Investigation of the Stress-induced Microcracking Processes in Crystalline Rocks through Simultaneous Acoustic Emission and Strain Monitoring

Sana Zafar¹, Ahmadreza Hedayat¹, and Omid Moradian²

¹Colorado School of Mines ²ETH Zürich

November 26, 2022

Abstract

We performed laboratory-scale experiments on Barre granite specimen with a single pre-existing flaw to study the microscopic processes that occur during the deformation of a brittle material such as granite at different stress levels from crack initiation to the failure of the specimen. Here, we focus on the evolution of the tensile and shear cracks as a function of stress under unconfined compression. Acoustic emission technique (AET) in combination with the two dimensional (2-D) digital image correlation (DIC) technique have been used to track the changes in the source mechanisms of the registered AE events, along with the development of strains around the flaw tips of a uniaxially loaded prismatic Barre granite specimen. The parametric analysis along with the moment tensor inversion of the AE signals were used to discuss the cracking levels and the cracking mechanisms. In particular, the microcracks observed through AE monitoring prior to specimen failure were presented in terms of their spatio-temporal evolution and linked with the changes in the inelastic strain component measured through the 2D-DIC along the localized area. The mode of deformation computed from the image based strain profiles, enabled direct comparison of the nucleation, growth and interaction of the microcracks with the AE monitoring technique.

Investigation of the Stress-induced Microcracking Processes in Crystalline Rocks through Simultaneous Acoustic Emission and Strain Monitoring

Sana Zafar¹, Ahmadreza Hedayat¹, Omid Moradian²

¹Department of Civil and Environmental Engineering, Colorado School of Mines, Golden, CO, USA
 ²Department of Earth Science, Swiss Federal Institute of Technology (ETH), Zurich, Switzerland

Key Points:

1

2

3

7

 prismatic Barre granite specimens. Mode of deformation observed through AE was explicitly correlated by the ratic component of DIC strain Mode of deformation obtained through AE and DIC showed a consistent near correlation with increasing levels of stress. 	3	Damage evolution was continuously observed	under unconfined compression on
 tic component of DIC strain Mode of deformation obtained through AE and DIC showed a consistent net)	prismatic Barre granite specimens.	
• Mode of deformation obtained through AE and DIC showed a consistent net	0	Mode of deformation observed through AE wa	as explicitly correlated by the nonelas-
0	L	tic component of DIC strain	
correlation with increasing levels of stress.	2	Mode of deformation obtained through AE an	d DIC showed a consistent near-linear
8	3	correlation with increasing levels of stress.	

Corresponding author: S. Zafar, sanazafar@mymail.mines.edu

14 Abstract

We performed laboratory-scale experiments on Barre granite specimen with a single pre-15 existing flaw to study the microscopic processes that occur during the deformation of a 16 brittle material such as granite at different stress levels from crack initiation to the failure 17 of the specimen. Here, we focus on the evolution of the tensile and shear cracks as a function 18 of stress under unconfined compression. Acoustic emission technique (AET) in combination 19 with the two dimensional (2-D) digital image correlation (DIC) technique have been used 20 to track the changes in the source mechanisms of the registered AE events, along with the 21 development of strains around the flaw tips of a uniaxially loaded prismatic Barre granite 22 specimen. The parametric analysis along with the moment tensor inversion of the AE 23 signals were used to discuss the cracking levels and the cracking mechanisms. In particular, 24 the microcracks observed through AE monitoring prior to specimen failure were presented 25 in terms of their spatio-temporal evolution and linked with the changes in the inelastic 26 strain component measured through the 2D-DIC along the localized area. The mode of 27 deformation computed from the image based strain profiles, enabled direct comparison of 28 the nucleation, growth and interaction of the microcracks with the AE monitoring technique. 29

30 1 Introduction

Cracking processes in rocks is a complex phenomena. Discrete creation and propagation 31 of microcrack causes the brittle failure of the rocks. Therefore an understanding of the 32 mechanics and mechanisms involved during rock fracture plays an important role in de-33 signing civil engineering structures and different rock breaking processes such as hydraulic 34 fracturing, drilling, and blasting etc (Z. T. Bieniawski, 1967). The microcracking processes 35 that eventually causes the failure of a material has been studied through fracture mechanics 36 which states that microcracks are created due to localized stress concentration caused by 37 the presence of pre-existing flaws. Griffith (1921) satisfactorily explained that the actual 38 stress needed to fracture a bulk material is less than the theoretical stress required to break 39 the atomic bonds of the material, this low fracture strength is due to the presence of a large 40 number of randomly distributed microscopic flaws in the material (Griffith, 1921; Z. Bieni-41 awski, 1967). Similarly, McClintock and Irwin (McClintock & Irwin, 1965) showed that the 42 material deforms inelastically before the crack propagation due to the displacement field 43 around the tips of the pre-existing flaw known as the fracture process zone. The process 44 zone consists of microcracks and with the increase in the load, these microcracks propagate 45 and coalesce to form macrocracks which leads to the failure of the material (Wawersik & 46 Fairhurst, 1970). These studies indicate that the presence of pre-existing flaw in a material 47 acts as a stress concentrator and the growth of the microcracks particularly from these stress 48 concentration areas causes the brittle fracture of the rock specimen (S. Peng & Johnson, 49 1972; Tapponnier & Brace, 1976; Kranz, 1983). Therefore, a better understanding of the 50 cracking mechanisms involved in rock damage around a pre-existing flaw is an essential pre-51 requisite to predict the macroscopic failure of the rock bearing structures. Although several 52 studies have been done on rock and rock-like specimens with an existing flaw by different 53 researchers (Bazant & Kazemi, 1990; Fortin et al., 2009; L. Wong & Einstein, 2009; Tal 54 et al., 2016; Li & Einstein, 2017), only limited knowledge about its fracture process under 55 unconfined compression has been obtained.

In order to study the evolution of microcracks in stressed rock specimen various direct microstructural observation techniques such as scanning electron microscope and optical microscope have been used (Brace et al., 1966; Reches & Lockner, 1994; Kranz, 1979). Tapponier and Brace (1976) investigated the progression of damage in Westerly granite specimens using scanning electron microscopy (SEM) analysis. The cracking processes and the increase in the crack density was observed as a function of stress. In their study, the evolution of shear cracks at microscopic level was overlooked because of the low magnification of the SEM. Fredrich et al. (1989) studied the micromechanical process of the brittle

to plastic transition in Carrara marble using optical and transmission electron microscopy 65 (Fredrich et al., 1989). Zhao et al., (1993) characterized the different stress-induced cracking 66 mechanisms by conducting real-time SEM observations on a marble plate with an inclined 67 pre-existing flaw (Zhao et al., 1993). They observed that the microcracks were mostly tensile in nature, with a few shear cracks (L. N. Y. Wong & Xiong, 2018). Wong and Einstien (2009) performed microscopic observations on double flawed specimen under uniaxial com-70 pression. They utilized the environmental scanning electron (ESEM) and SEM imaging 71 techniques to study the microscopic behavior and further linked the microscopic damage 72 to the macroscopic failure of the rock specimen. As per their observation, the coalescence 73 between the two pre-existing flaws took place through the evolution of a number of tensile 74 microcracks. Cheng et al. (2018) investigated the progression of damage in marble speci-75 men containing en echelon flaws using an optical microscope. Their study investigated the 76 development of tensile and shear cracks at the microscopic and the macroscopic scale as a 77 function of stress (Y. Cheng & Wong, 2018). Although these techniques provided useful in-78 formation about the internal microstructure of the rock material subjected to loading, they 79 failed to provide a continuous observations of the cracking processes without pausing the load or interfering with the loading process (Chang & Lee, 2004; Paterson & Wong, 2005). 81 Therefore, as a continuous measurement technique, acoustic emission (AE) monitoring in 82 combination with the 2D-digital image correlation (DIC) was used in this study to analyze 83 the damage processes in real time (Moore & Lockner, 1995; Crider, 2015; Moradian et al., 2016; Ghamgosar et al., 2017; Tarokh et al., 2017). The AE technique is considered as one 85 of the most widely used methods for non-destructive monitoring (Guo et al., 2017; Hampton 86 et al., 2018; Xu & Zhang, 2018; Lin et al., 2019) because of its ability to detect the dynamic 87 motions in the material whereas most of the other methods like ultrasonic testing have the ability to detect the existing geometrical defects. The 2D-DIC is also the most extensively 89 used non-contact optical method for displacement and strain field measurement in real-time 90 (Pan et al., 2009; Hedayat et al., 2014; Sutton et al., 2009; Shirole et al., 2019). 91

Various experimental studies have been conducted over the past few years using the AE 92 and 2D-DIC techniques in combination for the damage characterization in rocks of various 93 geometries under different loading conditions (Lin & Labuz, 2013; Lin et al., 2014; Kao et 94 al., 2016; Guo et al., 2017; Dong et al., 2017; J.-L. Cheng et al., 2017; Li & Einstein, 2017; 95 Lin et al., 2019). Based on these researches, it can be concluded that both the techniques in combination have the capability of detecting the damage initiation and evolution in the 97 rock specimen. AE detects the source location and mechanism of the AE events while DIC 98 provides the strain field related to the deformation of the material. Hence, the combination 99 of AE and DIC can provide a detailed evaluation of the fracturing process in rocks from 100 microscopic to macroscopic scale. Lin and Labuz(2013), Lin et al.(2014), Zhang et al.(2015) 101 and Lin and Labuz(2019) all adopted the above-mentioned techniques to study the fracture 102 process zone in a three point bending test on a pre-notched rock specimen. They used 103 the two techniques to identify the size of the fracture process zone at the tips of the notch 104 and to distinguish the regions accommodated with the process zone and the actual crack 105 propagation within the rock specimen. Kao et al. (2016) characterized the spalling near a 106 free surface in laboratory experiments on rocks using AE and DIC (Kao et al., 2016). They 107 analyzed the damage based on the AE locations and compared it with the inelastic strain 108 measurements obtained through DIC. However, the cracking mechanism was not studied 109 in their work. Li and Einstien, 2017 conducted four point bending experiments on a pre-110 notched granite specimen to observe the process zone development and crack propagation using AE and DIC techniques. Based on their observations they defined the extent of process 112 zone and crack front. In most of these studies on rocks the major focus was to identify the 113 damage zones based on AE source locations and the DIC strain and displacement field 114 115 measurements. However the evaluation of the mode of deformation from the two techniques and their correlation has been rarely reported. 116

