al., 2015; Andrews & Webb, 2018; M. Rugenstein, Caldeira, 81 & Knutti, 2016; Proistosescu & Huybers, 2017; Dong et al., 2019, 2020) and also through-82 out the historical time period (Paynter & Frölicher, 2015; Gregory & Andrews, 2016; Ar-83 mour, 2017; Marvel et al., 2018; Dessler, 2020). The tropical Pacific, the relative warm-84 ing of midlatitude or global oceans to the West Pacific warm pool, the North Atlantic, 85 and the mid- and high latitudes have all been suggested to influence global feedbacks (e.g. 86 Winton et al., 2010; Trossman et al., 2016; Andrews & Webb, 2018; Dong et al., 2020; 87 Zelinka et al., 2020). The mechanism most often invoked is the dependence of lower tro-88 pospheric stability on the ratio of local and remote SSTs. Regions warming faster than 89 the West Pacific warm pool? which sets the temperature of the free troposphere through 90 deep convection ? show a reduced lower tropospheric stability, a decrease in low-cloud 91 coverage, and thus, a strong cloud and net radiative effect at the top of the atmosphere 92 (e.g. Zhou et al., 2016; Ceppi & Gregory, 2017). In the CMIP6 models, the shortwave 93 cloud feedbacks in the extratropics appear to be more important for the nonconstancy 94 of  $\lambda$  than clouds in the tropics (Zelinka et al., 2020; Bacmeister et al., 2020), but the rel-95 atively short record of global cloud observations makes it difficult to assess cloud mod-96 eling against the observations (Loeb et al., 2020). Other studies highlight the dependence 97 of feedbacks on temperature and radiative forcing (Meraner et al., 2013; Rohrschneider 98 et al., 2019; Bloch-Johnson et al., 2021). 99

The nonconstancy of  $\lambda$  implies that the forcing definition in Eq. (2) is ambiguous. This is particularly apparent for strong temperature responses, when  $\lambda T$  more strongly affects the determination of the value of F. Here the magnitude and time-dependence of  $\lambda$  are particularly important. Larson and Portmann (2016) demonstrated for instance that  $\lambda$  obtained from regressions in the first 20 yr time period of abrupt4xCO<sub>2</sub> gives higher forcing estimates compared to regressions in 150 yr time period. This is one of several reasons why Forster et al. (2016) recommends fixed-SST methods instead of regression
 methods to determine the forcing.

We explore how an alternative definition of effective forcing with a time-scale de-108 pendent  $\lambda$  differs from estimates by F13. To compute these alternative estimates, we de-109 compose the temperature response assuming it responds linearly to the forcing, and we 110 demonstrate that the linear temperature response to the new forcing is close to the mod-111 elled temperature response in future scenarios for most CMIP5 models. By a linear re-112 sponse, we mean the temperature response determined from a linear non-homogeneous 113 system of differential equations, whose solution can be expressed as a convolution be-114 tween a Green's function and the forcing. Our results suggest that this forcing estimate 115 appears more appropriate for estimating temperature responses using linear response mod-116 els than previous estimates. 117

Our method is an iterative routine, starting with the F13 estimate of forcing, then computing the linear response to this forcing, which is further used to compute a new forcing estimate, etc., until convergence to a final forcing estimate is obtained. Theory and methods are described in Section 2, and the results are shown in Section 3. In Section 4, we discuss the assumptions made in our method, and how it compares to other forcing estimates, before we conclude in Section 5.

#### <sup>124</sup> 2 Theory and methods

The time-scale dependence of  $\lambda$  is analysed by making use of the same decomposition as in Proistosescu and Huybers (2017), hereafter PH17. While PH17 use the method to better understand estimates of climate sensitivity, we are interested in the intersect of the fit with the vertical axis, the initial radiative imbalance. We also estimate parameters using a different approach, mainly because our method simplifies the comparison to methods based on single regression estimates in Gregory plots. The equations that will be presented in this section provide interpretations of the different  $\lambda$ 's that may appear in a Gregory plot, as well as interpretations of "forcing estimates" based on regressions on decadal to centennial time scales. The method is based on the assumption that the temperature response can be decomposed into a sum of K components  $T = \sum_{n=1}^{K} T_n$ , where each component is the exponential temperature response to the forcing on the time scale  $\tau_n$  [yrs],

$$T_n(t) = c_n \exp(-t/\tau_n) * F(t).$$
(3)

The \* denotes a convolution, and the factors  $c_n \left[\frac{Km^2}{W}\right]$  are the amplitudes of the tem-125 perature responses per unit forcing. As further explained in the next subsection, this tem-126 perature decomposition can be interpreted as either approximating different global-scale 127 processes (such as mixed-layer versus deep ocean responses to forcing) or as regions re-128 sponding with different pace to the forcing (such as the tropics in general versus regions 129 of upwelling or deep ocean convection).  $c_n$  therefore depends on both the feedbacks and 130 thermal inertia associated with different regions, and the fraction of the global area in-131 volved in the response at time scale  $\tau_n$ . 132

Furthermore, the method assumes that constant feedback parameters  $\lambda_n$  exist, with n = 1, ..., K associated with each time scale, such that the terms in Eq. (1) can be decomposed into the following sums:

$$N(t) = \sum_{n=1}^{K} N_n(t) = F(t) + \sum_{n=1}^{K} \lambda_n T_n(t) = F(t) + \lambda(t)T(t)$$
(4)

By rewriting Eq. (4), PH17 noted that the time-variation of  $\lambda(t)$  can be explained as a weighted average of the feedbacks associated with different components  $T_n(t)$  of the global temperature:

$$\lambda(t) = \frac{\sum_{n=1}^{K} \lambda_n T_n(t)}{\sum_{n=1}^{K} T_n(t)}$$
(5)

<sup>133</sup> We note that in a 4xCO<sub>2</sub> experiment, we define the forcing to be a constant, and <sup>134</sup> the slope  $\lambda(t)$  must be interpreted as the slope of a line drawn between the fixed forc-<sup>135</sup> ing F and a point (T(t), N(t)). This slope may differ from a linearization around a point <sup>136</sup> (T(t), N(t)) by regressing a range of points (see discussion on feedback definitions in M. A. A. Ru-<sup>137</sup> genstein and Armour (2021)).

Armour et al. (2013) suggested a similar decomposition, but interpreted the components as locally constant feedbacks multiplied by local temperatures with different time evolution. However, recent studies suggest that non-local feedbacks are also important (Andrews et al., 2015; Zhou et al., 2016; Dong et al., 2019; Bloch-Johnson et al., 2020), meaning that temperature changes in one region, and in particular the West Pacific, can influence feedbacks globally.

#### <sup>144</sup> 2.1 Linear model and response

A simple model of temperature changes in the climate system can be constructed by considering different boxes or components that store and exchange energy. If assuming that all anomalous heat fluxes are linearly related to temperature anomalies in the system, the heat uptake in all boxes can be written into a linear non-homogeneous system

$$\mathbf{C}\frac{d\mathbf{T}(t)}{dt} = \mathbf{K}\mathbf{T}(t) + \mathbf{F}(t) \tag{6}$$

By choosing the vector of temperature change components  $\mathbf{T}$  to be K-dimensional, 145 the system describes K components that will respond on K different time scales, and 146 the vector  $\mathbf{F}$  the atmospheric forcing acting directly on each component. The vector  $\mathbf{F}$ 147 could in principle contain different forcings in different regions. The heat capacities  $\left[\frac{Wyr}{m^2K}\right]$ 148 associated with each component are along the diagonal of the diagonal  $K \times K$  matrix 149  $\mathbf{C}$ , and coefficients for heat exchange between components and heat loss to the atmo-150 sphere  $\left[\frac{W}{m^2 K}\right]$  constitute the matrix **K**. The left-hand side of this equation describes the 151 heat uptake of each component, and the sum of all heat uptakes must equal the net ra-152 diative imbalance N. In this sum of all components, all fluxes between components can-153 cel out, and the sum reduces to Eq. (4). 154

Linear systems like this have been widely studied, often using one, two or three boxes 155 (e.g. Geoffroy, Saint-Martin, Olivié, et al., 2013; Fredriksen & Rypdal, 2017). Symmet-156 ric matrices K will describe diffusive heat fluxes depending on the temperature differ-157 ence between two boxes, and feedback parameters will appear on its diagonal. Non-symmetric 158 parts may be due to the dependence of temperature anomalies in one box only. For in-159 stance change in sinking processes due to temperature anomalies in the North Atlantic 160 regarded as one box, may by mass continuity induce horizontal mass and hence energy 161 fluxes from adjacent ocean basins regarded as other boxes, independent of the temper-162 ature change in these boxes. K may also incorporate heat fluxes to the deep ocean if as-163 suming they can be modelled as linear functions of temperature components (e.g. Held 164 et al., 2010; Geoffroy, Saint-Martin, Olivié, et al., 2013). 165

By applying the method variation of parameters, it can be shown that the solution to Eq. (6) is (see the supporting information):

$$\mathbf{T}(t) = \int_{-\infty}^{t} e^{(t-s)\mathbf{C}^{-1}\mathbf{K}} \mathbf{C}^{-1}\mathbf{F}(s) \, ds, \tag{7}$$

showing that the temperature at time t is a response to the forcing experienced at all previous times s. If the matrix  $\mathbf{C}^{-1}\mathbf{K}$  has only negative eigenvalues,  $-1/\tau_n$ , the solu-

tion for each temperature component  $T_k(t)$  will be a weighted sum of K exponential responses to the global average forcing F with time scales  $\tau_n$  (where the weights  $\beta_n$  are determined by eigenvalues, eigenvectors, and heat capacities),

$$T_k(t) = \int_{-\infty}^t \sum_{n=1}^K \beta_n e^{(s-t)/\tau_n} F(s) \, ds$$
(8)

Furthermore, the global surface temperature is a weighted average of the components  $T_k(t)$ :

$$\overline{T(t)} = \sum_{n=1}^{K} c_n \int_{-\infty}^{t} e^{(s-t)/\tau_n} F(s) \, ds$$
(9)

where we define the new weights  $c_n$  to be an area-weighted average of the weights  $\beta_n$ . If the forcing is not the same in all regions, Eq. (9) is still valid if the regional forcings are scaled versions of the global average forcing. We recognize Eq. (9) as a convolution between a Green's function G(t) and a forcing F(t), consistent with Eq. (3): T(t) = G(t) \*  $F(t) = \int_{-\infty}^{t} G(t-s)F(s)ds$ , with  $G(t) = \sum_{n=1}^{K} G_n(t) = \sum_{n=1}^{K} c_n \exp(-t/\tau_n)$ , assuming negative eigenvalues.

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#### 2.2 Estimating linear response in abrupt 4xCO<sub>2</sub> experiments

To simplify the estimation of parameters of these responses (time scales  $\tau_n$  and am-173 plitudes  $c_n$ ), we start by fixing the time scales, such that T and N depend linearly on 174 the remaining parameters  $c_n$ . We find that the exact choice of time scales is not impor-175 tant, as long as we choose them well separated, and within the range of expected time 176 scales. Annual time scales are important over land and shallow ocean areas, while decadal 177 and centennial time scales are particularly important in ocean regions with mixing to 178 the deeper oceans, and hence higher thermal inertia. Following PH17, we use three dif-179 ferent time scales. They find three time scales to be the smallest number that well de-180 scribes the temperature responses. In addition as explained later, we will assume the ex-181 istence of a fourth time scale explaining slower temperature responses than can be ob-182 served in the records studied in this paper. 183

We analyse data from 21 CMIP5 models, available at https://esgf-node.llnl 184 .gov/projects/cmip5/. The variables used are global annual averages of surface air tem-185 peratures (tas), and net top-of-atmosphere radiation, computed as the difference between 186 incoming shortwave radiation and outgoing longwave and shortwave radiation (rsdt - rlut 187 - rsut). To minimize the effect of possible model drifts, the temperature T(t) and the 188 variables used to compute the net top of atmosphere radiation N(t) time series are de-189 fined as deviations from linear trends in the corresponding time period of the control run 190 (trend values for the  $abrupt4xCO_2$  period are given in Table S1, and are very small). With 191 this definition we also avoid non-zero means of N(t) in equilibrium, which is the case for 192 many models (Forster et al., 2013). 193

The shortest time scale  $\tau_1$  is chosen to be a random number between 1 and 6 years, 194 the second time scale  $\tau_2$  is a random factor between 5 and 10 multiplied by  $\tau_1$ , and the 195 third is a randomly chosen time scale between 80 and 1000 years. The random choice 196 is done 1000 times for each model, and finally, for each model, we keep the set of  $\tau_n$  with 197 the best (least squares) fit to the modelled temperature evolution for 150 years after an 198 abrupt quadrupling of  $CO_2$ . The resulting parameters are dependent on the length of 199 the time series used. If using longer time series the longest time-scale responses may change 200 the most, but these are also the least important for our 21st century analyses. 201

The temperature response for these step-forcing experiments can be found by computing the integrals in Eq. (9) with a constant forcing  $F_{4xCO_2}$  for t > 0. This integral results in

$$T_{4\text{xCO}_2}(t) = \sum_{n=1}^{K} a_n (1 - e^{-t/\tau_n})$$
(10)

where  $a_n = c_n \tau_n F_{4\text{xCO}_2}$  is the equilibrium temperature of each component, and the equilibrium climate sensitivity (ECS) is defined as  $\frac{1}{2} \sum_{n=1}^{K} a_n$  (equilibrium response to a dou-202 203 bling of  $CO_2$ ). 204

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The expression for N is derived as:

$$N_{4xCO_{2}}(t) = F_{4xCO_{2}} + \sum_{n=1}^{K} (\lambda_{n}T_{n}(t))$$
  
=  $F_{4xCO_{2}} + \sum_{n=1}^{K} (\lambda_{n}a_{n}(1 - e^{-t/\tau_{n}}))$   
=  $F_{4xCO_{2}} + \sum_{n=1}^{K} \lambda_{n}a_{n} - \sum_{n=1}^{K} \lambda_{n}a_{n}e^{-t/\tau_{n}}$   
=  $-\sum_{n=1}^{K} \lambda_{n}a_{n}e^{-t/\tau_{n}}$ 

where we in the last step set that  $F_{4xCO_2} + \sum_{n=1}^{K} \lambda_n a_n = 0$ , due to the constraint that  $N \to 0$  when  $t \to \infty$ . Introducing the notation that  $b_n = -a_n \lambda_n$  gives us  $N_{4xCO_2}(t) = \sum_{n=1}^{K} N_n(t) = \sum_{n=1}^{K} b_n e^{-t/\tau_n}$ , and  $F_{4xCO_2} = -\sum_{n=1}^{K} \lambda_n a_n = \sum_{n=1}^{K} b_n$ . 206 207 208

The parameters  $a_n$ ,  $b_n$  could be found using linear regression, but that does some-209 times violate the physical assumption that these should have the same sign as the forc-210 ing. Therefore we have used the non-negative least squares algorithm to ensure positive 211 parameters. This is used only for finding  $a_n$ , and the resulting temperature responses 212 are shown in Figure 1 b). This method could in principle also have been used to find  $b_n$ , 213 but this does not seem to provide a sufficiently good fit on the short scales. Instead,  $\lambda_n$ 214 are determined in a Gregory plot, and then used to compute  $b_n = -\lambda_n a_n$ . 215

#### 2.3 Algorithm for estimating $\lambda_n$

The  $\lambda_n$ ,  $n = 1, \ldots, K$  are all determined from linear fits in a Gregory plot, as 217 shown in Figure 1 a). We start with estimating  $\lambda_3$  corresponding to time scale  $\tau_3$ , then 218 we estimate  $\lambda_2$ , and finally  $\lambda_1$ . We assume that the sum  $\sum_{n=1}^{3} a_n$  underestimates the 219 equilibrium response, since the sum excludes the response on the multi-millennial scale 220  $\tau_4$ . However, we assume  $\tau_4$  is so large that we can make the following approximations 221 for  $t \leq 150$  years: 222

$$T_4(t) = a_4 \left( 1 - e^{-t/\tau_4} \right) \approx 0$$
 (11)

$$N_4(t) = b_4 e^{-t/\tau_4} \approx b_4$$
 (12)

Hence  $T(t) \approx \sum_{n=1}^{3} T_n(t)$  and  $N(t) \approx b_4 + \sum_{n=1}^{3} N_n(t)$ , where  $b_4$  could be interpreted 223 as a constant heat flux going into the deeper oceans, hereby not leading to surface warm-224 ing on short time scales. We made the somewhat arbitrary choice of setting  $\tau_4 = 5000$ 225 years, and assume  $\lambda_4 = \lambda_3$ . The results are not sensitive to the choice of  $\tau_4$  as long as 226 the approximations in Eqs. (11) and (12) hold. In the 150 year long runs considered in 227 this paper, we have no information about  $\lambda_4$ , but longer runs show that the feedback pa-228 rameter changes little on the longer time scales (M. Rugenstein et al., 2020). 229

**Determining**  $\lambda_3$ : We consider only temperatures larger than the equilibrium tem-230 perature of the first two components, such that  $T_1(t) + T_2(t) \approx a_1 + a_2$ , and we have: 231  $N(t) \approx -\lambda_3(a_3 - T_3(t)) + b_4$ . The total temperature is therefore approximated by  $T(t) \approx$ 232

 $a_1 + a_2 + T_3(t), \text{ resulting in } N(t) \approx -\lambda_3(a_1 + a_2 + a_3 - T(t)) + b_4. \text{ This shows that } N \text{ is}$ approximately a linear function of T with slope  $\lambda_3$  for  $T > a_1 + a_2$ . Therefore,  $\lambda_3$  is computed by linear regression of these points, and the equilibrium temperature found by following this line until N = 0. This equilibrium estimate should be higher than  $\sum_{n=1}^{3} a_n$ , and the difference is  $a_4$ . Whenever the unphysical result  $a_4 < 0$  is obtained, we exclude the chosen time scales from our analysis.

