Demonstration and Analysis of Rarefied Particle Motions on Hillslopes

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Abstract

During the last century, descriptions of sediment transport on the surface of Earth have been mostly deterministic and strongly influenced by concepts from continuum mechanics. The assumption that particle motions on hillslopes and in rivers satisfy the continuum hypothesis has provided an important foundation for this topic. Recent studies, however, have recognized that bed load and hillslope sediment transport conditions often are rarefied and do not satisfy continuum assumptions, therein pointing to the need for new ways of describing particle motions and transport. The problem of rarefied sediment transport is probabilistic in nature, and emerging methods for describing particle motions hark back to the pioneering work of Einstein (1938), who conceptualized bed load transport as a probabilistic problem. Here we provide a data set of particle travel distances and supplemental high-speed videos of particle-surface collisions collected during laboratory experiments to assess a theoretical formulation of the probabilistic physics of rarefied particle motions and deposition on rough hillslope surfaces. The formulation is based on a description of the kinetic energy balance of a cohort of particles treated as a rarefied granular gas, and a description of particle deposition that depends on the energy state of the particles. Both laboratory and field-based measurements are consistent with a generalized Pareto distribution of travel distances and predicted variations in behavior associated with the balance between gravitational heating and frictional cooling by particle-surface collisions. These behaviors vary from a truncated distribution associated with rapid thermal collapse to an exponential distribution representing approximately isothermal conditions to a heavy-tailed distribution associated with net heating of particles. The transition to a heavy-tailed distribution likely involves an increasing conversion of translational to rotational kinetic energy leading to larger travel distances with decreasing effectiveness of collisional friction. The analysis points to the need for further clarity concerning how particle size and shape in concert with surface roughness influence the extraction of particle energy and the likelihood of deposition.

Demonstration and analysis of rarefied particle motions on hillslopes

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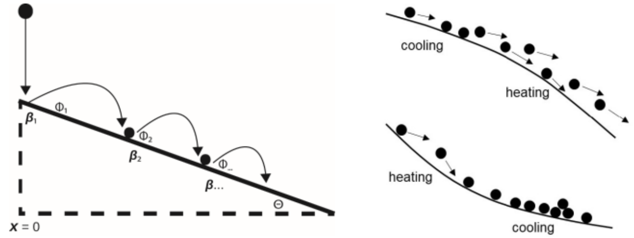
Theory

Consider the motion of a sediment particle tumbling down a rough surface inclined at an angle, having started this motion at position x = 0.



The potential energy of the particle, with a maximum value at the initial $x = 0$, is converted to kinetic energy (hereafter referred to as gravitational heating) as it moves downslope. Frictional cooling acts to counter this heating as the particle bounces down the rough surface. The particle is disentrained, or deposited, at the point on the surface where heating is overcome by cooling through collisional friction. **The distances that particles travel following entrainment directly reflect the probabilistic mechanics of motion and deposition.**

The ratio of gravitational heating to frictional cooling is defined by the Kirkby number,

$$Ki = \frac{4 \tan \varphi \, S}{\langle \beta_x \rangle},$$

where φ denotes the expected reflection angle of particles following collision with the surface, S denotes the surface slope, and β_x denotes the proportion of the translational particle energy E_p extracted by the collision, namely,

$$\beta_x = -\frac{\Delta E_p}{E_p} \, .$$

If we write the Kirkby number as $Ki = {}^{S}/_{\mu}$, then

$$\mu = \frac{\langle \beta_x \rangle}{4 \tan \varphi}$$

may be considered a friction coefficient. In turn, for a given initial average particle energy E_{a0} , a characteristic length scale of deposition L_c can be defined as