This study attempts to provide an insight into the stress-induced cracking processes involved during deformation of brittle rock specimen containing a pre-existing flaw through extending

the available experimental observations. In this study, the cracking processes were monitored 119 simultaneously with an 8 channel acoustic emission system and 2D-digital image correlation 120 technique under unconfined compression. The experimental observation based on cumulative 121 AE hits and cumulative AE energy helped to analyze the stress thresholds corresponding 122 to different stages of cracking and the moment tensor analysis of the AE sources provided 123 in-depth knowledge about the cracking processes involved during this period. The 2D-DIC 124 strain measurement approach was used to monitor the evolution of damage in terms of the 125 nonelastic strain component with increasing levels of stress. One of the new major finding 126 of the present study was to obtain the mode of deformation from the DIC strain profiles and 127 relate it with the cracking mechanisms obtained through the moment tensors of AE in real 128 time. Independent measurements of tensile and shear deformation with increasing levels of 129 loading from the two techniques provided a unique opportunity to correlate the changes in 130 the cracking processes explicitly with the damage in the rock specimen. 131

¹³² 2 Experimental Design

133 2.1 Material

This study for the characterization of the damage process was performed on Barre granite 134 (BG) specimen with a single pre-existing flaw. BG is crystalline in nature and obtained 135 from the south-west region of Burlington, Vermont (USA) (Nasseri et al., 2010; Iqbal & Mohanty, 2007). This rock is a representative of the Earth's crust and one of the most 137 extensively studied rocks with a rich literature (e.g., (Goldsmith et al., 1976; S. S. Peng, 138 1975; Kranz, 1979; Morgan et al., 2013; Moradian et al., 2016)). BG is a gray granodiorite 139 with its grain size ranging from 0.2mm to 3.0mm (medium-fine grained). It has an average 140 grain size of 0.87mm (Nasseri et al., 2010; Iqbal & Mohanty, 2007). BG has a very consistent 141 mineral composition which consists of about 65% felspar (average grain size of 0.95), 27%142 quartz (average grain size of 0.94mm), 9% biotite (average grain size of 0.83mm) (Iqbal & Mohanty, 2007; Nasseri et al., 2010; Dai & Xia, 2010). It has a density of 2.66 gm/cm^3 with 144 a porosity of 0.59% (Iqbal & Mohanty, 2007). Barre granite specimen in its intact form has 145 the following average properties: Young's modulus=58 GPa (Shirole, Walton, & Hedayat, 146 2020), uniaxial compressive strength=170 MPa (Zafar et al., 2020), average compressional 147 P- wave velocity = 4000 m/s (Moradian et al., 2016). Same block of Barre granite was used 148 to prepare the prismatic specimens (152 mm x 76 mm x 25 mm) and the pre-existing flaws 149 were cut by OMAX water jet. The flaw was cut throughout the thickness of the specimen. 150 The flaw length and the its inclination angle with respect to the horizontal axis is 25mm and 45° , respectively (Figure 1a). For the purpose of experimental result presentation, three 152 representative specimens labelled as 'BG-1', 'BG-2' and 'BG-3' are discussed in this paper 153 from a comprehensive series of tests. 154

¹⁵⁵ 2.2 Loading System Set-up

A computer-controlled servo-hydraulic loading machine was used for conducting unconfined 156 compression experiments on three Barre granite specimen with an existing flaw. The loading 157 was applied in displacement controlled mode at a displacement rate of 1 μ m/s. The dis-158 placement was controlled through three Linear Variable Differential Transformers (LVDTs). 159 These LVDTs recorded the overall axial deformation of the rock specimen. The displacement 160 control mode helped in controlling the deformation of the rock specimen closed to their uni-161 axial compressive strength to protect the AE sensors from damage. Proper synchronization 162 was ensured among the three systems during the test. 163

164 2.3 2D-DIC Setup

Digital image correlation (DIC) technique is the most commonly used and widely accepted 165 non-destructive, non-contacting optical deformation and strain measurement approach that 166 can be utilized for the evaluation of the complex behavior of geomaterials (Pan et al., 2009; 167 Hedayat et al., 2014; Sutton et al., 2009; Bruck et al., 1989). This technique evaluates full-168 field displacement and full-field strains to sub-pixel accuracy by comparing the reference 169 image (without mechanical loading) to the image corresponding to strained state (under 170 mechanical loading). Because of its sub-pixel resolution, accurate DIC measurements detect 171 optically invisible cracks. The 2D-DIC technique is preferred over other optical methods be-172 cause of its simple experimental set-up and specimen preparation and also provides accurate 173 displacement and strain field measurement (Hedayat & Walton, 2017). 174

In 2D-DIC, the image taken before applying the load is known as the reference image and 175 this reference image is then used for comparing the images acquired throughout the loading. In order to compare the images acquired in the stressed state with the reference image, 177 the specimen surface should have a unique random gray intensity pattern. After acquiring 178 the digital images, DIC uses a correlation function between the images, to compare the 179 acquired image with the reference image. An area of interest (AOI) is first specified within 180 the image and further divided into small group of pixels known as subsets (Schwartz et al., 181 2013). These subsets are separated from each other through the step size. In 2D-DIC, the 182 correlation functions such as zero normalized cross-correlation (ZNCCD) or zero normalized 183 sum of squared difference (ZNSSD) (Pan et al., 2009) is used for tracking the subsets between the reference and the deformed images. The reason behind the selection of a square subset 185 over a single pixel is that it provides a wider variation in the intensity of the gray scale 186 values which makes it more identifiable from other subsets in the deformed image. To get 187 the accurate measurement in 2D-DIC and to track the changes in the reference and the 188 deformed images, each subset is defined by a unique distribution of gray-scale values known 189 as the speckle pattern. The 2D-DIC technique locates the subsets in the deformed image, 190 initially defined in the reference image assuming that the gray-scale values in the subset remains preserved even after the deformation (Bourcier et al., 2013). A correlation algorithm 192 is then implemented between the subsets of the reference and the deformed images to find 193 the optimal matching between the coordinates. The coordinates of the extremum position 194 of the correlation coefficient defines the new position of the deformed subset with respect 195 to the reference subset. The difference between the position of the reference subset and the 196 deformed subset gives the displacement vector (Hedayat et al., 2014). The procedure is then 197 repeated for all the virtual grid lines in a systematic manner to obtain the displacement along 198 the surface of the specimen at various stages of deformations. The numerical differentiation 199 of the displacement field along the specimen surface gives the strain field measurement in 200 real time. The strain field calculation is based on the standard Lagrangian approach of the 201 continuum mechanics. The strains around the grid points are averaged across an area (filter 202 size) to get a continuous strain profile along the surface of the specimen. 203

The proper implementation of the DIC technique comprises of three consecutive steps: (i) specimen preparation and experimental set-up, (ii) image acquisition, and (iii) processing the acquired images using correlation algorithm. To obtain a high-quality gray-scale distribution for accurate DIC measurements, the planar surface of the specimen was cleaned and a unique random speckle pattern was created by using a multi-color paint (Rust-Oleum) to paint the surface of the specimen (Sutton et al., 2009).

Grasshopper (Point Grey) charged coupled device (CCD) camera with a Fujinon lens of focal length 35mm was used for acquiring the digital images. The aperture, focus and polarization of the lens were operated manually. The Fly Capture SDK software was used to control the field of view from the camera, brightness, and the rate of image acquisition (Shirole et al., 2019). Before each test, dust on the polarizing lens and reflections on the surface specimen were minimized. In our experimental setup, the CCD camera was kept at a distance of 1000 mm from the planar surface of the specimen to avoid the error due to

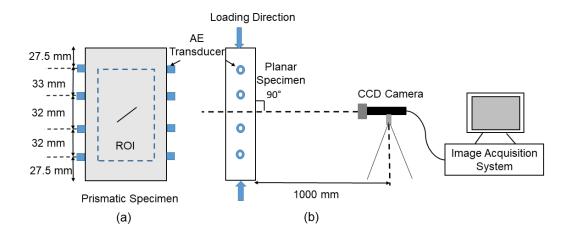


Figure 1. Schematic diagram of the (a) Prismatic specimen with a pre-existing flaw (flaw length is 25mm and the flaw inclination angle with respect to the horizontal axis is 45°) with the location of the AE sensors, the dashed blue line shows the region of interest, (B) 2D-DIC setup with the CCD camera and the image acquisition system

the out of plane deformations below the prescribed limit of $\Delta z/zc\sim 10^{-4}$ (Modiriasari et al., 2017). The surface of the specimen was kept perpendicular to the optical axis of the camera and the plane of the camera parallel to the planar surface of the specimen. The images were captured at a rate of 10 frames per second for these experiments. A polarizing lens with a conjugate polarizer was used for better illumination. The DIC component of the setup is shown in Figure 1. The whole surface (152 x 76 mm²) of the specimen was imaged with the camera.

The digital images captured during the experiments were analyzed using the VIC-2D soft-224 ware licensed by Correlated Solutions to extract the displacement and strain along the 225 specimen surface using the correlation criterion. The analysis in the software requires the selection of the appropriate region of interest (ROI) in the reference image (Figure 1). For 227 DIC measurements a virtual grid is formed on the reference image, which requires subset 228 size and step size as an input to the software. According to previous research, (Hedayat 229 et al., 2014; Shirole et al., 2019), a step size of 5 pixels and a subset size of 15 pixels were 230 selected. This provided sufficient overlap between the subsets and required less computa-231 tion time for the image analysis. The correlation procedure was executed by applying a 232 constant magnification factor to convert the pixels to their respective physical dimensions (Hedayat et al., 2014; Shirole et al., 2019). In these experiments, by comparing the pixel 234 measurements from the fixed field of view of the camera to the physical dimension of the 235 entire specimen surface, a constant magnification factor of 90μ m/pixel was used that is each 236 pixel in the digital image is equal to 90μ m in the physical dimension. 237

238 2.4 AE Setup

Acoustic Emission (AE) was implemented to track the spatiotemporal changes in the registered AE events around the flaw tips of the uniaxially loaded prismatic specimen. The experiment was instrumented with eight piezoelectric AE sensors mounted on the sides of the specimen (Figure 1a). The piezoelectric sensors Nano 30 from Mistras Group, Inc. were used in the study to record the AE signals. The Nano-30 AE sensor has a frequency response over the range of 125-750kHz with a resonant frequency of 300kHz. The miniature size of the sensor makes it easy to mount in small and tight spaces. They were attached on the sides along the longitudinal axis of the specimen with epoxy (produced by Hardman, Royal
Adhesives and Sealants). The epoxy was in contact with the sensor for 9 hours and the
velocity measured by Pencil Lead Break test was documented after every 1.5 hour interval.
The efficiency of the coupling was verified by the pencil lead break (PLB) and the auto
sensor test (AST).