**Determining**  $\lambda_2$ : First we subtract our estimates of  $T_3(t)$ ,  $T_4(t)$  and  $N_3(t)$ ,  $N_4(t)$ 239 from the time series T(t) and N(t), respectively. We then obtain estimates of  $T_1(t)$  + 240 241  $T_2(t)$  and  $N_1(t)+N_2(t)$ , and these points are the dark gray dots in Figure 1a). For  $a_1 < t < 1$  $T_1(t) + T_2(t) < a_1 + a_2, T_1(t) + T_2(t)$  is approximately  $a_1 + T_2(t)$ , and should equal 242 the equilibrium value  $a_1 + a_2$  when  $N_1(t) + N_2(t) = 0$ . In this range,  $N_1(t) + N_2(t) \approx$ 243  $-\lambda_2(a_2-T_2(t))$ , approximately linearly related to  $T_1(t)+T_2(t)$ . Therefore,  $\lambda_2$  is esti-244 mated using a least-squares algorithm forcing the linear fit to go through the point  $(a_1 +$ 245  $a_2, 0).$ 246

247 **Determining**  $\lambda_1$ : We subtract estimates of  $(T_2(t), N_2(t))$  from the dark gray dots 248 to obtain estimates of  $T_1(t)$  and  $N_1(t)$  (light gray dots in Figure 1). We have now  $N_1(t) \approx$ 249  $-\lambda_1(a_1-T_1(t))$ , and we can, as previously, use least squares to compute  $\lambda_1$ , forcing the 250 linear fit to pass the point  $(a_1, 0)$ .

In the least squares fits, we also include an upper time limit to the set of points 251 to be included in the calculation. This limit is set to the first time step after reaching 252 99% of the equilibrium temperature of the component of interest. In this way, our slope 253 is associated with the response on the particular time scale  $\tau_n$ , and little influenced by 254 the fluctuations around the equilibrium values. Changing this limit to e.g. 90% or 95%255 has only minor effects on the results. Feedback parameters associated with fluctuations 256 around the base state, or more precisely, radiative restoring coefficients are studied in 257 several papers (Colman & Power, 2010; Colman & Hanson, 2013; Lutsko & Takahashi, 258 2018; Bloch-Johnson et al., 2020). Depending on the model, they can be similar or dif-259 ferent from those associated with the final fluctuation after a quadrupling of  $CO_2$  (M. Ru-260 genstein et al., 2020), and they may also differ from feedbacks associated with forced re-261 sponses (e.g. Zhou et al., 2015; Dessler & Forster, 2018). 262

When all  $a_n$ ,  $\lambda_n$  are estimated, we compute  $b_n = -\lambda_n a_n$  and we finally have our estimate of  $F_{4xCO_2} = \sum_{n=1}^{4} b_n$ . That is, the sum of the initial radiative imbalance of all 4 components.

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#### 2.4 New estimates of effective forcing time series

Using our parameter estimates from the previous subsections, we can for any experiment use the global mean evolutions of T(t) and N(t) to compute a new estimate of the effective forcing as follows:

- 1. Compute F(t) using F13's method (a single estimate of  $\lambda$ ), and take this as the initial estimate of the effective forcing.
- 272 2. Use this forcing estimate and amplitudes  $c_n = \frac{a_n}{\tau_n F_{4xCO_2}}$  estimated from 4xCO<sub>2</sub> 273 experiments to compute the components  $T_n(t)$  from Eq. (3) by performing convolution integrals.
  - 3. A new estimate of F(t) can then be computed as:

$$F(t) = N(t) - \sum_{n} \lambda_n T_n(t)$$
(13)

4. Repeat steps 2-3 until convergence of F(t). We have used 20 iterations.

We demonstrate how the method can be applied to study the forcing for 1% CO<sub>2</sub> experiments, the historical period and the four representative concentration pathways (RCPs) RCP2.6, RCP4.5, RCP6.0 and RCP8.5.

Table 1. Estimated parameters, where we define  $F_{2x}$  and  $T_{2x}$  to be half the forcing and equilibrium temperature estimated for a quadrupling of CO<sub>2</sub>. The parameters in parentheses  $(-\lambda)$ ,  $(F_{2x})$  and  $(T_{2x})$  are estimated from a single linear regression over years 1-150 in a Gregory plot. The results differ slightly from the numbers reported from the Gregory method by Andrews et al. (2012), possibly because of minor differences in the way global annual average values are constructed. For one model (GFDL-ESM2G) the best fit consists of two exponential responses, where we estimate  $a_2 = 0$  and report  $\lambda_2 = b_2/a_2$  as 'NaN'.

	$ au_1$	$ au_2$	$ au_3$	$-\lambda_1$	$-\lambda_2$	$-\lambda_3$	$(-\lambda)$	$F_{2\mathbf{x}}$	$(F_{2\mathbf{x}})$	$T_{2\mathbf{x}}$	$(T_{2\mathbf{x}})$
ACCESS1-0	2.43	12.79	231.10	1.30	1.12	0.56	0.78	3.72	2.97	4.33	3.83
ACCESS1-3	1.13	5.80	150.10	1.46	1.30	0.56	0.82	3.60	2.89	4.12	3.53
CanESM2	2.86	26.39	279.11	1.30	1.01	0.91	1.04	4.24	3.83	3.83	3.69
CCSM4	1.04	5.52	197.28	1.32	1.77	0.90	1.18	4.02	3.47	3.19	2.94
CNRM-CM5	1.45	10.71	392.15	1.38	1.09	1.22	1.14	3.87	3.71	3.20	3.25
CSIRO-Mk3-6-0	1.62	11.29	308.98	1.86	1.12	0.41	0.63	3.94	2.58	4.94	4.08
GFDL-CM3	3.28	32.58	98.81	1.21	0.80	0.63	0.75	3.61	2.99	4.24	3.97
GFDL-ESM2G	2.98	17.50	291.97	1.76	NaN	0.90	1.29	3.65	3.09	2.67	2.39
GFDL-ESM2M	1.03	5.77	240.02	1.52	1.58	1.22	1.38	3.58	3.36	2.52	2.44
GISS-E2-H	1.56	10.43	186.27	2.02	1.83	1.40	1.65	4.21	3.81	2.39	2.31
GISS-E2-R	1.51	10.61	232.40	2.98	1.02	1.42	1.79	5.09	3.78	2.25	2.11
HadGEM2-ES	1.01	8.39	367.62	1.96	0.89	0.35	0.63	4.02	2.90	5.91	4.61
inmcm4	1.02	5.65	597.43	1.90	1.48	1.28	1.43	3.18	2.98	2.14	2.08
IPSL-CM5A-LR	1.72	16.54	163.83	1.03	0.84	0.58	0.75	3.43	3.10	4.55	4.13
IPSL-CM5B-LR	1.21	8.01	80.30	2.39	1.11	0.91	1.02	3.64	2.64	2.68	2.60
MIROC-ESM	1.78	11.32	266.35	1.96	0.92	0.68	0.91	5.37	4.26	5.21	4.67
MIROC5	2.77	15.17	89.28	1.72	1.43	1.36	1.52	4.38	4.13	2.80	2.72
MPI-ESM-LR	1.81	9.20	202.56	1.30	1.50	0.86	1.13	4.53	4.09	3.91	3.63
MPI-ESM-MR	1.02	6.23	158.54	2.27	1.45	0.94	1.18	5.15	4.07	3.67	3.46
MRI-CGCM3	1.42	11.61	233.73	2.22	1.34	0.96	1.25	4.05	3.24	2.76	2.60
NorESM1-M	1.75	9.34	273.12	1.87	1.52	0.78	1.11	3.88	3.10	3.17	2.80
Model mean	1.73	11.73	231.26	1.73	1.28	0.90	1.12	4.04	3.38	3.50	3.20
Standard deviation	0.69	6.58	115.35	0.45	0.30	0.31	0.31	0.56	0.50	1.02	0.78

#### 279 **3 Results**

The results of the linear response fit for T(t) and N(t) following an abrupt qua-280 drupling of  $CO_2$  are given for the model NorESM1-M in Figure 1, and the estimated pa-281 rameters are listed in Table 1. We note from Figure 1a) that both the forcing and equi-282 librium temperature estimates are higher than when obtained from a straight line fit. 283 The narrow spread of the light blue lines also indicate that the choice of time scales is 284 of little importance, and hence not affecting the overall conclusions. Similar plots are 285 shown for the other models listed in Table 1 in the Supporting information. The uncer-286 tainty in both the forcing estimate and ECS estimate vary substantially from model to 287 model. Models with a rapid initial warming, such as GISS-E2-R, have fewer points con-288 straining the regression estimate for the shortest time scale, implying larger uncertainty 289 of the forcing. 290

An overview of all our estimates of the 4xCO<sub>2</sub> forcing is presented in Figure 2. In addition, we compare our forcing estimates to regression estimates done for years 1-20 and years 1-150. In all except one model, the 1-20 year regression gives a higher estimate

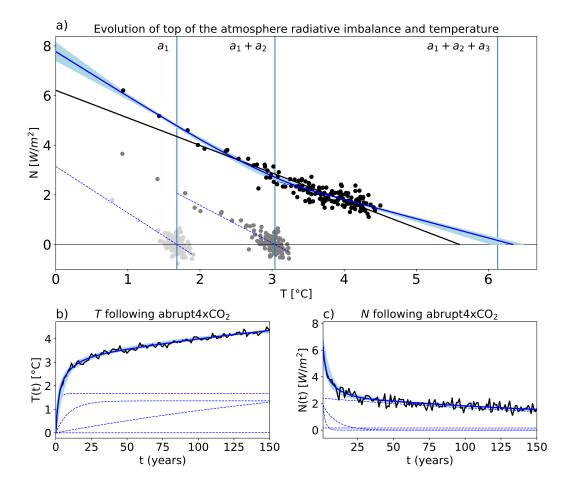


Figure 1. Results for NorESM1-M: a) The black dots and line is a conventional Gregory plot, the light blue lines (recognized as light-blue shading) are our fits to the black points with 1000 different choices of time scales, and the dark blue fit is when using the best (least squares) fits for the temperature in b). Vertical blue lines are the sums of equilibrium temperatures  $\sum_{n=1}^{m} a_n$ , m = 1, 2, 3. The dark (light) gray dots are N vs. T after subtracting components associated with the third (and second) time scales, and the dashed blue lines are fits to these dots. b) The black curve is the climate model temperature output, and the light blue curves are best fits to the modelled temperature using 1000 different choices of time scales. The dark blue curve is the best fit, and the dashed blue curves are the individual components due to the four time scales which are summed to obtain this fit. c) As panel b), but for the change in net top of the atmosphere radiation.

than the 1-150 year regression. And in all but two models, our best forcing estimate is even higher than estimates obtained from regression of years 1-20. The fixed-SST 4xCO<sub>2</sub> forcing estimates reported by Andrews et al. (2012) are higher than regression-based estimates over 150 years for most of the models where this is available, but smaller than our new forcing estimates.

Using global annual means of N(t) and T(t) from the coupled models, we continue by testing the algorithm described in Section 2.4 for 1% CO<sub>2</sub> experiments. In these experiments we expect a linearly increasing forcing, because to first order, for small increases in CO<sub>2</sub> the forcing depends logarithmically on the CO<sub>2</sub> concentration (Myhre et al., 1998)

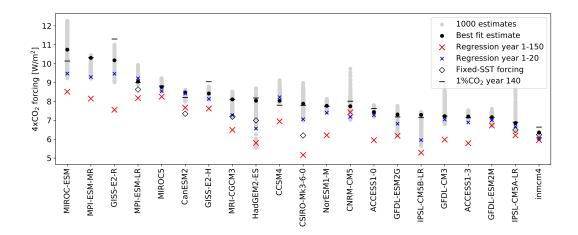


Figure 2. A summary of the  $4xCO_2$  forcing estimates made in this paper, to provide an overview of their uncertainties and how they compare to regression estimates. The 1%CO<sub>2</sub> estimates are the linear fits to the estimated 1% CO<sub>2</sub> forcing time series evaluated in year 140, the time of quadrupling (except for the models GFDL-ESM2G and GFDL-ESM2M, where the estimates are instead twice the doubling estimates in year 70). Fixed-SST estimates are taken from Andrews et al. (2012) for the models where these are available.

(see limitations of this discussed in Byrne and Goldblatt (2014); Etminan et al. (2016); 303 Gregory et al. (2015); Bloch-Johnson et al. (2021)). A linear increase is indeed what we 304 observe for NorESM1-M in Figure 3, for both the initial and the new forcing estimate. 305 For the new estimate we note a high consistency between the climate model tempera-306 ture output and the linear response to the forcing. This result suggests that our method 307 can successfully construct forcing estimates that well predicts the surface temperature 308 responses. Results for other models are similar, and are shown in the supporting infor-309 mation. After 140 years of 1% increase the CO<sub>2</sub> concentration is quadrupled, and the 310 linear fit to the 1% CO<sub>2</sub> forcing time series evaluated in year 140 is yet another estimate 311 of  $F_{4xCO_2}$ , which we include in Figure 2. For most models this estimate is close to our 312 best estimates determined from  $abrupt4xCO_2$  experiments. 313

Next we apply the algorithm to the historical and RCP experiments to compute 314 forcing estimates for the time period 1850 - 2100. Our new forcing estimate for the his-315 torical and RCP8.5 experiment for NorESM1-M diverges from the forcing estimate us-316 ing a single feedback parameter when approaching the end of the 21st century (Figure 317 4a). The difference is about 2  $W/m^2$  in 2100, and smaller differences are seen during the 318 historical period. As a result, the sum of the linear temperature responses we compute 319 by convolving with the two forcing estimates according to Eq. (3) also diverge (dashed 320 curves in Figure 4b), reaching a difference of almost 1 degree in year 2100. We note that 321 the linear response to our new forcing (dashed blue curve) is remarkably close to the cli-322 mate model temperature output, indicating that our alternative forcing definition and 323 linear response assumption is the better approximation for this model. This result holds 324 also for the other RCP scenarios (see Figures S109 - S111 in the supporting information). 325

By computing the time-varying feedback parameter  $\lambda(t)$  using Eq. (5), we find a generally higher magnitude than the single estimate of  $\lambda$ . During the historical period the global temperature response is often close to 0, causing high fluctuations in the estimated  $\lambda(t)$ . The estimate becomes more stable for the future scenarios, where we find a slowly decreasing magnitude of  $\lambda(t)$ , consistent with a higher weighting of the slow responses. For all years in the experiment, the magnitude of  $\lambda(t)$  is still considerably higher

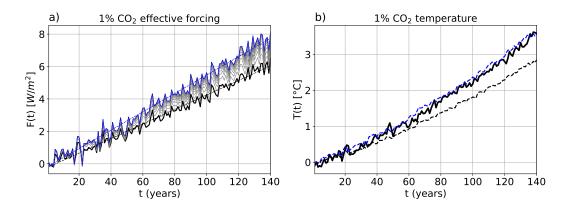


Figure 3. Results for NorESM1-M: a) The black curve is the forcing computed as in F13, using a single and constant value of  $\lambda$ . The gray curves are the iterations of the algorithm described in section 2.4, using three different  $\lambda$ 's, and the blue curve the new forcing obtained by convergence after 20 iterations. The dashed lines are linear fits to the initial and final forcing estimates. b) The thick black curve is the modelled temperature change, and the black and blue dashed curves the linear responses to the black and blue curves in a), applying the same response function as estimated in Figure 1 b).

than the single regression estimate, hence the term  $-\lambda(t)T(t)$  gives a higher contribution to the forcing estimate. This effect on the forcing is however only visible when the temperature response is strong.

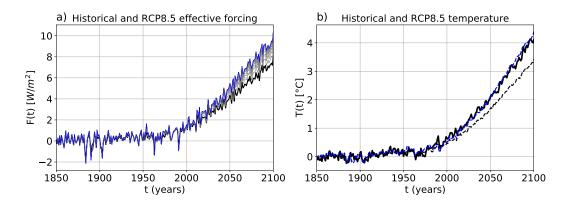


Figure 4. Similar to Figure 3, but for NorESM1-M historical and RCP8.5 experiment.