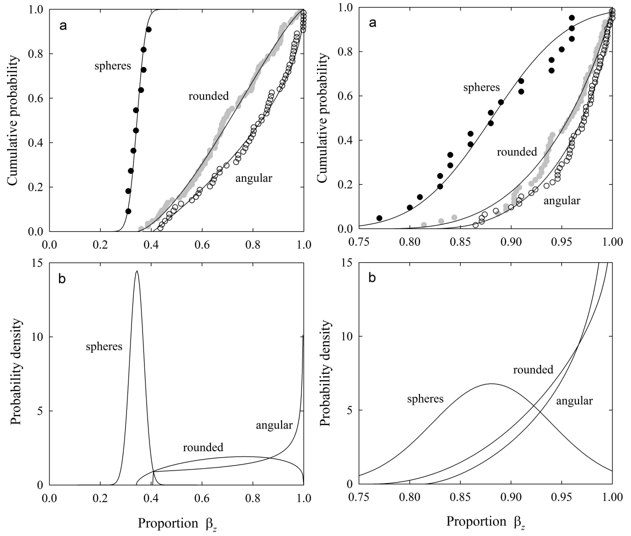
$$L_c = \alpha \frac{E_{a0}}{mg\mu \cos \theta} \, ,$$

where the factor α modulates this length scale, likely in relation to particle size, angularity and mode of motion (e.g., translational versus rotational).

Demonstration and Analysis

[1] Drop experiments aimed at demonstrating the basis for treating the proportion of energy extraction , β_x, as a random variable.

- ❑ Varied angularity, size, and drop height of particles
- ❑ Varied surface roughness on which particles were dropped



Plots of (a) cumulative distribution of β_z for glass spheres fit to a Gaussian distribution and rounded and angular gravel particles fit to a beta distribution, and (b) associated probability density functions of fitted distributions. Left set of plots is for collisions on hard slate, and right set of plots is for collisions on a rough concrete surface.

[2] Particle travel distance experiments using laboratory-scale hillslope with a concrete surface and a pendulum catapult launching device.

- ❑ Varied angularity of particles
- ❑ Varied surface slope

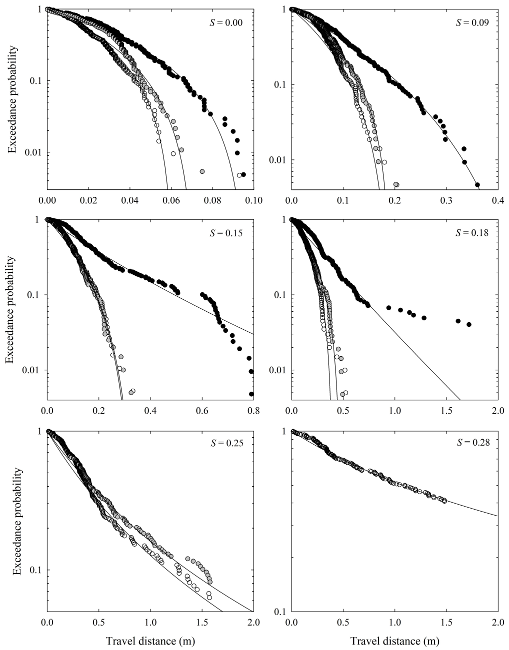
Data from travel distance experiments are consistent with the generalized Pareto distribution obtained through a formal energy balance in Furbish et al. 2019 *(in prep).*

The fitting of distribution shape, A, and scale, B, parameters provides estimates of frictional factor μ and scaling factor α where the latter can be defined as

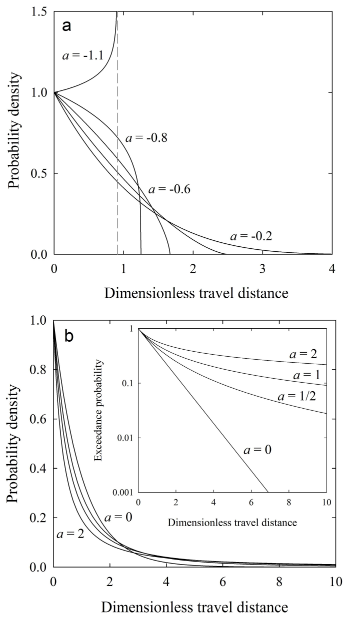
$$\alpha = \frac{B\gamma mg\mu \cos \theta}{E_{a0}}$$

with γ denoting the ratio of the average energy to the harmonic mean energy.

Particle angularity affects energy extraction and travel distances.



Plots of exceedance probability versus travel distance over six values of slope S showing **angular** (open circles), **rounded** (black circles) and small (gray circles) particles together with fitted distributions (lines).



Plots of dimensionless probability density function $f_{\hat{x}}(\hat{x})$ versus dimensionless travel distance $\hat{x}$ for scale parameter b = 1 and shape parameters (a) a < 0 and (b) a > 0.