In this experimental-setup, 2/4/6 PAC preamplifiers were used to amplify the output voltage of the AE sensors by 20dB in order to improve the detection efficiency of the sensors for recording. The sampling frequency was 5 MHz with a sample length of 15k and a pre-trigger of 256 μ s. Eight channel board and system from the MISTRAS Group, Inc. was used as a part of the AE data acquisition system. The system was controlled by real-time operating software AEwin where the peak definition time (PDT), hit definition time (HDT) and hit lockout time (HLT) were set as 200, 800 and 350 μ s, respectively. The maximum duration was taken as 3ms.

All the AE signals (waveforms) were recorded and further analyzed for the source localization 259 and source type characterization using the moment tensor inversion method reported in Li et 260 al.(2019)(Li et al., 2019). The source location was based on the first arrival of the P-waves. 261 The arrival time picking was done using the Akaike information criterion (AIC) (Maeda, 262 1985; Kurz et al., 2005). Locations were determined using a constant P-wave velocity 263 field model for a minimum distance error of 3 mm and optimized using "fmincon" function 264 in MATLAB. While AE source location analysis helped to describe the spatiotemporal evolution of damage, AE source mechanism analysis and their dependence on the stress state of rocks enables detailed insight into the cracking processes at the microlevel. 267

A generalized relationship between the seismic sources and the elastic waves is summarized by (Richards & Aki, 1980; Ohtsu, 1991). Thus, the AE waves can be represented by

$$u_{ix,t} = G_{ip,q}(x, y, t)m_{pq} * S(t)$$
(1)

where $u_{ix,t}$ is the displacement at crack location x, $G_{ip,q}$ (x,y,t) is the spatial derivative 268 of Green's function, which describes the response of the medium to a disturbance, m is 269 the moment tensor, the asterisk denotes the convolution operation and S(t) represents the 270 source time function. The moment tensor inversion analysis based on Simplified Green's 271 function for Moment tensor Analysis (SiGMA) procedure (Ohtsu, 1995) was used to identify 272 the source mechanism and their evolution in these experiments. This method selects the 273 compressional (P) wave portion from the full-space Green's function when applied to an isotropic and homogeneous material. It is a quantitative approach in which the source is 275 represented by a moment tensor matrix (m) (Eq. 1) which is a 3x3 matrix. The elements 276 of the matrix describes the forces acting on the source (Graham et al., 2010). Each element 277 in the matrix denotes one of the 9 double-couples acting at the source. Since the matrix is 278 symmetric, it contains six independent elements. The diagonal elements represent tensile or 279 compressional couples and the off diagonal elements represent the shear couples. The SiGMA 280 procedure uses the simplified form of Eq.(1) to determine the six independent components of 281 the moment tensor by solving a set of linear equations in terms of the first motion amplitude 282 A(x) as shown in Eq.(2), as follows: 283

$$\mathbf{A}(x) = \frac{C_s Re(t, r)}{R} \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}$$
(2)

where A(x) is the amplitude of the first motion observed at the sensor location x. C_s is the coefficient of calibration for the sensor, R is the distance from the sensor to the AE source. Vector r is the direction vector of R and Re(t,r) denotes the reflection coefficient (Ohtsu, 1995). These values can be obtained through the source localization. Thus, source location is required to perform the moment tensor analysis. Once the moment tensor of a source has been determined, the cracking mechanism is classified using the eigenvalues of the moment tensor (Ohtsu, 1991). The moment tensor is decomposed to its eigenvalues to split the tensor into an isotropic component (ISO), a deviatoric component 'Compensated Linear Vector Dipole' (CLVD) and a double-couple (DC) component. X and Y denotes the maximum shear and CLVD components respectively, giving a DC part (X,0,-X), a CLVD part (Y,-0.5Y,-0.5Y) and the isotropic part in direction, Z. The values of X,Y and Z calculated from the following equations are used to determine the shear and tensile crack ratios:

$$\frac{\lambda_1}{\lambda_1} = X + Y + Z$$

$$\frac{\lambda_2}{\lambda_1} = 0 - \frac{Y}{2} + Z$$

$$\frac{\lambda_3}{\lambda_1} = -X - \frac{Y}{2} + Z$$
(3)

where λ_1 , λ_2 and λ_3 are the maximum, intermediate and minimum eigenvalues. In the SiGMA procedure, the values of X, Y and Z gives the shear, deviatoric and isotropic component of the source, respectively. An AE source with X < 40% and Y+Z > 60% is classified as a tensile crack, X < 40% is typically considered as a shear crack and 40% < X < 60% is classified as a mixed mode crack (Ohtsu, 1995).

From the eigenvalue analysis of the moment tensor, three eigenvectors e1, e2, e3 can also be obtained. The eigenvector analysis of the moment tensor provides the orientation of the cracks. l and n represents the cracking motion vector and the vector normal to the crack surfaces which can be evaluated using equation (4):

$$e1 = l + n$$

$$e2 = l \times n$$

$$e3 = l - n$$
(4)

In case of a tensile crack, the cracking motion vector l is parallel to the normal vector n and for shear cracks the two vectors are usually perpendicular.

In this study, the above procedure was used not only to classify the different AE events as shear, tensile, or mixed mode crack but also to identify their orientation at different levels of cracking. As for the source localization in three dimensions, there are four unknowns (x, y, z, and t) which require the detection of the AE signals by minimum of four channels but the moment tensor has six independent components. Hence, in these experiments, the source localization was also done for a minimum of six channels.

306 3 Results and Discussion

307

3.1 Crack Initiation (CI) and Crack Damage (CD)

Brittle rock is a heterogeneous material made up of various inherent microstructures. Nu-308 merous experimental results demonstrate that the microcracking in rock is effected by these 309 internal heterogeneities (Brace et al., 1966; Martin & Chandler, 1994; Eberhardt et al., 310 1998). Considering the fact that Barre granite is a brittle rock (quartz content is 27%) and 311 hence the failure is caused by the initiation, growth and coalescence of microcracks created 312 due to material heterogeneity under compression. Direct observation of these microcracks 313 have revealed that the primary mechanism of deformation in brittle rocks is local tensile 314 cracking which is due to the extensile strains (Lajtai, 1974; Tapponnier & Brace, 1976; 315 Kranz, 1983; Moore & Lockner, 1995), in which the cracks are oriented parallel to the di-316 rection of the major principal stress (Wulff et al., 1999; Martin & Chandler, 1994; Moore 317

& Lockner, 1995). As the load applied to the specimen increases, a complex heterogeneous 318 combination of tensile and shear stresses gets concentrated at the tips of the pre-existing 319 flaws. Various experimental studies reveal that brittle fracture in compression is due to the development of the extensile microstresses. The macroscopic failure takes place due to 321 the interaction of these tensile microcracks close to the tips of the pre-existing flaws. How-322 ever, tensile cracks are not solely responsible for the overall failure of the material (Lajtai, 323 1974). Therefore, shear failure mechanism caused by the compressive stress concentration 324 becomes active at later stages of the cracking process (Griffith, 1921; Lajtai, 1974). Once 325 sufficient number of extensile cracks are formed, they start to interact, at this stage (crack 326 damage stress threshold) the shear (frictional) cracking becomes dominant (Brace et al., 327 1966; Tapponnier & Brace, 1976; Martin & Chandler, 1994; Hajiabdolmajid et al., 2002; Jian-po et al., 2015). Martin and Chandler (1994) studied that the rock strength is made up 329 of two components: friction and cohesion. The cohesive component is the primary strength 330 component at early stages of loading and gets destroyed by the tensile cracking. Once suf-331 ficient damage has accumulated, the cohesion strength gets reduced and frictional strength 332 component gets mobilized (Hajiabdolmajid et al., 2002). During this stage, high structural 333 changes to the specimen takes place, with an increase in the density of microcracks by about 334 sevenfold (Hallbauer et al., 1973). 335

The procedure adopted in this study for the quantification of the tensile and shear cracks accumulated in the rock specimen throughout the loading are described in the subsequent sections. The section deals with the cracking levels and the cracking mechanisms obtained through the experimental observations, it further illustrates the methodology used for the selection of the strain metrics evaluated from the 2D-DIC strain measurements to quantify the nonelastic damage into the rock specimen.