Repeating the analysis in Figure 4 for all models and RCP scenarios shows that 335 the method presented here works well for many models, but not all (Figures in support-336 ing information). A summary of these results are given in Figure 5, where panel a) com-337 pares the mean estimated forcing over years 2091-2100 using the two different methods. 338 The names of the scenarios are constructed to reflect the intended forcing in the end of 339 the 21st century (van Vuuren et al., 2011), and these forcing levels are also shown for 340 comparison. We find that model estimates using F13's method are centered at lower val-341 ues, while our new forcing estimates are centered close to or slightly above the intended 342 levels. However, the intended forcing is difficult to prescribe as it depends on model-specific 343 fast adjustments, so we can only expect these to be approximate values. The GISS-E2-344

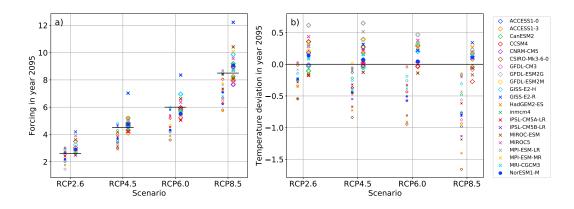


Figure 5. Estimated year 2095 forcing (a) and temperature difference between the result of the linear response and the climate model output (b). For each scenario, the left points show results using F13's method, and the right points show results using our method. Values in year 2095 are computed by averaging over the ten years 2091-2100. The forcing levels 2.6, 4.5, 6.0 and  $8.5 \text{ W/m}^2$  are also shown for reference in a) as horizontal black lines.

R model might be considered as an outlier, and its response to  $abrupt4xCO_2$  is also visually different from the other models.

Consistent with the increase in forcing level, we observe an increase in the estimated 347 linear temperature responses in panel b). The linear responses to F13 forcing are mostly 348 lower than the climate model temperature output, and the responses to our new forc-349 ing are scattered around, with a center slightly above. Some deviation from the climate 350 model temperature is expected due to internal variability, and to assess this expected 351 uncertainty, we refer to the model spread of the Community Earth System Model Large 352 Ensemble (CESM-LE) (Kay et al., 2015). Here 40 model simulations for the historical 353 + RCP8.5 scenarios from the same model show a model spread of around 0.4 K, which 354 is attributed to internal variability. 355

Using F13 forcing, the linear response is within these uncertainties for only a few models. For the new forcing, more models are within this uncertainty range than outside. There are also other uncertainties to consider, e.g. associated with our parameter estimation method, probably making the expected uncertainty interval larger than 0.4 K. The uncertainty due to internal variability is also model-dependent (Olonscheck et al., 2020), hence it is difficult to identify models where our linear response hypothesis and forcing estimation method fail.

We note also that the uncertainty of the future scenario forcing estimates is strongly related to the uncertainty of the  $4xCO_2$  forcing, since both are highly influenced by  $\lambda_1$ (the inter-model correlation between our  $4xCO_2$  and RCP8.5 forcing is 0.82). This is particularly apparent for the GISS-E2-R model, where the response of the first few years is so abrupt that forcing estimates, and hence linear responses, are uncertain with both our and F13's estimation method.

In the two models CNRM-CM5 and MIROC5 the two forcing estimates are very similar, because the feedback is close to constant for all years. For these models we find also that the forcing estimate based on a single feedback parameter gives a slightly better estimate of the linear response. So if the global feedback in fact is constant for all years considered here, using all years in the regression should give a more certain estimate of the feedback parameter, and therefore also more certain forcing estimates.

For the three models GFDL-ESM2G, GFDL-ESM2M, and inmcm4 we find that 375 our method is performing less well (see Figures in the Supporting information). The rea-376 son is probably linked to the almost constant  $4xCO_2$  temperature responses over years 377  $\sim 20-70, \sim 20-60$  and  $\sim 20-120$ , respectively. Our linear response with exponen-378 tially relaxing temperatures always predicts continuously increasing temperatures, which 379 therefore poorly approximates these  $4xCO_2$  global temperatures. The flattening of the 380 response could possibly be linked to changes in the ocean circulation, e.g. a slowdown 381 of the Atlantic meridional overturning circulation. In that case, linear systems with com-382 plex eigenvalues giving oscillatory responses could be an alternative solution. Hence, we 383 will not disregard linear response in these results, but leave further testing of including 384 oscillations in the responses to future studies. 385

#### 386 4 Discussion

For most  $abrupt4xCO_2$  experiments the Gregory plot follows a convex curve, hence 387 our forcing estimates are mostly higher than those found from simple regression anal-388 yses over 150 years (Andrews et al., 2012), or using only the first 20 years (Andrews et 389 al., 2015; Larson & Portmann, 2016). As suggested by PH17, this convexity could be ex-390 plained by considering different feedback parameters associated with the different time 391 scales of the responses. The time-scale dependence of the feedback parameter could be 392 due to feedbacks varying in strength at different time scales, or it could be regionally dif-393 ferent feedbacks weighted differently with time in the global average when the pattern 394 of surface warming evolves. Since it is likely a combination of these circumstances, an 395 interpretation of our parameters could be summarized into:  $\lambda_1$ : Average of annual-scale 396 feedbacks in regions with strong annual-scale responses,  $\lambda_2$ : Average of decadal-scale feed-397 backs in regions with strong decadal-scale responses,  $\lambda_3$ : Average of centennial-scale feed-398 backs in regions with strong centennial-scale responses. Or as we come back to later, this 399 description could also be considered an approximation of feedbacks changing with cli-400 mate state. 401

The fixed-SST estimation method does not include time-variation and uncertain-402 ties in the feedback parameter. Instead, extra model simulations are made with SSTs 403 fixed to climatological values, and the top of atmosphere radiative imbalance is diagnosed. A drawback of this method is that atmospheric and land surface temperatures are al-405 lowed to change. Hence the global temperature anomaly is not 0 when the radiative im-406 balance is diagnosed, and the forcing estimate is therefore contaminated with fast feed-407 back processes associated with land warming. The fixed-SST estimates should be more 408 comparable to our radiative imbalance after some months of adjustments of T(t) and N(t), 409 and Figure 2 shows that they are indeed lower than our estimates for the models where 410 they are available. 411

Ideally the fixed-SST method should be extended to fix the land surface temper-412 atures also, in order to provide a consistent framework where forcing and feedbacks are 413 well separated. Due to technical difficulties, this has only been done for one complex global 414 climate model so far (Andrews et al., 2021). As discussed by Andrews et al. (2021), sev-415 eral methods have been suggested to correct fixed-SST estimates to account for effects 416 of land temperature changes. One could for instance extrapolate the estimate to T =417 0 using Eq. (2) given that we know the feedback parameter, or use radiative kernels (Richardson 418 et al., 2019). Richardson et al. (2019) call these estimates Adjusted Effective radiative 419 forcing, and find also these to be the best predictors for global surface temperatures be-420 cause they have the efficacies closest to 1. 421

Efficacy factors are introduced to correct for differences in how strong the climate response is to different forcing agents, due to e.g differences in rapid adjustments, or effects of a forcing being concentrated in certain regions. Forcing in experiments considered in this study are dominated by CO<sub>2</sub>, a well-mixed greenhouse gas. Other forcings

present during the historical period and future scenarios could be more spatially inho-426 mogeneous, e.g. aerosols, and contribute to different spatial patterns of the response. We 427 neglect this effect when applying the parameters estimated for  $abrupt4xCO_2$  experiments 428 to other experiments, and assume the regional patterns to evolve similarly for different 429 experiments. During the historical period, a changing feedback parameter will only re-430 sult in weak changes in our forcing estimate since the temperature responses are still rel-431 atively weak. But if applying our method to strong forcings other than  $CO_2$ , the pos-432 sible effect of efficacies should be investigated first. 433

434 When estimating a time-varying forcing, an alternative to fixing the SSTs to climatological values (as employed in RFMIP) is to prescribe the SSTs to e.g. the simu-435 lated historical values from the coupled model (as employed in AerChemMIP). These 436 methods produce relatively similar results (Forster et al., 2016), and will both have a lower 437 uncertainty than regression-based estimates. Regression-based estimates are influenced 438 by changes in T(t) arising due to internal variability, e.g. El Niño events, which could 439 drive changes in N(t). In prescribed-SST methods the temperature-driven changes in 440 N(t) is subtracted, resulting in a reduced noise level in the forcing estimate (Forster et 441 al., 2013). 442

The theory described in this paper does not include an explicit temperature-dependence 443 of the feedback parameter (Rohrschneider et al., 2019; Bloch-Johnson et al., 2021), since 444 it is assumed that Eq. (6) is linear and **K** is independent of temperature. However, our 445 estimation algorithm does not clearly distinguish between a time-scale dependence and 446 a temperature-dependence of the feedbacks, since these dependencies are intrinsically linked. 447 In particular, the strong temperature responses to  $4xCO_2$  is invoked on the long time 448 scales, where the responses to the shorter time scales have already been realised, hereby 449 affecting the feedback parameters if they have temperature dependence. If the  $4xCO_2$ 450 responses have temperature-dependent feedbacks, the model needed to explicitly explain 451 them becomes nonlinear, and our linear approach may perform less well in providing re-452 sponses to other scenarios with weaker or stronger temperature responses than that of 453  $4xCO_2$ . We believe this only causes smaller errors in the temperature responses stud-454 ied here, but it is a potential explanation for our forcing and responses for the future sce-455 narios being slightly overestimated. 456

Linear response theory is widely used to describe responses of climate variables. 457 If a forcing record is known, linear response is a computationally cheap tool to estimate 458 e.g. temperature responses compared to running a fully coupled climate model. Many 459 studies assume a Green's function taking a certain form, with unknown parameters that 460 need to be estimated. For box models taking the form of Eq. (6) the Green's function 461 is a sum of exponential functions, but power-laws with fewer parameters have also been 462 used with success (Rypdal & Rypdal, 2014; Fredriksen & Rypdal, 2017). Linear responses 463 to RCP forcing are often studied using a non-parametric approach developed by Good 464 et al. (2011). In the supporting information we show how this method relates to our lin-465 ear model. This method was used in Good et al. (2013) to find the response to RCP sce-466 narios using the forcing computed by F13. They use this to simulate only differences be-467 tween RCP scenarios, while we attempt to simulate the full temperature evolution since 468 the historical runs started until year 2100. Another difference to our approach is that 469 we obtain a smoother estimate of the expected response to forcing, with fluctuations only 470 coming from the forcing, while the responses of Good et al. (2013) are themselves influ-471 enced by internal variability. 472

Larson and Portmann (2016) note that the non-parametric model written in matrix form:  $\mathbf{Y} = \mathbf{X}\Delta\mathbf{F}/F_0$  can be inverted to estimate the forcing increments  $\Delta\mathbf{F}$ , which can further be summed up to find the forcing time series. In this equation  $\mathbf{Y}$  is a vector of the time evolution of a climate variable, and  $\mathbf{X}$  is a matrix containing the same variable in the abrupt4xCO<sub>2</sub> experiment. Their resulting forcing estimate depends also on the forcing estimate  $F_0$  from the abrupt4xCO<sub>2</sub> experiment, which introduces a po-

tential source of bias in the estimate. Internal variability from  $\mathbf{X}$  and  $\mathbf{Y}$  can lead to a 479 very noisy estimate, but some of this is removed when they replace the original  $abrupt4xCO_2$ 480 time series with a fitted exponential response. With our method we also greatly reduce 481 the influence of internal variability from the experiment where the forcing is to be estimated by smoothing it with our linear response to the estimated forcing. So we can 483 say that there is a trade-off between a noisy estimate and having more parameters to be 484 estimated. The method by Larson and Portmann (2016) is treated as an alternative to 485 the F13 method, but here we show how the F13 method and the linear response can be 486 put into one framework. While Larson and Portmann (2016) can demonstrate that their 487 method is not directly dependent of a changing feedback parameter, our method also has 488 the power to explain why this can be the case. 180

#### 490 5 Conclusions

The method presented here cleanly separates between forcing and responses to forc-491 ing, where the estimated parameters from  $abrupt4xCO_2$  experiments are used to deter-492 mine forcing and surface temperature responses for other experiments. The resulting RCP 493 forcing estimates at the end of the 21st century is closer to the target levels than pre-494 vious estimates by F13. Our high forcing estimates are strongly influenced by the high 495 magnitude of the feedback parameter  $\lambda_1$  at annual time scales. Unfortunately this value is uncertain, as it depends crucially on the first few years of adjustment. Using more ensemble members of abrupt4xCO<sub>2</sub> experiments may help constrain the estimate of  $\lambda_1$  (M. Ru-498 genstein, Gregory, et al., 2016). More members would also constrain regression estimates 499 of forcing in general (Forster et al., 2016). 500

Forcing based on fixed-SST methods is often higher than the regression estimate 501 over 150 years (Andrews et al., 2012; Tang et al., 2019), has a smaller uncertainty and 502 is more computationally efficient (Forster et al., 2016). However, these forcing estimates 503 are only available for a few models and scenarios in CMIP5. They will be available for 504 more models and scenarios in CMIP6 (Smith et al., 2020), but far from all. The forc-505 ing estimation method presented here could therefore be a valuable supplement in the 506 cases where fixed-SST forcing is unknown, particularly for models where a linear rela-507 tion between N and T is a poor approximation. Improved forcing estimates could help 508 to quantify the dependency of forcing value on  $CO_2$  concentration in studies comparing e.g. 0.5x, 2x, 4x, 8x CO<sub>2</sub>, and temperature dependence of feedbacks (Bloch-Johnson 510 et al., 2021). 511

Putting forcing, linear responses, and nonconstancy of the global feedback parameter into a unified framework provides also an important insight into why the traditional regression-based forcing estimates may be too low. Furthermore, it suggests how these methods can be improved to provide better forcing estimates, resolving the problems caused by assuming a constant feedback parameter in regression-based methods (Forster et al., 2016).

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The CMIP5 data are available at https://esgf-node.llnl.gov/projects/cmip5/. The forcing estimates from this paper will be stored in https://dataverse.no/, and can be accessed through https://doi.org/10.18710/IHUVTB. Our python code will be permanently stored in zenodo (link will be inserted when paper is accepted. The code is currently available at https://github.com/Hegebf/CMIP5-forcing). We thank Timothy

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<sup>759</sup> https://doi.org/10.1038/ngeo2828

# Supporting Information for "Estimating radiative forcing with a nonconstant feedback parameter and linear response"

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# Contents of this file

- 1. Text S1 to S2  $\,$
- 2. Figures S1 to S111
- 3. Table S1  $\,$

**Introduction** This document repeats Figures 1, 3 and 4 for all models and available RCP scenarios. For a description of the figures, see the NorESM1-M figures in the main manuscript.

Text S1 and S2 elaborates some of the mathematics needed to 1) derive the temperature response, and 2) understand the relationship between our linear model and the nonparametric linear models considered in other papers.

Table S1 in the end lists the piControl trend values used when subtracting linear trends from the variables of the  $abrupt4xCO_2$  experiment.

# Text S1: Deriving temperature response

To show that Eq. (7) is the solution of Eq. (6), we start by rewriting to:

$$\frac{d\mathbf{T}(t)}{dt} = \mathbf{C}^{-1}\mathbf{K}\mathbf{T}(t) + \mathbf{C}^{-1}\mathbf{F}(t)$$

:

We consider first the homogeneous problem

$$\frac{d\mathbf{T}(t)}{dt} = \mathbf{AT}(t)$$

where  $\mathbf{A} = \mathbf{C}^{-1}\mathbf{K}$ . The matrix of possible solutions  $\mathbf{x}_i(t)$  to this problem is the fundamental matrix

$$\mathbf{\Phi}(t) = [\mathbf{x}_1(t) \mid \mathbf{x}_2(t) \mid \ldots \mid \mathbf{x}_n(t)].$$

 $e^{\mathbf{A}t}$  is a fundamental matrix when **A** consists of constant coefficients, since

$$\frac{d\mathbf{\Phi}(t)}{dt} = \frac{de^{\mathbf{A}t}}{dt} = \mathbf{A}e^{\mathbf{A}t} = \mathbf{A}\mathbf{\Phi}(t).$$

According to the variation of parameters formula for first-order linear systems  $\frac{d\mathbf{x}}{dt} = \mathbf{P}(t)\mathbf{x} + \mathbf{f}(t)$ , a particular solution is given by

$$\mathbf{x}_p(t) = \mathbf{\Phi}(t) \int \mathbf{\Phi}(t)^{-1} \mathbf{f}(t) dt$$

(see e.g. Edwards and Penney (2007)). For our problem, this means that the particular solution is

$$\mathbf{x}_p(t) = e^{\mathbf{A}t} \int e^{-\mathbf{A}t} \mathbf{C}^{-1} \mathbf{F}(t) dt.$$

Given an initial value  $\mathbf{T}(0) = \mathbf{T}_0$ , the full solution can be written as

$$\mathbf{T}(t) = e^{\mathbf{C}^{-1}\mathbf{K}t}\mathbf{T}_0 + \int_0^t e^{(t-s)\mathbf{C}^{-1}\mathbf{K}}\mathbf{C}^{-1}\mathbf{F}(s) \ ds,$$

or alternatively, if we know the full history of the system,

$$\mathbf{T}(t) = \int_{-\infty}^{t} e^{(t-s)\mathbf{C}^{-1}\mathbf{K}} \mathbf{C}^{-1}\mathbf{F}(s) \, ds.$$
  
November 5, 2021, 9:27am

# Text S2: Relation to non-parametric impulse-response models

To find the relation between linear model considered here and linear models considered in e.g. Larson and Portmann (2016); Good, Gregory, and Lowe (2011); Good, Gregory, Lowe, and Andrews (2013), we start with the general equation from the end of Section 2.1:  $T(t) = \int_{-\infty}^{t} G(t-s)F(s)ds$ . As noted by Hasselmann, Sausen, Maier-Reimer, and Voss (1993), such a convolution integral can also describe a general climate state variable  $\Phi(t)$  that responds linearly to a forcing:

$$\Phi(t) = \int_0^t G(t-s)F(s)ds \tag{1}$$

assuming F(t) = 0 for  $t \le 0$ . If the forcing takes the form of a unit step-function, which is 0 for  $t \le 0$  and 1 for t > 0, the climate response is:

$$R(t) = \int_0^t G(t-s)ds$$

and  $\frac{dR}{dt} = G(t)$ . By performing an integration by parts, we note that Eq. (1) can be rewritten to

$$\Phi(t) = \int_0^t \frac{dF}{ds} R(t-s)ds \tag{2}$$

where the additional term R(0)F(t) - R(t)F(0) = 0 because R(0) = 0 and F(0) = 0. Discretizing this integral using time steps of years results in the same type of sum used by Larson and Portmann (2016); Good et al. (2011, 2013):

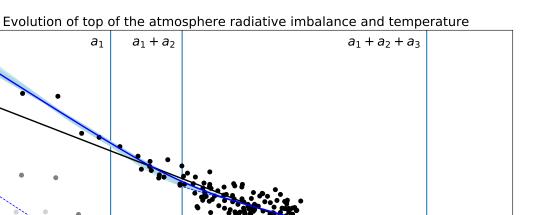
$$\Phi_i = \sum_{j=0}^i \Delta F_j R_{i-j} \tag{3}$$

If using a response to a step-forcing  $\Delta F_s$  instead of the unit response, the response needs to be normalized by  $\Delta F_s$ .

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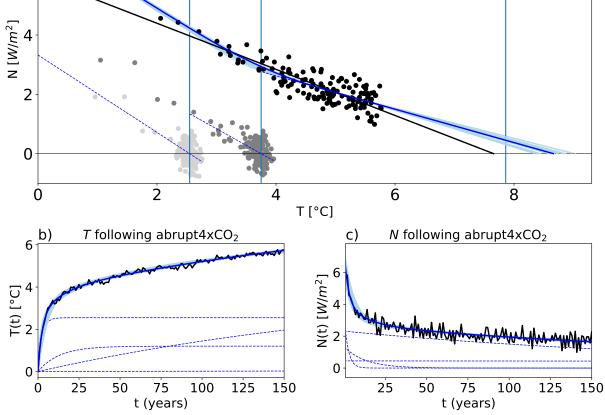


Figure S1. As Figure 1, but for the model ACCESS1-0.

 $a_1$ 

 $a_1 + a_2$ 

a)

8

6

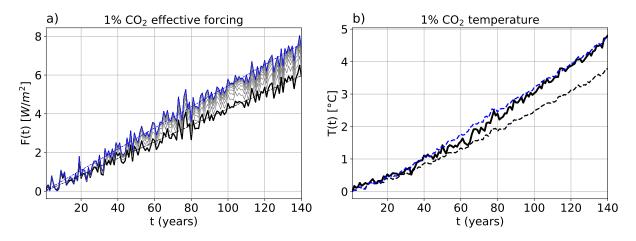


Figure S2. As Figure 3, but for the model ACCESS1-0.

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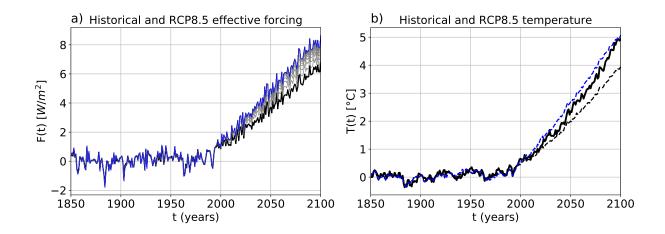


Figure S3. As Figure 4, but for the model ACCESS1-0.

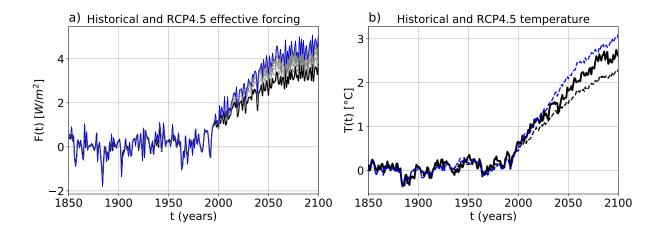


Figure S4. As Figure 4, but for the model ACCESS1-0 and experiment RCP4.5.

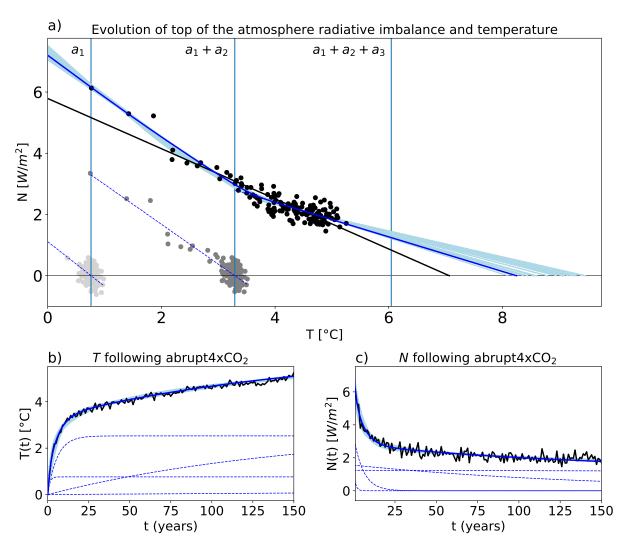


Figure S5. As Figure 1, but for the model ACCESS1-3.

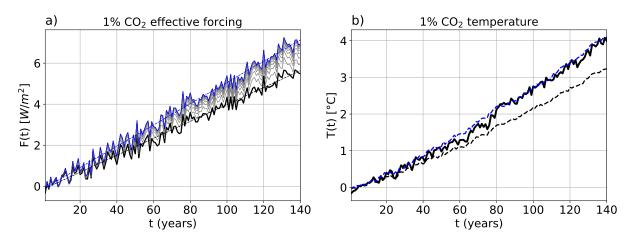


Figure S6. As Figure 3, but for the model ACCESS1-3.

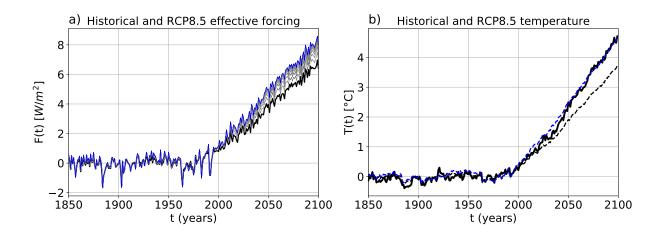


Figure S7. As Figure 4, but for the model ACCESS1-3.

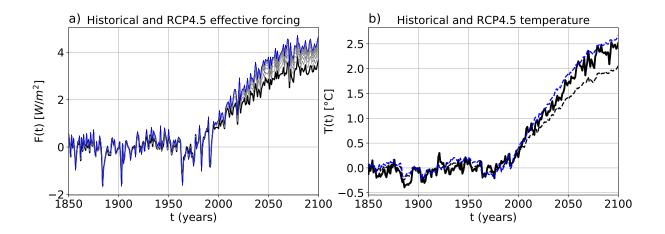


Figure S8. As Figure 4, but for the model ACCESS1-3 and experiment RCP4.5.

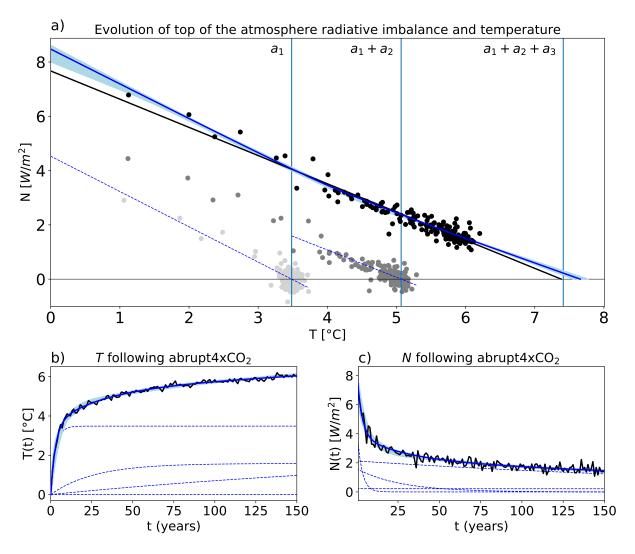


Figure S9. As Figure 1, but for the model CanESM2.

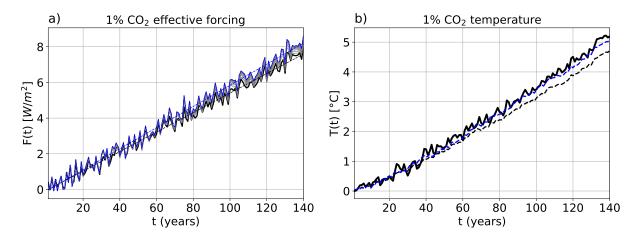


Figure S10. As Figure 3, but for the model CanESM2.

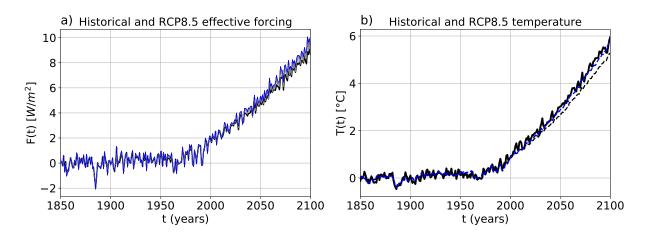


Figure S11. As Figure 4, but for the model CanESM2.

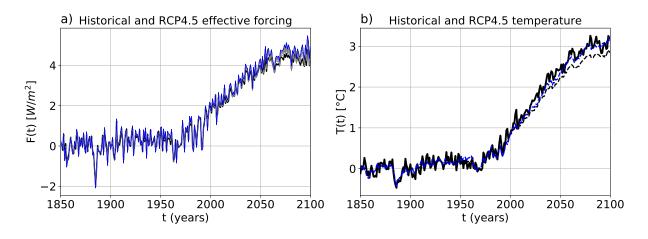


Figure S12. As Figure 4, but for the model CanESM2 and experiment RCP4.5.

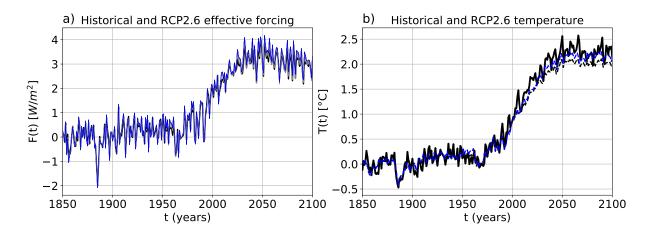


Figure S13. As Figure 4, but for the model CanESM2 and experiment RCP2.6.



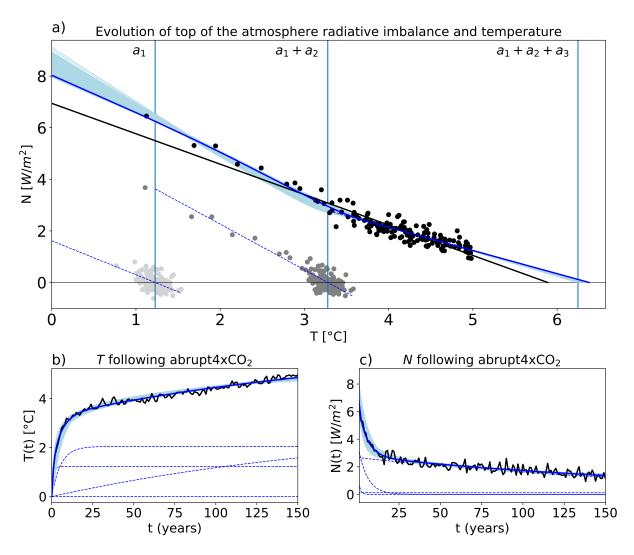


Figure S14. As Figure 1, but for the model CCSM4.

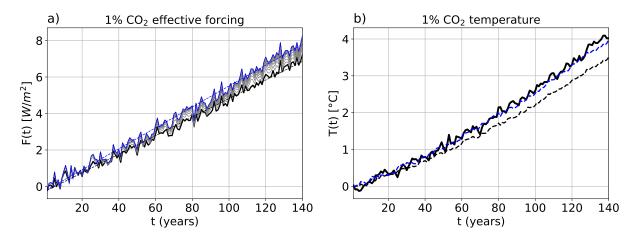


Figure S15. As Figure 3, but for the model CCSM4.

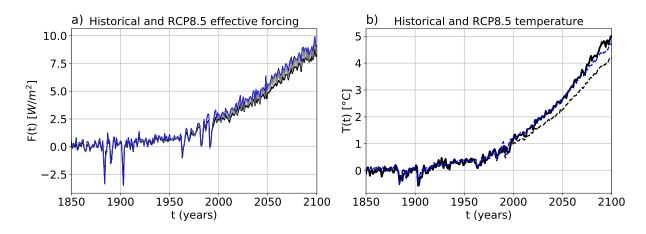


Figure S16. As Figure 4, but for the model CCSM4.

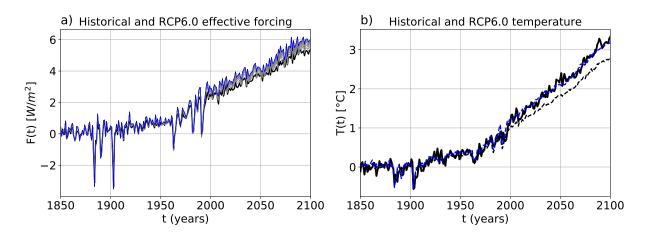


Figure S17. As Figure 4, but for the model CCSM4 and experiment RCP6.0.

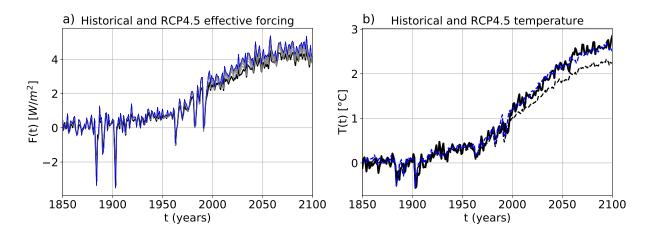


Figure S18. As Figure 4, but for the model CCSM4 and experiment RCP4.5.

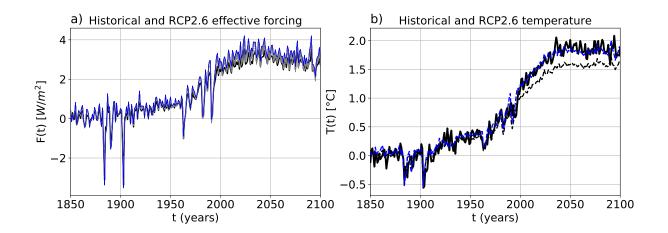


Figure S19. As Figure 4, but for the model CCSM4 and experiment RCP2.6.

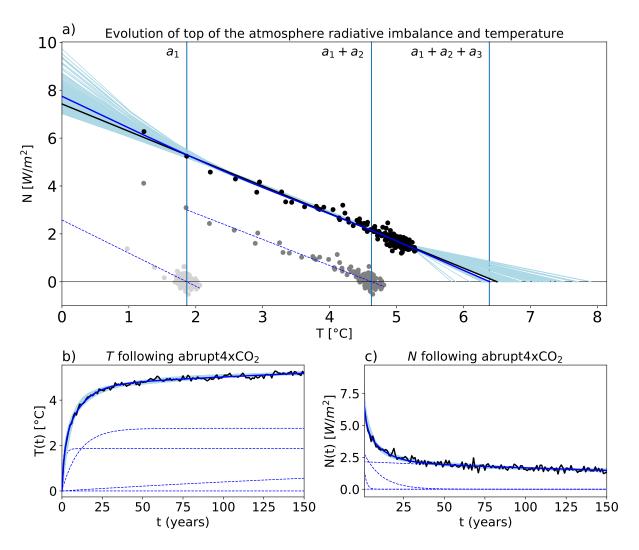


Figure S20. As Figure 1, but for the model CNRM-CM5.