342 343

3.1.1 Observation of Crack Initiation (CI) and Crack Damage (CD) using AE Signatures

As shown in Figure 2 (a & b), AE signatures were observed around the flaw tips in the 344 specified region of interest, in sync with the 2D-DIC measurements. Several studies revealed 345 that the AE hits acquired throughout the loading corresponds to the increasing number of 346 microcracks and the energy of the signal denotes the magnitude of the cracking sources in 347 materials (Lockner, 1993; Moradian et al., 2016). Therefore, to investigate the cracking 240 levels, common parametric features of the AE waveform such as hits and energy emitted by the seismic sources were analyzed as a function of the normalized stress. The two important 350 components in the brittle rock fracture that is crack initiation (CI) and crack damage (CD) 351 thresholds were identified (Eberhardt et al., 1998). 352

Figure 2 (a & b) shows the rate and cumulative plots of the AE hits and AE energy as 353 a function of applied stress normalized by the peak strength. The trend of the changes 354 in the cumulative plots of AE hits due to increasing level of stress are consistent for all 355 the specimen in the region of interest. Figure 2a shows that the initiation of significant 356 AE activity in the cumulative hits plot occurred at 37-41% of the peak strength. This 357 behavior has been detected in all the three rock specimen. Therefore, this point can be 358 linked as the crack initiation point among the cracking levels. This cracking level detected 359 by AE monitoring is consistent with the findings of several other studies corroborating it as the crack initiation (Pestman & Van Munster, 1996; Nicksiar & Martin, 2013). However, cumulative AE energy plot (Figure 2b) does not show any significant change in the trend 362 at this stage (CI), which is consistent with the fact that microcracks have very low AE 363 energy (Kim et al., 2015). When the load is further increased beyond crack initiation, 364 365 it does not lead to the failure but the cracks become stable after propagating to some fraction of its initial length, known as the stable growth of the cracks. This can be seen by 366 the constant increase in the cumulative hits plot. The cumulative AE energy plot is also 367 constant and does not show any significant change in this region. This constant increase in the AE signal parameters indicate 'stable crack growth.' When the load reaches 85-90% of 369

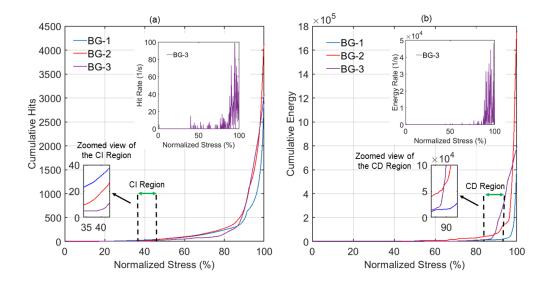


Figure 2. (a) Variation of cumulative AE hits as a function of normalized stress for the rock specimens BG-1,BG-2 and BG-3 at the specified ROI. The insets show the zoomed view of the CI region and the variation in the hit rate throughout loading for BG-3,(b) Variation of cumulative AE energy as a function of normalized stress for the rock specimen BG-1,BG-2 and BG-3 at the specified ROI. The insets show the zoomed view of the CD region and the variation of the energy rate throughout loading for BG-3.

the peak strength, the cumulative plots of AE hit and AE energy shows a sharp increase. 370 This indicates the accumulation of microcracks to macrocracks, which is confirmed by the 371 high amount of AE energy released at this stage. Therefore, the rise in the AE energy 372 release can be called as 'macro-crack initiation'. Other researchers (Eberhardt et al., 1998; 373 M. Diederichs, 2003; Nicksiar & Martin, 2012) have called this point as the 'crack damage'. 374 At low levels of loading (0-30%) of the failure stress), the AE parameters does not show 375 any activity, whereas it has been found in previous studies that some AE signals can be 376 seen in the initial loading stages due to crack closure and elastic deformation (Scholz, 1968; 377 Eberhardt et al., 1998; Moradian et al., 2016). However, in these experiments, the analysis was mainly focused from the microcrack initiation to the failure of the sample, so the levels 379 prior to the crack initiation was adopted to be quiet by setting a high threshold in the 380 AE settings (~ 70 dB). Analysis of the digital images also did not reveal any significant 381 information about the stages prior to crack initiation. 382

3.1.2 Observation of Crack Initiation (CI) and Crack Damage (CD) using 2D-DIC

The 2D-DIC technique was used to measure the strain along the specimen surface in the region of interest (ROI) and the damage was characterized based on the non-elastic strain measurements. The evolution of the non-elastic strain components in the rock specimen represents the initiation and growth of the microcracks. Therefore, in the present study, the inelastic components of the tensile and shear strain values were evaluated at various levels of loading for the analysis of the different mode of deformation.

³⁸³ 384

301 3.1.3 Crack Initiation (CI) using Tensile Strain Measurements

As the damage progresses, strain accumulation takes place in the specimen. When the 392 specimen is loaded under unconfined compression, major principal strain (ϵ_{11}) is caused 393 along the longitudinal axis of the specimen, while the minor principal strain (ϵ_{22}) results 394 due to the material expanding in the lateral direction (Poisson's effect). Based on the fact 395 that the concentration of the local tensile stress at the tip of the flaw is the primary mode 396 of deformation in brittle rocks (Moore & Lockner, 1995), the minor principal strain (ϵ_{22}) 307 (extensile strain) distribution was studied in the specific region of interest for the three rock specimen to understand the effect of heterogeneity on the microcracking behavior. At low levels of loading (20%) of the failure stress), the strain distribution showed a small 400 standard deviation (0.4 m ϵ). As the stress level increases, the spread in the histogram and 401 the standard deviation also increases. At 60% of the failure stress the standard deviation 402 in strain distribution is around 1.1 m ϵ and goes up to 3 m ϵ at 95% of the failure stress. 403 This indicates the increased heterogeneity in the strain field due to strain localization at 404 higher levels of damage. As the stress level increases the histogram shift towards the left which shows the higher concentration of the extensile strains. The spread in the histogram also indicates the increase in the number of DIC grid points with higher magnitudes of the 407 tensile strain. As shown in Figure 3 some of the DIC grid points show compressive strain in 408 the tensile strain field which can be due to the heterogeneous distribution of strain at the 409 pixel scale (M. S. Diederichs, 1999; Shirole et al., 2019). 410

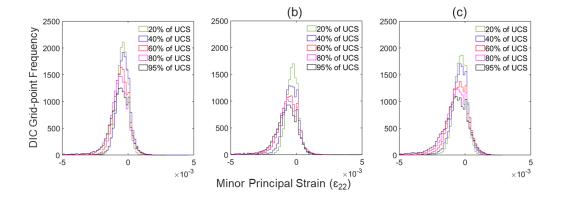


Figure 3. Histograms showing the minor principal strain distribution in the ROI for three prismatic Barre granite specimens: (a) BG-1,(b) BG-2,(c) BG-3. Negative (-) strains represent extension

In order to obtain the distribution of the localized tensile strain in the ROI, an apparent tensile strain (ϵ_{AT}^T), similar to the method adopted by (Song et al., 2013; Shirole et al., 2019), is used:

$$\epsilon_{AT}^{T} = \left| \sum_{i=1}^{N} \left\langle \epsilon_{22,i} \right\rangle \right| \tag{5}$$

where $\epsilon_{22,i}$ represents the minor principal strain at the ith DIC grid point, and N is the total number of DIC grid points in the specified region of interest. For the computation of the apparent tensile strain, only negative strain values are taken into account. This apparent tensile strain contains both the elastic and inelastic component of the tensile strain present in the rock specimen.

As it is known that acoustic emission is released due to microcracking which leads to the irreversible increase in the rock volume, known as dilatancy. This increase in volume is

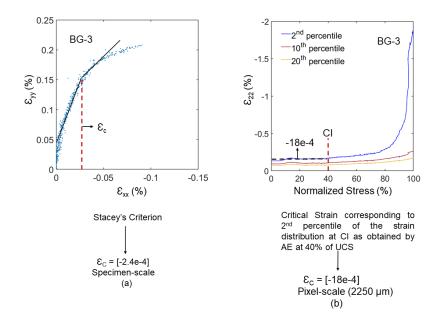


Figure 4. Procedure adopted for the evaluation of the critical threshold of tensile strain (ϵ_c) from DIC strain measurement (a) at specimen-scale (Stacey, 1981), (b) at pixel-scale (M. S. Diederichs, 1999; Shirole et al., 2019)

caused by the tensile opening. (Brace et al., 1966; Scholz, 1968; Sondergeld & Estey, 1982). 421 Several experimental studies have shown that the initiation of the microcracking can be 422 seen by a significant rise in the AE activity which is considered as the crack initiation 423 stress (CI) (Eberhardt et al., 1999; Moradian et al., 2016) as shown in Figure 2. Similarly, 424 the regions accumulated with the tensile microcracking are expected to be characterized 425 by the tensile strain above some threshold limit at which the cracking initiates i.e. the plastic deformation takes place. With this in mind, and to establish the correlation between 427 the microcracking observed through AE and DIC strain measurements, it is important to 428 compute the nonelastic strain component which governs the plastic deformation in the rock 429 specimen. To evaluate the nonelastic component, the critical tensile strain limit for Barre 430 granite was estimated. The procedure adopted for the critical strain limit calculation is 431 shown in Figure 4. 432

⁴³³ Due to the heterogeneous nature of rocks, the scale of strain measurement is also considered ⁴³⁴ in order to calculate the critical threshold value of the tensile strain (ϵ_c). The heterogeneity ⁴³⁵ in rocks create spatially uneven strain response to the applied stress, which causes irregular ⁴³⁶ fluctuations in the strain distribution from the specimen scale to pixel scale. To accurately ⁴³⁷ measure the critical limit of tensile strain the analysis was performed at both the specimen-⁴³⁸ scale and pixel-scale (Figure 4).

At the specimen-scale, Stacey (1981) strain criterion was used to evaluate the critical thresh-439 old value of the tensile strain. It has been established that the initiation of extensile micro-440 cracks in rocks is highlighted by the deviation in the slope of vertical strain (ϵ_{yy}) vs. lateral 441 strain (ϵ_{xx}) plot from linear to non-linear. Figure 4a shows the plot of vertical strain vs. 442 lateral strain for BG-3 in which the change in the slope can be observed corresponding to 443 -2.4e-4 strain value. Therefore, the critical tensile strain limit (ϵ_c) at the specimen scale was 444 estimated as -2.4e-4 (negative strain(-) represents extension). Since the DIC strain measure-445 ments are based on pixel-scale, the critical limit of tensile strain needs to be calculated at 446 the pixel-scale. At the pixel-scale, the value of ϵ_c was calculated based on CI stress thresh-447

old obtained through AE and was considered as 40% of the UCS. The 2nd, 10th and 20th 448 percentiles of strain distribution were plotted as a function of the normalized stress (Figure 449 4b) for BG-3. The second percentile of the strain distribution was chosen for the evaluation of CI threshold because of the following two reasons: (i) the strain distribution for second percentile followed a trend similar to the evolution of the acoustic emission (Eberhardt et 452 al., 1998; Moradian et al., 2016; Shirole et al., 2019), and (ii) at CI stress level, very few 453 tensile cracks were formed, which signifies that very few pixels have tensile strain value 454 greater than the critical limit. As shown in Figure 4b, the pixel-scale value of the critical 455 tensile strain limit was estimated as -18e-4. After the evaluation of the critical limit of the 456 tensile strain (ϵ_c), the nonelastic apparent tensile strain (ϵ_{AT}^{NT}) was determined as follows: 457

$$\epsilon_{AT}^{NE} = \bigg| \sum_{i=1}^{N} \langle \epsilon_{22,i} \rangle \bigg|, \text{where } \epsilon_{22,i} \le \epsilon_c \tag{6}$$

The nonelastic apparent tensile strain component obtained through the above procedure was further utilized as a strain metrics to quantify the tensile deformations in the ROI region and to explicitly evaluate the extent of damage with the evolution of tensile cracks observed through the moment tensor analysis.