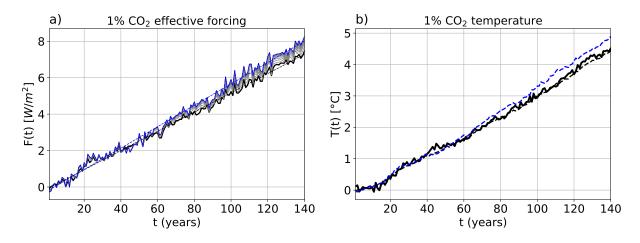


Figure S21. As Figure 3, but for the model CNRM-CM5.

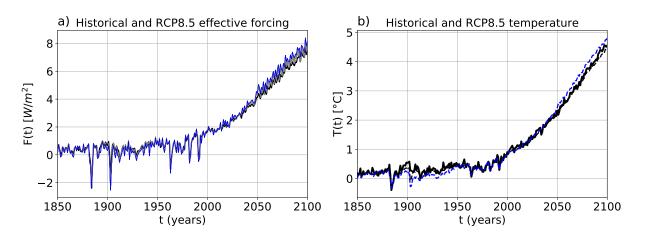


Figure S22. As Figure 4, but for the model CNRM-CM5.

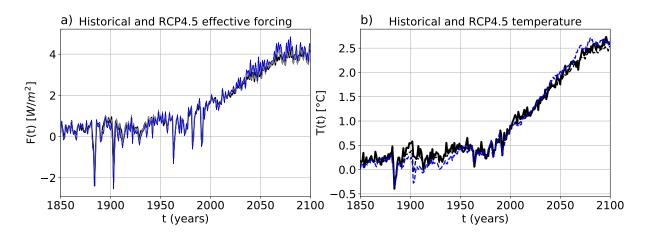


Figure S23. As Figure 4, but for the model CNRM-CM5 and experiment RCP4.5.

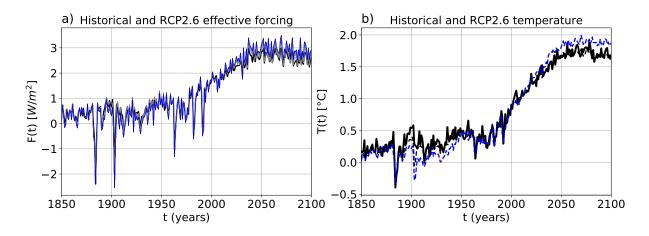


Figure S24. As Figure 4, but for the model CNRM-CM5 and experiment RCP2.6.

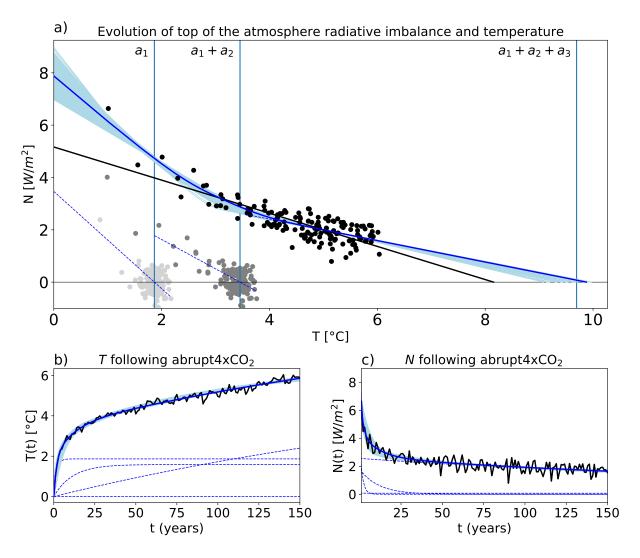


Figure S25. As Figure 1, but for the model CSIRO-Mk3-6-0.

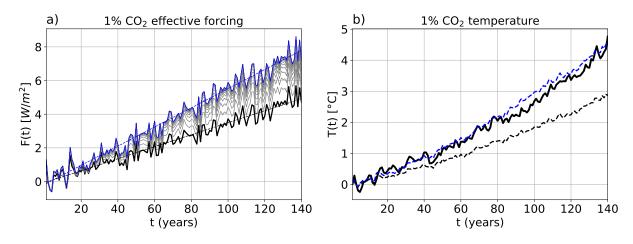
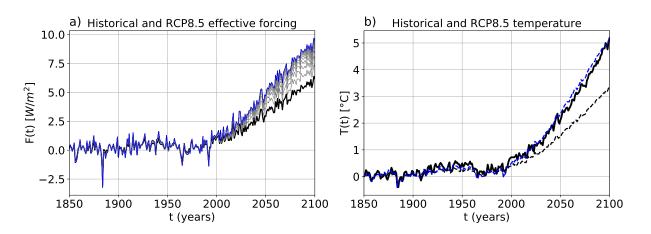


Figure S26. As Figure 3, but for the model CSIRO-Mk3-6-0.



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Figure S27. As Figure 4, but for the model CSIRO-Mk3-6-0.

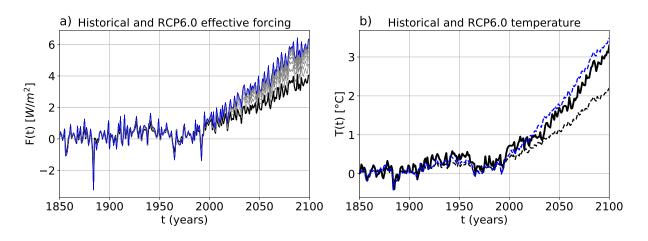


Figure S28. As Figure 4, but for the model CSIRO-Mk3-6-0 and experiment RCP6.0.

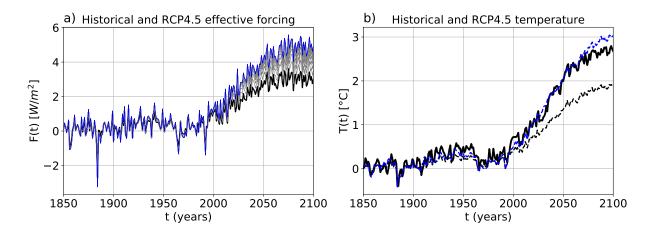


Figure S29. As Figure 4, but for the model CSIRO-Mk3-6-0 and experiment RCP4.5.

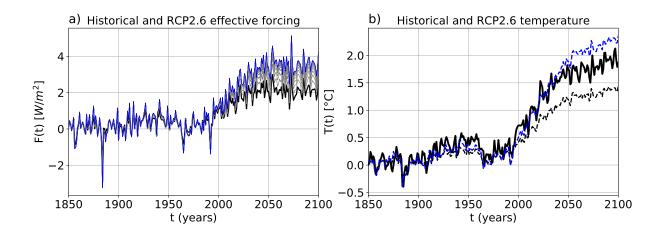


Figure S30. As Figure 4, but for the model CSIRO-Mk3-6-0 and experiment RCP2.6.

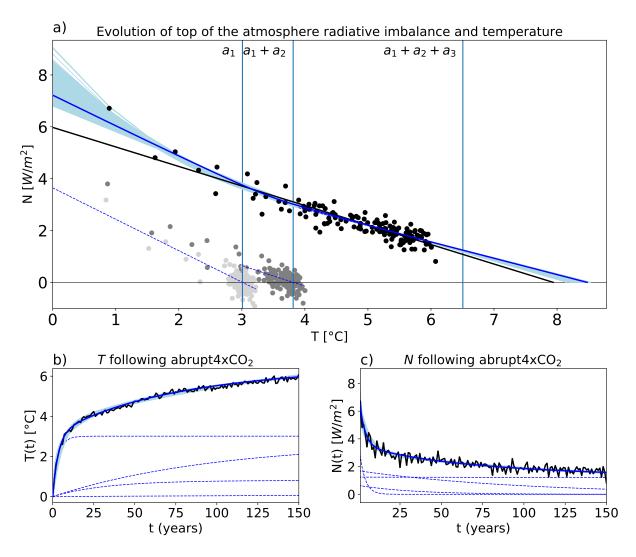


Figure S31. As Figure 1, but for the model GFDL-CM3.

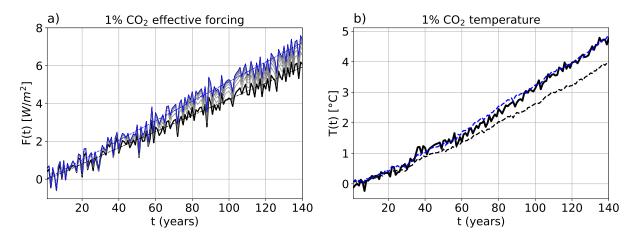


Figure S32. As Figure 3, but for the model GFDL-CM3.

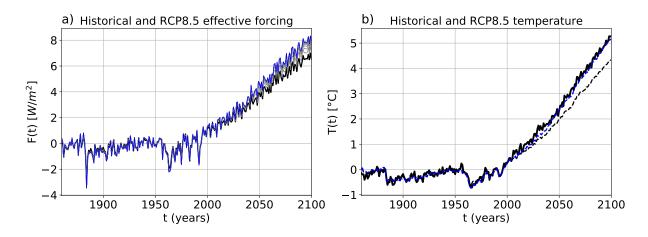


Figure S33. As Figure 4, but for the model GFDL-CM3.

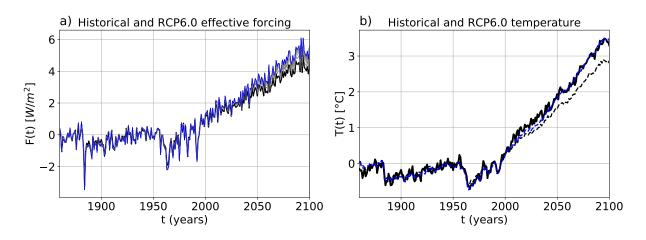


Figure S34. As Figure 4, but for the model GFDL-CM3 and experiment RCP6.0.

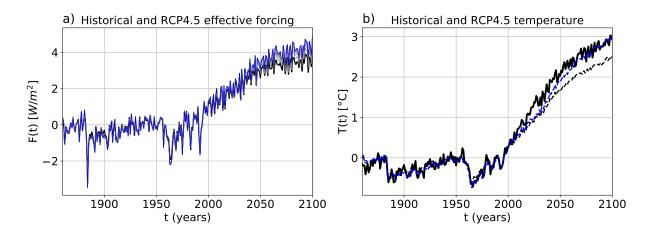


Figure S35. As Figure 4, but for the model GFDL-CM3 and experiment RCP4.5.

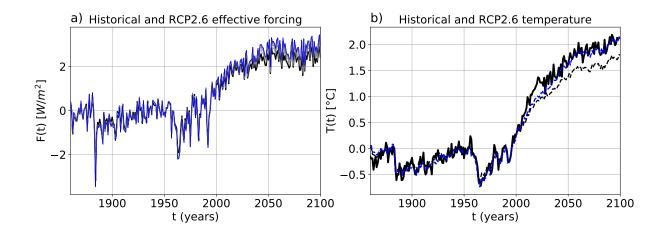


Figure S36. As Figure 4, but for the model GFDL-CM3 and experiment RCP2.6.

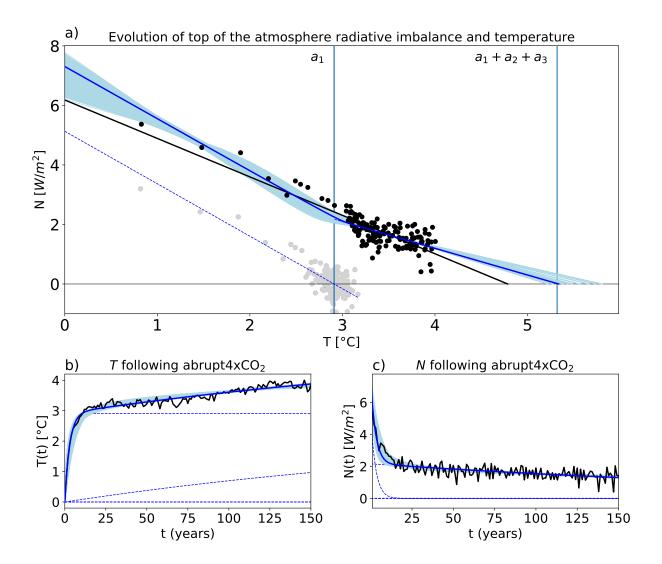
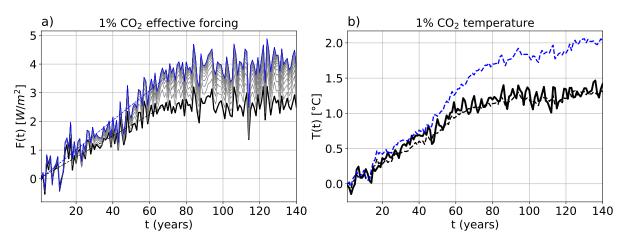


Figure S37. As Figure 1, but for the model GFDL-ESM2G.



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Figure S38. As Figure 3, but for the model GFDL-ESM2G. Note that in this model  $CO_2$  increases only until we reach a doubling after 70 years.

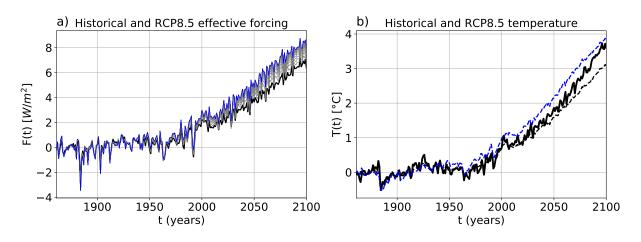


Figure S39. As Figure 4, but for the model GFDL-ESM2G.

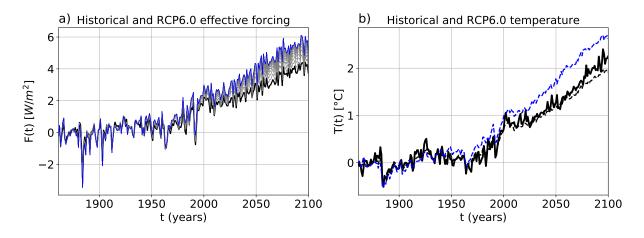


Figure S40. As Figure 4, but for the model GFDL-ESM2G and experiment RCP6.0.

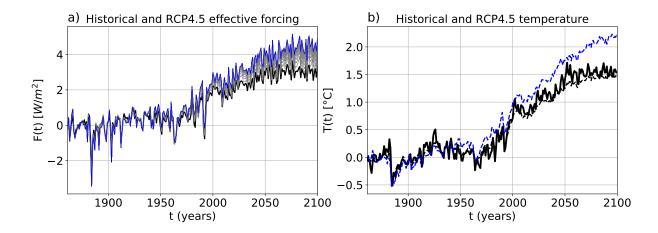


Figure S41. As Figure 4, but for the model GFDL-ESM2G and experiment RCP4.5.

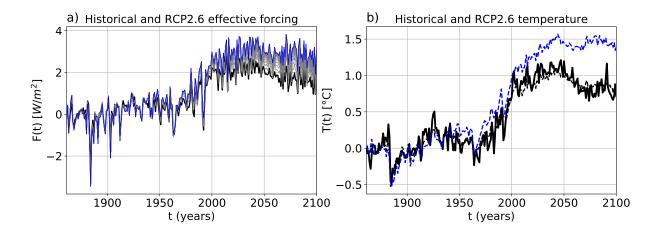


Figure S42. As Figure 4, but for the model GFDL-ESM2G and experiment RCP2.6.

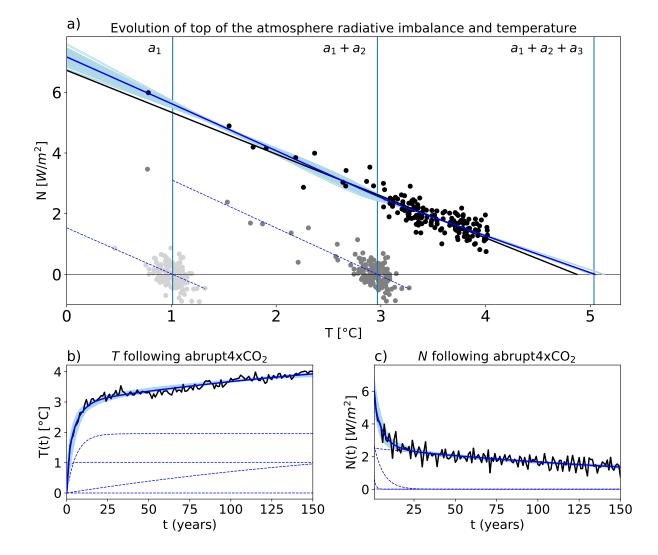


Figure S43. As Figure 1, but for the model GFDL-ESM2M.



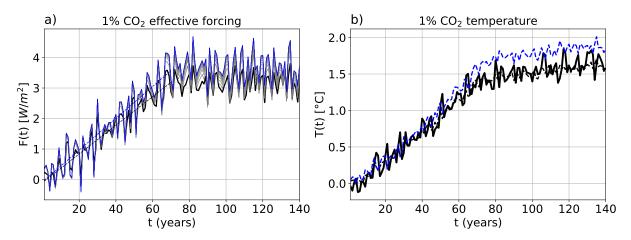


Figure S44. As Figure 3, but for the model GFDL-ESM2M. Note that in this model  $CO_2$  increases only until we reach a doubling after 70 years.

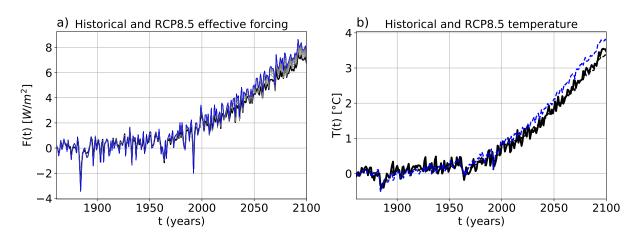


Figure S45. As Figure 4, but for the model GFDL-ESM2M.

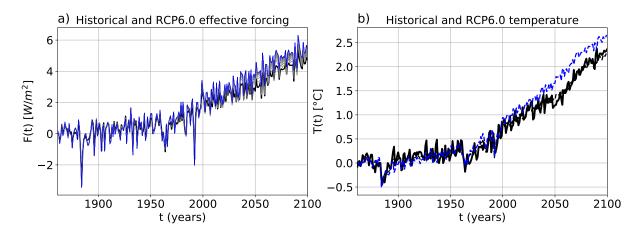


Figure S46. As Figure 4, but for the model GFDL-ESM2M and experiment RCP6.0.

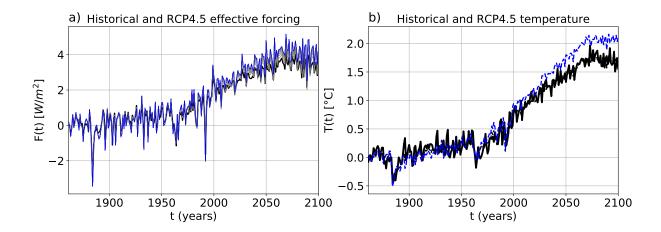


Figure S47. As Figure 4, but for the model GFDL-ESM2M and experiment RCP4.5.

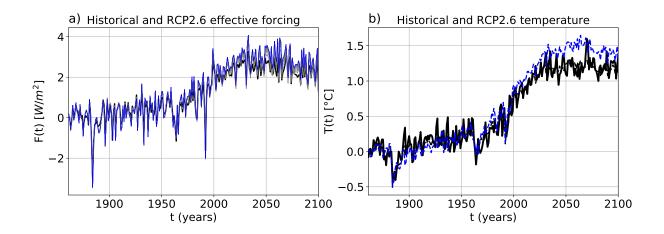


Figure S48. As Figure 4, but for the model GFDL-ESM2M and experiment RCP2.6.

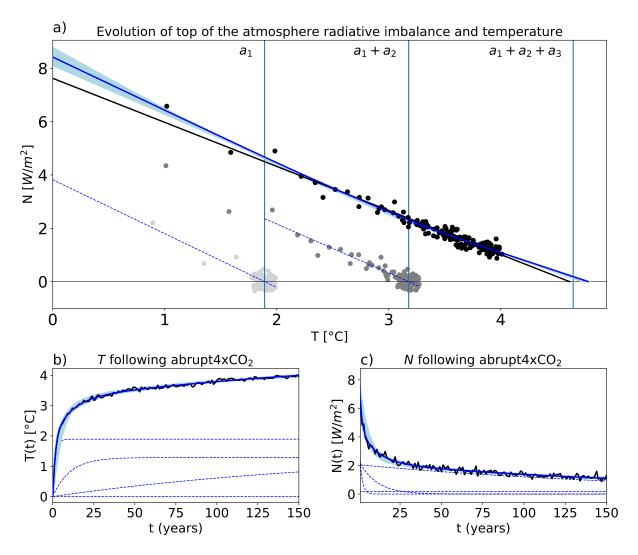


Figure S49. As Figure 1, but for the model GISS-E2-H.

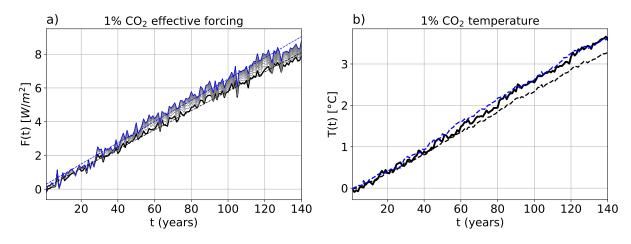


Figure S50. As Figure 3, but for the model GISS-E2-H.

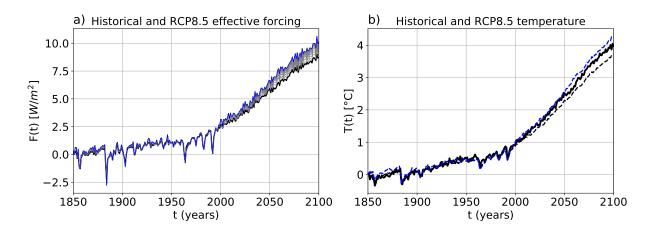


Figure S51. As Figure 4, but for the model GISS-E2-H.

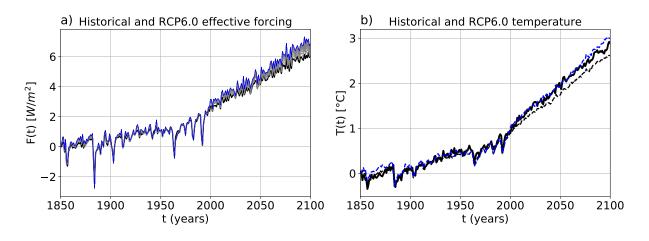


Figure S52. As Figure 4, but for the model GISS-E2-H and experiment RCP6.0.

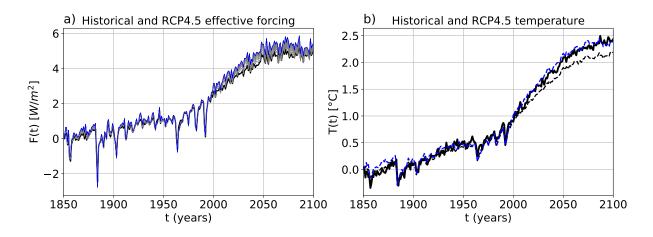
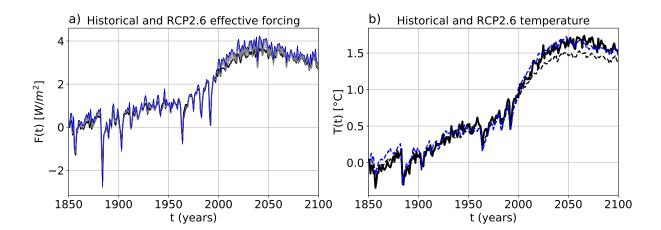


Figure S53. As Figure 4, but for the model GISS-E2-H and experiment RCP4.5.



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Figure S54. As Figure 4, but for the model GISS-E2-H and experiment RCP2.6.



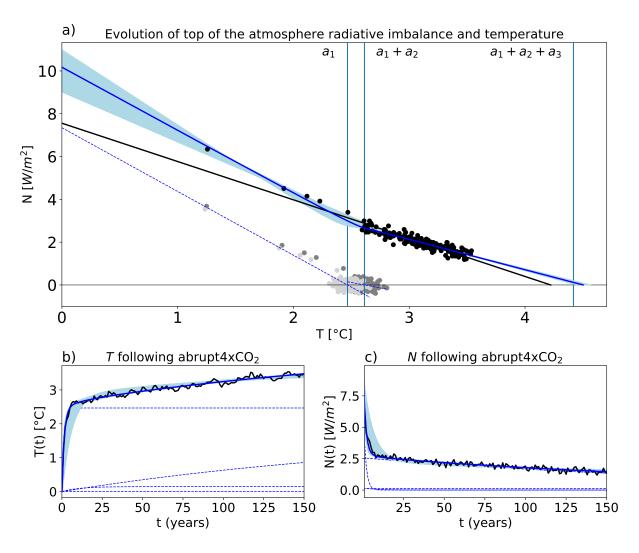


Figure S55. As Figure 1, but for the model GISS-E2-R.

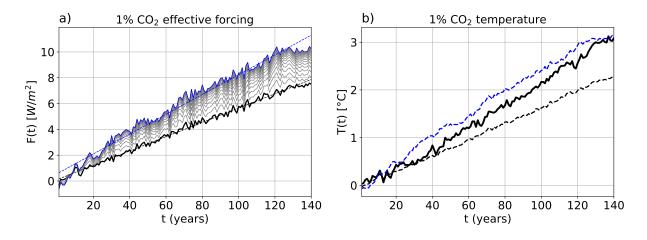


Figure S56. As Figure 3, but for the model GISS-E2-R.

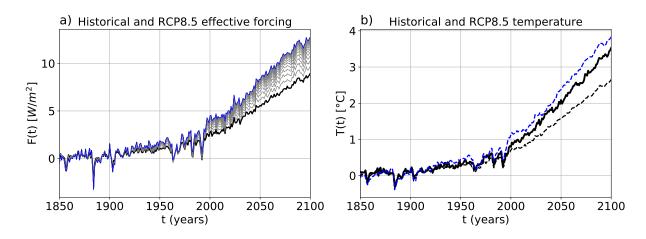


Figure S57. As Figure 4, but for the model GISS-E2-R.

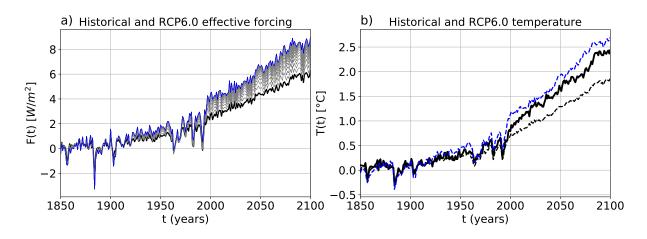


Figure S58. As Figure 4, but for the model GISS-E2-R and experiment RCP6.0.

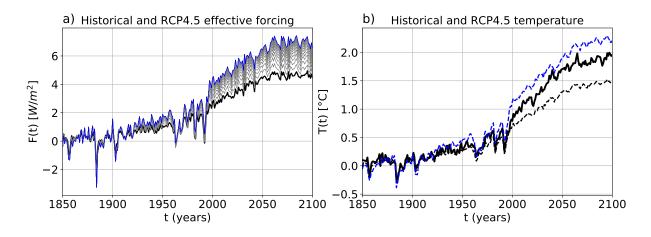


Figure S59. As Figure 4, but for the model GISS-E2-R and experiment RCP4.5.

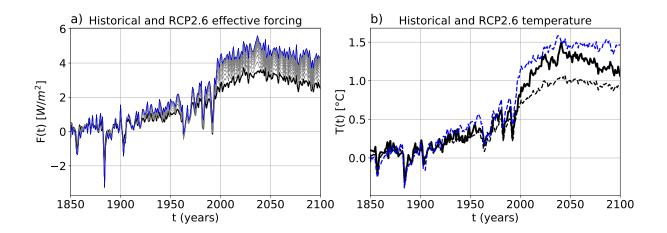


Figure S60. As Figure 4, but for the model GISS-E2-R and experiment RCP2.6.

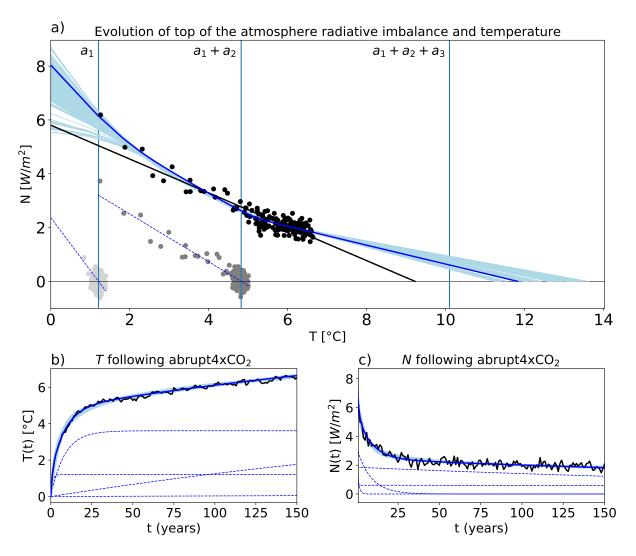


Figure S61. As Figure 1, but for the model HadGEM2-ES.

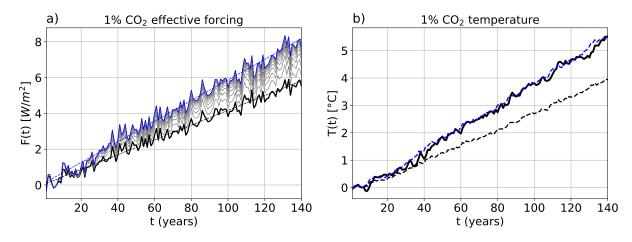


Figure S62. As Figure 3, but for the model HadGEM2-ES.

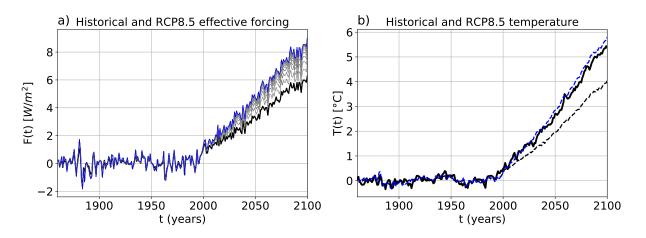


Figure S63. As Figure 4, but for the model HadGEM2-ES.

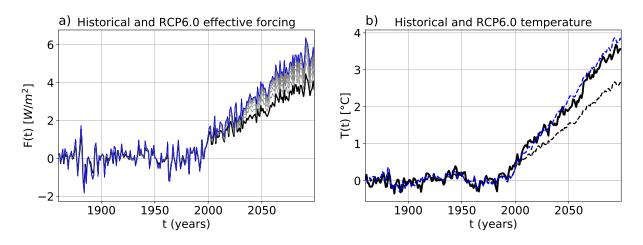


Figure S64. As Figure 4, but for the model HadGEM2-ES and experiment RCP6.0.

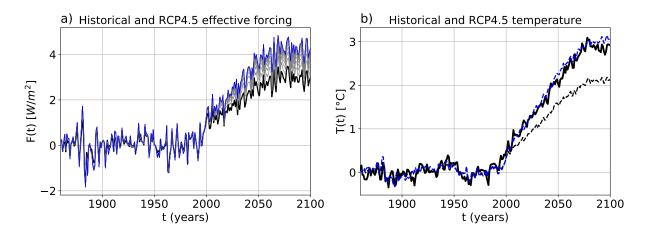


Figure S65. As Figure 4, but for the model HadGEM2-ES and experiment RCP4.5.

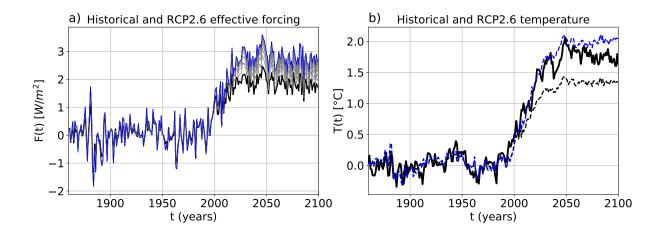


Figure S66. As Figure 4, but for the model HadGEM2-ES and experiment RCP2.6.

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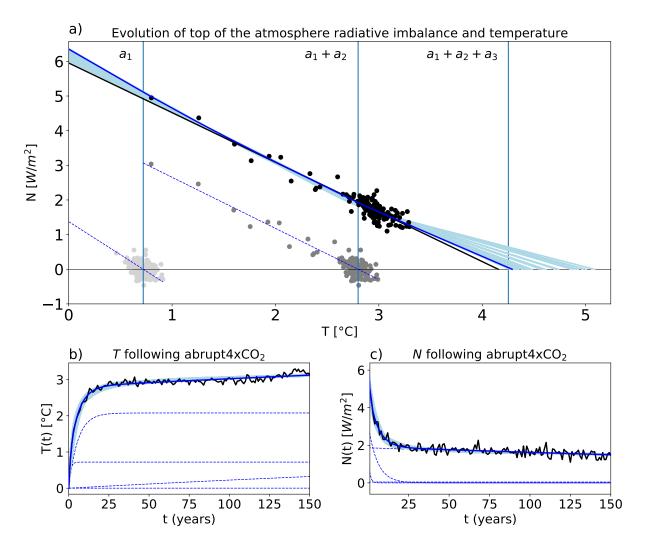


Figure S67. As Figure 1, but for the model inmcm4.