3.1.4 Crack Damage (CD) using Shear Strain Measurements

462

The nonelastic component of apparent shear strain was obtained using an approach similar 463 to the quantification of the nonelastic apparent tensile strain component. As shear yield 464 usually takes place along the plane of maximum shear strain (γ_{max}) (Jian-po et al., 2015; 465 Shirole, Hedayat, & Walton, 2020), therefore maximum shear strain measurements were 466 considered in this study for shear damage characterization. Figure 5 shows the maximum 467 shear strain distribution at different levels of stress in the ROI. It is evident from the figure that as the stress on the rock specimen increases, the histogram shifts towards the right which indicates the increasing magnitude of the maximum shear strain. An increase in the 470 heterogeneity of the maximum shear strain field can be observed at higher levels of stress 471 which is consistent with the fact that shear cracking in rock dominates only beyond the 472 crack damage (CD). 473

$$\gamma_{max} = \left| \left(\epsilon_{11} - \epsilon_{22} \right) / 2 \right| \tag{7}$$

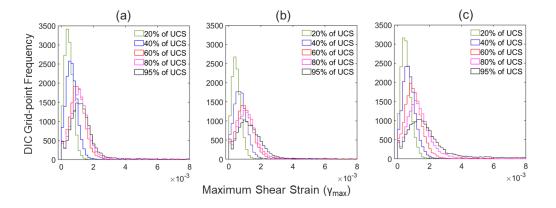


Figure 5. Histograms showing the maximum shear strain distribution in the ROI for three prismatic Barre granite specimens: (a) BG-1,(b) BG-2,(c) BG-3.

In this study, the damage due to tension has been characterized at pixel-scale measurements 474 obtained through DIC. For the quantification of shear damage in the ROI, the non-elastic 475 shear strain component needs to be evaluated at the pixel-scale. To estimate the pixel-scale 476 non-elastic shear strain component, a critical threshold value of shear strain (γ_c) has been 477 evaluated. As shear cracking in rocks primarily takes place beyond the CD stress level, 478 therefore the rock specimen is expected to have less number of shear cracks as obtained 479 through AE observations (Shirole, Hedayat, & Walton, 2020). Therefore, it is expected 480 that at CD, very few pixels exhibit values of γ_{max} greater than γ_c . This shows that the 481 critical shear strain limit (γ_c) should correspond to a small maximum shear strain (γ_{max}) 482 distribution percentile. Figure 6 explains the procedure adopted for the determination of 483 γ_c . The appropriate percentile distribution was chosen based on the fact that since shear 484 cracking in rocks accelerates beyond CD stress level, the γ_{max} percentile corresponding to 485 γ_c should show a deviation from linearity beyond CD threshold. To determine the value 486 of γ_c , several percentiles of γ_{max} were plotted as a function of normalized stress (Figure 487 6). The strain distribution in the ROI follows a linear trend up to a certain stress level and 488 then deviates at higher magnitudes of stress ($\sim 80\%$ of the failure stress and above) which 489 is consistent with the AE observations as shown in figure (2) for the rock specimen BG-3. 490 Figure 6 shows that the data deviates from the linear trend at $\sim 80\%$ of the UCS and above. 491 This deviation from linearity can be associated with the unstable growth of the microcracks 492 (Moradian et al., 2016). The 5th percentile of the γ_{max} strain variation does not show any 493 significant change in the slope, but the 0.5^{th} percentile and 0.1^{st} percentile showed a distinct 494 deviation from linearity at around 80% of the failure stress. The 0.5^{th} percentile strain-field 495 was chosen for the evaluation of (γ_c) because of two reasons (i) it followed a trend similar 496 to the AE observations, (ii) it is consistent with the findings of the numerical model for 497 granitic rocks proposed by (Sinha et al., 2020) which states that 0.3% -0.5% of grains in the 498 micro-mechanical model showed shear damage at CD. 499

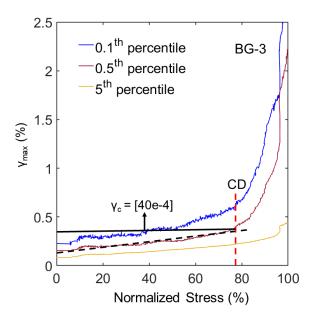


Figure 6. Procedure followed to obtain the pixel-scale critical shear strain limit (γ_c). The critical limit for maximum shear strain was obtained as (40e-4) corresponding to the 0.5^{th} percentile of the maximum shear strain variation (Shirole, Hedayat, & Walton, 2020)

The critical limit of the maximum shear strain (γ_c) obtained through the above procedure 500 was (40e-4), which is greater than the critical limit of the tensile strain ($\epsilon_c = -18e - 5$) 501 and also consistent with the previous observations that damage in brittle rocks initiate in tension prior to shear (M. Diederichs et al., 2004). These observations are also consistent 503 with the CWFS (cohesion weakening and frictional strengthening) model proposed by Ha-504 jiabdolmajid et al. (2002). After the determination of the specific γ_c , procedure similar to 505 the quantification of tensile damage was adopted to evaluate the total apparent shear strain 506 and the non-elastic component of shear strain as shown in equation 8 and 9. The strain 507 described in the above sections are considered as the apparent one, because in some cases 508 it does not hold true due to the evolution of damage (Song et al., 2013) 509

$$\gamma_{AS}^{T} = \left| \sum_{i=1}^{N} \gamma_{max,i} \right| \tag{8}$$

$$\gamma_{AS}^{NE} = \left| \sum_{i=1}^{N} \left\langle \gamma_{max,i} \right\rangle \right|, \text{ where } \gamma_{max,i} \ge \gamma_c \tag{9}$$

In equation 8 and 9, γ_{max} represents the maximum shear strain across the ROI at the N number of DIC grid points which in this case is 15770. Therefore, the apparent maximum shear strain can be defined as the summation of the maximum shear strain across the specified region of interest. The non-elastic component is evaluated by considering the maximum shear strain greater than the estimated critical value at the DIC grid points.

515 3.2 Crack Source Mechanisms

As discussed in the previous section that the progression of tensile and shear damage in rocks can be better understood on the basis of the non-elastic component of tensile and shear strain measurements. Therefore this section deals with the spatiotemporal distribution of the different crack source mechanism obtained through the 2D-DIC and AE techniques and their correlation.

Scholz (1968) (Scholz, 1968) reported that AE signals recorded during a rock fracture ex-521 periment, shows a rate of occurrence that can be correlated with the nonelastic stress-strain 522 behavior of the rock. He conducted experiments on Westerly granite under unconfined com-523 pression and correlated the inelastic volumetric strain with the rate of AE, however the 524 mechanism of cracking was not discussed in the study. A similar technique has been applied 525 in this study where the occurrence of AE events were correlated with the DIC strain mea-526 surements with a major emphasis on the cracking mechanism. In particular, the evolution of 527 different crack types obtained through moment tensor analysis were observed with increas-528 ing levels of nonelastic strain components (tensile and shear). Understanding this behavior is important to observe the extent of shear and tensile deformation in the rock specimen 530 throughout the damage. With this in mind, the evolution of tensile and shear cracks and 531 the calculated non-elastic component of tensile and shear strain has been plotted as a func-532 tion of loading in the ROI for each rock specimen (Figure 7). The detailed description is 533 provided in the following subsections. 534

⁵³⁵ 3.2.1 Temporal Evolution of Crack Mechanisms

Figure 7a shows a consistent relationship between the evolution of the tensile cracks detected by AE and the non-elastic component of tensile strain detected by DIC, throughout the loading for all the three rock specimen. In uniaxial compression test, the CI threshold denotes the initiation of microcracks (also obtained from AE) in the form of stable extensile microcracks governed by the non-elastic tensile deformation in rocks (Lockner, 1993). This is quite evident from Figure 7a which shows a rise in the evolution of the tensile cracks in the CI region. The nonelastic tensile strain component which is a metric of damage around

the flaw tips, also shows a significant rise around 30-40% of UCS. However, the rise in the

AE crack mechanism occurred earlier than the strain values, this can be due to the fact that

the DIC is related only to the surface strain measurements, whereas AE accounts for the deformation in the rock volume.

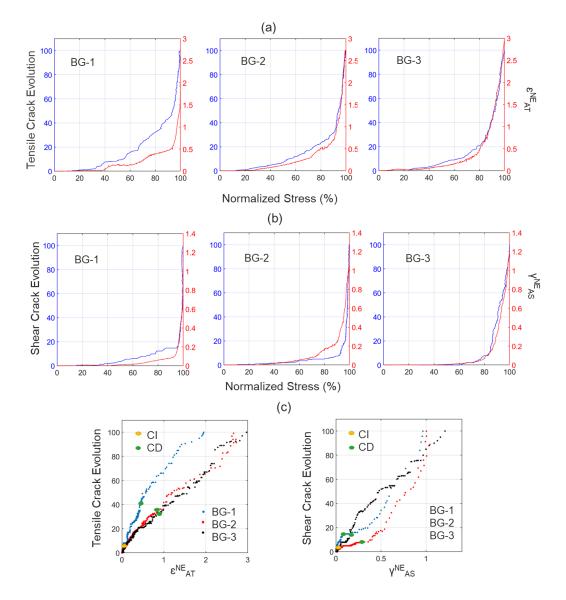


Figure 7. (a) Variation of tensile crack evolution (AE) and Non-elastic component of tensile strain (DIC) in the ROI for three Barre granite specimens, (b) Variation of shear crack evolution (AE) and Non-elastic component of maximum shear strain (DIC) in the ROI for three Barre granite specimens, (c) Correlation between the type of crack and nonelastic strain component in the ROI for the three rock specimens, the CI and CD has been distinguished by the yellow and green circles on the map.