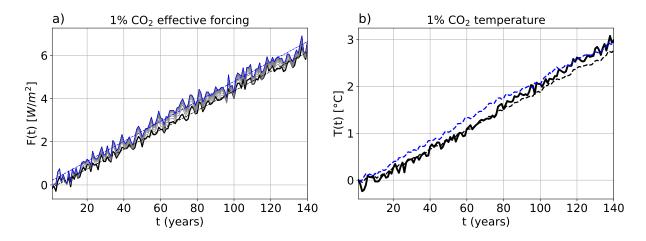


Figure S68. As Figure 3, but for the model inmcm4.

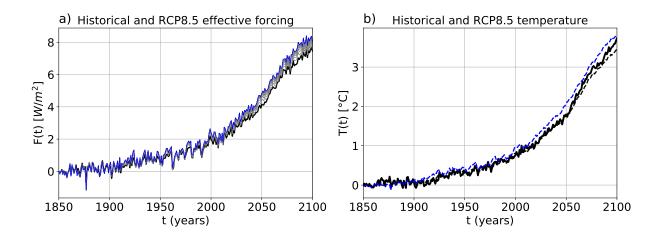


Figure S69. As Figure 4, but for the model inmcm4.

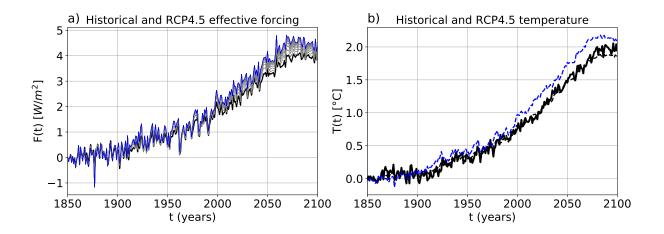


Figure S70. As Figure 4, but for the model inmcm4 and experiment RCP4.5.

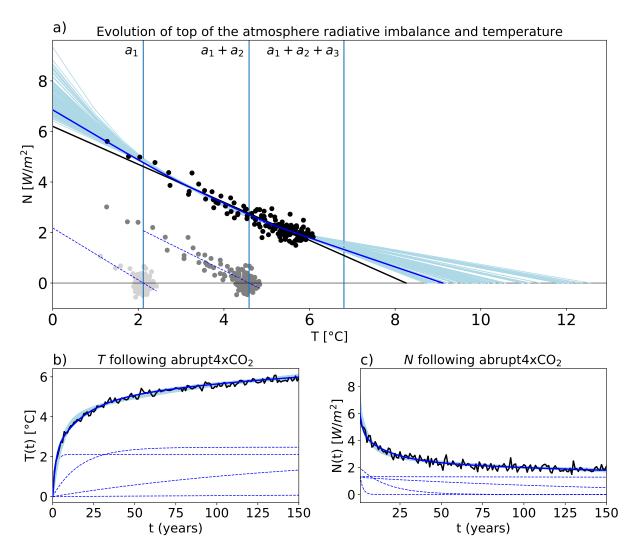


Figure S71. As Figure 1, but for the model IPSL-CM5A-LR.

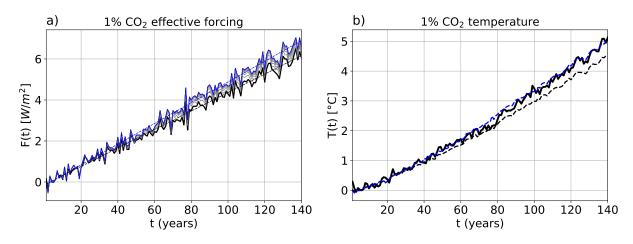


Figure S72. As Figure 3, but for the model IPSL-CM5A-LR.

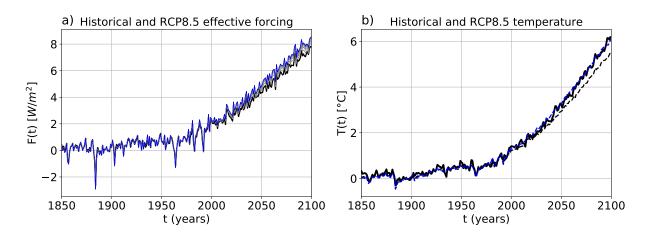


Figure S73. As Figure 4, but for the model IPSL-CM5A-LR.

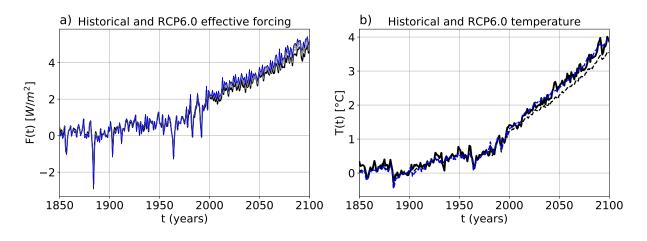


Figure S74. As Figure 4, but for the model IPSL-CM5A-LR and experiment RCP6.0.

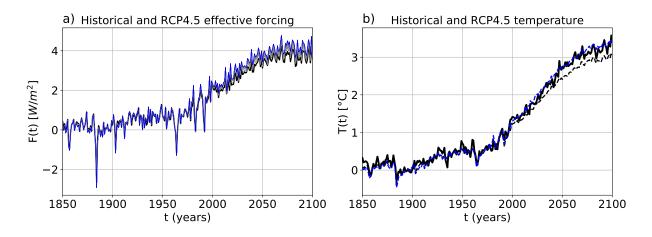


Figure S75. As Figure 4, but for the model IPSL-CM5A-LR and experiment RCP4.5.

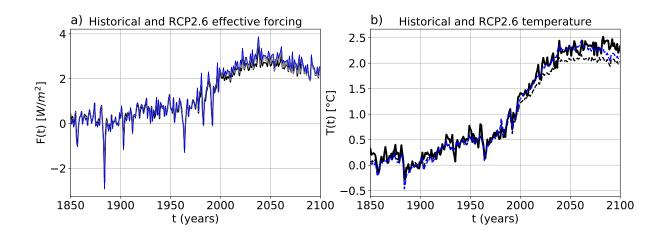


Figure S76. As Figure 4, but for the model IPSL-CM5A-LR and experiment RCP2.6.

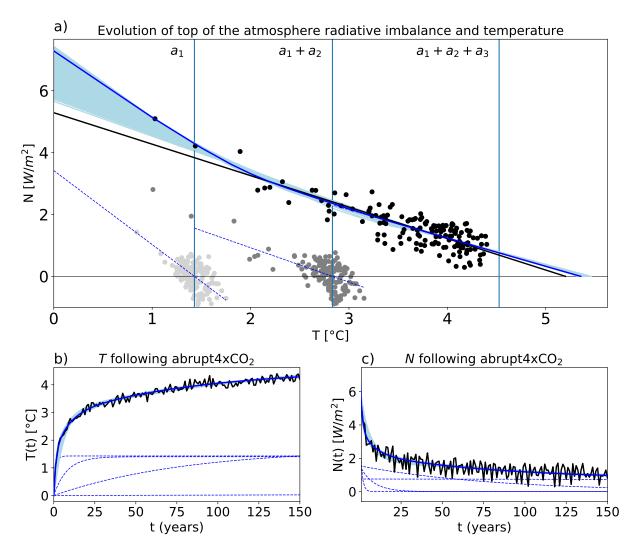


Figure S77. As Figure 1, but for the model IPSL-CM5B-LR.

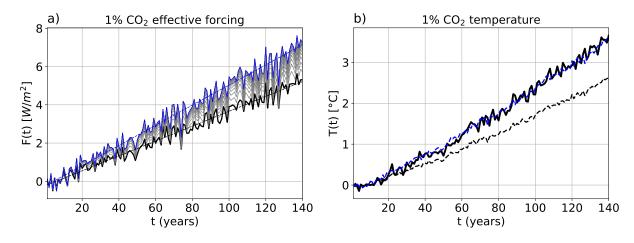
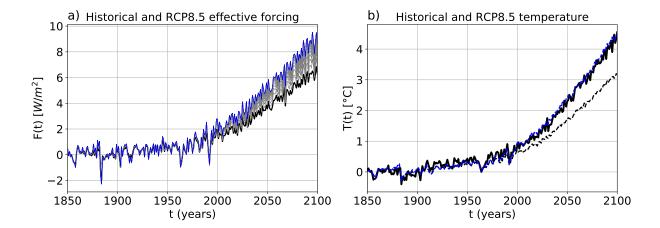


Figure S78. As Figure 3, but for the model IPSL-CM5B-LR.





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Figure S79. As Figure 4, but for the model IPSL-CM5B-LR.

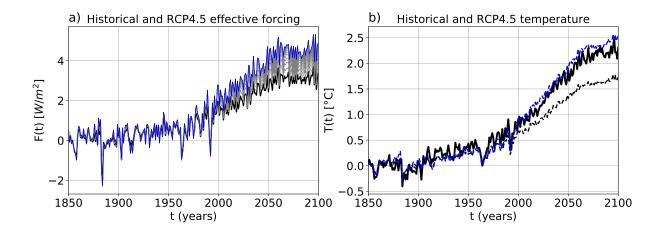


Figure S80. As Figure 4, but for the model IPSL-CM5B-LR and experiment RCP4.5.

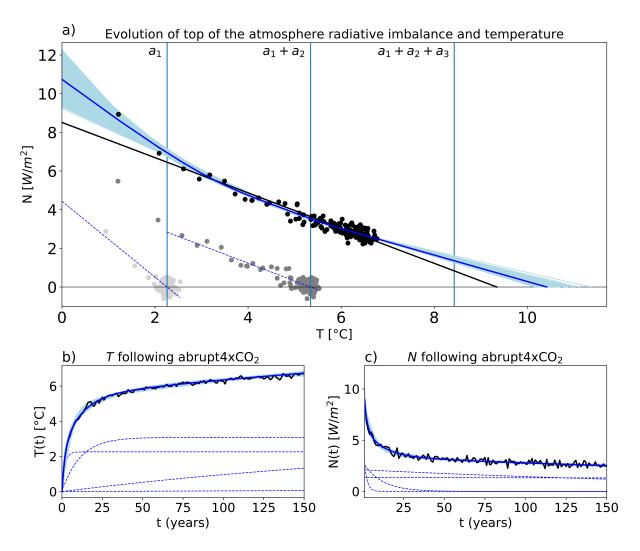


Figure S81. As Figure 1, but for the model MIROC-ESM.

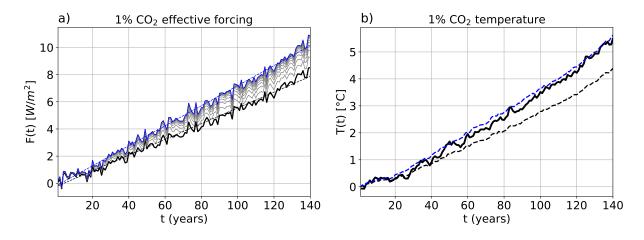


Figure S82. As Figure 3, but for the model MIROC-ESM.



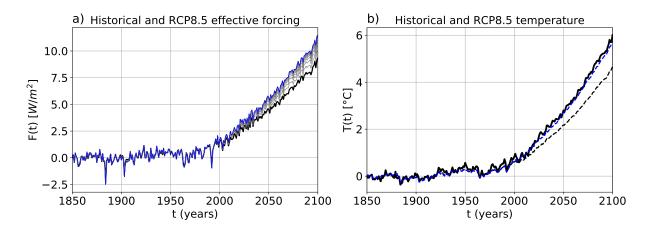


Figure S83. As Figure 4, but for the model MIROC-ESM.

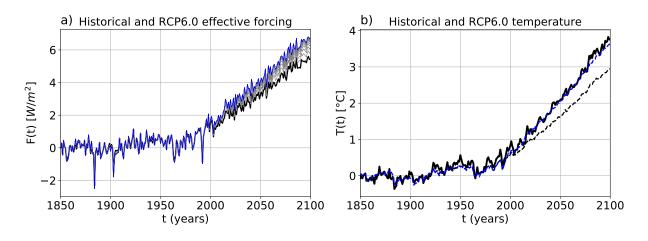


Figure S84. As Figure 4, but for the model MIROC-ESM and experiment RCP6.0.

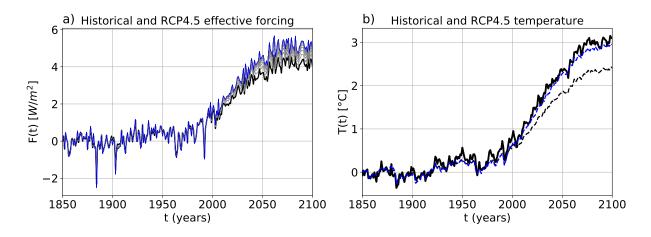


Figure S85. As Figure 4, but for the model MIROC-ESM and experiment RCP4.5.

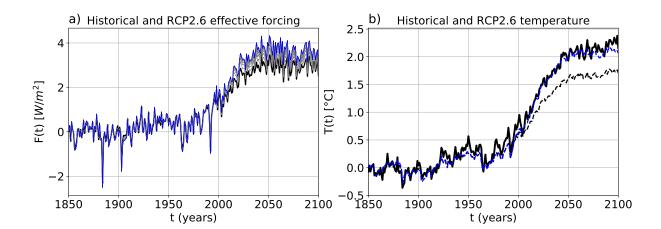


Figure S86. As Figure 4, but for the model MIROC-ESM and experiment RCP2.6.

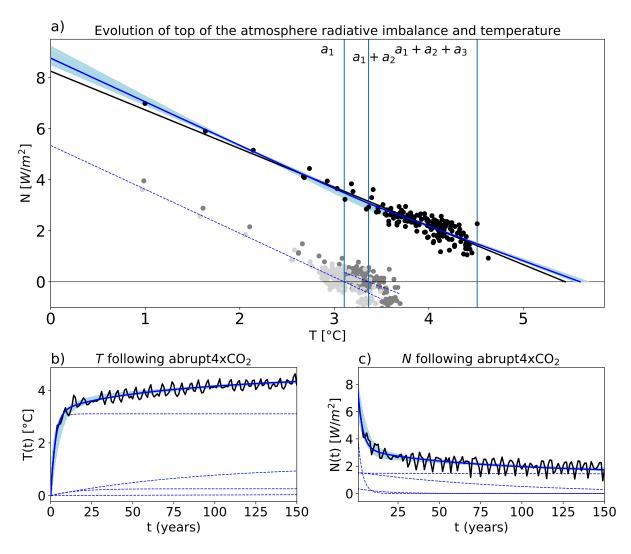


Figure S87. As Figure 1, but for the model MIROC5.

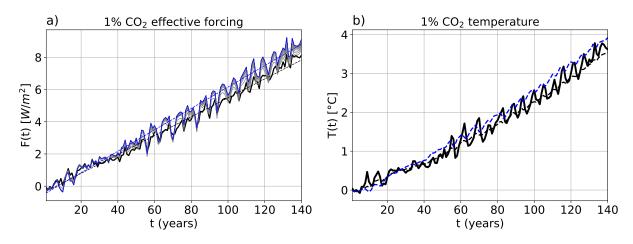


Figure S88. As Figure 3, but for the model MIROC5.

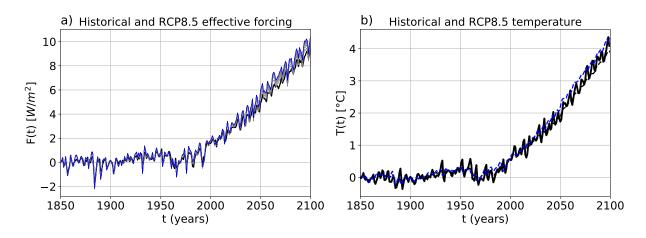


Figure S89. As Figure 4, but for the model MIROC5.

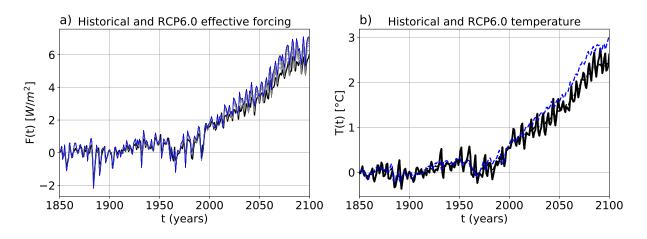


Figure S90. As Figure 4, but for the model MIROC5 and experiment RCP6.0.

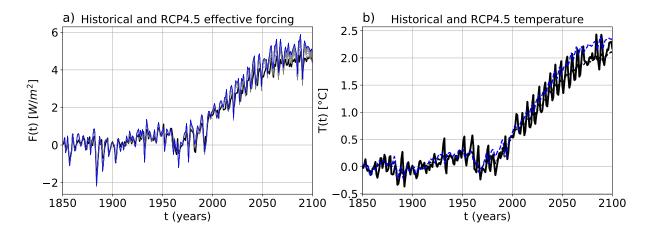


Figure S91. As Figure 4, but for the model MIROC5 and experiment RCP4.5.

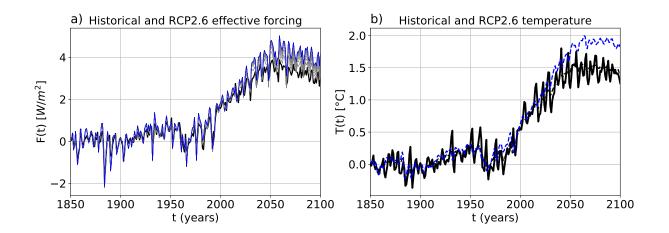


Figure S92. As Figure 4, but for the model MIROC5 and experiment RCP2.6.

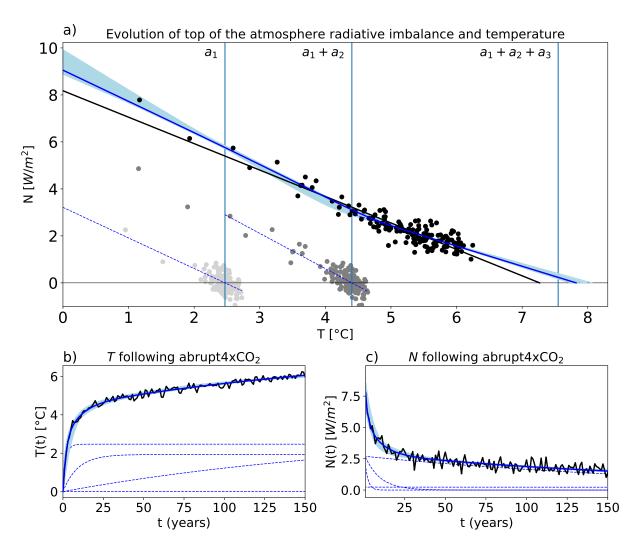


Figure S93. As Figure 1, but for the model MPI-ESM-LR.

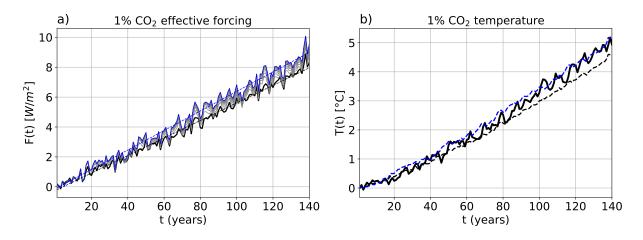


Figure S94. As Figure 3, but for the model MPI-ESM-LR.

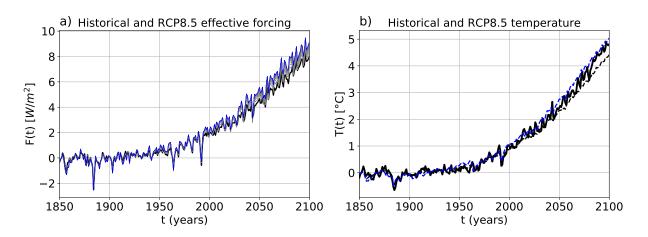


Figure S95. As Figure 4, but for the model MPI-ESM-LR.

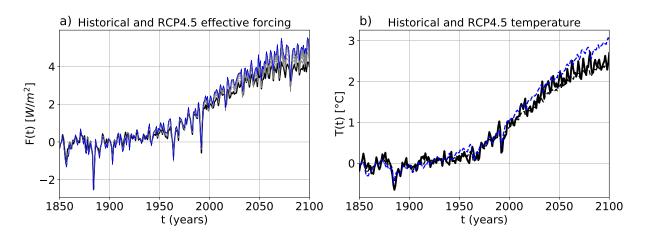


Figure S96. As Figure 4, but for the model MPI-ESM-LR and experiment RCP4.5.

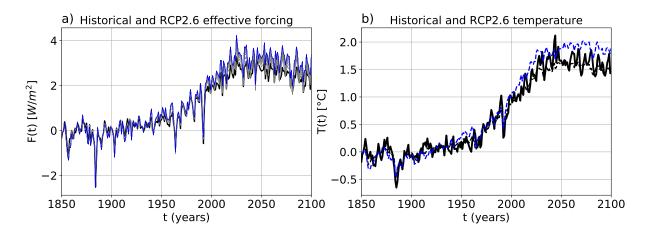


Figure S97. As Figure 4, but for the model MPI-ESM-LR and experiment RCP2.6.

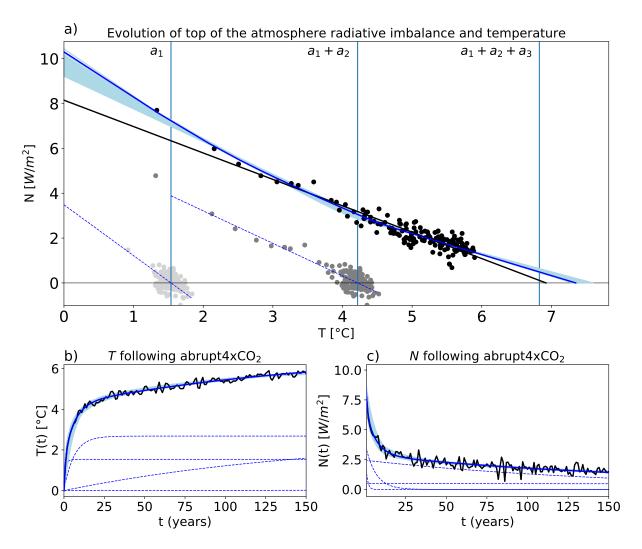


Figure S98. As Figure 1, but for the model MPI-ESM-MR.

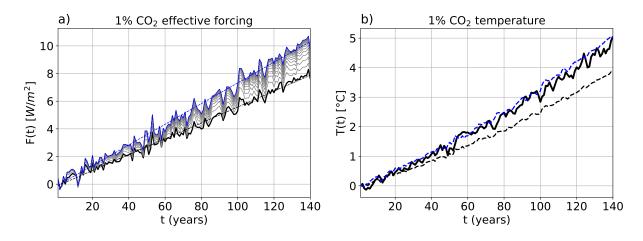


Figure S99. As Figure 3, but for the model MPI-ESM-MR.



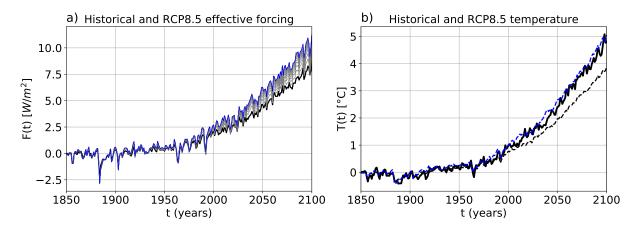


Figure S100. As Figure 4, but for the model MPI-ESM-MR.

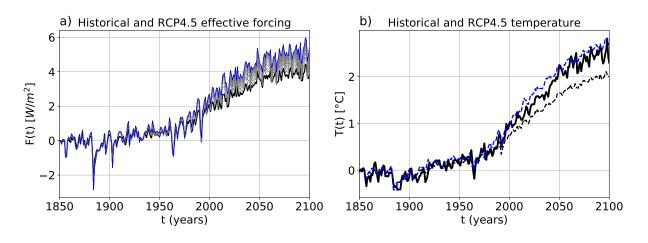


Figure S101. As Figure 4, but for the model MPI-ESM-MR and experiment RCP4.5.

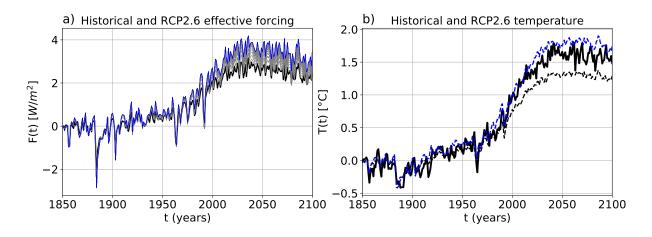


Figure S102. As Figure 4, but for the model MPI-ESM-MR and experiment RCP2.6.

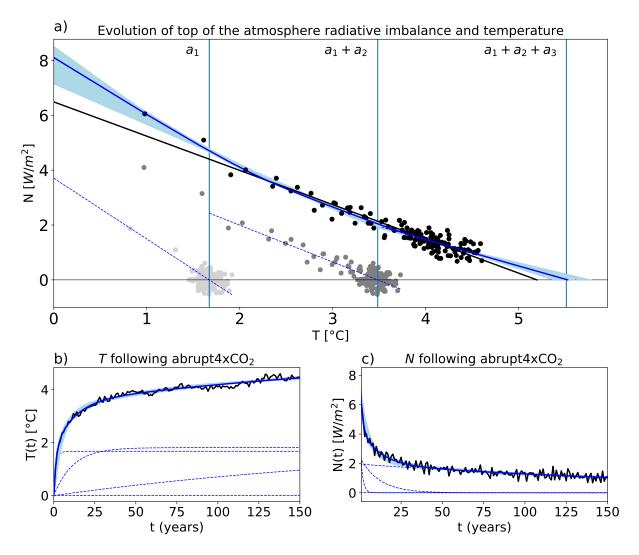


Figure S103. As Figure 1, but for the model MRI-CGCM3.

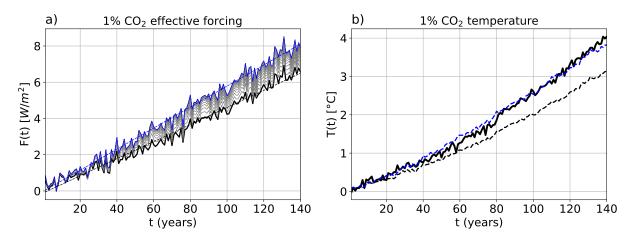


Figure S104. As Figure 3, but for the model MRI-CGCM3.

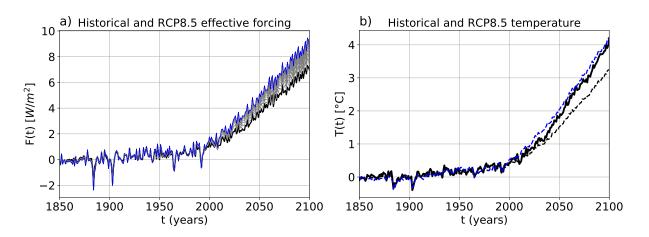


Figure S105. As Figure 4, but for the model MRI-CGCM3.

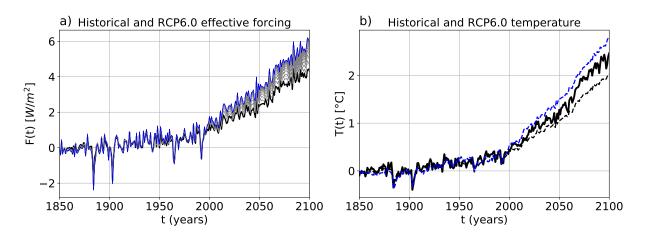


Figure S106. As Figure 4, but for the model MRI-CGCM3 and experiment RCP6.0.

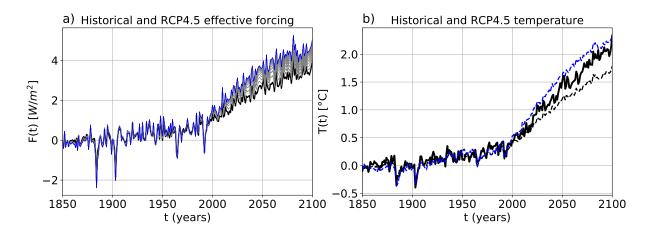
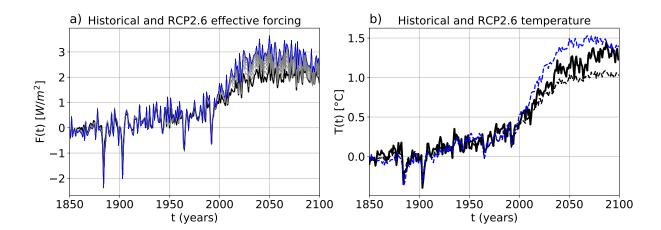


Figure S107. As Figure 4, but for the model MRI-CGCM3 and experiment RCP4.5.



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Figure S108. As Figure 4, but for the model MRI-CGCM3 and experiment RCP2.6.

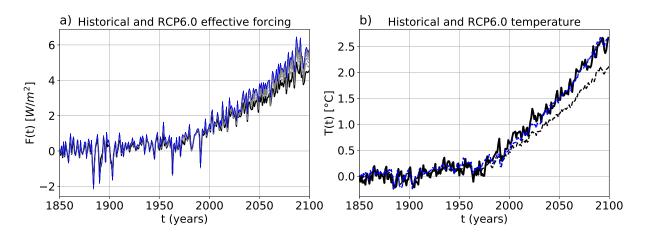


Figure S109. As Figure 4 with the model NorESM1-M, but for the experiment RCP6.0.

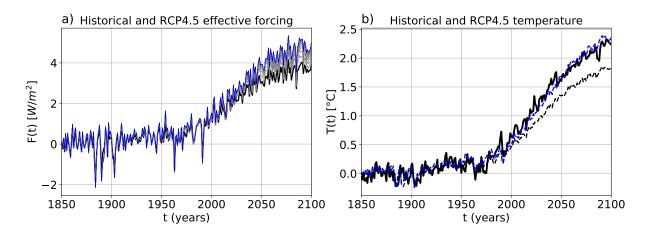


Figure S110. As Figure 4 with the model NorESM1-M, but for the experiment RCP4.5.

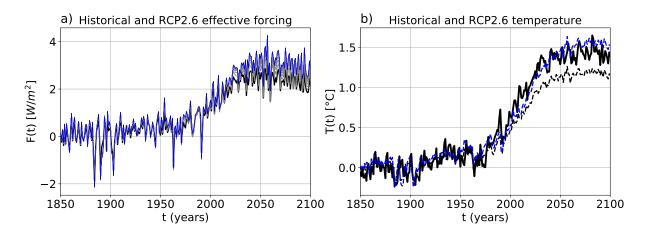


Figure S111. As Figure 4 with the model NorESM1-M, but for the experiment RCP2.6.

**Table S1.** The piControl trends computed over the 150 year period after branching of the abrupt4xCO<sub>2</sub> experiment. Temperature trends have units °C per year, and the top of atmosphere radiation components have units  $W/m^2$  per year.

radiation components have units $W/m^2$ per year.				
	$\Delta T/year$	$\Delta rlut/year$	$\Delta rsdt/year$	$\Delta rsut/year$
ACCESS1-0	1.05e-03	1.52e-03	1.66e-06	-1.31e-03
ACCESS1-3	4.76e-04	1.14e-03	1.79e-07	-6.73e-04
CanESM2	1.80e-04	2.02e-04	9.70e-17	-2.18e-04
CCSM4	-5.03e-04	-4.86e-04	-1.28e-15	7.58e-04
CNRM-CM5	1.16e-03	1.61e-03	-1.09e-06	-9.32e-04
CSIRO-Mk3-6-0	6.71e-04	8.82e-04	-3.76e-09	-1.09e-03
GFDL-CM3	8.09e-04	1.51e-03	-1.29e-15	-9.08e-04
GFDL-ESM2G	-1.04e-03	-1.86e-03	0.00e+00	1.93e-03
GFDL-ESM2M	-1.11e-04	-3.93e-04	0.00e+00	-3.21e-04
GISS-E2-H	8.76e-04	9.77e-04	-1.02e-15	-6.38e-04
GISS-E2-R	5.33e-04	7.95e-04	-3.75e-16	-6.00e-04
HadGEM2-ES	-2.84e-04	1.06e-04	0.00e+00	1.67e-04
inmcm4	-7.63e-04	-1.18e-03	1.48e-06	1.03e-03
IPSL-CM5A-LR	-3.86e-04	-3.81e-04	1.02e-10	1.01e-03
IPSL-CM5B-LR	1.36e-03	2.84e-03	-2.39e-10	-1.62e-03
MIROC-ESM	6.67e-04	7.70e-04	-2.85e-06	-9.82e-05
MIROC5	-3.60e-04	-2.79e-04	4.66e-10	6.02e-04
MPI-ESM-LR	-5.64e-05	2.76e-04	-8.26e-07	-1.28e-04
MPI-ESM-MR	2.89e-05	8.32e-05	-1.40e-07	-1.91e-05
MRI-CGCM3	2.93e-04	1.07e-03	2.75e-06	-4.39e-04
NorESM1-M	-4.72e-04	-9.77e-04	1.14e-10	3.36e-04
min	-1.04e-03	-1.86e-03	-2.85e-06	-1.62e-03
max	1.36e-03	2.84e-03	2.75e-06	1.93e-03