546

547 When the load is increased further (from CI to CD), the nonelastic tensile strain increases

with the increasing number of tensile cracks. The curve shows an accelerated increase in the nonelastic tensile strain around the CD region (80%-90% of UCS) which is consistent with

the fact that the damage in brittle rocks primarily occur due to the local tensile stresses 550 even if the rocks are subjected to compressive stress field. From these observations, it can 551 be concluded that the nonelastic component of the tensile strain is a good representative 552 of the tensile crack evolution in the rock specimen and it can be used to quantify the 553 initiation, growth and coalescence of the extensile microcracking in rocks. Figure 7c shows 554 the correlation between the cumulative number of tensile cracks and non-elastic tensile strain 555 component evaluated through DIC full-field strain measurements in the ROI. In particular, 556 a near linear correlation can be seen between the tensile crack and non-elastic tensile strain 557 field. At higher levels of stress the trend becomes non-linear because of the loss in correlation 558 of strain measurements. This observation is consistent with the findings of many direct 559 microscopic observational approaches (e.g. optical microscope, scanning electron microscopy (SEM) etc.) which indicate that crack initiation takes place in tensile mode at the flaw tips 561 at early stages of loading in the rock specimen. 562

Since the nonelastic apparent maximum shear strain (γ_{AS}^{NE}) can be used as a metric for the 563 characterization of shear damage in rocks, the evolution of shear cracks and the nonelastic 564 component of maximum shear strain was plotted as a function of normalized stress for the three rock specimen in the ROI (Figure 7b). The curves show a sharp increase around 566 the CD threshold (80% - 90%) and above. These observations are consistent with many 567 experimental and numerical evaluation which states that the microcracks induced by the 568 tensile strain begin to coalesce beyond the CD stress threshold and causes the shear-strain 569 induced microcracks to dominate (M. Diederichs, 2003; M. Diederichs et al., 2004; J. Peng 570 et al., 2017). These results also indicate that shear strain at the pixel scale represents a 571 more local level of shear damage and thus can be correlated with the AE crack mechanism. 572 A similar observation can be seen in figure 7c, which shows the ratio of the shear cracks 573 with respect to total number of shear cracks at each stress level as a function of nonelastic 574 maximum shear strain. The trend shows a near linear correlation, although the number of 575 shear cracks between the CI and CD limit is very low. After the CD, an increase in the 576 number of shear cracks can be seen. 577

The results in the present study shows that with the evolution of shear cracks, the nonelastic component of shear strain increases by seven to ninefolds from the CD limit to the failure of the specimen. These observations are consistent with the conclusions of several other studies such as Martin and Chandler (1994), Diederichs et al. (2003), Martin et al. (2010), Sinha and Walton (2020) and the CWFS (cohesion weakening and frictional strengthening) model proposed by Hajiabdolmajid et al. (2002).

The consistent correlation between the temporal evolution of tensile and shear cracks with 584 the nonelastic DIC strain components provides a better understanding about the mechanism 585 of microfracture accumulation and failure in brittle rocks. The results suggest that the 586 formation of macrocrack involves the existence of both tensile and shear microcracks but 587 the proportion of their evolution is different as the damage progresses. As shown in figure 588 7a the evolution of tensile microcracks and nonelastic tensile strain shows an increasing trend between the CI and CD which can be associated with the tensile opening at the 590 macroscale. Once the tensile cracking has occured at the macroscale, the ratio of shear crack 591 and nonelastic shear strain dominates (beyond CD) this could be due to the separation 592 caused by the tensile opening in the rock specimen. Thus, the increasing trend in the 593 evolution of shear cracks (from CD to the failure of the rock specimen) can be interpreted as 594 the shear macrocrack formation. Although few shear microcracks can be observed between 595 the CI and CD (figure 7b) this can be interpreted as the widening of the fracture process 596 zone (L. N. Y. Wong & Xiong, 2018). In addition to this, the present experimental results also confirms explicitly an interesting finding that the strain metrics applied in the present 598 study can be used as an effective tool to identify the various cracking levels in rock damage 599 in combination with the AE monitoring. 600

⁶⁰¹ 3.2.2 Spatial Distribution of Crack Mechanisms

The evolution of the mode of fracture through AE was obtained using the moment tensor 602 inversion technique. In order to analyze the mode of deformation from DIC strain measure-603 ments, the normal and shear component of strain were calculated by adopting a procedure 604 similar to the method proposed by Tal et al. (2016). To analyze the strain in the ROI, 605 the principal strain component was calculated for all the grid points in the specific region 606 (ROI). In order to get the damage features (microcracks), only those grid points in the ROI 607 were selected in which the difference between the principal strain component was larger than 0.01 (1%) (Tal et al., 2016) and where more than 5 grid points remained (filter size is 5, strain resolution of DIC was $2250\mu\epsilon$). This filtering of the strain map was done to 610 eliminate the noise with a conservative approach so as to ensure that the damage features 611 obtained through the image based strain profiles after filtering had minimum measurement 612 errors. These damage features were considered as the microcracks and their orientation was 613 obtained through visual inspection. Once the orientation was identified, the normal and 614 shear components of the strains were calculated by resolving the values perpendicular and 615 parallel to the crack trend. Observations from the two techniques, that is the evolution of the tensile and shear cracks through moment tensors of AE and the mode of deformation 617 obtained through image based strain profiles from DIC were plotted at different levels of 618 stress. Figures 8 and 9 shows the spatial distribution of the AE cracks obtained through 619 the moment tensor analysis and the normal and shear component of the strain obtained 620 through DIC in the ROI for BG-3. The evolution of tensile crack is compared with the 621 normal component of strain (Figure 8) and the shear crack evolution is compared with the 622 shear component of strain along the crack trend (Figure 9). 623

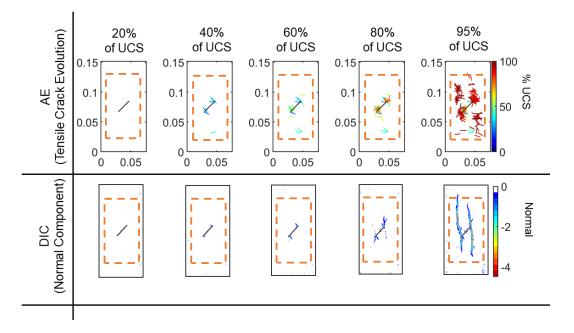


Figure 8. Comparison between the mode of deformation obtained through AE (tensile cracks) and DIC (normal component of strain) at different levels of stress in the ROI for BG-3.

Figure 8 shows that the tensile cracks initiated close to the flaw tips at around 40% of UCS which is the CI for BG-3. At the same stress level, the normal mode of deformation can also be seen in the strain maps. Few microcracks through AE can be seen in the lower region of the specimen which is not evident in the DIC strain maps, this could be due to the damage

along the depth of the specimen which cannot be observed in the image analysis. Moreover,

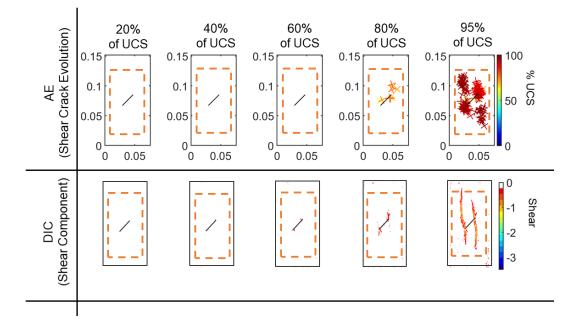


Figure 9. Comparison between the mode of deformation obtained through AE (shear cracks) and DIC (shear component of strain) at different levels of stress in the ROI for BG-3

as the loading increases, the microcracks propagate subparallel along the direction of the
 major principal stress. As expected, the strain concentration at the flaw tips increases with
 increasing levels of stress.

Figure 9 shows the evolution of the shear cracks from the AE moment tensor analysis and 632 the shear component of strain resolved parallel to the crack trend at different levels of stress 633 for BG-3. At initial stages (40-50%) of the failure stress) no shear cracking can be seen 634 through both the techniques. The shear cracks initiated at higher levels of stress (60-80% of)635 the failure stress), which is very close to the CD stress threshold of the rock specimen. The 636 evolution of shear microcracks between the CI and CD is associated with the widening of 637 the fracture process zone. Similarly, some shear strain concentration can be seen at 60% of the failure stress which can be due to the evolution of few shear microcracks in between CI 639 to CD. The results of the shear component of strain measurements are very consistent with 640 the shear crack evolution at later stages of loading. Near failure (95% of UCS) both modes 641 of deformation (tensile and shear) was observed, but the shear mode is more dominant. 642

These observations experimentally illustrate the relationship between the mode of defor-643 mation obtained through image based strain profiles and the process of microfracturing 644 obtained through the moment tensors of AE. The results are consistent with the previous 645 observations (Tapponnier & Brace, 1976; Paterson & Wong, 2005) that showed that micro-646 cracks initiate in tension in low porosity brittle rocks further extending to shear damage. 647 Although it has been previously observed that the nonelastic strain component shows a con-648 sistent relationship with the tensile and shear damage in rocks, these results further confirms 649 that the overall deformation in rocks subjected to various levels of stress is a combination of both (normal and shear) modes of deformation. 651

652 4 Conclusions

In this study, the temporal-spatial evolution of stress-induced microcracks and the proportion of different modes of cracking around the flaw tip was studied for Barre granite specimen with an existing flaw under unconfined compression. The moment tensor analysis
was employed on the AE waveforms for the evaluation of the crack mechanism in rocks
subjected to unconfined compression and the non-elastic tensile and shear strain component
was computed from the 2D-DIC strain measurements. Based on the results from the previous studies that AE represents the inelastic deformation (microcracking) in the rocks, the
damage mechanism obtained through AE was linked with the non-elastic tensile and shear
strain field evaluated from the DIC strain monitoring.

The 2D-DIC strain maps showed that as the load increases the heterogeneity in the strain 662 field increases due to the accumulated damage in the rock microstructure. Based on the 663 fact that the damage process in rocks initiate in tension, the tensile strain distribution was 664 analyzed in the ROI. Similarly, to analyze the evolution of shear cracks, the distribution 665 of shear strain was observed through the maximum apparent shear strain in the ROI. In 666 particular, the nonelastic tensile strain (ϵ_{AT}^{NE}) and the nonelastic shear strain (γ_{AS}^{NE}) distribution was analyzed in the specific region of interest. To obtain the non-elastic component 667 668 of strains, the critical limit for tensile (ϵ_c) and shear strain (γ_c) were computed above which the strain was considered as non-elastic. The results showed that the evolution of tensile cracks obtained through the moment tensor analysis and the non-elastic apparent tensile 671 strain followed a consistent trend throughout the loading for all the experiments. Similar 672 observation was seen for the evolution of shear cracks and non-elastic shear strain. 673

To analyze the mode of deformation from the DIC based strain profiles, the normal and 674 shear components of strain along the damage features were computed. Using filtering tech-675 niques to the 2D-DIC strain data, linear damage features (microcracks) and their orientation 676 were obtained from the image based strain profiles. Once the linear features and their ori-677 entation was identified, the strain field was resolved into the normal and shear components 678 along the crack length. The study showed a consistent trend between the AE and DIC 679 observations in the ROI for the shear and normal deformations. In particular, tensile defor-680 mation was observed throughout the loading initiating from the CI stress threshold while 681 shear deformation dominated closer to the peak stress.

683 ACKNOWLEDGMENTS

This research is supported by the U.S.Department of Energy, Office of Basic Energy Sciences, under Award Number DE-SC0019117. Interested readers can obtain the data related to this research article from https://figshare.com/s/56776f9041c56a3f29bc.

687 References

- Bazant, Z. P., & Kazemi, M. (1990). Determination of fracture energy, process zone length
 and brittleness number from size effect, with application to rock and concrete. Inter *national Journal of fracture*, 44(2), 111–131.
- Bieniawski, Z. (1967). Mechanism of brittle fracture of rock: part ii—experimental stud ies. In International journal of rock mechanics and mining sciences & geomechanics
 abstracts (Vol. 4, pp. 407–423).
- Bieniawski, Z. T. (1967). Mechanism of brittle fracture of rock: part i—theory of the
 fracture process. In International journal of rock mechanics and mining sciences &
 geomechanics abstracts (Vol. 4, pp. 395–406).
- Bourcier, M., Bornert, M., Dimanov, A., Héripré, E., & Raphanel, J. (2013). Multiscale
 experimental investigation of crystal plasticity and grain boundary sliding in synthetic
 halite using digital image correlation. Journal of Geophysical Research: solid earth,
 118(2), 511–526.
- Brace, W., Paulding Jr, B., & Scholz, C. (1966). Dilatancy in the fracture of crystalline
 rocks. Journal of Geophysical Research, 71(16), 3939–3953.
- Bruck, H., McNeill, S., Sutton, M. A., & Peters, W. (1989). Digital image correlation using
 newton-raphson method of partial differential correction. *Experimental mechanics*,

705

29(3), 261-267.

- Chang, S.-H., & Lee, C.-I. (2004). Estimation of cracking and damage mechanisms in rock
 under triaxial compression by moment tensor analysis of acoustic emission. *International Journal of Rock Mechanics and Mining Sciences*, 41(7), 1069–1086.
- Cheng, J.-L., Yang, S.-Q., Chen, K., Ma, D., Li, F.-Y., & Wang, L.-M. (2017). Uniaxial
 experimental study of the acoustic emission and deformation behavior of composite
 rock based on 3d digital image correlation (dic). Acta Mechanica Sinica, 33(6), 999–
 1021.
- Cheng, Y., & Wong, L. N. Y. (2018). Microscopic characterization of tensile and shear
 fracturing in progressive failure in marble. Journal of Geophysical Research: Solid
 Earth, 123(1), 204–225.
- Crider, J. G. (2015). The initiation of brittle faults in crystalline rock. Journal of Structural Geology, 77, 159–174.
- Dai, F., & Xia, K. (2010). Loading rate dependence of tensile strength anisotropy of barre granite. *Pure and applied geophysics*, 167(11), 1419–1432.
- Diederichs, M. (2003). Manuel rocha medal recipient rock fracture and collapse under low
 confinement conditions. Rock Mechanics and Rock Engineering, 36(5), 339–381.
- Diederichs, M., Kaiser, P., & Eberhardt, E. (2004). Damage initiation and propagation in
 hard rock during tunnelling and the influence of near-face stress rotation. International
 Journal of Rock Mechanics and Mining Sciences, 41 (5), 785–812.
- Diederichs, M. S. (1999). Instability of hard rockmasses, the role of tensile damage and
 relaxation (Unpublished doctoral dissertation). University of Waterloo.
- Dong, W., Wu, Z., Zhou, X., Wang, N., & Kastiukas, G. (2017). An experimental study on crack propagation at rock-concrete interface using digital image correlation technique.
 Engineering fracture mechanics, 171, 50–63.
- Eberhardt, E., Stead, D., Stimpson, B., & Read, R. (1998). Identifying crack initiation and
 propagation thresholds in brittle rock. *Canadian geotechnical journal*, 35(2), 222–233.
- Eberhardt, E., Stimpson, B., & Stead, D. (1999). Effects of grain size on the initiation and
 propagation thresholds of stress-induced brittle fractures. *Rock mechanics and rock engineering*, 32(2), 81–99.
- Fortin, J., Stanchits, S., Dresen, G., & Gueguen, Y. (2009). Acoustic emissions monitor ing during inelastic deformation of porous sandstone: comparison of three modes of
 deformation. Pure and Applied Geophysics, 166(5-7), 823–841.
- Fredrich, J. T., Evans, B., & Wong, T.-F. (1989). Micromechanics of the brittle to plastic transition in carrara marble. Journal of Geophysical Research: Solid Earth, 94(B4), 4129–4145.
- Ghamgosar, M., Erarslan, N., & Williams, D. (2017). Experimental investigation of fracture process zone in rocks damaged under cyclic loadings. *Experimental Mechanics*, 57(1), 97–113.
- Goldsmith, W., Sackman, J., & Ewerts, C. (1976). Static and dynamic fracture strength
 of barre granite. In International journal of rock mechanics and mining sciences &
 geomechanics abstracts (Vol. 13, pp. 303–309).
- Graham, C. C., Stanchits, S., Main, I. G., & Dresen, G. (2010). Comparison of polarity
 and moment tensor inversion methods for source analysis of acoustic emission data.
 International journal of rock mechanics and mining sciences (Oxford, England: 1997),
 47(1), 161.
- Griffith, A. A. (1921). Vi. the phenomena of rupture and flow in solids. *Philosophical trans- actions of the royal society of london. Series A, containing papers of a mathematical or physical character, 221* (582-593), 163–198.
- Guo, M., Alam, S. Y., Bendimerad, A. Z., Grondin, F., Rozière, E., & Loukili, A. (2017).
 Fracture process zone characteristics and identification of the micro-fracture phases in recycled concrete. *Engineering Fracture Mechanics*, 181, 101–115.
- Hajiabdolmajid, V., Kaiser, P. K., & Martin, C. (2002). Modelling brittle failure of rock.
 International Journal of Rock Mechanics and Mining Sciences, 39(6), 731–741.
- Hallbauer, D., Wagner, H., & Cook, N. (1973). Some observations concerning the micro-

760	scopic and mechanical behaviour of quartzite specimens in stiff, triaxial compression
761	tests. In International journal of rock mechanics and mining sciences \mathcal{C} geomechanics
762	abstracts (Vol. 10, pp. 713–726).
763	Hampton, J., Gutierrez, M., Matzar, L., Hu, D., & Frash, L. (2018). Acoustic emission char-
764	acterization of microcracking in laboratory-scale hydraulic fracturing tests. Journal
765	of Rock Mechanics and Geotechnical Engineering, 10(5), 805–817.
766	Hedayat, A., Pyrak-Nolte, L. J., & Bobet, A. (2014). Detection and quantification of slip
767	along non-uniform frictional discontinuities using digital image correlation. Geotech-
768	nical Testing Journal, 37(5), 786–799.
769	Hedayat, A., & Walton, G. (2017). Laboratory determination of rock fracture shear stiffness
770	using seismic wave propagation and digital image correlation. Geotechnical Testing
771	Journal, 40(1), 92–106.
772	Iqbal, M., & Mohanty, B. (2007). Experimental calibration of isrm suggested fracture
773	toughness measurement techniques in selected brittle rocks. Rock Mechanics and Rock
774	Engineering, $40(5)$, 453.
775	Jian-po, L., Yuan-hui, L., Shi-da, X., Shuai, X., & Chang-yu, J. (2015). Cracking mecha-
776	nisms in granite rocks subjected to uniaxial compression by moment tensor analysis
777	of acoustic emission. Theoretical and Applied Fracture Mechanics, 75, 151–159.
778	Kao, CS., Tarokh, A., Biolzi, L., & Labuz, J. F. (2016). Inelastic strain and damage in
779	surface instability tests. Rock Mechanics and Rock Engineering, $49(2)$, 401–415.
780	Kim, JS., Lee, KS., Cho, WJ., Choi, HJ., & Cho, GC. (2015). A comparative
781	evaluation of stress-strain and acoustic emission methods for quantitative damage
782	assessments of brittle rock. Rock Mechanics and Rock Engineering, 48(2), 495–508.
783	Kranz, R. L. (1979). Crack growth and development during creep of barre granite. In
784	International journal of rock mechanics and mining sciences \mathcal{E} geomechanics abstracts
785	(Vol. 16, pp. 23–35).
786	Kranz, R. L. (1983). Microcracks in rocks: a review. <i>Tectonophysics</i> , 100(1-3), 449–480.
787	Kurz, J. H., Grosse, C. U., & Reinhardt, HW. (2005). Strategies for reliable automatic
788 789	onset time picking of acoustic emissions and of ultrasound signals in concrete. Ultrasonics, $43(7)$, 538–546.
790	Lajtai, E. (1974). Brittle fracture in compression. International Journal of Fracture, 10(4),
791	525-536.
792	Li, B. Q., da Silva, B. G., & Einstein, H. (2019). Laboratory hydraulic fracturing of granite:
793	acoustic emission observations and interpretation. Engineering Fracture Mechanics,
794	209, 200-220.
795	Li, B. Q., & Einstein, H. H. (2017). Comparison of visual and acoustic emission observa-
796	tions in a four point bending experiment on barre granite. Rock Mechanics and Rock
797	$Engineering, \ 50(9), \ 2277-2296.$
798	Lin, Q., & Labuz, J. F. (2013). Fracture of sandstone characterized by digital image
799	correlation. International Journal of Rock Mechanics and Mining Sciences, 60, 235–
800	245.
801	Lin, Q., Wan, B., Wang, Y., Lu, Y., & Labuz, J. F. (2019). Unifying acoustic emission and
802	digital imaging observations of quasi-brittle fracture. Theoretical and Applied Fracture
803	Mechanics, 103, 102301.
804	Lin, Q., Yuan, H., Biolzi, L., & Labuz, J. F. (2014). Opening and mixed mode fracture pro-
805	cesses in a quasi-brittle material via digital imaging. Engineering Fracture Mechanics,
806	<i>131</i> , 176–193.
807	Lockner, D. (1993). The role of acoustic emission in the study of rock fracture. In In-
808	ternational journal of rock mechanics and mining sciences & geomechanics abstracts $(V_1 \downarrow 20 = -222, 200)$
809	(Vol. 30, pp. 883–899).
810	Maeda, N. (1985). A method for reading and checking phase times in autoprocessing system
811	of seismic wave data. $Zisin$, 38, 365–379.
812	Martin, C., & Chandler, N. (1994). The progressive fracture of lac du bonnet granite. In
813	International journal of rock mechanics and mining sciences & geomechanics abstracts (Vol. 21, pp. $642, 650$)
814	(Vol. 31, pp. 643–659).

- McClintock, F. A., & Irwin, G. (1965). Plasticity aspects of fracture mechanics. In *Fracture toughness testing and its applications*. ASTM International.
- Modiriasari, A., Bobet, A., & Pyrak-Nolte, L. J. (2017). Active seismic monitoring of crack initiation, propagation, and coalescence in rock. *Rock Mechanics and Rock Engineering*, 50(9), 2311–2325.
- Moore, D. E., & Lockner, D. (1995). The role of microcracking in shear-fracture propagation in granite. *Journal of Structural Geology*, 17(1), 95–114.
- Moradian, Z., Einstein, H. H., & Ballivy, G. (2016). Detection of cracking levels in brittle rocks by parametric analysis of the acoustic emission signals. *Rock Mechanics and Rock Engineering*, 49(3), 785–800.
- Morgan, S. P., Johnson, C. A., & Einstein, H. H. (2013). Cracking processes in barre granite: fracture process zones and crack coalescence. *International Journal of Fracture*, 180(2), 177–204.
- Nasseri, M., Grasselli, G., & Mohanty, B. (2010). Fracture toughness and fracture roughness
 in anisotropic granitic rocks. Rock Mechanics and Rock Engineering, 43(4), 403–415.
- Nicksiar, M., & Martin, C. (2012). Evaluation of methods for determining crack initiation
 in compression tests on low-porosity rocks. *Rock Mechanics and Rock Engineering*, 45(4), 607–617.
- Nicksiar, M., & Martin, C. (2013). Crack initiation stress in low porosity crystalline and
 sedimentary rocks. *Engineering Geology*, 154, 64–76.
- Ohtsu, M. (1991). Simplified moment tensor analysis and unified decomposition of acoustic
 emission source: application to in situ hydrofracturing test. Journal of Geophysical
 Research: Solid Earth, 96 (B4), 6211–6221.
- ⁸³⁸ Ohtsu, M. (1995). Acoustic emission theory for moment tensor analysis. Research in ⁸³⁹ Nondestructive Evaluation, 6(3), 169–184.
- Pan, B., Qian, K., Xie, H., & Asundi, A. (2009). Two-dimensional digital image correlation
 for in-plane displacement and strain measurement: a review. *Measurement science and technology*, 20(6), 062001.
- Paterson, M. S., & Wong, T.-f. (2005). Experimental rock deformation-the brittle field.
 Springer Science & Business Media.
- Peng, J., Wong, L. N. Y., & Teh, C. I. (2017). Influence of grain size heterogeneity
 on strength and microcracking behavior of crystalline rocks. Journal of Geophysical
 Research: Solid Earth, 122(2), 1054–1073.
- Peng, S., & Johnson, A. (1972). Crack growth and faulting in cylindrical specimens of
 chelmsford granite. In *International journal of rock mechanics and mining sciences & geomechanics abstracts* (Vol. 9, pp. 37–86).
- Peng, S. S. (1975). A note on the fracture propagation and time-dependent behavior of rocks
 in uniaxial tension. In *International journal of rock mechanics and mining sciences & geomechanics abstracts* (Vol. 12, pp. 125–127).
- Pestman, B., & Van Munster, J. (1996). An acoustic emission study of damage development
 and stress-memory effects in sandstone. In *International journal of rock mechanics and mining sciences & geomechanics abstracts* (Vol. 33, pp. 585–593).
- Reches, Z., & Lockner, D. A. (1994). Nucleation and growth of faults in brittle rocks.
 Journal of Geophysical Research: Solid Earth, 99(B9), 18159–18173.
- Richards, P. G., & Aki, K. (1980). Quantitative seismology: Theory and methods. Freeman.
- Scholz, C. (1968). Microfracturing and the inelastic deformation of rock in compression.
 Journal of Geophysical Research, 73(4), 1417–1432.
- Schwartz, E., Saralaya, R., Cuadra, J., Hazeli, K., Vanniamparambil, P. A., Carmi, R.,
 Kontsos, A. (2013). The use of digital image correlation for non-destructive and
 multi-scale damage quantification. In Sensors and smart structures technologies for *civil, mechanical, and aerospace systems 2013* (Vol. 8692, p. 86922H).
- Shirole, D., Hedayat, A., & Walton, G. (2019). Experimental relationship between compressional wave attenuation and surface strains in brittle rock. Journal of Geophysical Research: Solid Earth, 124 (6), 5770–5793.
- Shirole, D., Hedayat, A., & Walton, G. (2020). Illumination of damage in intact rocks by

870 871	ultrasonic transmission-reflection and digital image correlation. <i>Journal of Geophysical Research: Solid Earth</i> , 125(7), e2020JB019526.
872	Shirole, D., Walton, G., & Hedayat, A. (2020). Experimental investigation of multi-scale
873	strain-field heterogeneity in rocks. International Journal of Rock Mechanics and Min-
874	ing Sciences, 127, 104212.
	Sinha, S., Shirole, D., & Walton, G. (2020). Investigation of the micromechanical damage
875	process in a granitic rock using an inelastic bonded block model (bbm). Journal of
876	Geophysical Research: Solid Earth, 125(3), e2019JB018844.
877	Sondergeld, C., & Estey, L. (1982). Source mechanisms and microfracturing during uniaxial
878	cycling of rock. <i>pure and applied geophysics</i> , 120(1), 151–166.
879	Song, H., Zhang, H., Fu, D., Kang, Y., Huang, G., Qu, C., & Cai, Z. (2013). Experimental
880	study on damage evolution of rock under uniform and concentrated loading condi-
881	
882	tions using digital image correlation. Fatigue & Fracture of Engineering Materials & Structures, 36(8), 760–768.
883	
884	Stacey, T. (1981). A simple extension strain criterion for fracture of brittle rock. In
885	International journal of rock mechanics and mining sciences & geomechanics abstracts $(N_{cl}, 18, p_{T}, 460, 474)$
886	(Vol. 18, pp. 469–474).
887	Sutton, M. A., Orteu, J. J., & Schreier, H. (2009). Image correlation for shape, motion and
888	deformation measurements: basic concepts, theory and applications. Springer Science
889	& Business Media.
890	Tal, Y., Evans, B., & Mok, U. (2016). Direct observations of damage during unconfined
891	brittle failure of carrara marble. Journal of Geophysical Research: Solid Earth, 121(3),
892	1584–1609.
893	Tapponnier, P., & Brace, W. (1976). Development of stress-induced microcracks in westerly
894	granite. In International journal of rock mechanics and mining sciences & geomechan-
895	ics abstracts (Vol. 13, pp. 103–112).
896	Tarokh, A., Makhnenko, R. Y., Fakhimi, A., & Labuz, J. F. (2017). Scaling of the fracture
897	process zone in rock. International Journal of Fracture, $204(2)$, $191-204$.
898	Wawersik, W., & Fairhurst, C. (1970). A study of brittle rock fracture in laboratory com-
899	pression experiments. In International journal of rock mechanics and mining sciences
900	\mathscr{C} geomechanics abstracts (Vol. 7, pp. 561–575).
901	Wong, L., & Einstein, H. (2009). Crack coalescence in molded gypsum and carrara mar-
902	ble: part 2—microscopic observations and interpretation. Rock Mechanics and Rock
903	Engineering, $42(3)$, 513–545.
904	Wong, L. N. Y., & Xiong, Q. (2018). A method for multiscale interpretation of fracture pro- cesses in carrara marble specimen containing a single flaw under uniaxial compression.
905	
906	Journal of Geophysical Research: Solid Earth, 123(8), 6459–6490.
907	Wulff, AM., Hashida, T., Watanabe, K., & Takahashi, H. (1999). Attenuation behaviour
908	of tuffaceous sandstone and granite during microfracturing. <i>Geophysical Journal In-</i>
909	ternational, 139(2), 395-409.
910	Xu, X., & Zhang, ZZ. (2018). Acoustic emission and damage characteristics of granite
911	subjected to high temperature. Advances in Materials Science and Engineering, 2018. Zefer S. Hadavat, A. & Mandian O. (2020). Evaluation of analyzinitiation and down
912	Zafar, S., Hedayat, A., & Moradian, O. (2020). Evaluation of crack initiation and dam-
913	age in intact barre granite rocks using acoustic emission. Geotechnical Earthquake
914	Engineering and Special Topics.
915	Zhao, Yh., Huang, Jf., & Wang, R. (1993). Real-time sem observations of the microfrac-
916	turing process in rock during a compression test. In International journal of rock
917	mechanics and mining sciences & geomechanics abstracts (Vol. 30, pp. 643–652).